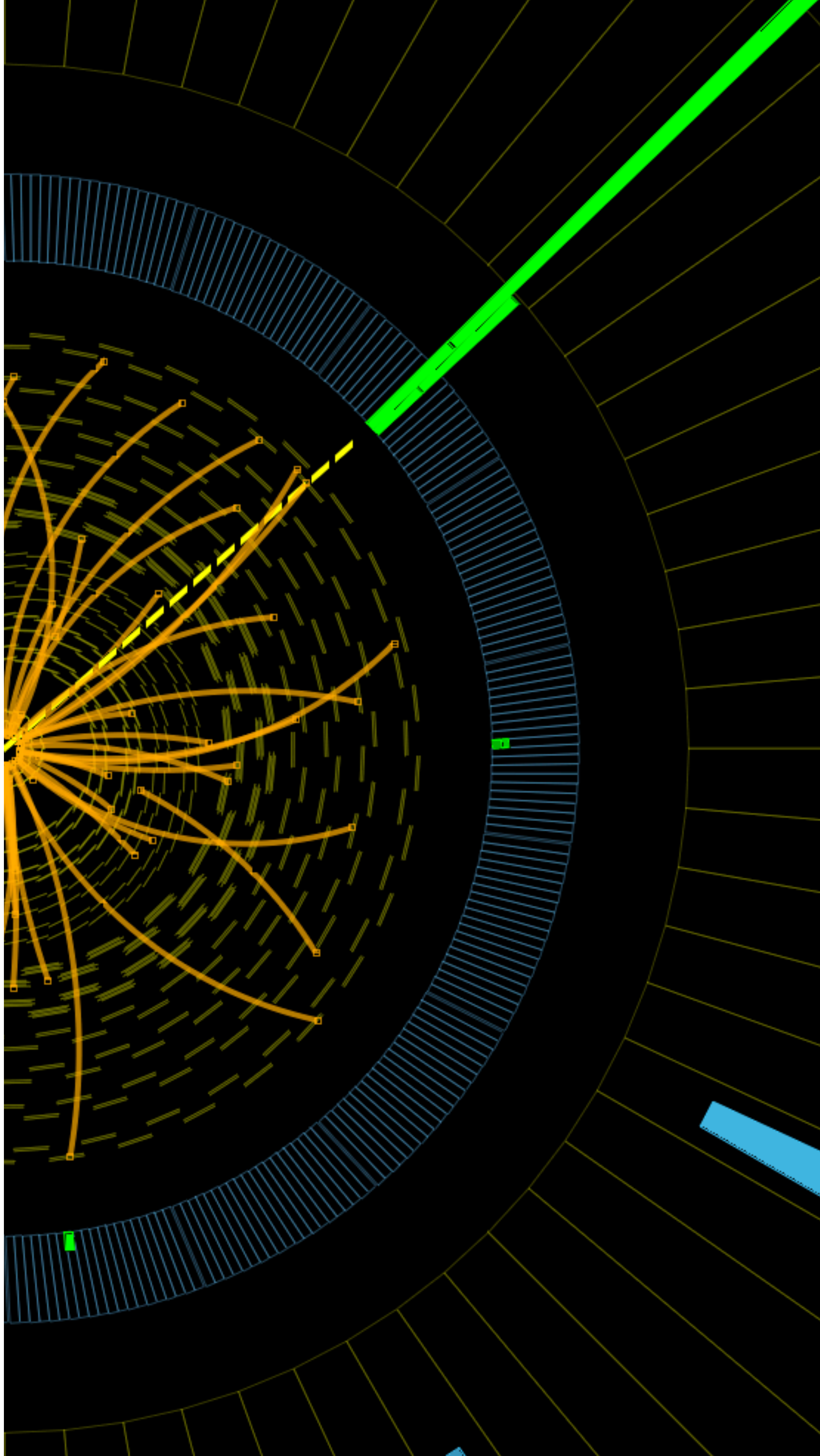
PROGRESS IN MULTI-LOOP CALCULATIONS (FOR TOP AND HIGGS PHYSICS)

