Final state interactions T-odd TMDs

Studying the hadron structure in Drell-Yan reactions

26-27 April 2010 CERN

Leonard Gamberg
Penn State University

OUTLINE

- **Transverse spin Effects in TSSAs**
- **Gauge links-Color Gauge Inv.-“T-odd” TMDs**
- **Limits in using Transverse Distortion and TSSAs**

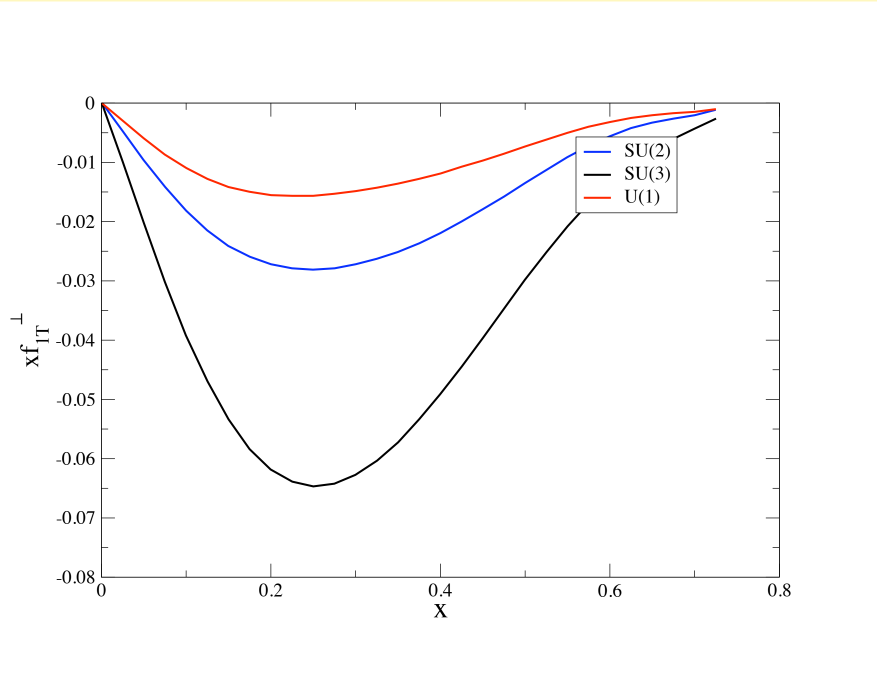
“QCD calc“ FSIs Gauge Links-Color Gauge Inv.“T-odd” TMDs

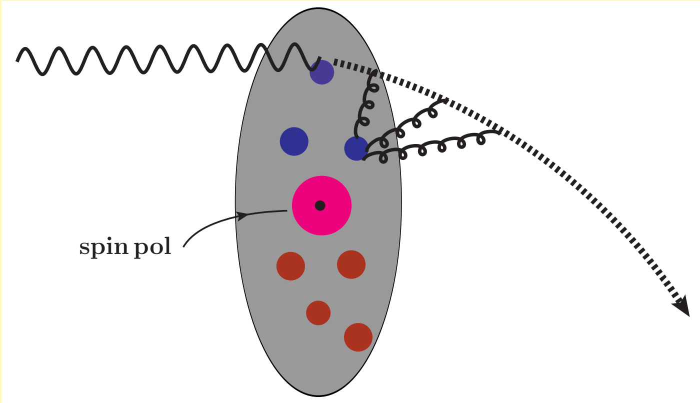
“Pheno” **-**Transverse Structure TMDs and TSSAs-**b** and **k** asymm

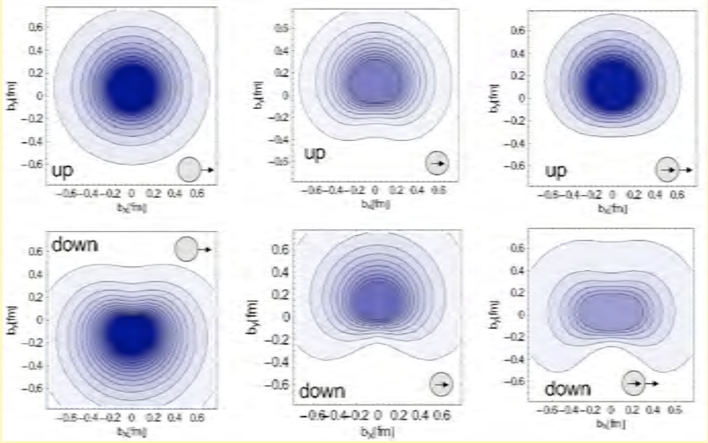
An improved dynamical approach for FSIs & model building

$f_{1T}^{\perp}(x, \mathbf{k}_{\perp}^{2})$

$\mathcal{E}(x, \mathbf{b}_{\perp}^{2})$







For Details see extra slides and

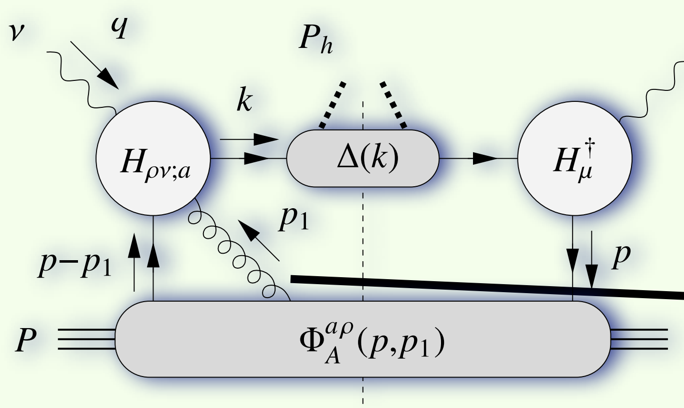
L.G. & Marc Schlegel

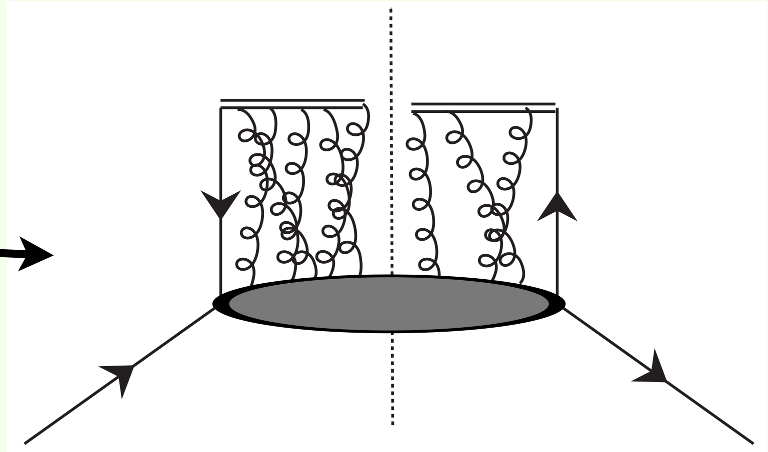
Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

T-Odd Effects From Color Gauge Inv. via Wilson Line

Gauge link determined re-summing gluon interactions btwn soft and hard
Efremov,Radyushkin *Theor. Math. Phys. 1981*
Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

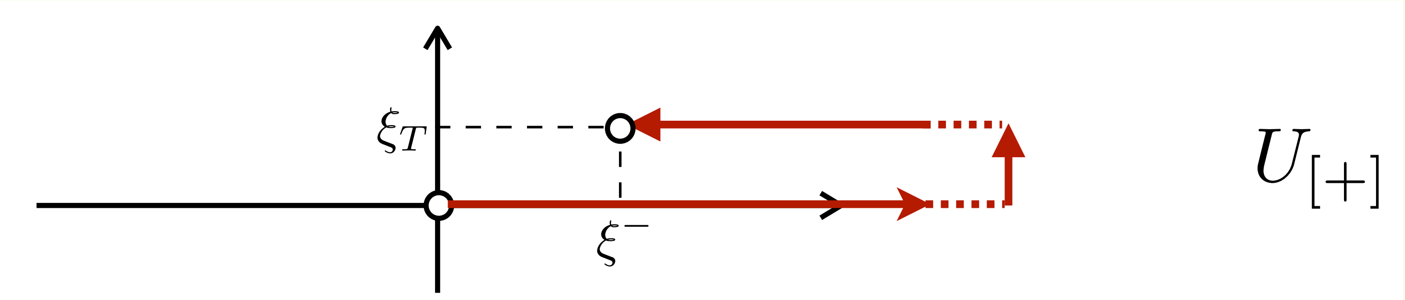




Summing gauge link with color
LG, M. Schlegel PLB 2010

- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}]}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

