



# Theory - round table (exclusive DY, GPDs)

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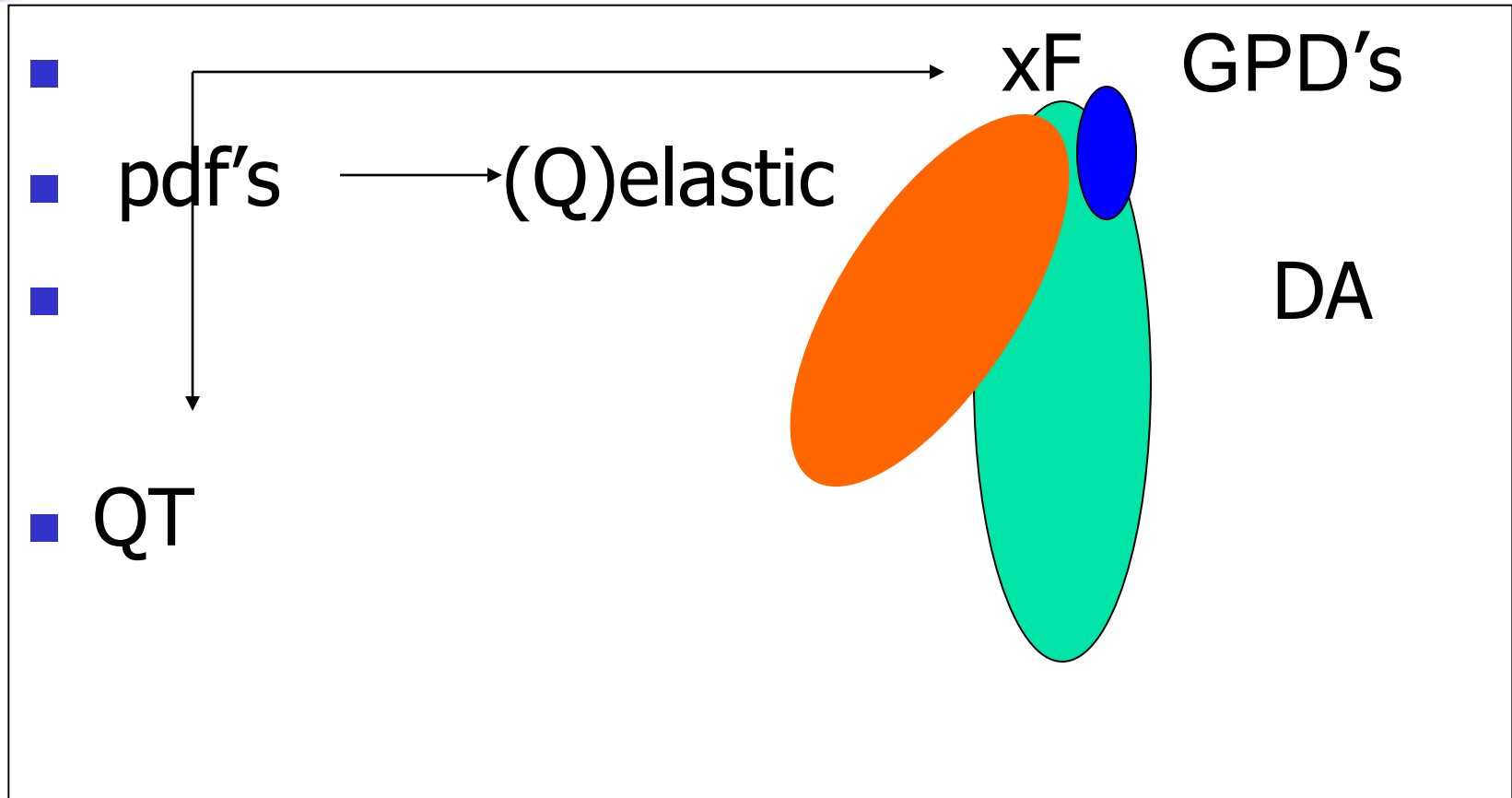
# Why exclusive limits of DY are interesting?



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- Possibility to study the inclusive-exclusive transition region and duality
- Access to the shape of pion Distribution Amplitude (simplest case of GPDs) – important because of BABAR puzzle and possible violation of (collinear) factorization due to flat DA (Radyushkin, Polyakov)
- Coherence of COMPASS program – study of GPDs in very different processes (crossing and analyticity in QCD)
- Theoretical interplay: TMDs  $\leftrightarrow$  GPDs
- Similarity in theoretical approaches

# Exclusive limits for DY



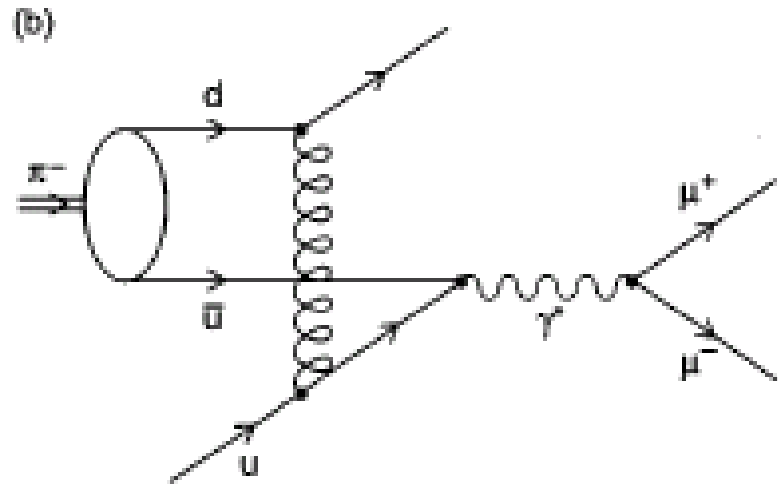
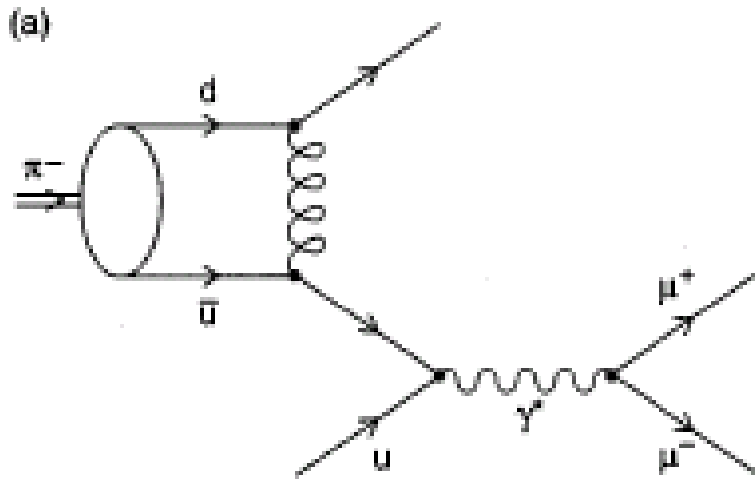


# Summed TDAs

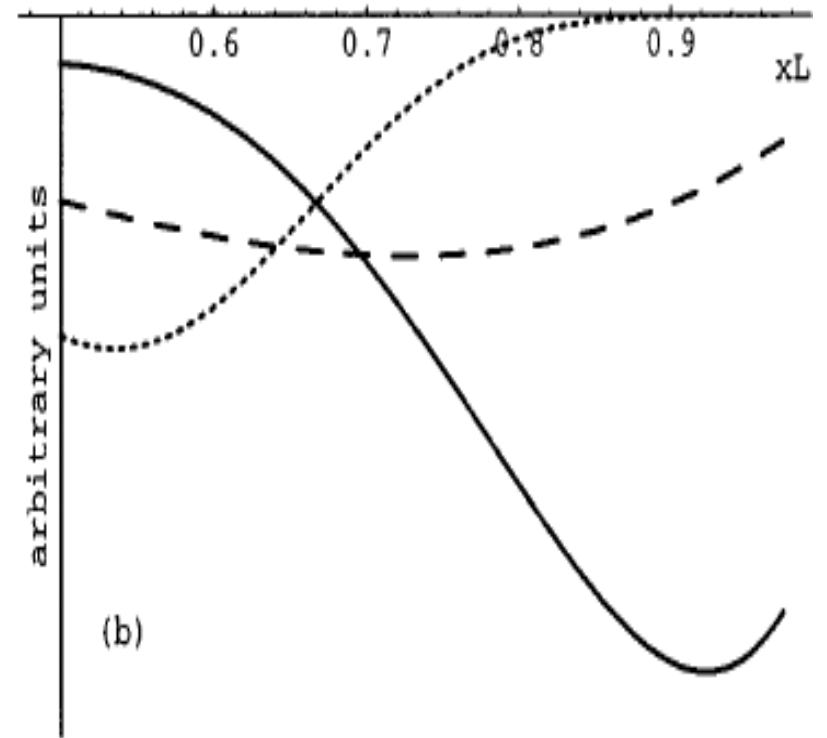
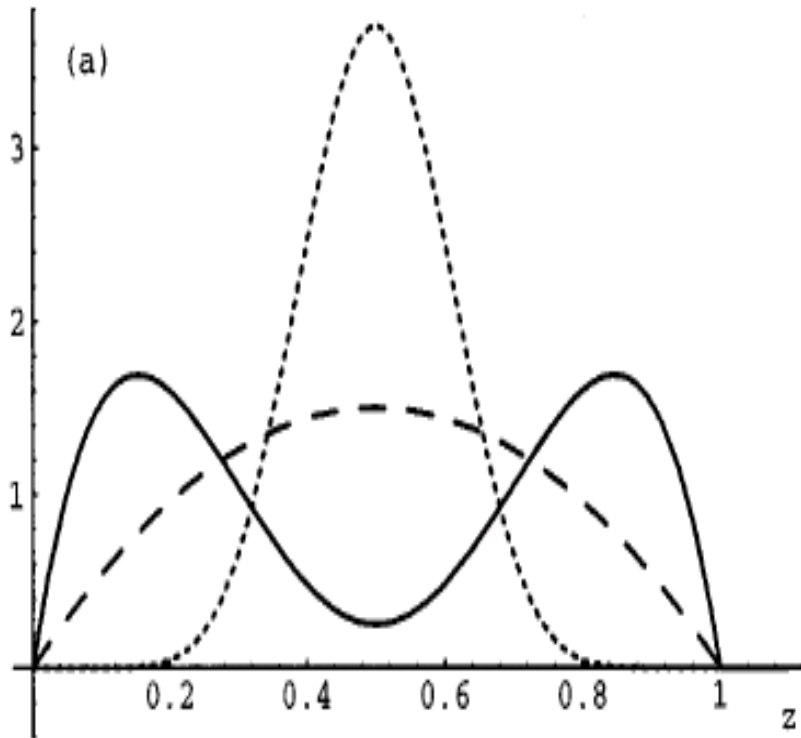
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- Low invariant mass – various resonances (and continuum) may contribute
- Same coefficient functions -> same scaling - added incoherently
- Similar situation -> in 2 vector mesons productions at L3 – various GDA's squared added (Anikin,Pire,OT)