ZPW 2020
Zürich, January 14th 2020
Lorenzo Tancredi
RSURF, University of Oxford



PRECISION QCD @ LHC

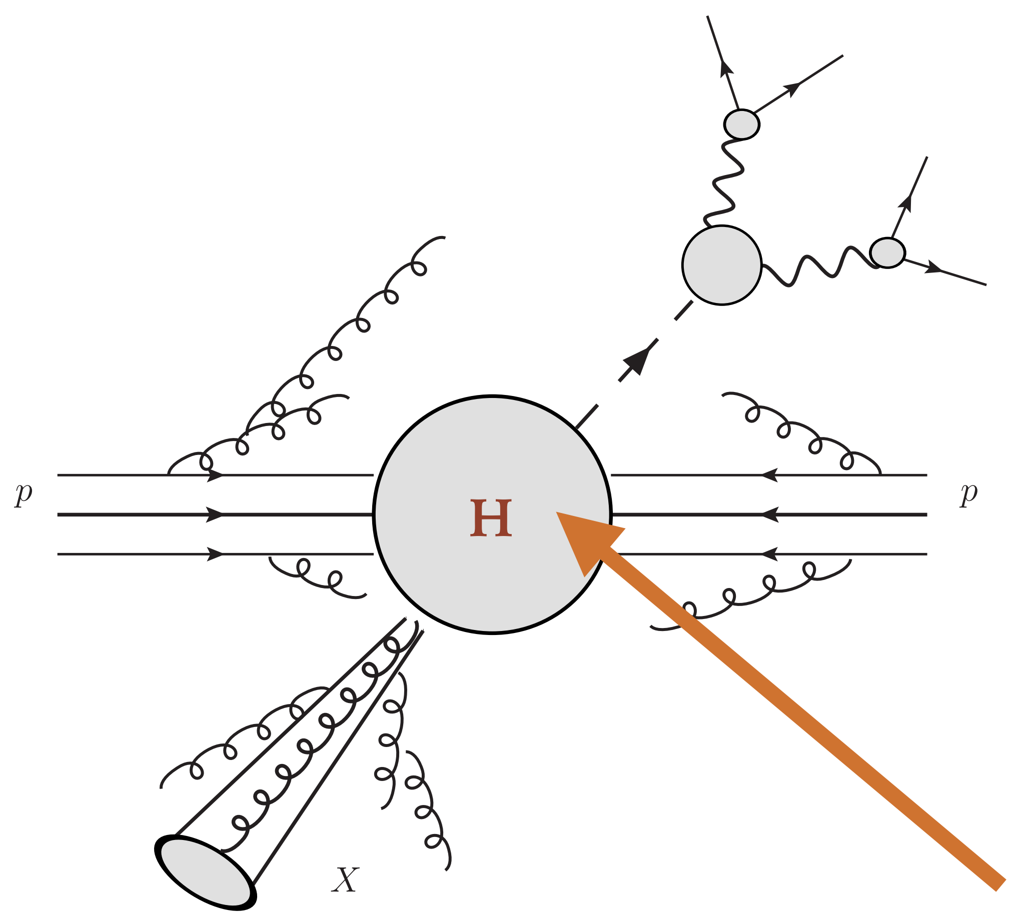
Precision @ the LHC, means (mainly) **precision in QCD**, in a very **dirty environment!**

$$pp \to HX \to l_1\bar{l}_1 + l_2\bar{l}_2 + X$$

Factorisation of long and short range physics

Non perturbative corrections

$\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right) \sim$ few percent?