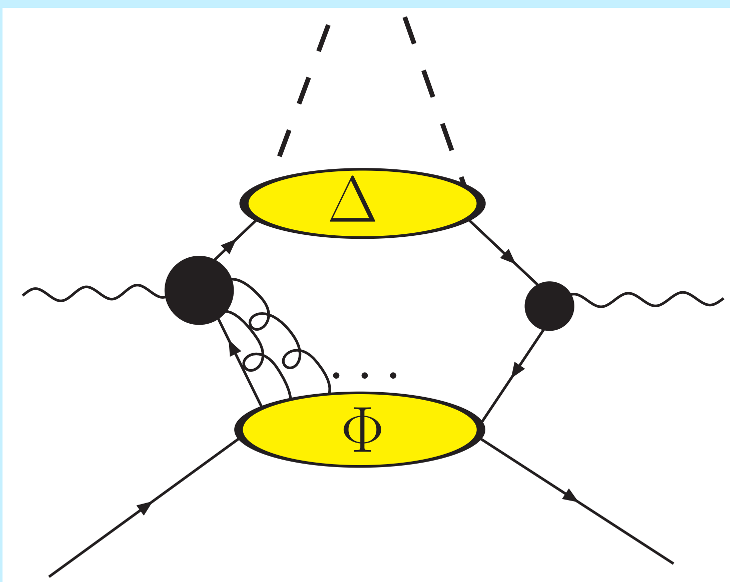


“Generalized Universality” Fund. Prediction of QCD

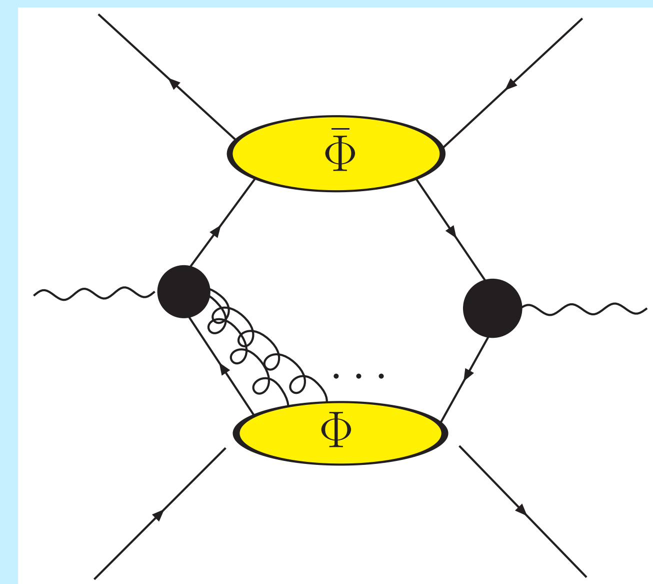
$$f^{\perp}_{1T_{SIDIS}}(x, k_T) = -f^{\perp}_{1T_{DY}}(x, k_T)$$

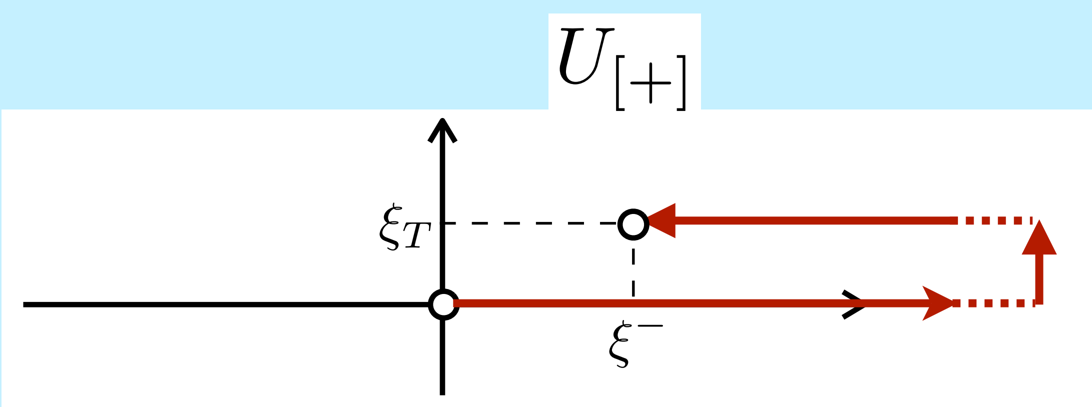
Compass, JPARC, E906-Fermi, RHIC II to test
EIC can pindown Sivers and Boer Mulders w/ higher luminosity
Talks of Denisov,Reimer,Abhay @ DIS 2010
talk of Alessandro CERN DY wksp

Process Dependence Collins PLB 02, Brodsky, Hwang, Schmidt NPB 02

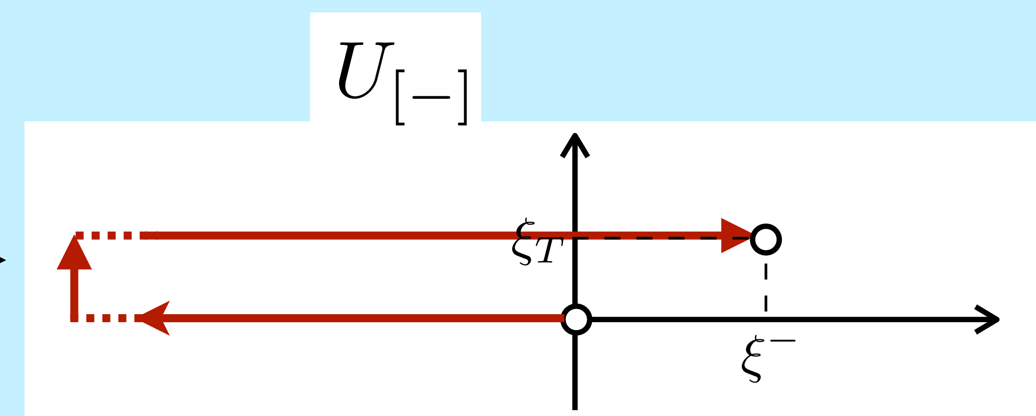


$$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$$





P&T



$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T)i\gamma^1\gamma^3$$

T-ODD Transverse Spin Transverse Momentum Correlations

Boer,Mulders PRD: 1998

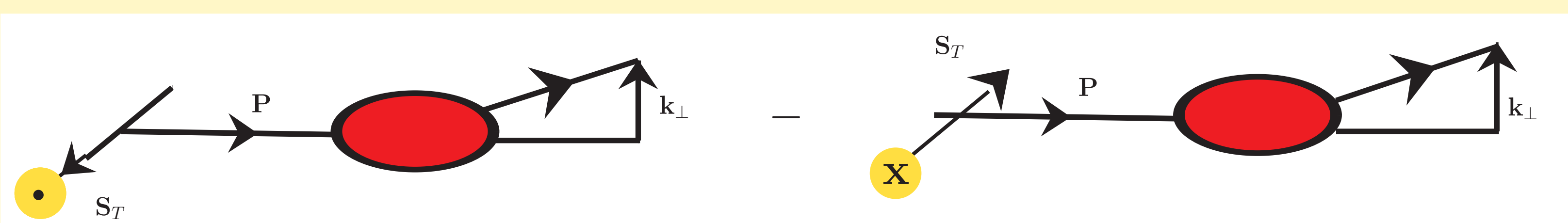
Correlation of transversely polarized *quark spin* with intrinsic $\boldsymbol{k}_\perp$

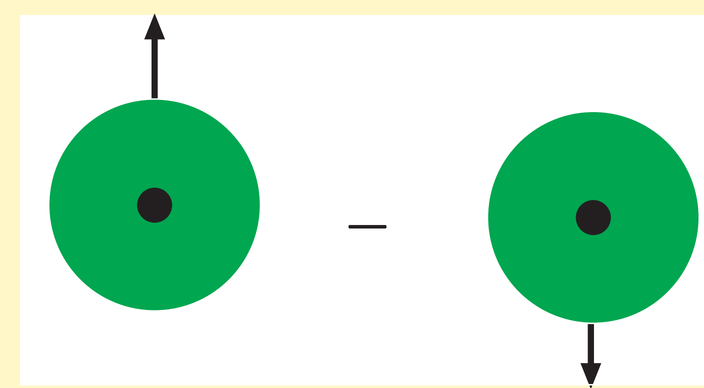
$\Rightarrow$ $$i s_T \cdot (\boldsymbol{k}_\perp \times \boldsymbol{P}) \to h_1^\perp(x, \boldsymbol{k}_\perp)$$

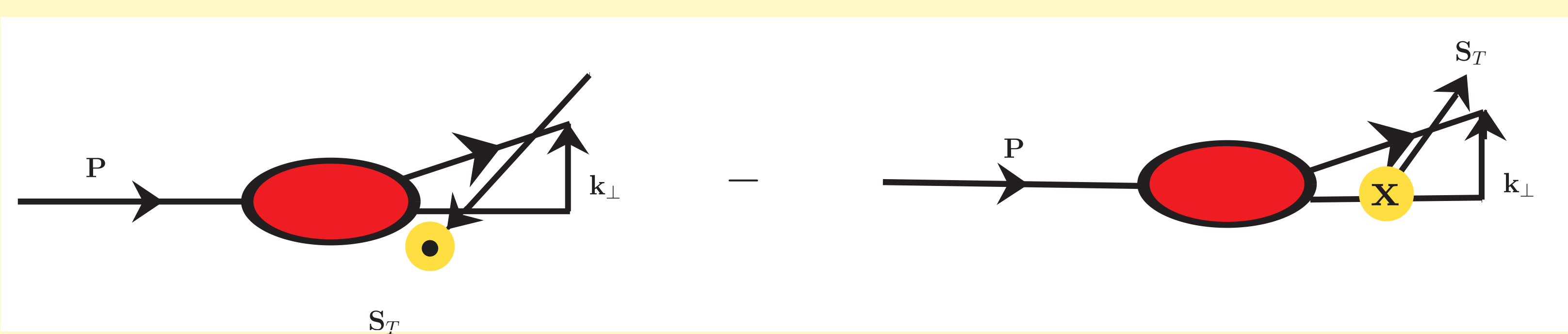
$\Rightarrow$ $$i S_T \cdot (\boldsymbol{k}_\perp \times \boldsymbol{P}) \to f_{1T}^\perp(x, k_\perp),$$

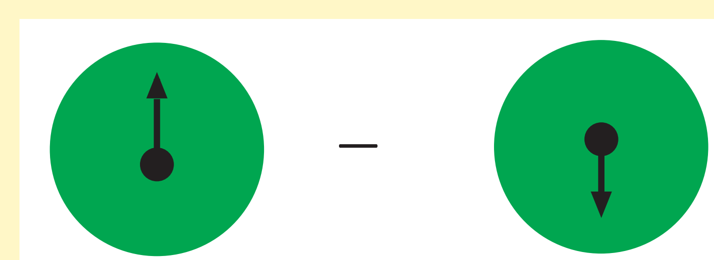
nucleon pol. \ quark pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs



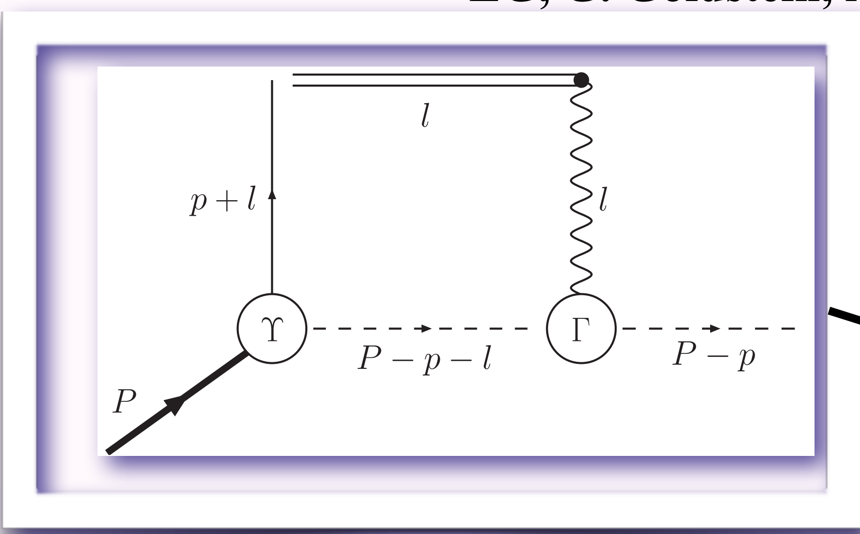






Studies FSIs in 1-gluon exchange approx.