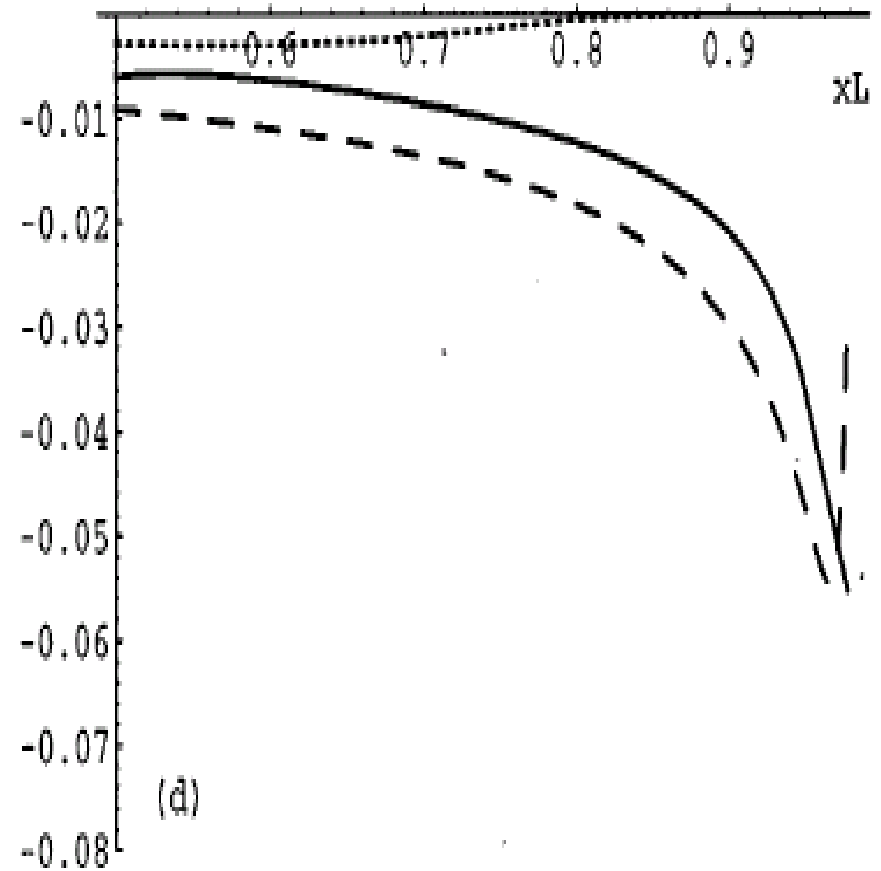
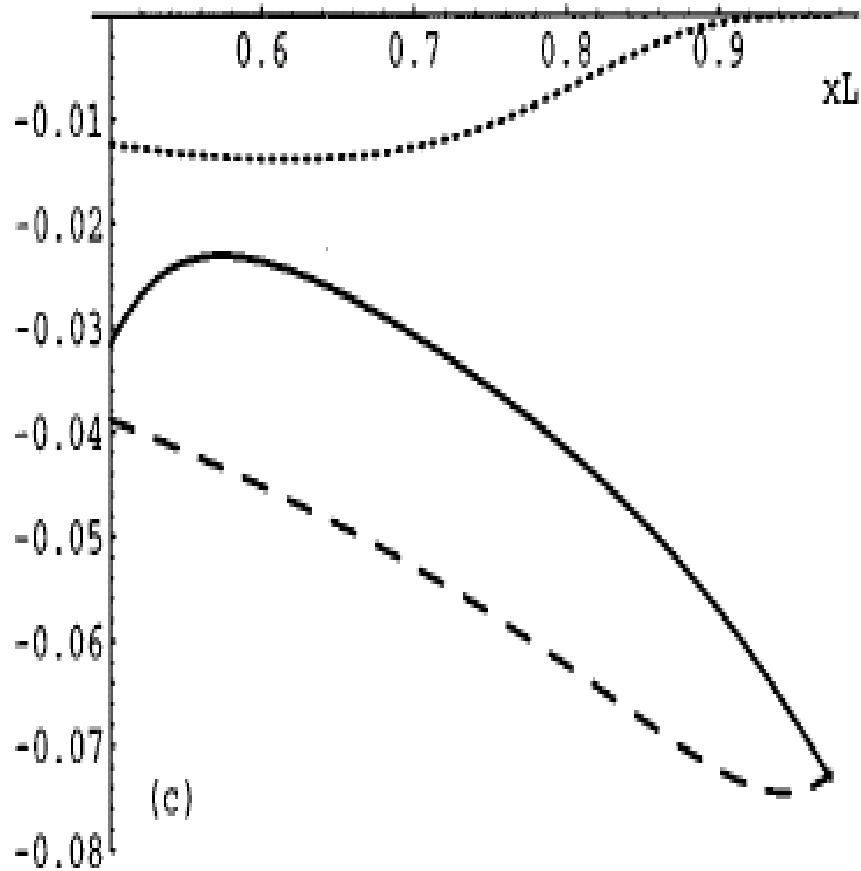
# Pion Light-cone Distribution in pion-(q)proton scattering



# Models for light-cone distributions and angular-weighted x-sections (Brandenburg, Mueller, OT; Bakulev, Stefanis, OT)



# Size of coefficients in angular distributions





# DA@COMPASS?

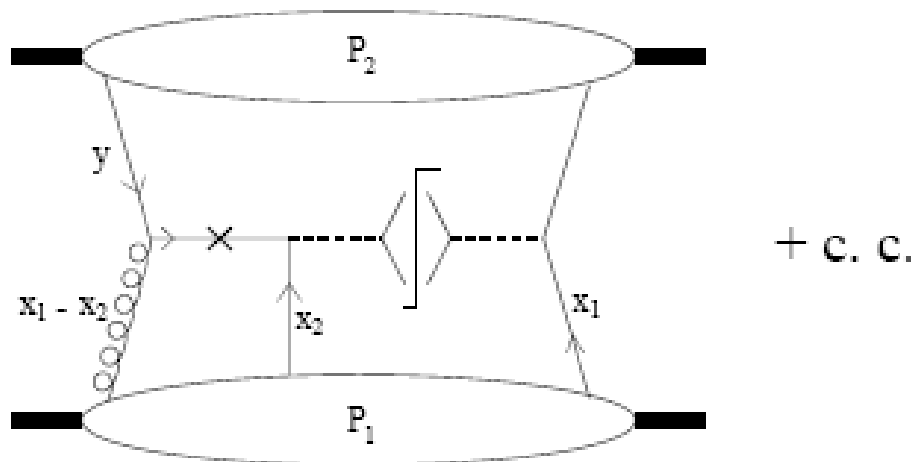
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- Suppression ( $\sim QT/Q$ ) of most pure longitudinal (Pire-Ralston) SSA – DA for transverse polarization?
- Transition pdf  $\leftrightarrow$  DA?
- DA in angular distribution for unpolarized target – how flat DA works?
- Pion exclusive electroproduction – flat DA – is similarity with BABAR behaviour observable?!