Precise determination of **parton content of proton**

PDFs Currently known at level $\sim$ few % for LHC

This Talk: focus on <u>HARD SCATTERING</u>

Aim to $\sim$ % precision

HARD SCATTERING IN PERTURBATIVE QFT

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2\left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \ldots$$

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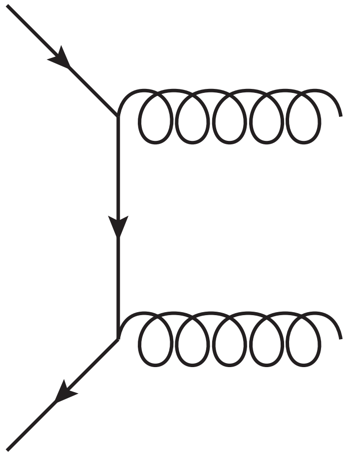

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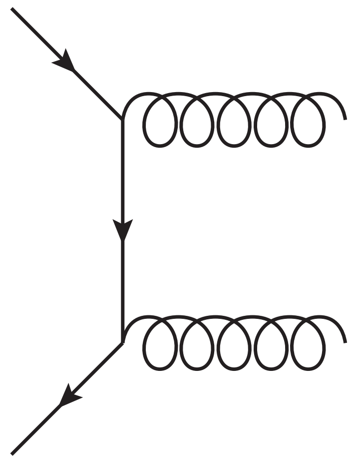

~ O(100%-50%)
precision

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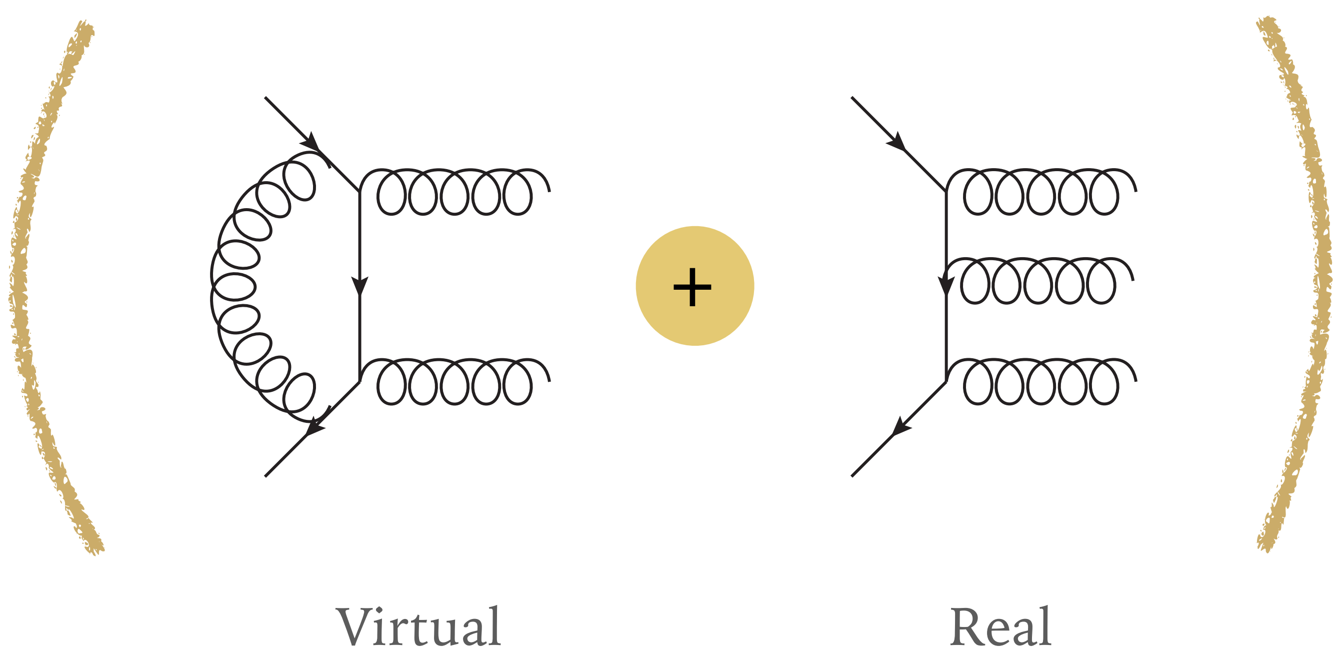

One-loop amplitudes well understood!