LG, G. Goldstein, M. Schlegel PRD 77 2008, Bacchetta Conti Radici PRD 78, 2008



Build the T-odd TMD PDF with Final State Interactions-- one gluon exchange approx of Gauge link

$$W_i(P,k,S) = -ie_q e_{dq} \int \frac{d^4 l}{(2\pi)^4} \frac{g_{ax}((p+l)^2)}{\sqrt{3}} \varepsilon^*_\sigma(P-p,\lambda)\mathcal{D}^{ax}_{\rho\eta}(P-p-l)$$

$$\times \frac{[g^{\sigma\rho} v\cdot(2P-2p-l) + (1+\kappa)(v^\sigma(P-p+l)^\rho + v^\rho(P-p-2l)^\sigma)]}{[l\cdot v + i0][l^2+i0][(l+p)^2 - m_q^2 + i0]}$$

$$\times \left[(\not p + \not l + m_q)\gamma_5\left(\gamma^\eta - R_g \frac{P^\eta}{M}\right)u(P,S)\right]_i,$$

$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^\perp(x,\vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

Many model calculations studying dynamics of FSIs
Brodsky, Hwang et al,
Pasquini et al,
Courtoy et al
....

Sivers Parameterizations and studies from FSIs

Anselmino et al. PRD 05, EPJA 08

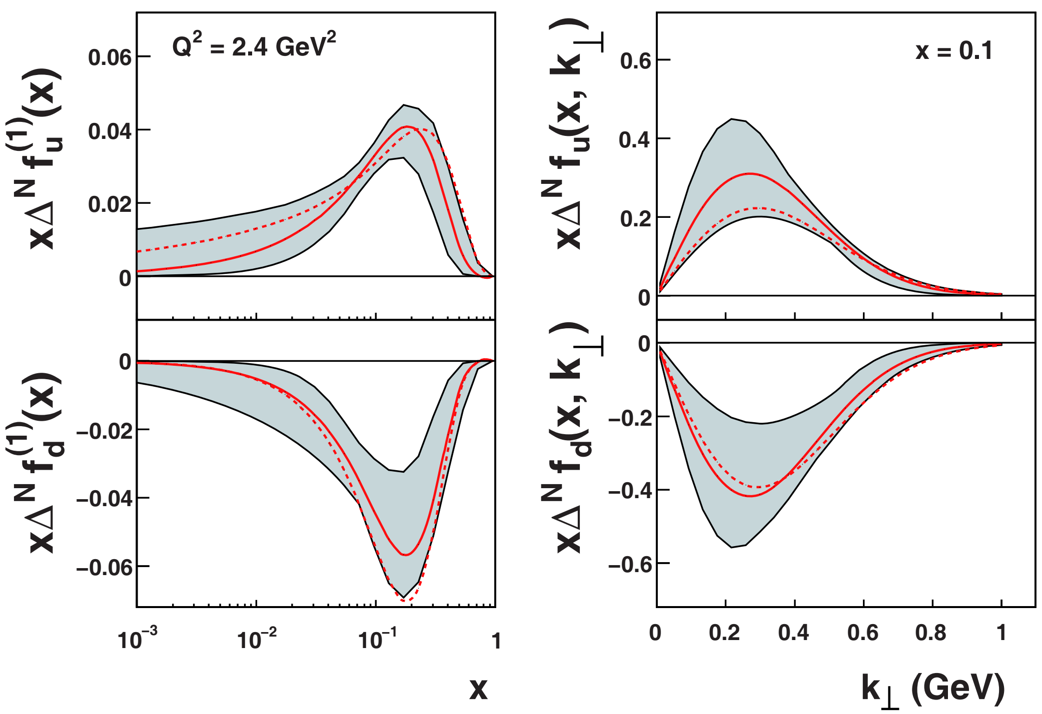


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Gamberg, Goldstein,Schlegel PRD 77, 2008

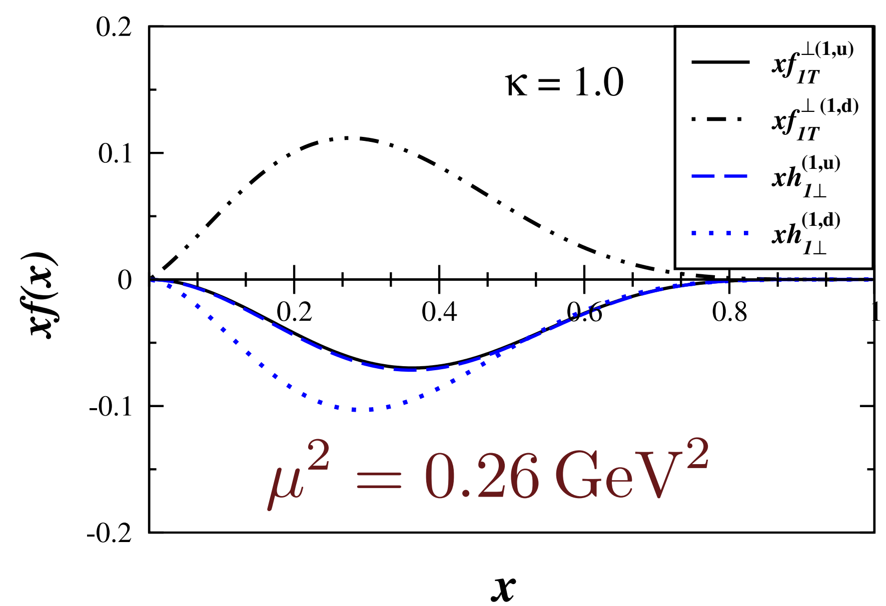
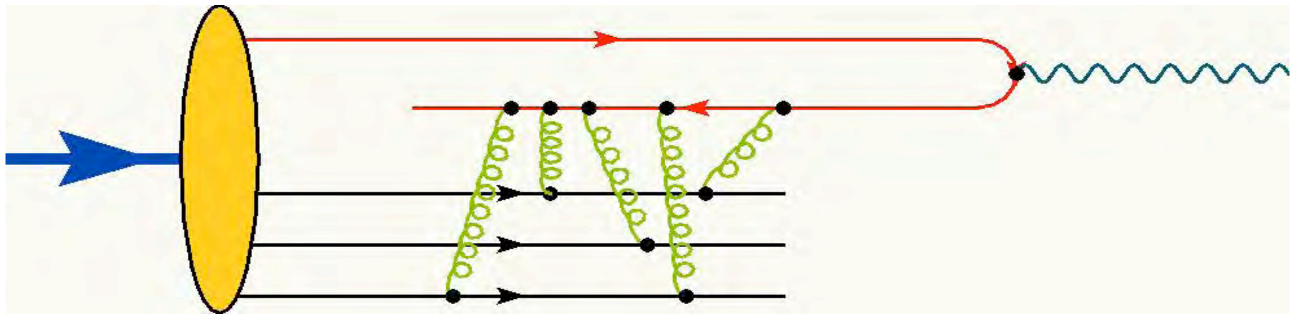


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

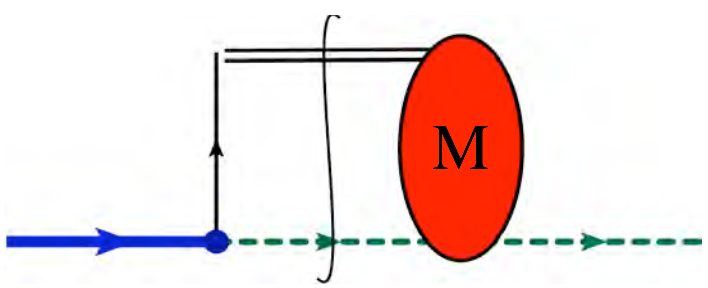
Explore connection calculate FSIs -Gauge Link



L.G. & Marc Schlegel
Phys.Lett.B685:95-103,2010 &
Mod.Phys.Lett.A24:2960-2972,2009

non-perturbative model of FSI.