# Duality for SSA in DY

- TM integrated DY with one transverse polarized beam (– unique SSA – gluonic pole (Hammon, Schaefer, OT))



$$A = g \frac{\sin 2\theta \cos \phi \left[ T(x, x) - x \frac{dT(x, x)}{dx} \right]}{M [1 + \cos^2 \theta] q(x)}$$

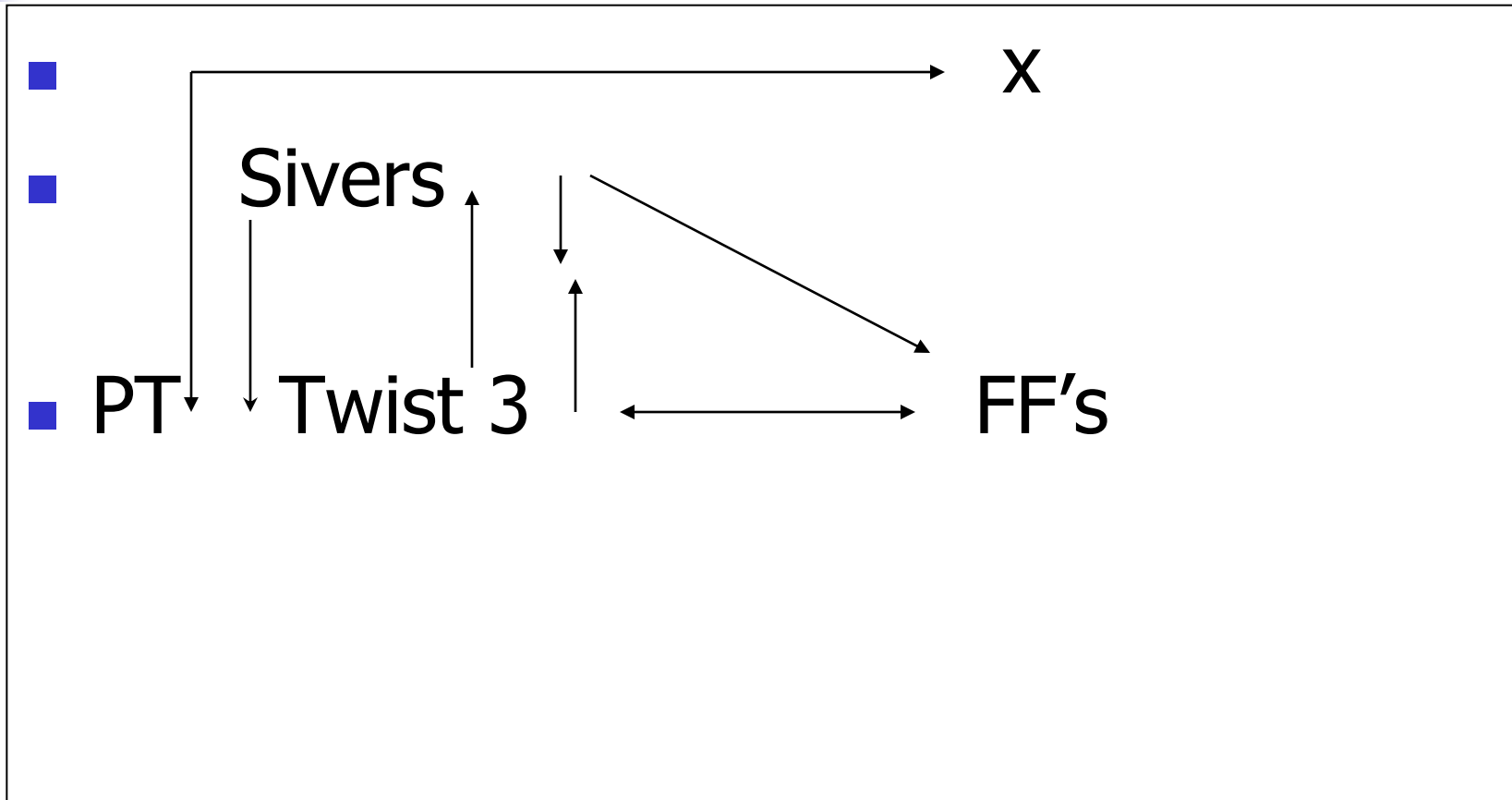


# SSA in exclusive limit

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- Proton-antiproton – valence annihilation  
- cross section is described by Dirac FF squared
- The same SSA due to interference of Dirac and Pauli FF's with a phase shift
- Exclusive large energy limit;  $x \rightarrow 1$  :  
 $T(x,x)/q(x) \rightarrow \text{Im } F_2/F_1$

# Kinematical domains for SSA's





# Discussions

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- GPDs – Bernard Pire
- Transition to exclusive –  
Andrea Bianconi



# Azimuthal Asymmetries

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Sensitive test of QCD – reflect the  
existence

of natural scattering PLANE

May be T-odd (talks of A.Efremov, A.  
Prokudin, S, Melis) and T-EVEN

# Important case: Drell-Yan process

- Angular distribution (leptons c.m. frame):

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \sim 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi .$$

- Lam –Tung relation (sum rule)

$$1 - \lambda - 2\nu = 0$$



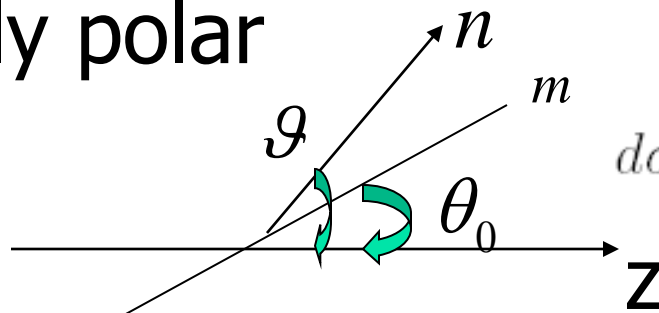
# Status of Lam-Tung relation

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- Holds at LO and (approximately) at NLO QCD
- violated by
- higher twists (Brodsky et al.),
- correlations of T-odd distributions(Boer),
- entanglement of quarks in QCD vacuum (Nachtmann)
- (and EXPERIMENTALLY).
- Physical origin?!

# Kinematic azimuthal asymmetry from polar one

Only polar



$$d\sigma \propto 1 + \lambda_0 (\vec{n}\vec{m})^2 = 1 + \lambda_0 \cos^2 \theta_{nm}$$

asymmetry with respect to  $m$ !

$$\cos \theta_{nm} = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos \phi$$

- azimuthal

angle appears with new

$$\lambda = \lambda_0 \frac{2 - 3 \sin^2 \theta_0}{2 + \lambda_0 \sin^2 \theta_0}$$

$$\nu = \lambda_0 \frac{2 \sin^2 \theta_0}{2 + \lambda_0 \sin^2 \theta_0}$$





# Generalized Lam-Tung relation

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- Relation between coefficients (high school math sufficient!)

$$\lambda_0 = \frac{\lambda + \frac{3}{2}\nu}{1 - \frac{1}{2}\nu}$$

- Reduced to standard LT relation for transverse polarization ( $\lambda_0 = 1$ )
- LT - contains two very different inputs: kinematical asymmetry+transverse polarization

# How realistic is “m-model”?- averaging procedure

- Theorem for averages

$$\frac{\int d\vec{x} \Delta\sigma(\vec{x})}{\int d\vec{x} \sigma(\vec{x})} = \frac{\Delta\sigma(\vec{x}_0)}{\sigma(\vec{x}_0)}$$

- Convenient tool for asymmetries analysis (e.g. Collins suppression with respect to Sivers – talks of A. Prokudin, S. Melis)
- For N-dimesinsional integration – N-1-dimensional “orbits”

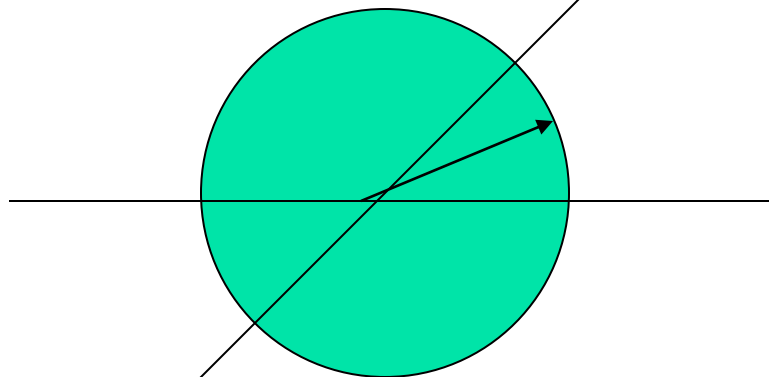
# Application for average in semi-inclusive Drell-Yan

- Choose

$$d\sigma = \sigma \lambda_0 (\vec{n} \vec{m})^2$$

- Average :

$$d\sigma \propto 1 + \lambda_0 (\vec{n} \vec{m}_0)^2$$



- Representative of "orbit" crossing scattering plane – cannot depend on  $n$

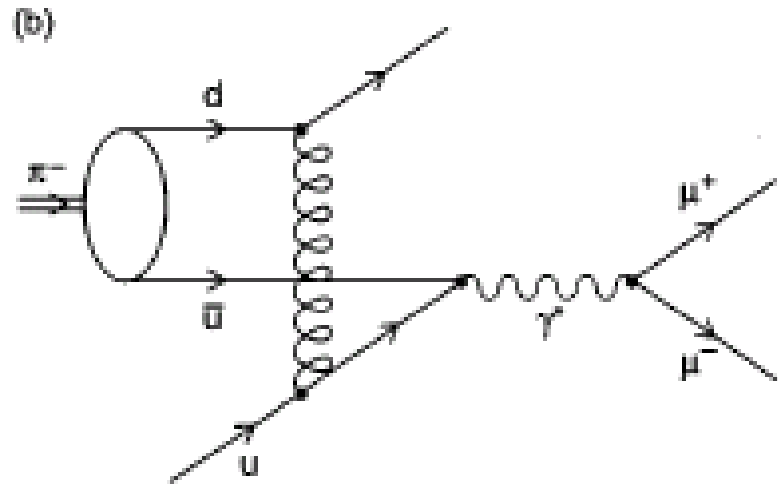
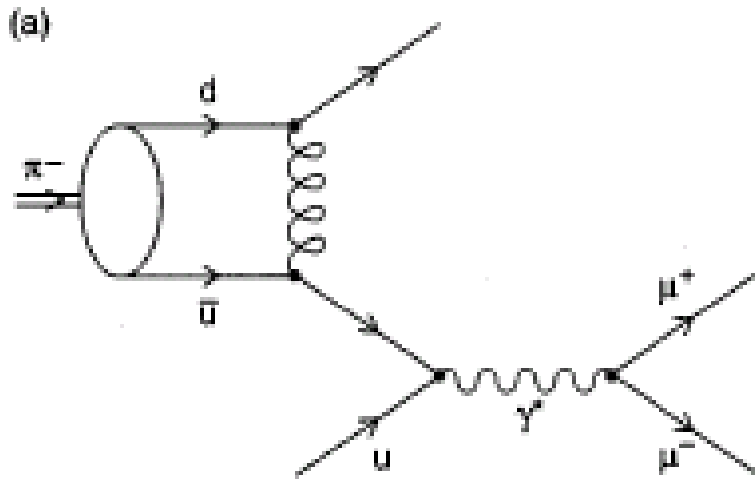