IR divergences under control

CS dipoles, FKS…

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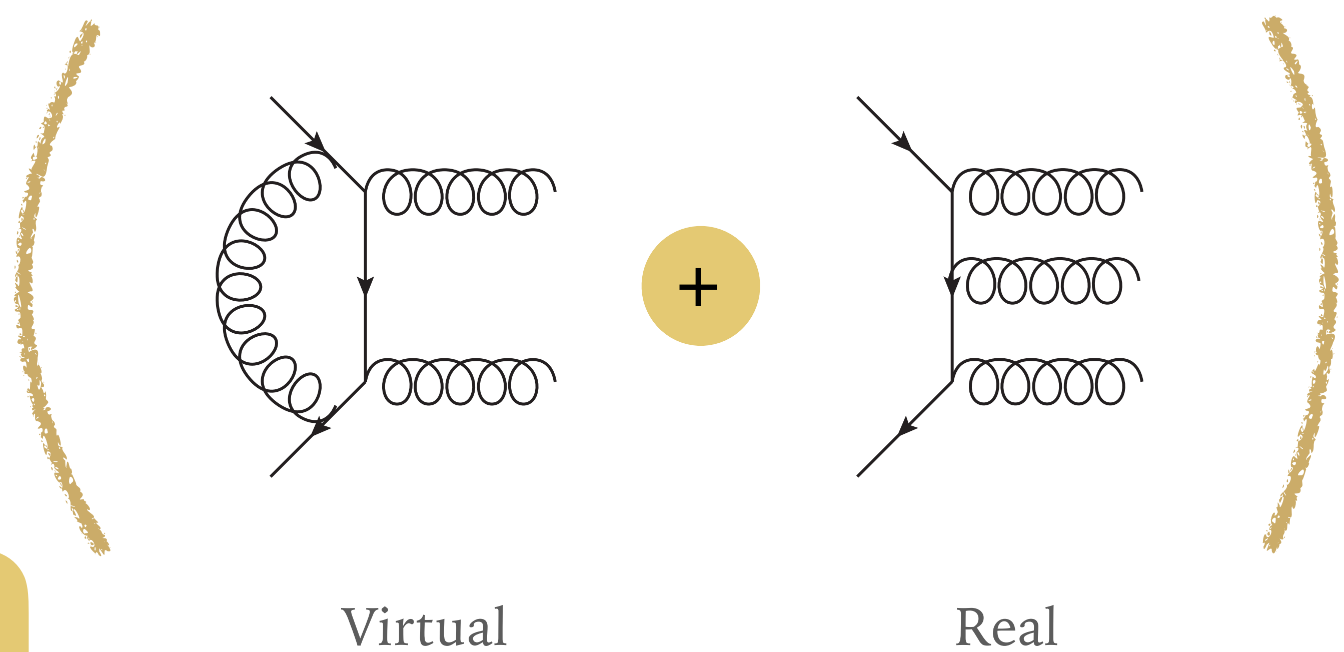

One-loop amplitudes well understood!

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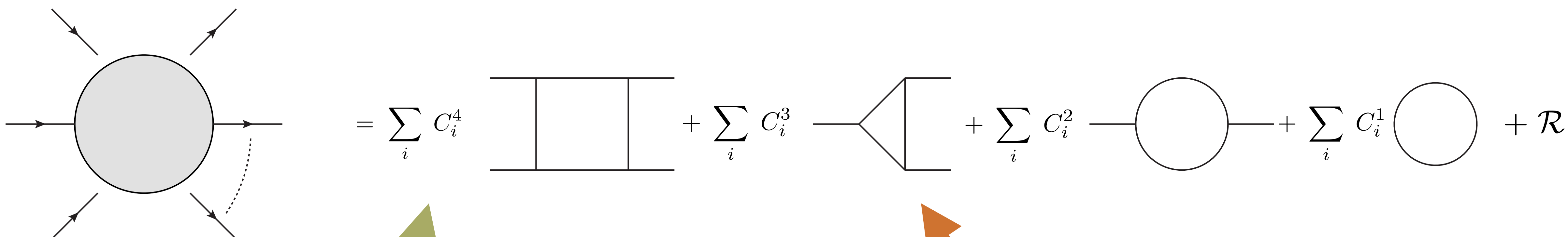
~ O(30%-10%) precision