W ~



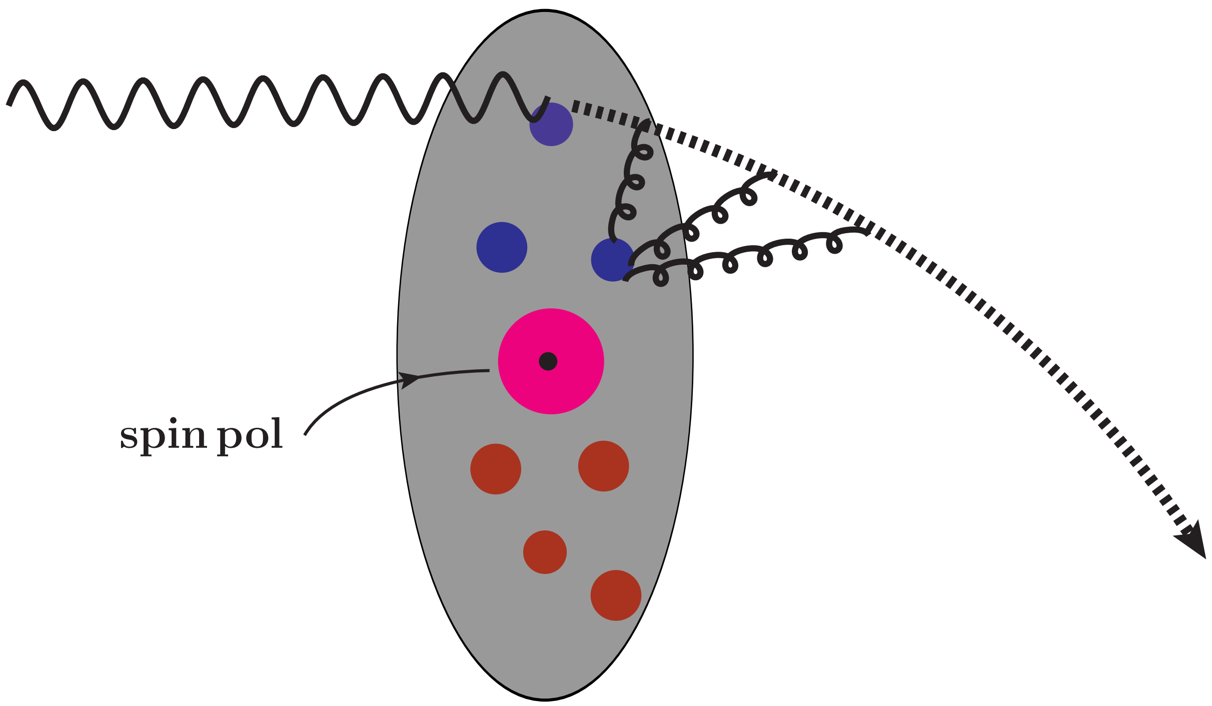
$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^{\perp}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

2+1 Dimensions Transverse Structure and TSSAs and TMDs

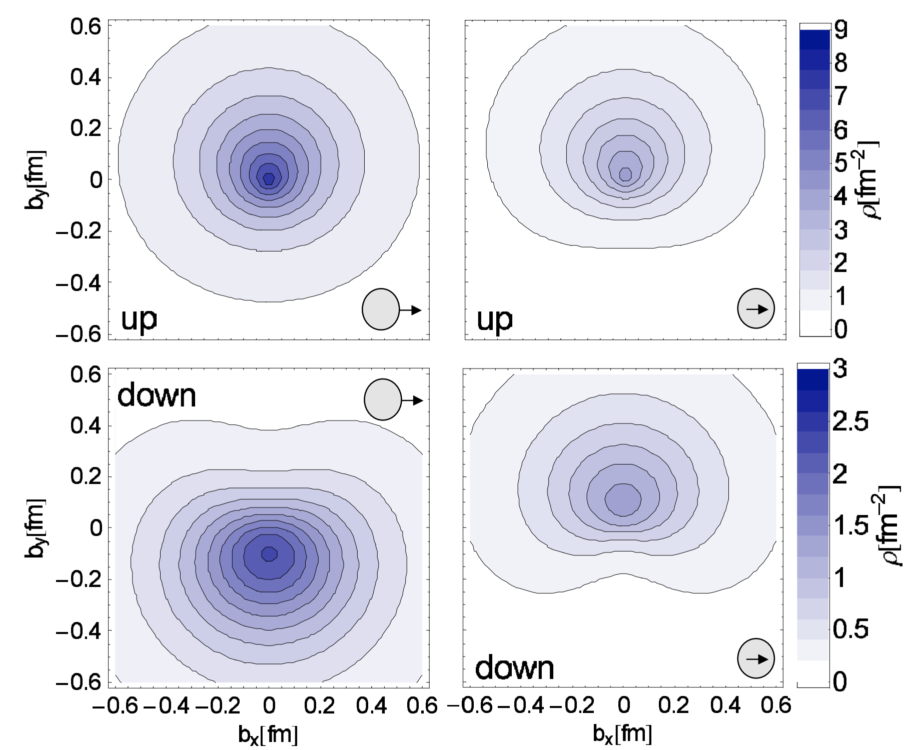
Intuitive picture of the Sivers asymmetry:

Spatial distortion in the transverse plane due to polarization

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]



Gockler et al. PRL07 x-moments of IP-GPDs



$$\vec{S}\cdot(\hat{P}\times\vec{k}_\perp)\, f_{1T}^{\perp}(x,\vec{k}_\perp^2)$$

$$\vec{S}\cdot(\hat{P}\times\vec{b})\, \left(\mathcal{E}(x,\vec{b}^2)\right)'$$

Spatial distortion + FSI lead to observable net effect

→ non-zero Left-Right (Sivers) asymmetry

"Spin-Orbit kinematics"

Analysis of correlators for TMDs and IP-GPDs similar forms

Burkhardt-02 PRD & ...
Diehl Hagler-05 EPJC,
Meissner, Metz, Goeke 07 PRD

$$\Phi^q(x, \vec{k}_T; S) = f_1^q(x, \vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2),$$

$$\mathcal{F}^q(x, \vec{b}_T; S) = \mathcal{H}^q(x, \vec{b}_T^2) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2)\right)',$$

$\mathbf{k}_T \leftrightarrow \mathbf{b}_T$ Not conjugates (!) and ...

$f_{1T}^{\perp}(x, \vec{k}_T^2)$ "Naive T-odd"

$\left(\mathcal{E}(x, \vec{b}_T^2)\right)'$ "Naive T-even"

FSIs needed.... Burkardt PRD 02 & NPA 04

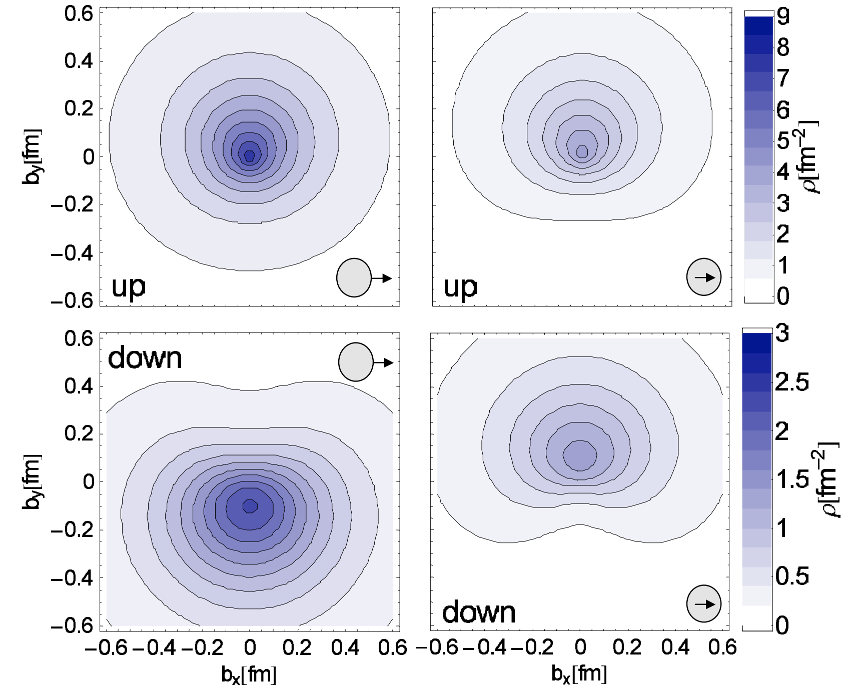
How do we test this further?

Used to predicting sign of TSSA-Sivers

Bukardt 02,04 NPA PRD

$$d_q^y = \frac{1}{2M}\int dx \int d^2\mathbf{b}_\perp \mathcal{E}_q(x,\mathbf{b}_\perp)$$

$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{F_{2,q}(0)}{2M} = \frac{\kappa}{2M}$$

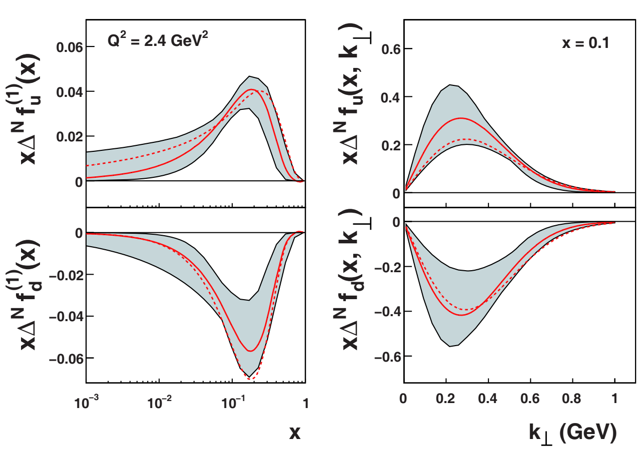


$$\kappa^p = 1.79\,, \quad \kappa^n = -1.91$$

$$\longrightarrow \kappa^{u/p} = 1.67\,, \quad \kappa^{d/p} = -2.03$$ w/ attractive interactions