# GLT relation -applicability

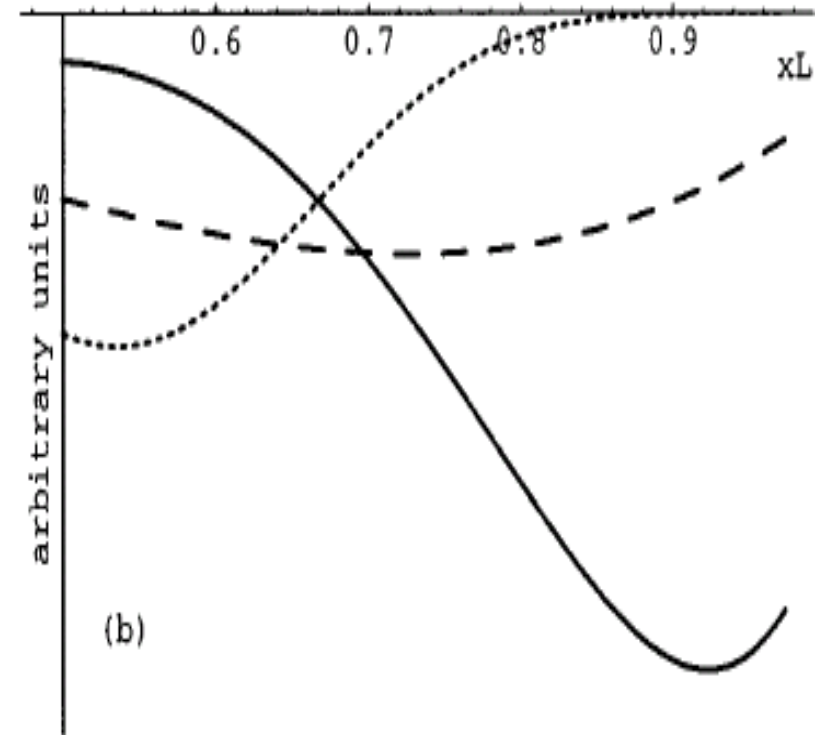
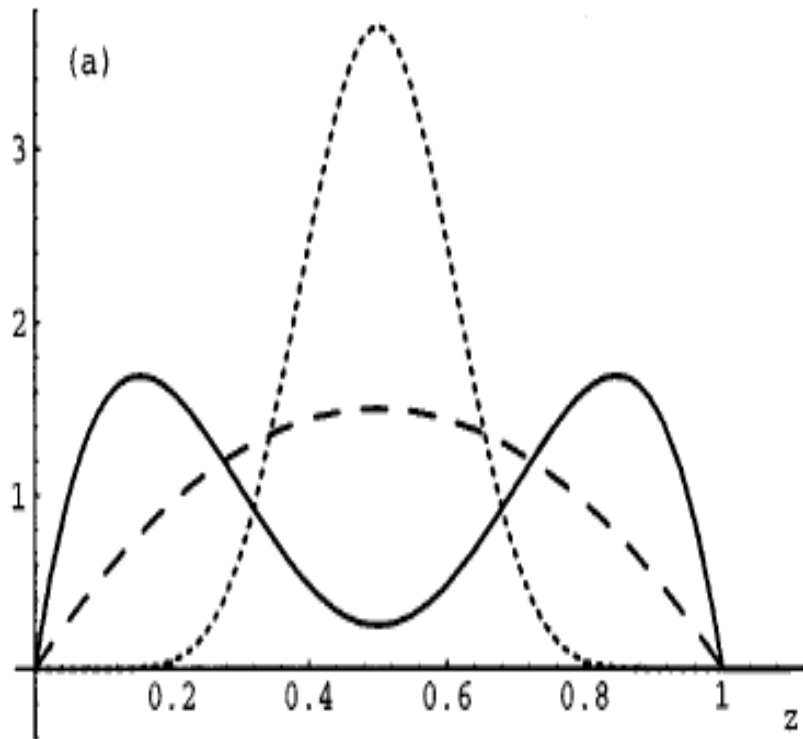
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- Appears for KINEMATICAL asymmetry in semi-inclusive process (only one physical plane exists)
- Violated if there is azimuthal asymmetry already in the subprocess (with respect to  $m$ ) – NLOQCD, HT.

# Pion Light-cone Distribution in pion-(q)proton scattering



# Models for light-cone distributions and angular-weighted x-sections





# Further studies

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- Various energy dependence of various sources of LT violation – DY at COMPASS, LHC (CMS)
- HT-updated pion distribution (Bakulev, Stefanis, OT)
- Simultaneous analysis with spin-dependent azimuthal asymmetries
- Heavy-ion collisions



# CONCLUSIONS

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Lam – Tung relation – different inputs

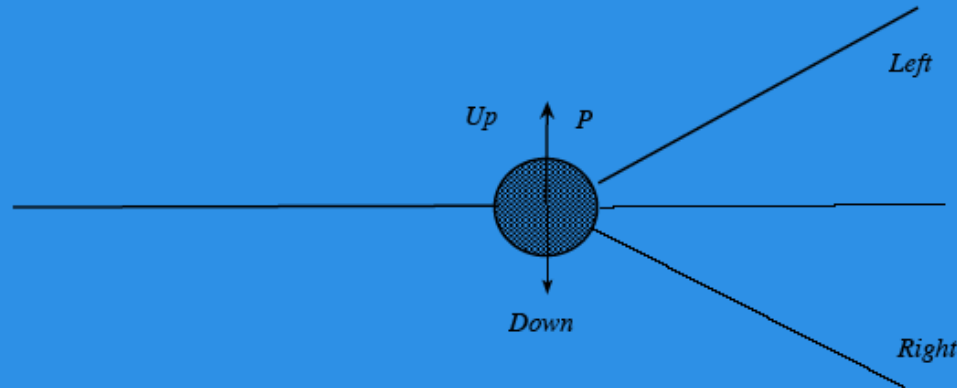
Separating of their role – generalization

DY at very different energy scale,  
Heavy-Ion collisions



# Non-relativistic Example

Simplest example - (non-relativistic) elastic pion-nucleon scattering  $\pi \vec{N} \rightarrow \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$   $\vec{n}$  is the normal to the scattering plane.

Density matrix:  $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$ ,

Differential cross-section:  $d\sigma \sim 1 + A(\vec{P}\vec{n})$ ,  $A = \frac{2\text{Im}(ab^*)}{|a|^2 + |b|^2}$



# Phases in QCD-I

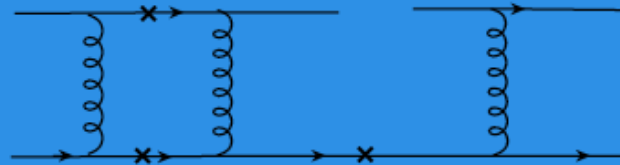
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- QCD factorization – soft and hard parts-
- Phases form soft (single-double relations requires NPQCD inputs), hard and overlap (relations possible)
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdaal (1960), found at JLAB recently):  
Kane, Pumplin, Repko (78) Efremov (78)

# Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like  $q - e$  scattering in DIS):

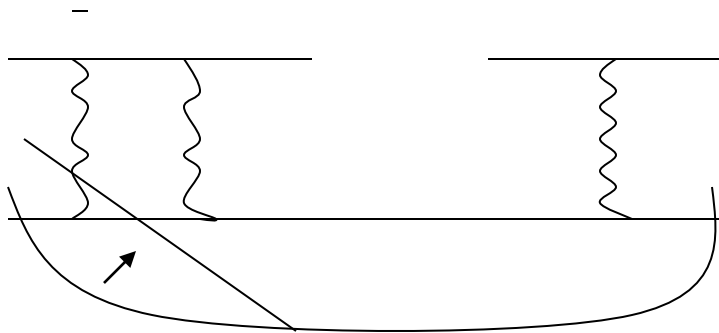


$$A \sim \frac{\alpha_S^{m_{PT}}}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# Short+ large overlap– twist 3

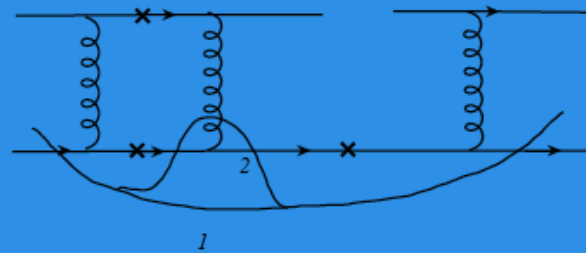
- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2  
– Born twist 3: Efremov, OT (85, Fermion poles); Qiu, Sterman (91, GLUONIC poles)

# Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop  $\rightarrow$  Born diagram

At Large distances - quark distribution  $\rightarrow$  quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of  $\alpha_S$  to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m_{PT}}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.