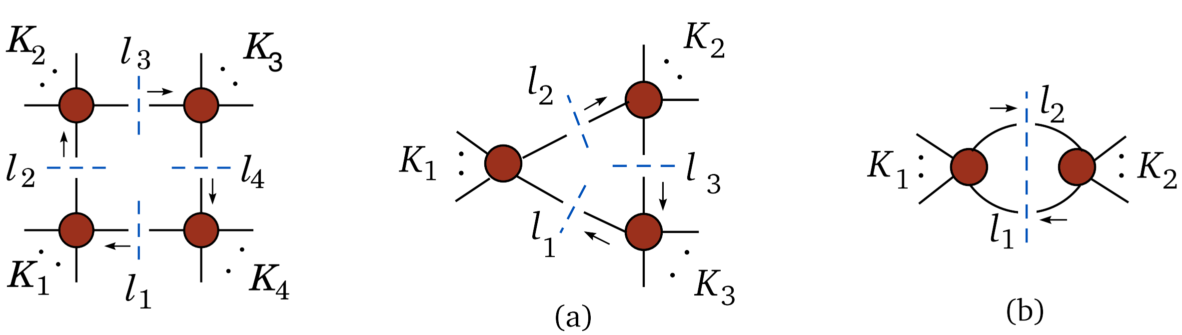
THE NLO REVOLUTION (ONE-LOOP VIRTUAL AMPLITUDES)

Unitarity @ 1 loop

[Ossola, Papadopoulos, Pittau, ’04]
[Bern Dixon, Kosover, ’05]
[Ellis, Kunszt, Melnikov, Zanderighi, ’08]

Every 1 loop amplitude can be decomposed in **boxes**, **triangles**, **bubbles** and **tadpoles**





All **Master integrals** known analytically in terms of simple functions:

logarithms, **di-logarithms**

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t}\, dt$$

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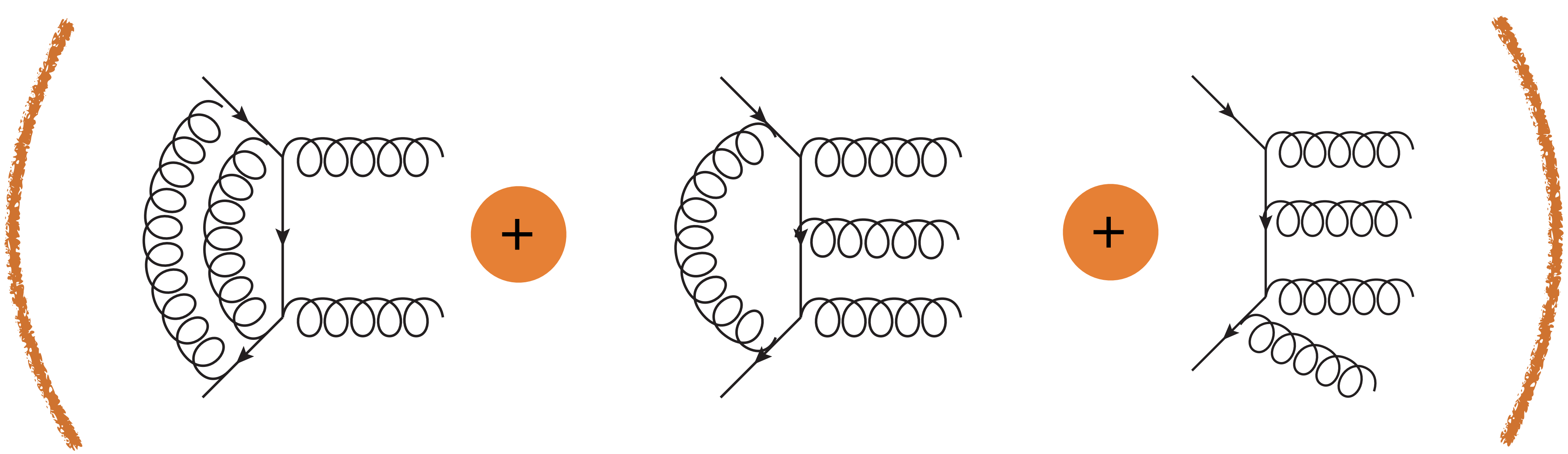