$$f_{1T}^{\perp(u)} = \text{neg} \quad \& \quad f_{1T}^{\perp(d)} = \text{pos}$$

Anselmino et al. PRD 05, EPJA 08



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Sivers

Gamberg, Goldstein, Schlegel PRD 77, 2008

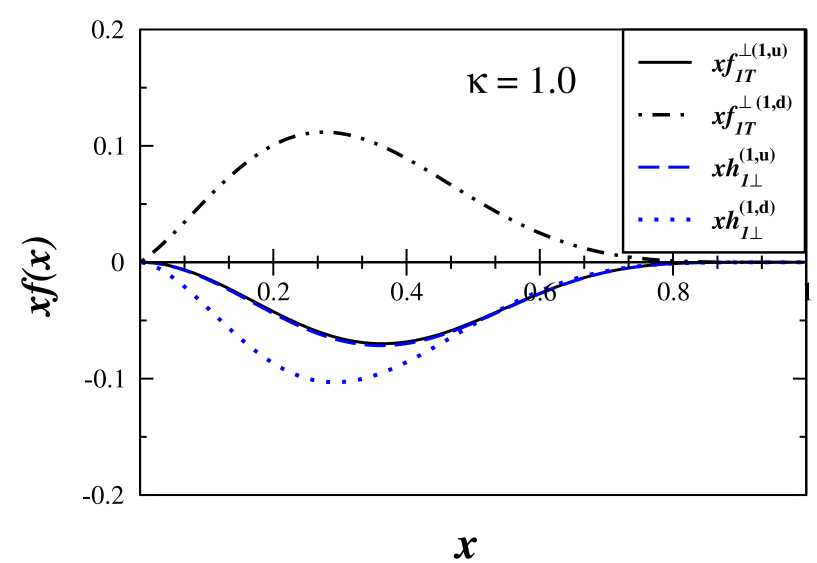


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

And for the ***PION*** Haegler et al PRL 07,08

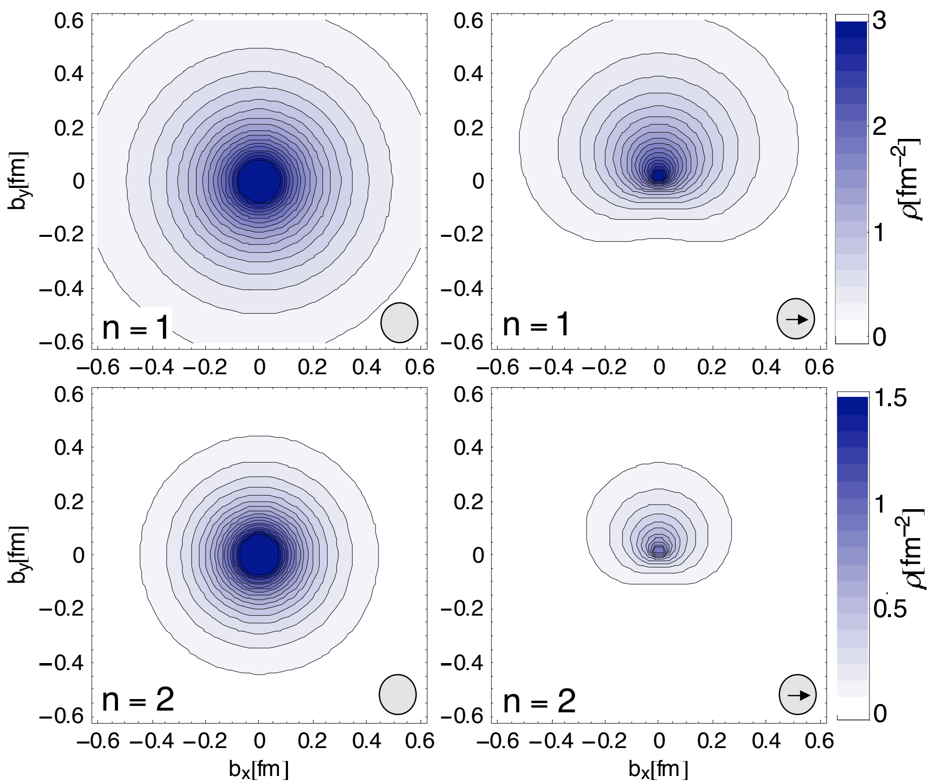


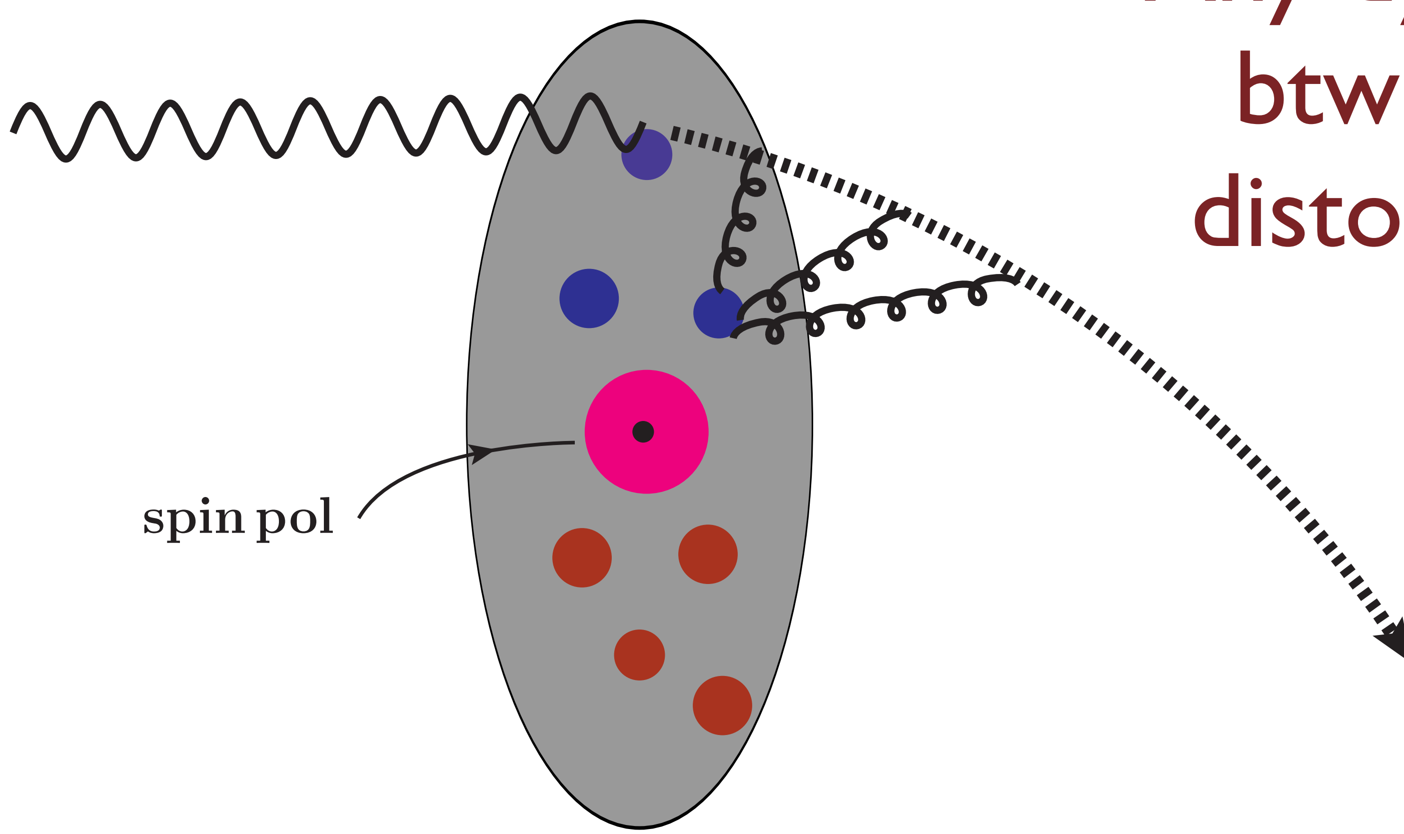
FIG. 6: The lowest two moments of the impact parameter densities of unpolarized (left) and transversely polarized (right) up-quarks in a π^+. The quark spin (inner arrow) is oriented in the transverse plane as indicated.

$$\rho^n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp)$$

$$= \frac{1}{2}\left[A^{\pi}_{n0}(b_\perp^2) - \frac{s^i_\perp \epsilon^{ij} b^j_\perp}{m_\pi} \frac{\partial}{\partial b_\perp^2} B^{\pi}_{Tn0}(b_\perp^2) \right]$$

$$\int_{-1}^{1} dx\, x^{n-1} H^\pi(x, \xi{=}0, b_\perp^2) = A^\pi_{n0}(b_\perp^2)\,,$$

$$\int_{-1}^{1} dx\, x^{n-1} E^\pi_T(x, \xi{=}0, b_\perp^2) = B^\pi_{Tn0}(b_\perp^2)$$



Any dynamical relation btwn impact space distortion and TSSA ???????