# Phases in QCD –large distances in fragmentation

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- Non-perturbative - positive variable
- Jet mass-Fragmentation function: Collins(92);Efremov,Mankiewicz, Tornqvist (92),
- Correlated fragmentation: Fracture function: Collins (95), O.T. (98).



# Phases in QCD-Large distances in distributions

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- Distribution :Sivers, Boer and Mulders – no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process: “Effective” or “non-universal” SH interactions by physical gluons – Twist-3 (Boer, Mulders, OT, 97)
- Brodsky (talk) -Hwang-Schmidt(talk) model:the same SH interactions as twist 3 but non-suppressed by Q: Sivers function – leading (twist 2).

# What is “Leading” twist?

- Practical Definition - Not suppressed as  $M/Q$
- However – More general definition: Twist 3 may be suppressed

as  $M/P_T$

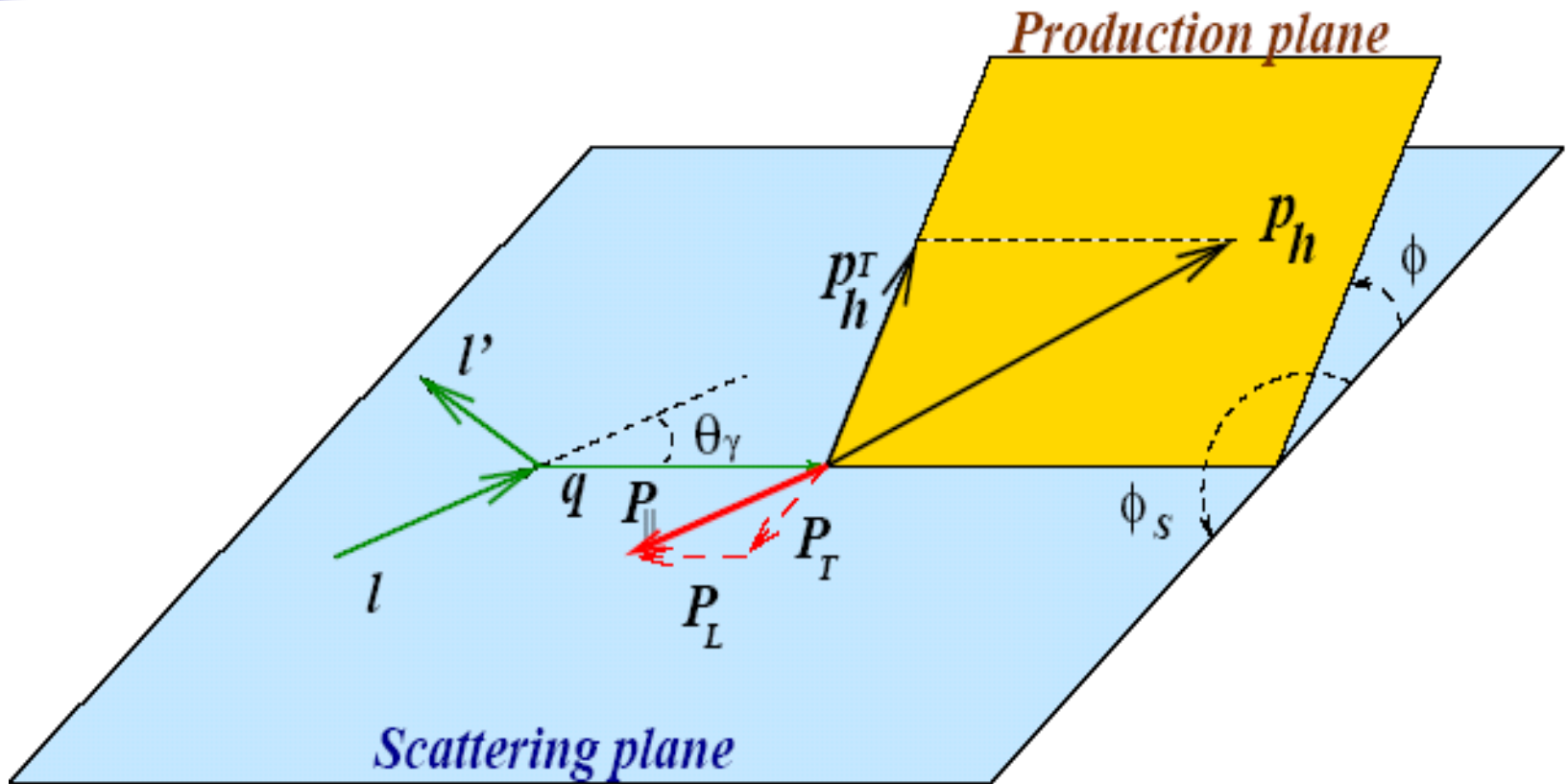
Twist 3 may contribute at leading order in  $1/Q$  !