Double Virtual Real Virtual Double Real

IR divergences more involved, ***another talk……***

Two-loop amplitudes open a ***new world***

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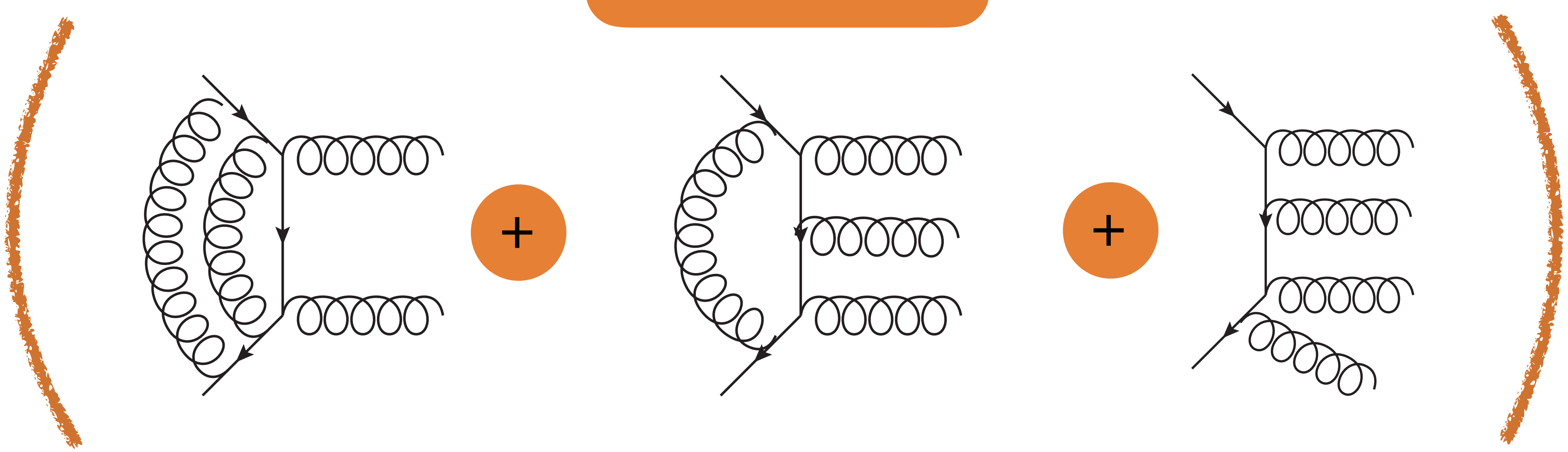
~ O(5%) precision



Double Virtual

Real Virtual

Double Real

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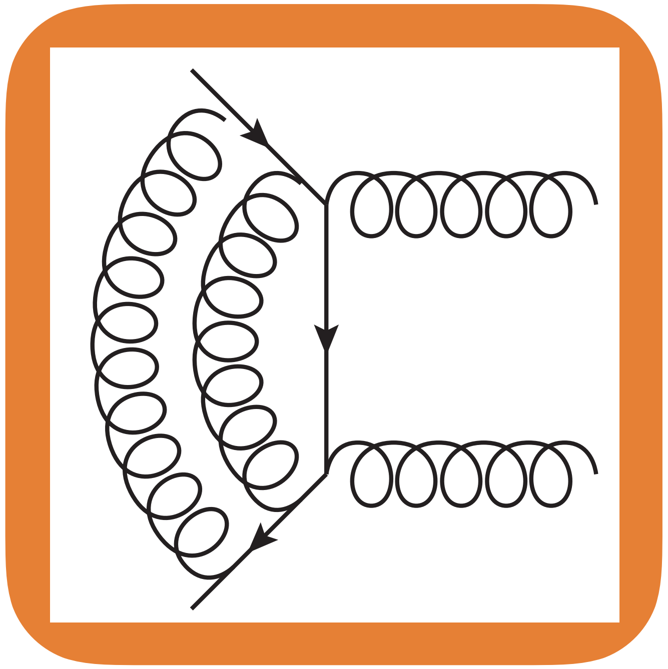