What observable to test this possible connection btnw TMD and Impact par. picture? Gluonic Pole ME

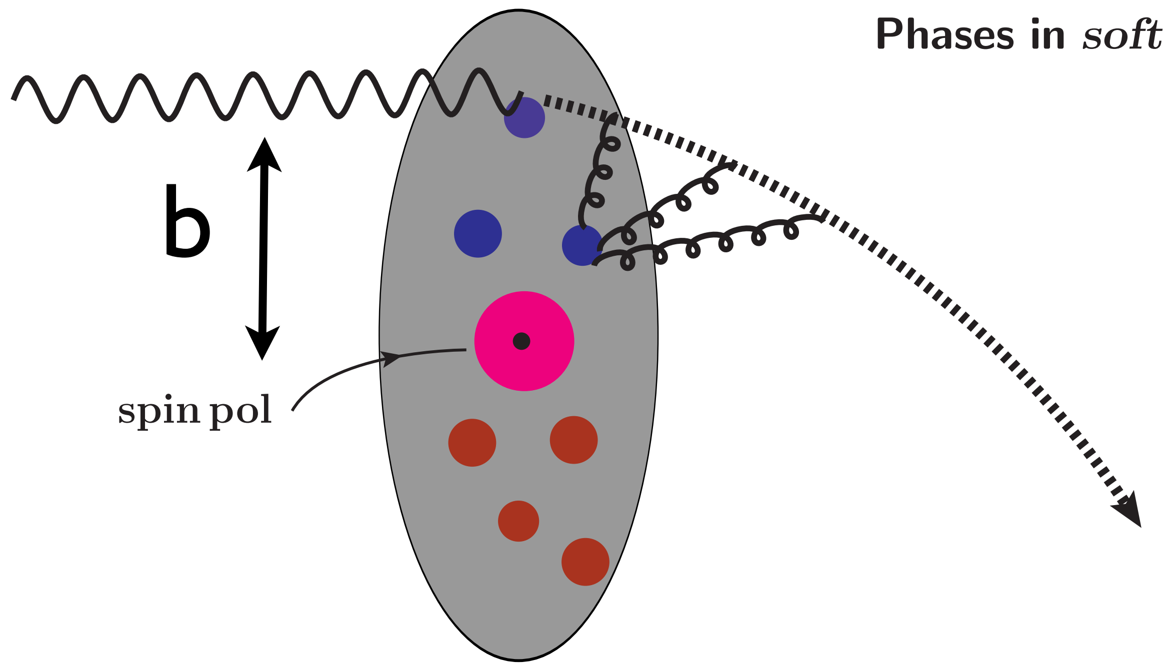
$$\langle k_T^i \rangle(x) = \int d^2 b_T \int \frac{dz^-}{2(2\pi)} \mathrm{e}^{ixP^+z^-} \langle P^+; \vec{0}_T; S_T | \bar{\psi}(z_1)\gamma^+[z_1; z_2] I^i(z_2) \psi(z_2) | P^+; \vec{0}_T; S_T \rangle$$

$$z_{1/2} = \mp \frac{z^-}{2} n_- + b_T$$

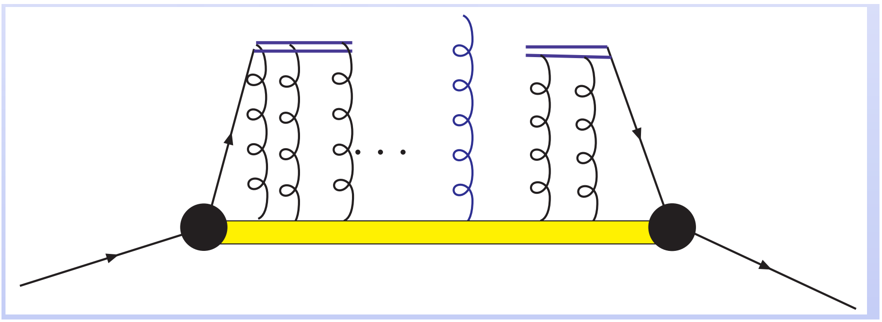
Impact parameter representation for GPD E

$$I^i(z^-) = \int dy^- \, [z^-; y^-] g F^{+i}(y^-) [y^-; z^-]$$

coll. "soft gluon pole" matrix element



Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982



Conjecture: factorization of final state interactions and spatial distortion:

$$\langle k_T^i \rangle = -M \epsilon_T^{ij} S_T^j \, f_{1T}^{\perp,(1)}(x) \simeq \int d^2 b_T \, \mathcal{I}^i(x, \vec{b}_T) \, \frac{\vec{b}_T \times \vec{S}_T}{M} \, \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$: Lensing Function = net transverse momentum

Also……

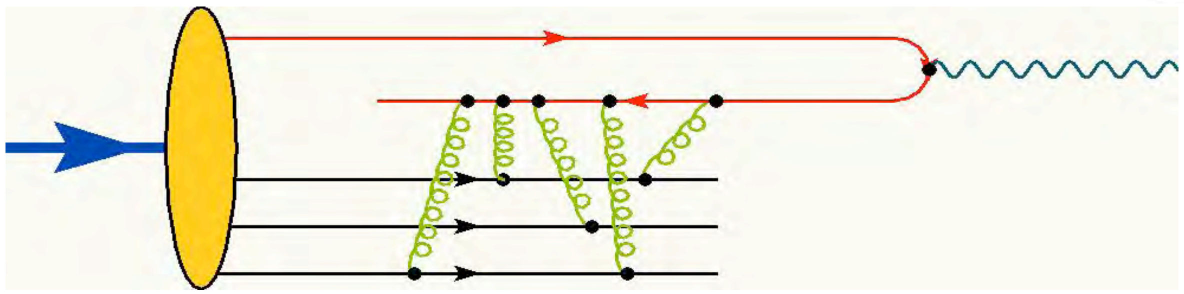
- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

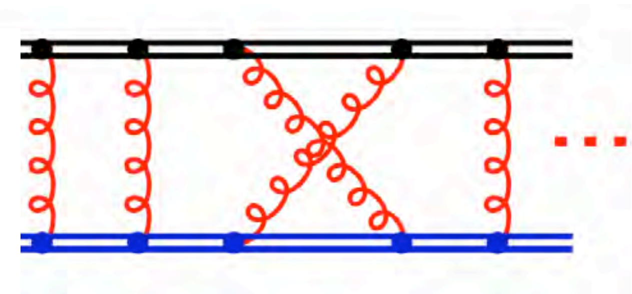
$$\langle k_T^i \rangle^j (x) = \int d^2 k_T \, k_T^i \, \frac{1}{2} \left(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S) \right)$$

$$\Rightarrow \quad -2M^2 \, h_1^{\perp,(1)}(x) \simeq \int d^2 b_T \, \vec{b}_T \cdot \vec{\mathcal{I}}(x, \vec{b}_T) \, \frac{\partial}{\partial b_T^2} \left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T \right)(x, \vec{b}_T^2)$$

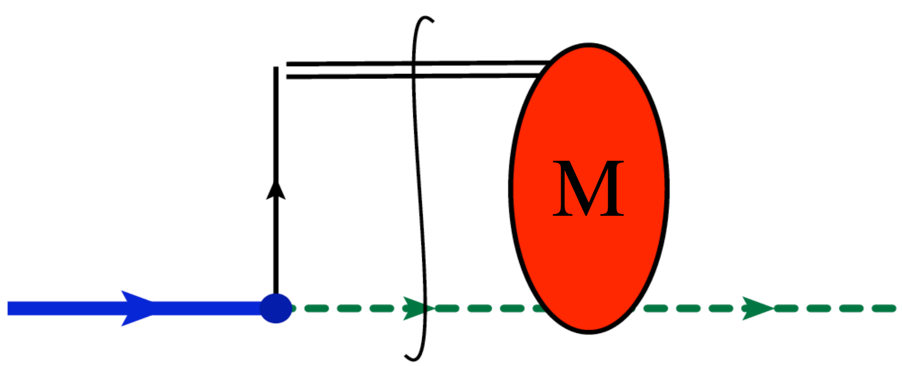
Diehl & Hagler EJPC (05), Burkardt PRD (04)

Sivers Function in this approach





- **Relativistic Eikonal models: Treat FSI non-perturbatively.**



- Still work within spectator framework, but *non-perturbative model of FSI*.
- In order to separate out GPDs, "cut" the diagram → "natural" picture of FSI.

$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int \frac{d^2 p_T}{(2\pi)^2}\, p_T^y\, I^y(x, |\vec{p}_T|)\, E^u\left(x, 0, -\frac{\vec{p}_T^{\,2}}{(1-x)^2}\right)$$

For Details see extra slides and

L.G. & Marc Schlegel

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Lensing Function

Express Lensing Function in terms of Eikonal Phase:

$$\mathcal{I}^i_{(N=1)}(x,\vec{b}_T) = \frac{1}{4}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[1+\cos\chi\left(\frac{|\vec{b}_T|}{1-x}\right)\right]$$

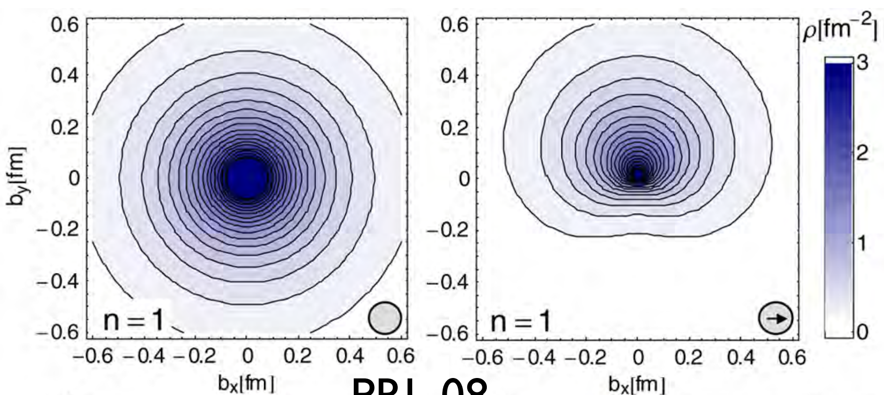
$$\mathcal{I}^i_{(N=2)}(x,\vec{b}_T) = \frac{1}{8}\frac{b^i_T}{|\vec{b}_T|}\chi'\left(\frac{|\vec{b}_T|}{1-x}\right)\left[3\left(1+\cos\frac{\chi}{4}\right)+\left(\frac{\chi}{4}\right)^2-\sin\frac{\chi}{4}\left(\frac{\chi}{4}-\sin\frac{\chi}{4}\right)\right]\left(\frac{|\vec{b}_T|}{1-x}\right)$$