Does this happen indeed?? – Explicit calculation for the case when  $Q \gg P_T$

May be interesting for experimental studies

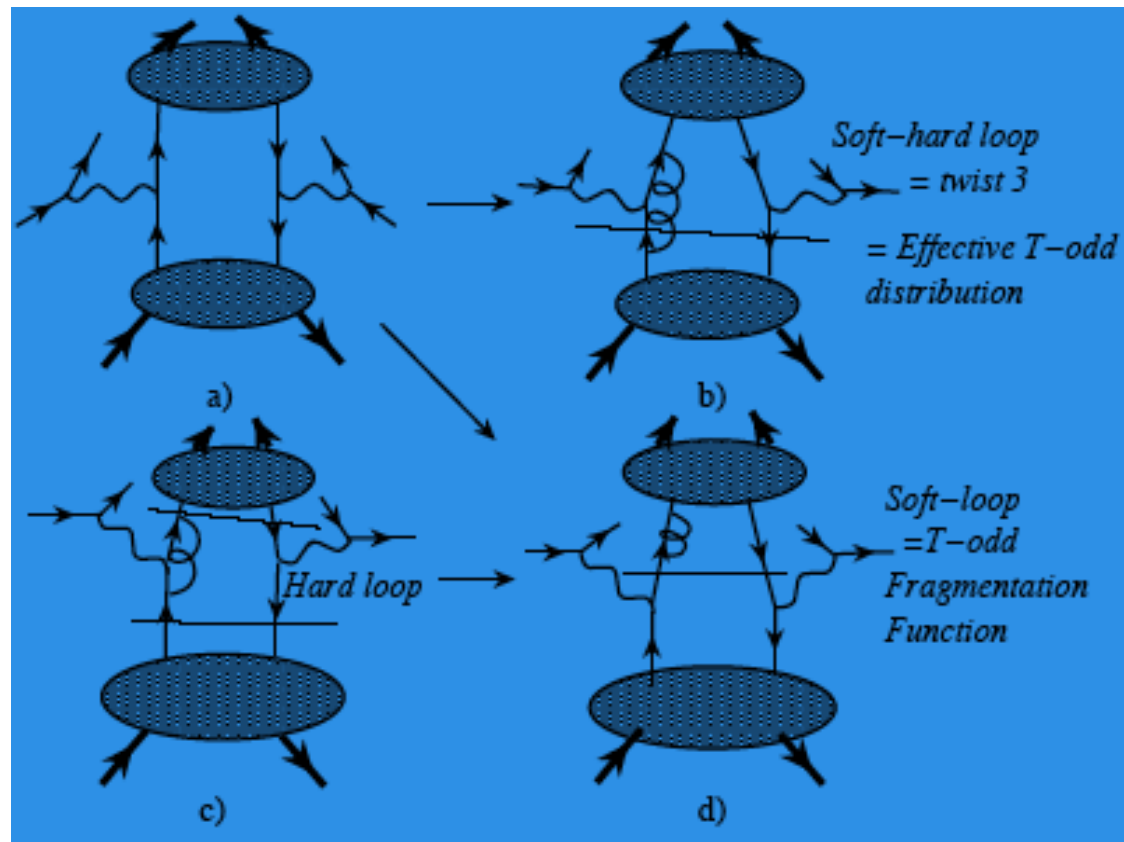


# Test ground for SSA : Semi-Inclusive DIS - kinematics

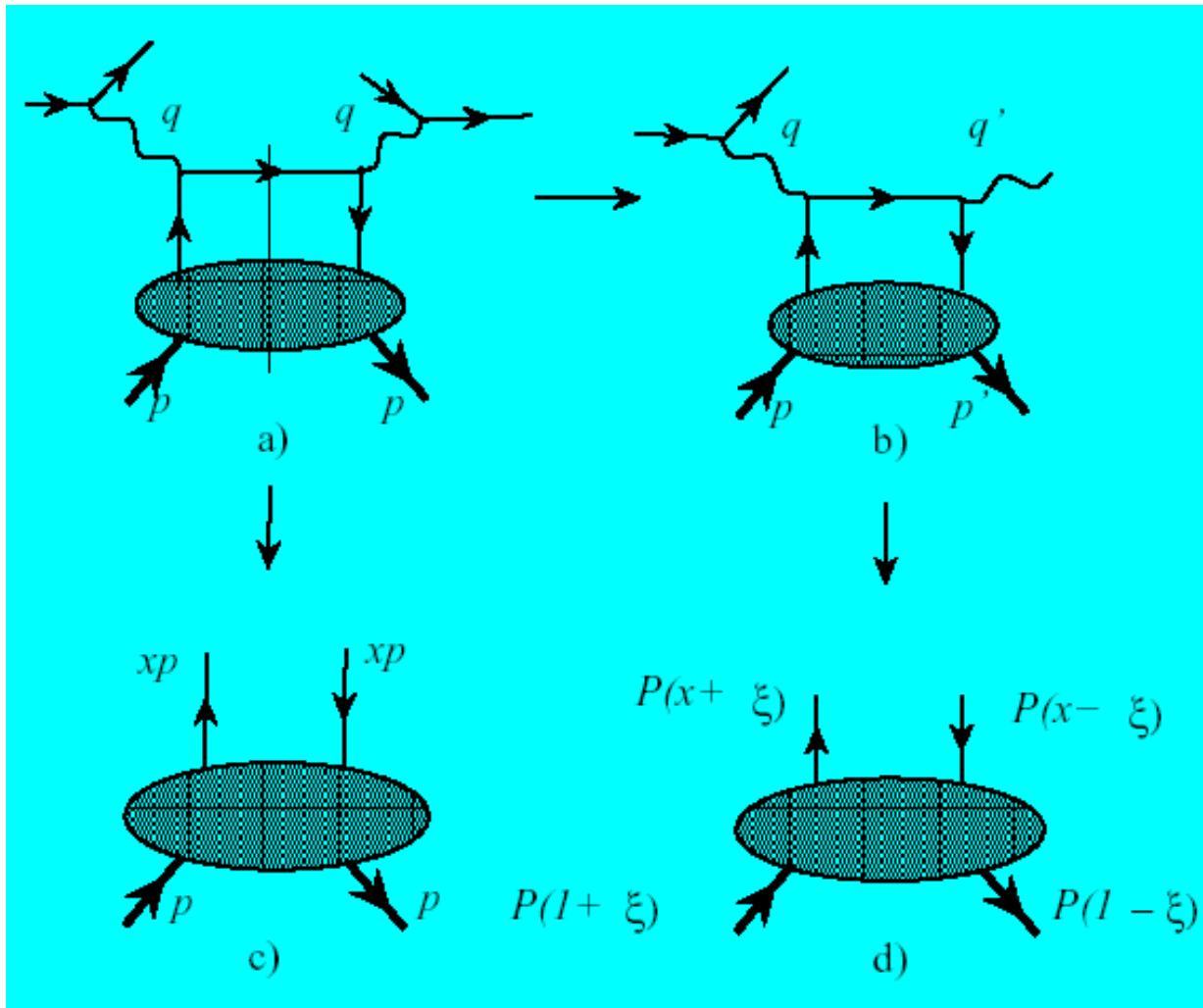


# Sources of Phases in SIDIS

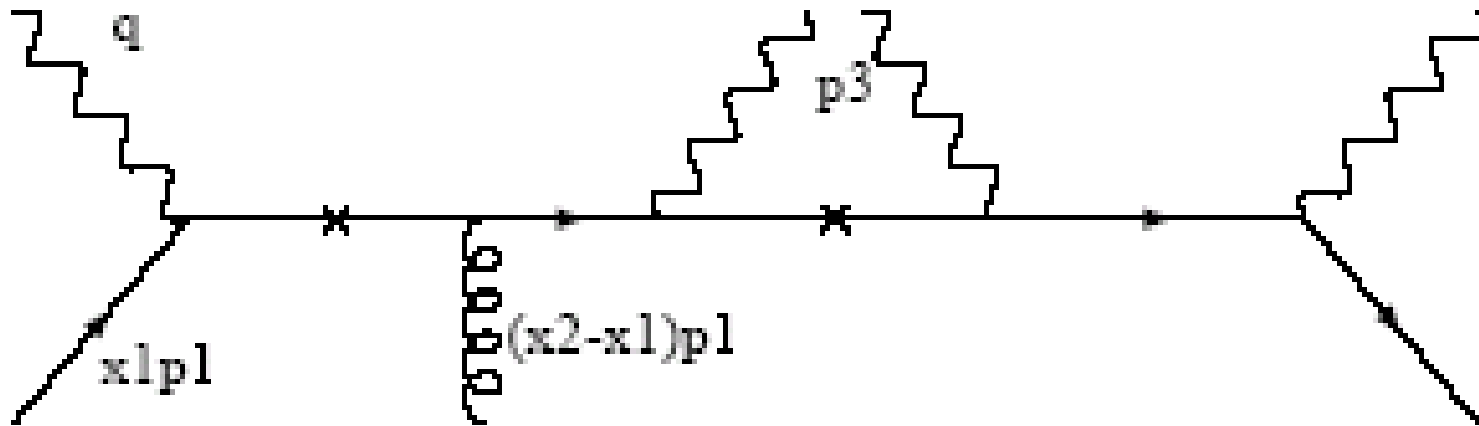
- a) Born - no SSA
- b) -Sivers (can be only effective)
- c) Perturbative
- d) Collins

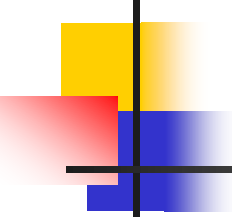


# Final Pion $\rightarrow$ Photon: SIDIS $\rightarrow$ SIDVCS (clean, easier than exclusive) - analog of DVCS



# Twist 3 partonic subprocesses for SIDVCS



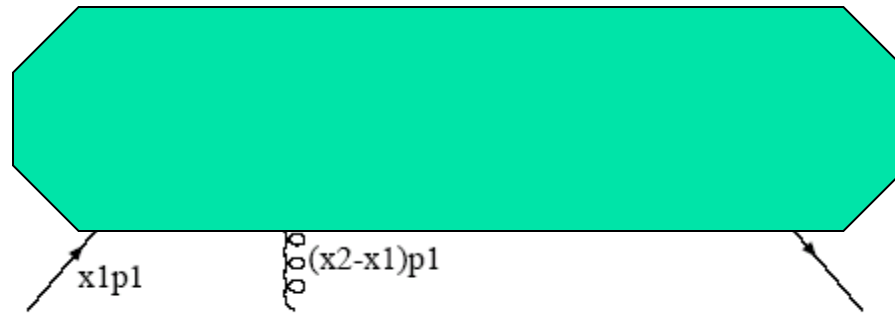


# Real and virtual photons - most clean tests of QCD

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- Both initial and final – real :Efremov, O.T. (85)
- Initial – quark/gluon, final - real : Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial - real, final-virtual (or quark/gluon) – Korotkiiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05, in preparation; smooth transition from fermionic to GLUONIC poles).

# Quark-gluon correlators



- Non-perturbative NUCLEON structure – physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta – quark momentum fractions are close to each other- gluonic pole; probed if :  
 $Q \gg P_T \gg M$

$$x_2 - x_1 = \delta = \frac{P_T^2 x_B}{Q^2 z}$$

# Cross-sections at low transverse momenta:

$$d\sigma_{total} = f(x_{Bj})8Q^2 \frac{x_{Bj}^2(1+(1-y)^2)(1+(1-z)^2)}{y^2z\delta} \quad (12)$$

$$d\sigma_{ax_1x_2} = b_A(x_{Bj}, x_2)8M_{PT} \frac{x_{Bj}(1+(1-y)^2)(2-z)}{y^2(1-z)\delta} s_T \sin(\phi_s^h) \quad (13)$$

$$d\sigma_{vx_1x_2} = b_V(x_{Bj}, x_2)8M_{PT} \frac{x_{Bj}(1+(1-y)^2)(1+(1-z)^2)}{y^2z(1-z)\delta} s_T \sin(\phi_s^h) \quad (14)$$

$$d\sigma_{a0x_2} = -b_A(0, x_2)8M_{PT} \frac{x_{Bj}^2(2(1-y)(1-2z) + y^2(1-z))}{y^2z^2\delta} s_T \sin(\phi_s^h)$$

(14) - non-suppressed for large Q if Gluonic pole exists=effective Sivers function; spin-dependent looks like unpolarized (soft gluon)

$$A \propto \frac{2M p_T \varphi_V(x_B)}{m_T^2 x_B q(x_B)} s_T \sin \phi_h^s$$



# Effective Sivers function

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- Needs (soft) talk of large and short distances
- Complementary to gluonic exponential, when longitudinal (unsuppressed by  $Q$ , unphysical) gluons get the physical part due to transverse link (Belitsky, Ji, Yuan)
- We started instead with physical (suppressed as  $1/Q$ ) gluons, and eliminated the suppression for gluonic pole.
- Advantage: use of standard twist-3 factorization, describing also T-EVEN DOUBLE Asymmetries – key for relating single and double asymmetries





# Twist 3 factorization (Efremov, OT '84, Ratcliffe, Qiu, Sterman)

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- Convolution of soft (S) and hard (T) parts

$$d\sigma_s = \int dx_1 dx_2 \frac{1}{4} Sp[S_\mu(x_1, x_2) T_\mu(x_1, x_2)]$$

- Vector and axial correlators: define hard process for both double ( $g_2$ ) and single asymmetries

$$T_\mu(x_1, x_2) = \frac{M}{2\pi} (\hat{p}_1 \gamma^5 s_\mu b_A(x_1, x_2) - i \gamma_\rho \epsilon^{\rho\mu sp_1} b_V(x_1, x_2))$$



# Twist 3 factorization -II

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- Non-local operators for quark-gluon correlators

$$b_A(x_1, x_2) = \frac{1}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \langle p_1, s | \bar{\psi}(0) \hat{n} \gamma^5 (D(\lambda_1) s) \psi(\lambda_2) | p_1, s \rangle,$$

$$b_V(x_1, x_2) = \frac{i}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \epsilon^{\mu s p_1 n} \langle p_1, s | \bar{\psi}(0) \hat{n} D_\mu(\lambda_1) \psi(\lambda_2) | p_1, s \rangle$$

- Symmetry properties (from T-invariance)

$$b_A(x_1, x_2) = b_A(x_2, x_1), \quad b_V(x_1, x_2) = -b_V(x_2, x_1)$$



# Twist-3 factorization -III

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- Singularities

$$b_A(x_1, x_2) = \varphi_A(x_1)\delta(x_1 - x_2) + b_A^r(x_2, x_1).$$

$$b_V(x_1, x_2) = \frac{\varphi_V(x_1)}{x_1 - x_2} + b_V^r(x_1, x_2)$$

- Very different: for axial – Wandzura-Wilczek term due to intrinsic transverse momentum
- For vector-GLUONIC POLE (Qiu, Sterman '91)
  - large distance background



# Sum rules

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- EOM + n-independence (GI+rotational invariance) –relation to (genuine twist 3) DIS structure functions

$$\int_0^1 x^n \bar{g}_2(x) dx = \int_0^1 x^n \left( \frac{n}{n+1} g_1(x) + g_2(x) \right) dx =$$
$$-\frac{1}{\pi(n+1)} \int_{|x_1, x_2, x_1-x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 \left[ \frac{n}{2} b_V(x_1, x_2) (x_1^{n-1} - x_2^{n-1}) + \right.$$
$$\left. b_A^r(x_1, x_2) \phi_n(x_1, x_2) \right], \quad \phi_n(x, y) = \frac{x^n - y^n}{x - y} - \frac{n}{2} (x^{n-1} - y^{n-1}), \quad n = 0, 2, \dots$$



# Sum rules -II

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- To simplify – low moments

$$\int_0^1 x^2 \hat{g}_2(x) dx = -\frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 b_V(x_1, x_2) (x_1 - x_2)$$

- Especially simple – if only gluonic pole kept:

$$\begin{aligned} \int_0^1 x^2 \bar{g}_2(x) dx &= -\frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \leq 1} dx_1 dx_2 \sum_f e_f^2 \varphi_V(x_1) \\ &= -\frac{1}{3\pi} \int_{-1}^1 dx_1 \sum_f e_f^2 \varphi_V(x_1) (2 - |x_1|) \end{aligned}$$

# Gluonic poles and Sivers function

- Gluonic poles – effective Sivers functions-Hard and Soft parts talk, but SOFTLY
- Supports earlier observations: Boer, Mulders, O.T. (1997); Boer, Mulders, Pijlman (2003).
- Implies the sum rule for effective Sivers function (soft=gluonic pole dominance assumed in the whole allowed  $x$ 's region of quark-gluon correlator)

$$x f_T(x) = \frac{1}{2M} T(x, x) = \frac{1}{4} \phi_v(x)$$

$$\int_0^1 dx x^2 \bar{g}_2(x) = \frac{4}{3\pi} \int_0^1 dx x f_T(x) (2-x)$$

# Compatibility of single and double asymmetries

- Recent extractions of Sivers function: Efemov(talk), Goeke, Menzel, Metz, Schweitzer(talk); Anselmino(talk), Boglione, D'Alesio, Kotzinian, Murgia, Prokudin(talks) – “mirror” u and d
- First moment of EGMMS = 0.0072 (0.0042 – 0.014) – courtesy P. Schweitzer
- Twist -3  $g_2$  (talk of J.P. Chen) - larger for neutron (0.0025 vs 0.0001) and of the same sign – nothing like mirror picture seen.
- Current status: Scale of Sivers function – seems to be reasonable, but flavor dependence seems to differ qualitatively.
- More work is needed: NLO corrections (happen to mix Collins and Sivers asymmetries! – work in progress), regular (beyond gluonic poles) twist 3 contribution, ...
- Inclusion of pp data, global analysis including gluonic (=Sivers) and fermionic poles



# CONCLUSIONS

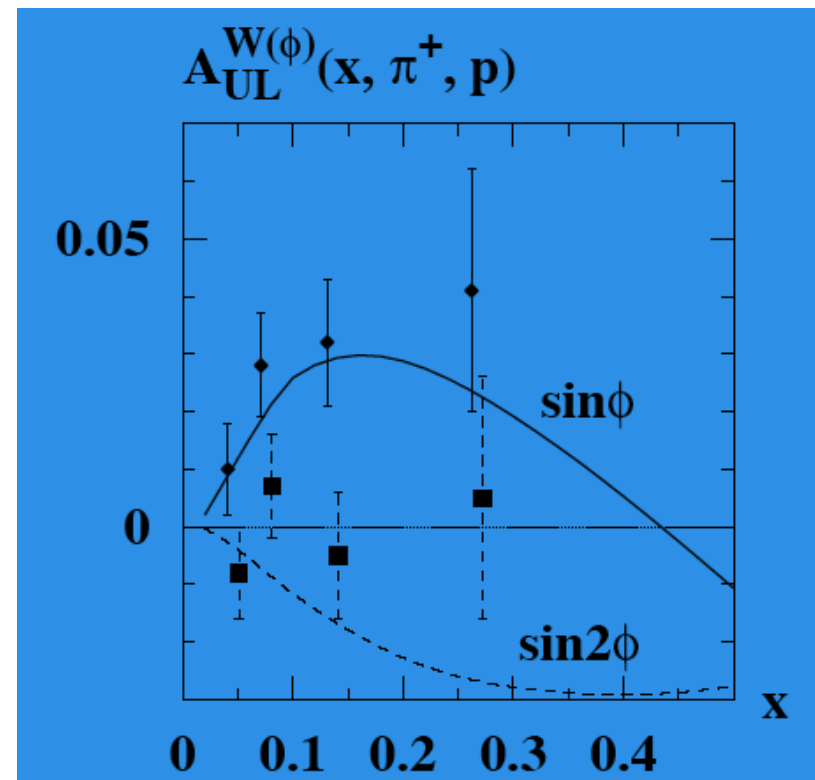
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- Relations of single and double asymmetries: phase should be known
- Semi-inclusive DVCS - new interesting hard process
- Low transverse momenta - effective twist 3 (but not suppressed as  $1/Q$ ) Sivers function (bounded by  $g_2$ ) – soft talk of large and short distances – supports earlier findings
- Rigorous QCD relations between single and double asymmetries: Sivers function – not independent! Double count (say, in PP at RHIC) should be avoided!
- Reasonable magnitude, but problems with flavor dependence. More experimental and theoretical studies on both sides required.



# Typical observable SSA in SIDIS

- Theory - Efremov, Goeke, Schweitzer
- Phase - from Collins function - extracted earlier from jets spin correlations qt LEP
- Spin of proton - transversity - from chiral soliton model





# Spin-dependent cross-section

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$$\begin{aligned}d\sigma^{\rightarrow} - d\sigma^{\leftarrow} = & \\ & M_{pT}b_A(0, x_2)(M_{A0}\sin(\phi_s) + N_{A0}\sin(\phi_s^h))s_T + \\ & M_{pT}b_A(x_1, x_2)(M_{A1}\sin(\phi_s) + N_{A1}\sin(\phi_s^h))s_T + \\ & M_{pT}b_V(0, x_2)(M_{V0}\sin(\phi_s) + N_{V0}\sin(\phi_s^h))s_T + \\ & M_{pT}b_V(x_1, x_2)(M_{V1}\sin(\phi_s) + N_{V1}\sin(\phi_s^h))s_T\end{aligned}$$

STRAIGHTFORWARD APPLICATION OF

TWIST 3 FACTORIZATION



# Properties of spin-dependent cross-section

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- Complicated expressions
- Sivers (but not Collins) angle naturally appears
- Not suppressed as  $1/Q$  provided gluonic pole exist
- Proportional to correlators with arguments fixed by external kinematics-twist-3 “partonometer”



# Experimental options for SIDVCS

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Natural extension of DVCS studies:  
selection of elastic final state –  
UNNECESSARY

BUT : Necessity of BH contribution also  
- interference may produce SSA



# Theoretical Implications

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- Twist - 3 SSA survive in Bjorken region provided gluonic poles exist
- The form of SSA - similar to the one provided by Sivers function
- Twist-3 (but non-suppressed as  $1/Q$ ) effective Sivers function is found
- Rigorously related to twist 3 part of structure function  $g_2$  - problems seen!
- New connection between different spin experiments

Pion from real photons –simple  
 expression for asymmetry  $A=$

$$\frac{b_A(0, x) - b_V(0, x)}{f(x)} \times$$

$$\times \frac{(1 - x_F)(C_F x_F - (x_F + 1)C_A/2)}{C_F(1 + x_F^2)} \frac{2M p_T}{m_T^2}$$

# Properties of pion SSA by real photons



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- Does not sensitive to gluonic poles
- Probe the specific (chiral) combinations of quark-gluon correlators
- Require (moderately) large  $P_T$  - may be advantageous with respect to DIS due to the specific acceptance.



# Pion beam + polarized target

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- Allows to study various ingredients of pion structure – rather different from nucleon
- Most fundamental one – pion-light cone distribution – manifested in SSA in DY:  
Brandenburg, Muller, O.T. (95)  
Where to measure?! COMPASS(Torino)?!!