$$\mathcal{I}^i_{(N=3)}(x,\vec{b}_T) = \text{numerics}$$

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

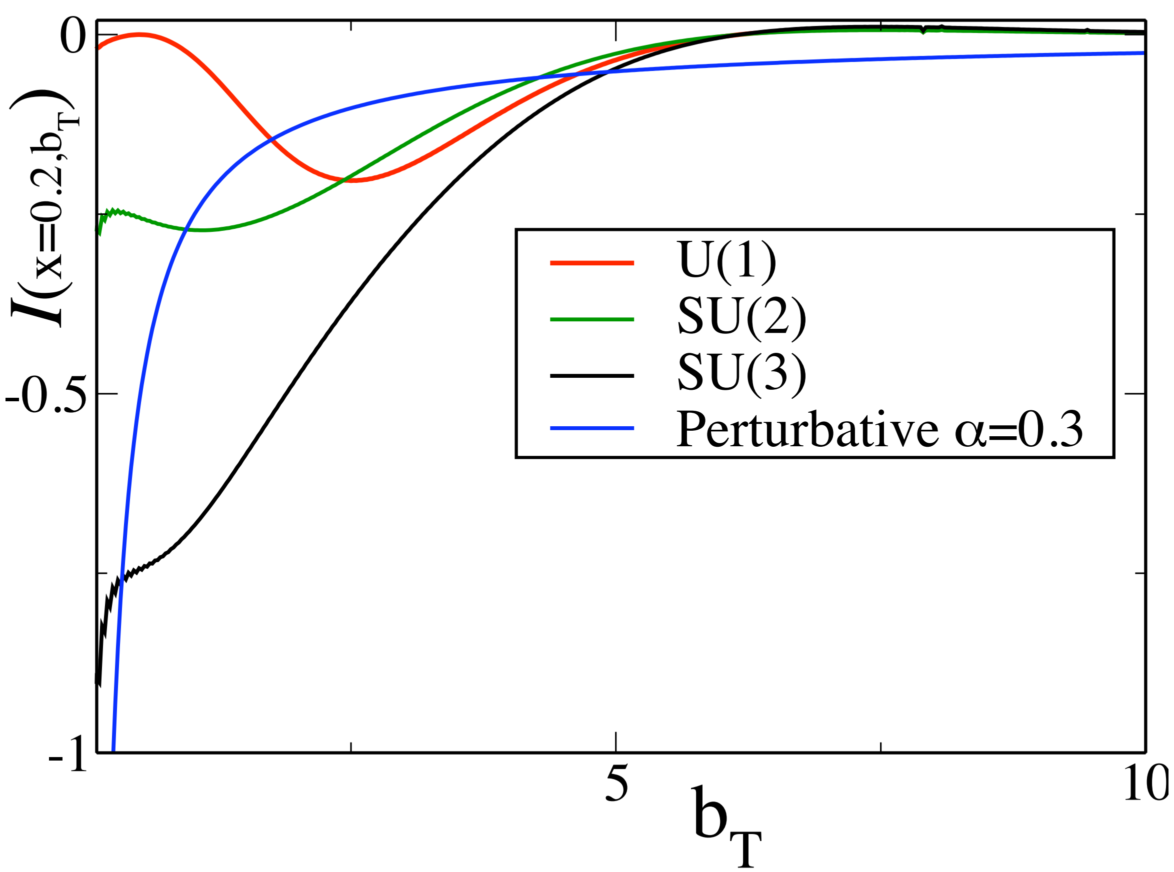
Spin Structure of the Pion



PRL 08

D. Brömmel,[1,2] M. Diehl,[1] M. Göckeler,[2] Ph. Hägler,[3]

FSI + distortion



FSIs are negative even with Color!

Prediction for Boer-Mulders Function of PION

L.G. & Marc Schlegel

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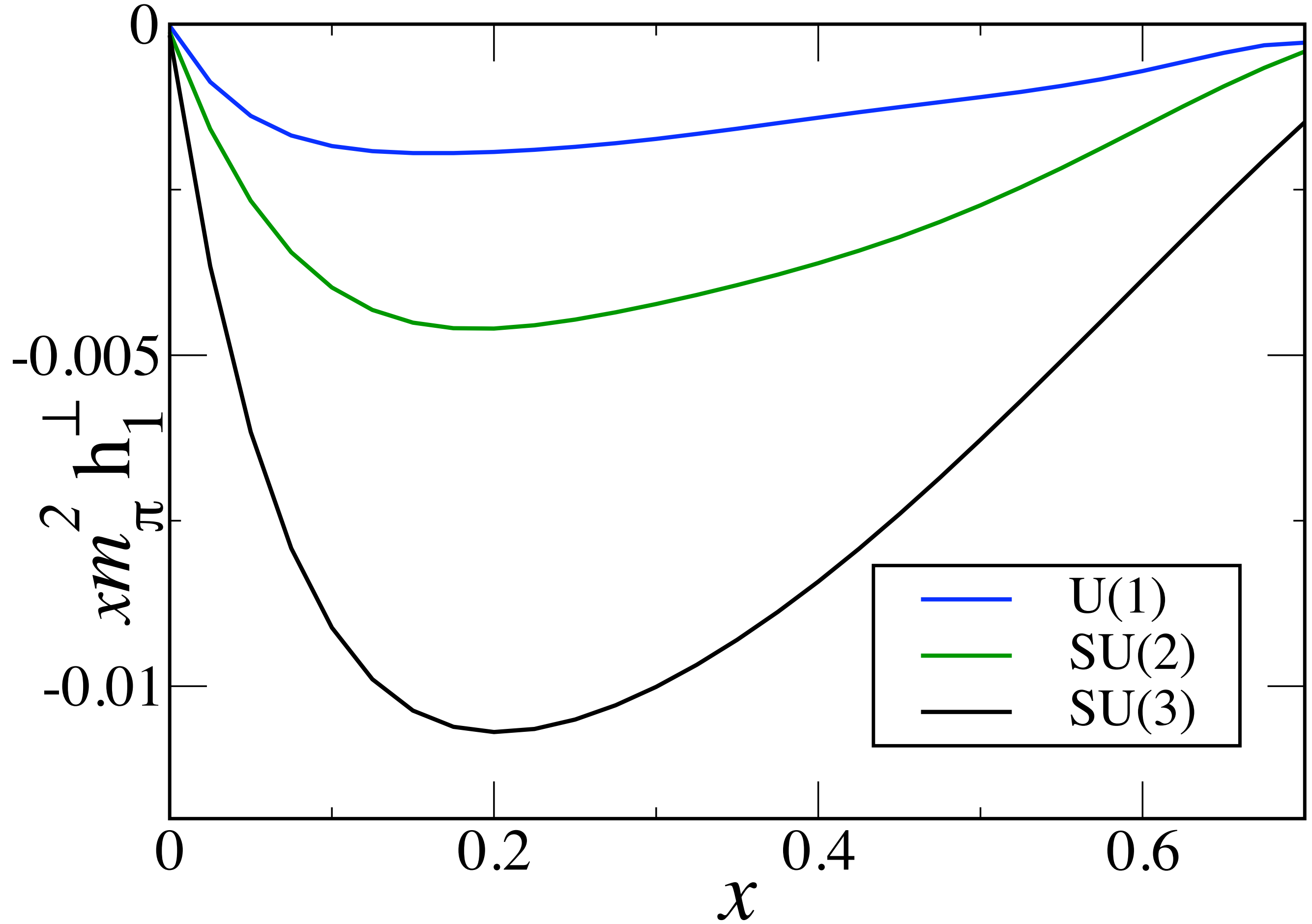
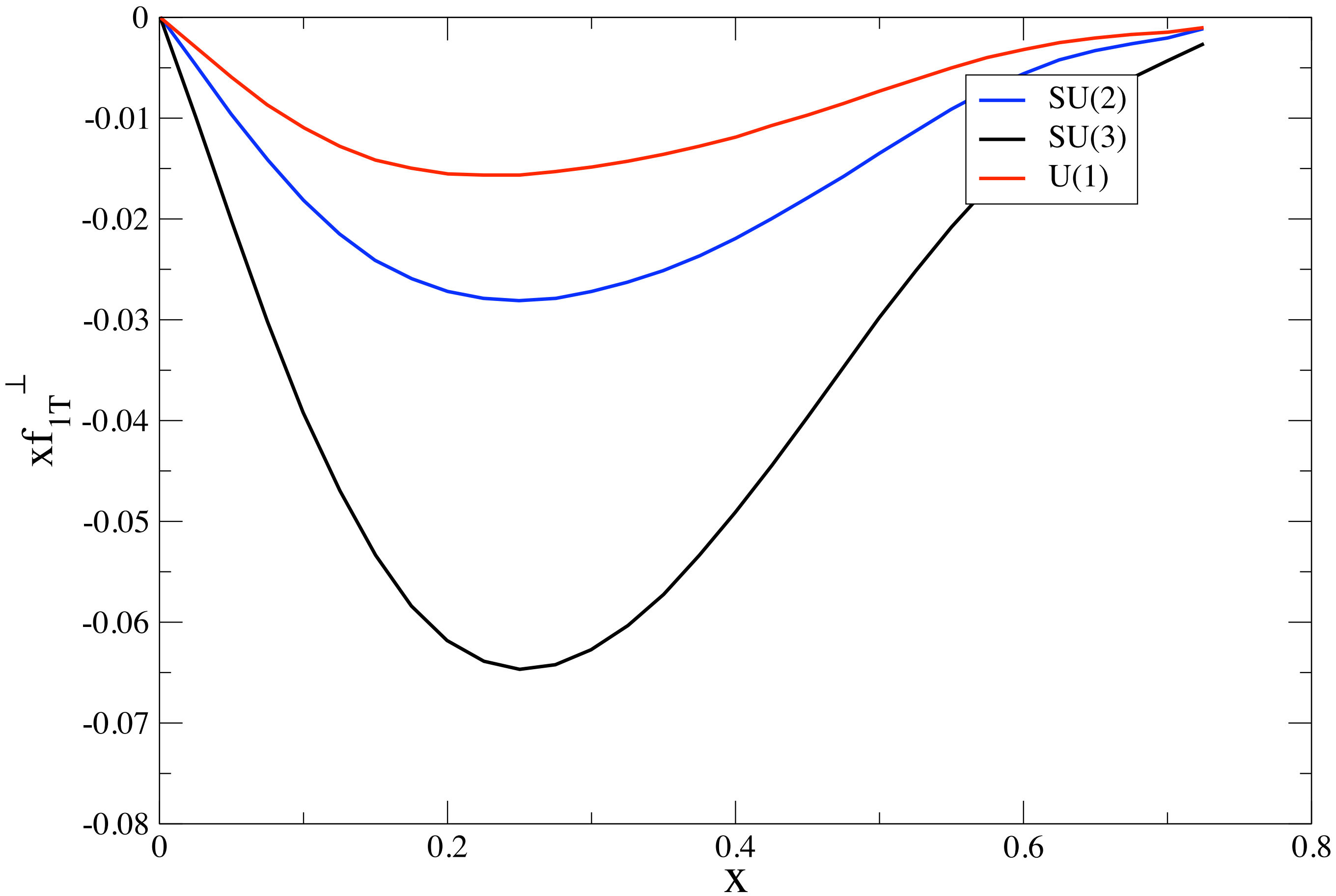


Relations produce a BM funct. approx equiv. to Sivers from HERMES

Expected sign i.e. FSI are negative

Answer will come from pion BM from COMPASS πN **Drell Yan**

Results for u-quark Sivers



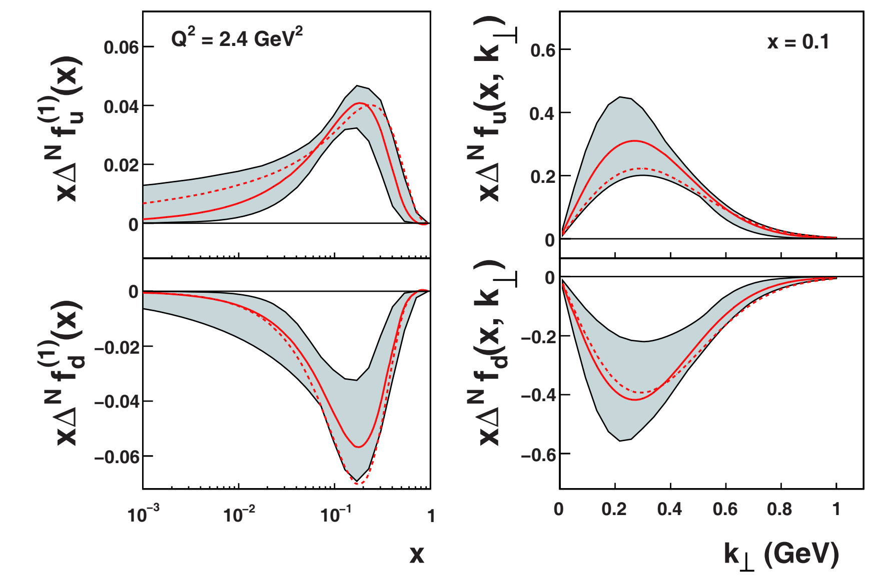


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

- Relations produce a Sivers effect 0.10-0.65
- Torino extraction ~ 0.05
- Color increase effect

To Do for Model Builders

- TMD part of the cos 2ϕ asymmetry using your Boer-Mulders functions (in pion and proton)
- Polarized asymmetry using the Boer-Mulders in the pion and the transversity in the proton
- Just to partially answer to the first question, when specialized to the Compass kinematics.
- And it would be great as well if you could say something on the last question.....About sea contributions..

Extra Slides

"Factorization" of Distortion and FSIs

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

$$\langle k_T^{q,i}(x)\rangle_{UT} = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\left[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\right]$$

Manipulate gauge link and trnsfm to $\vec{b}$ space

1) $$\langle k_T^{q,i}(x)\rangle_{UT} = \frac{1}{2}\int d^2\vec{b}_T \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+,\vec{0}_T;S|\bar{\psi}(z_1)\gamma^+\mathcal{W}(z_1;z_2)I^{q,i}(z_2)\psi(z_2)|P^+,\vec{0}_T;S\rangle$$

2) $$\mathcal{F}^{q[\Gamma]}(x,\vec{b}_T;S) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+,\vec{0}_T;S|\bar{\psi}(z_1)\Gamma\mathcal{W}(z_1;z_2)\psi(z_2)|P^+,\vec{0}_T;S\rangle, \qquad \Gamma \equiv \gamma^+$$

Comparing expressions difference is additional factor, $I^{q,i}$ and integration over $\vec{b}$

3) $$\langle k_T^{q,i}(x)\rangle_{UT} \simeq \int d^2\vec{b}_T\, I^{q,i}(x,\vec{b}_T)\mathcal{F}^q(x,\vec{b}_T;S).$$

FLOW CHART for calculation of Boer Mulders

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

$$2m_\pi^2 h_1^{\perp(1)}(x) \simeq \int d^2 b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x, \vec{b}_T)\, \frac{\partial}{\partial \vec{b}_T^2} \mathcal{H}_1^\pi(x, \vec{b}_T^2),$$

$$I^i(x, \vec{q}_T) = \frac{1}{N_c} \int \frac{d^2 p_T}{(2\pi)^2} (2p_T - q_T)^i \left(\Im[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\delta\beta}^{\alpha\delta} (|\vec{p}_T|)$$

$$\left((2\pi)^2 \delta^{\alpha\beta} \delta^{(2)}(\vec{p}_T - \vec{q}_T) + \left(\Re[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\gamma\alpha}^{\beta\gamma} (|\vec{p}_T - \vec{q}_T|)\right).$$

$$\left(\mathrm{M}^{\mathrm{eik}}\right)_{\delta\beta}^{\alpha\delta} (x, |\vec{q}_T + \vec{k}_T|) = \frac{(1-x)P^+}{m_s} \int d^2 z_T\, \mathrm{e}^{-i\vec{z}_T\cdot(\vec{q}_T + \vec{k}_T)} \quad (20)$$

$$\times\left[\int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\, \mathrm{e}^{-i\alpha\cdot u} \left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta} \left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}\right].$$

COLOR Integral

$$f_{\alpha\beta}(\chi) \equiv \int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\, \mathrm{e}^{-i\alpha\cdot u} \left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta} \left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}$$

$$f_{\alpha\beta}(\chi) = \sum_{n=1}^{\infty} \frac{(i\chi)^n}{(n!)^2} \sum_{a_1=1}^{N_c^2-1} \cdots \sum_{a_n=1}^{N_c^2-1} \sum_{P_n} (t^{a_1} \ldots t^{a_n} t^{a_{P_n(1)}} \ldots t^{a_{P_n(n)}})_{\alpha\beta}\,.$$

Reality Check

Parm. of GTMD correlator hermiticity parity time-reversal

from Andreas Metz INT talk

$$(x, \xi, \vec{k}_T, \vec{\Delta}_T)$$

$$W^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GTMD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0}$$

- Projection onto GPDs and TMDs

$$F^q = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=z_T=0}$$

$$= \int d^2\vec{k}_T \, W^q$$

$$\Phi^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{TMD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0}$$

$$= W^q \Big|_{\Delta=0}$$

GTMD-Wigner Function Correlator

- Parameterization of GTMD-correlator

Miessner Metz & Schlegel JHEP 2008 & 2009

Example:

$$W^{q\,[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

$\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

- Implications for potential nontrivial relations
  - Relations of second type

$$E(x,0,\vec{\Delta}_T^2) = \int d^2\vec{k}_T\left[-F_{1,1}^e + 2\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,F_{1,2}^e + F_{1,3}^e\right)\right]$$

$$f_{1T}^\perp(x,\vec{k}_T^2) = -F_{1,2}^o(x,0,\vec{k}_T^2,0,0)$$

These Have Different Mothers

$$\int d^2\vec{b}_T \, \mathcal{H}^q(x, \vec{b}_T^2) = \int d^2\vec{k}_T \, f_1^q(x, \vec{k}_T^2) = \int d^2\vec{k}_T \, \mathrm{Re}\Big[F_1^q(x, 0, \vec{k}_T^2, 0, 0)\Big]$$

$$f_{1T}^{\perp}(x, \vec{k}_T^2; \eta) = -F_{1,2}^{o}(x, 0, \vec{k}_T^2, 0, 0; \eta)$$

$$E(x, \xi, t) = \int d^2\vec{k}_T \left[- F_{1,1}^{e} + 2(1-\xi^2) \left(\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} \, F_{1,2}^{e} + F_{1,3}^{e} \right) \right]$$

$\longrightarrow$ No model-independent nontrivial relation between E and $f_{1T}^{\perp}$ possible

$\longrightarrow$ Relation in spectator model due to simplicity of the model

$\longrightarrow$ No information on numerical violation of relation

$\longrightarrow$ Likewise for nontrivial relation involving $h_1^{\perp}$

However is approximate relation good for phenomenological approach for model builders

Weighted Azimuthal asymm. corresponds to transv. moments of correlator: projects gluonic pole ME

$$\Phi_\partial^{\alpha[\mathcal{U}]}(x) = \int d^2k_T k_T^\alpha \Phi^{[\mathcal{U}]}(x, k_T).$$

Decomposes $\Phi_\partial^{\alpha\,[\mathcal{U}]}(x) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[\mathcal{U}]}\,\pi\Phi_G^\alpha(x,x),$

T-even → $\tilde{\Phi}_\partial^\alpha(x)$

T-odd → $\pi\Phi_G^\alpha(x,x)$

$$\langle k_T^{q,i}(x)\rangle_{UT} = \int d^2k_T\, k_T^i\, \frac{1}{2}\left[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\right]$$

$$A_{UT}^{P_{h\perp}/M}\sin(\phi - \phi_S)(x,z) = \frac{(-2)\sum_a e_a^2 x f_{1T}^{\perp(1)a}(x) D_1^a(z)}{\sum_a e_a^2 x f_1^a(x) D_1^a(z)}$$

Weighted Cross Sections contain ETQS Functions LINK BTW TWO Pictures!

Qiu-Sterman:PLB 1991, 1999, Koike et al. PLB 2000. . . 2007,

Ji,Qiu,Vogelsang,Yuan:PR 2006,2007. . .