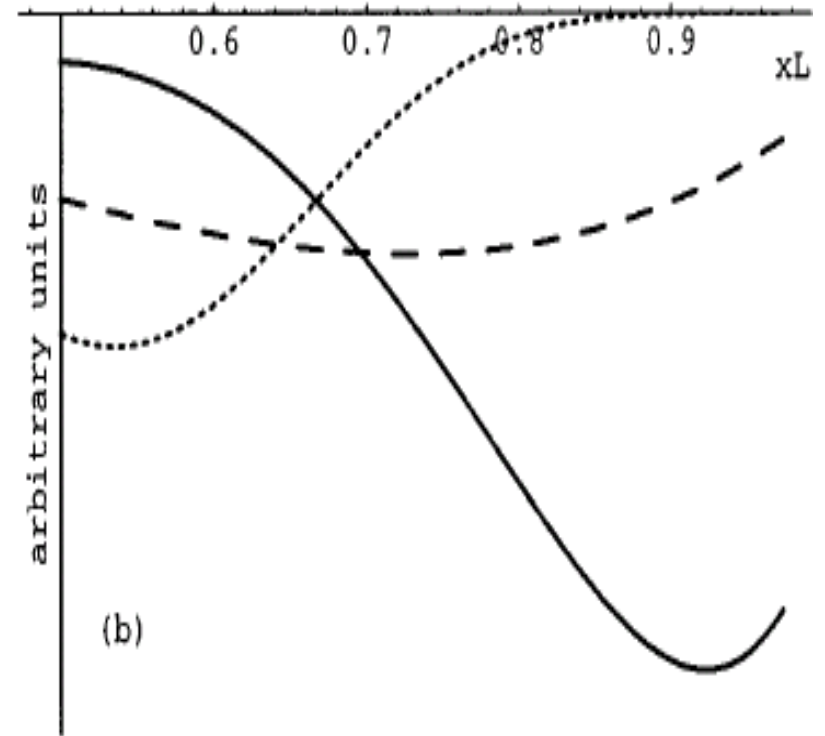
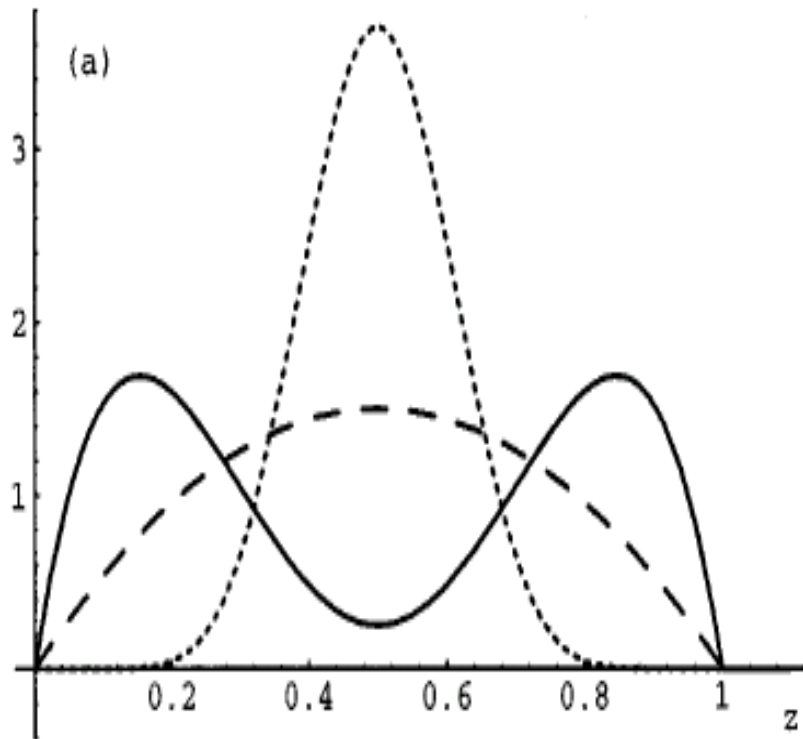
# Simplest case-longitudinal polarization- “partonometer”

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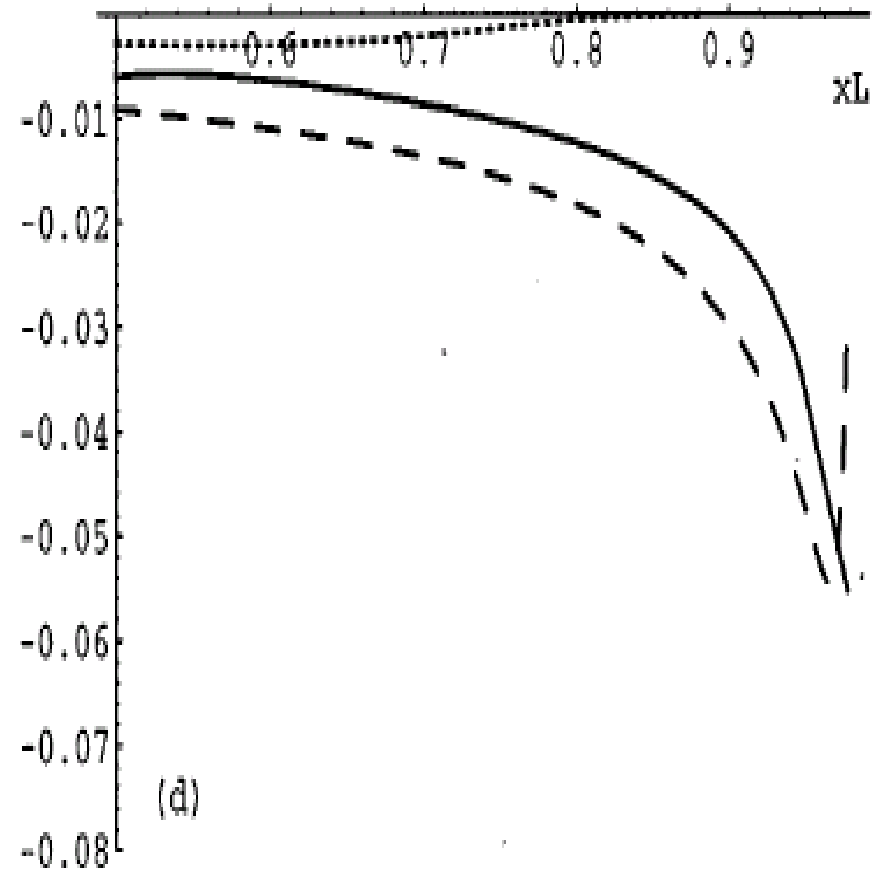
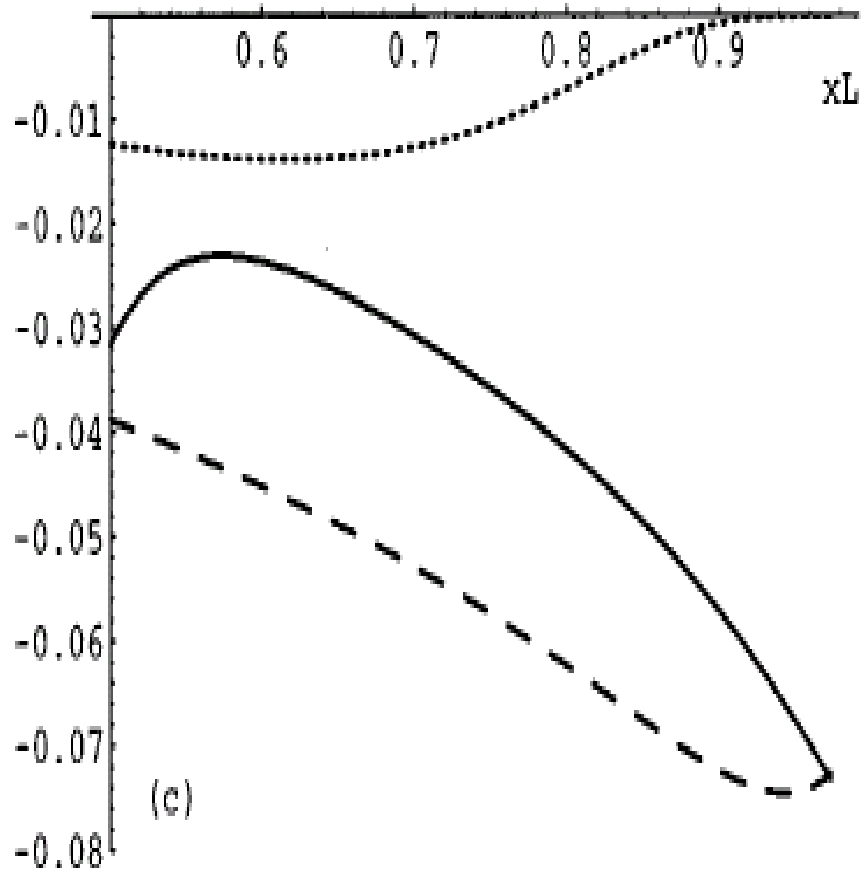
- Two extra terms in angular distribution, proportional to longitudinal polarization

$$\overline{\mu} \sin 2\theta \sin \phi + \frac{\nu}{2} \sin^2 \theta \sin 2\phi$$

# Models for light-cone distributions and angular-weighted x-sections



# Size of coefficients in angular distributions





# Transverse polarization

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- Much more complicated – many contributions
- Probe of transversity (X Boer T-odd effective distribution), Sivers function, twist-3 correlations, pion chiral-odd distributions)



# CONCLUSIONS-I

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- (Moderately) high Pions SSA by real photons – access to quark gluon correlators
- Real photons SSA: direct probe of gluonic poles, may be included to DVCS studies



# CONCLUSIONS-II

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- Pion beam scattering on polarized target – access to pion structure
- Longitudinal polarization – sensitive to pion distribution
- Transverse polarization – more reach and difficult