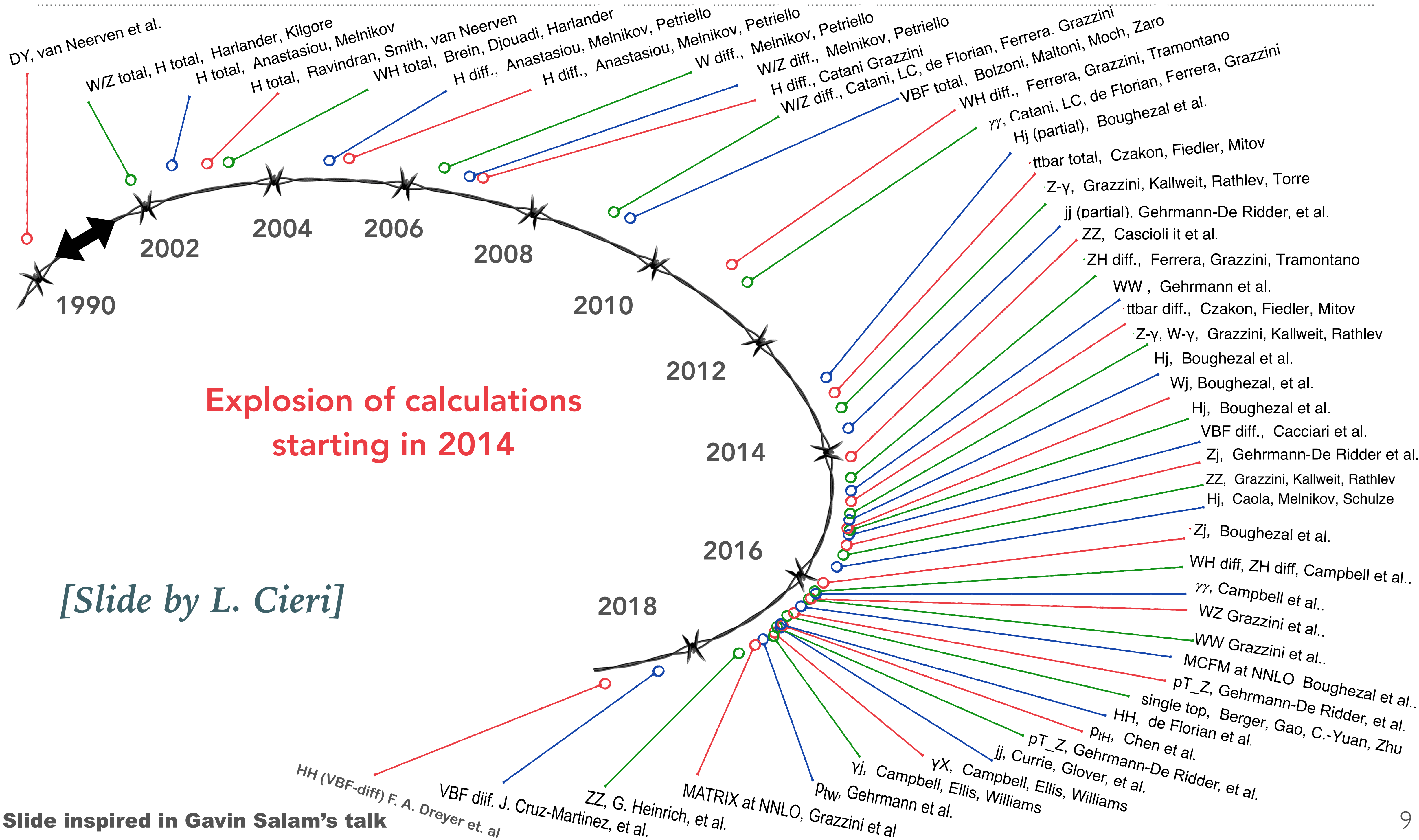
Multiloop amplitudes are often the **BOTTLENECK** of these calculations:

This Talk!

A lot of developments…

A NNLO REVOLUTION?

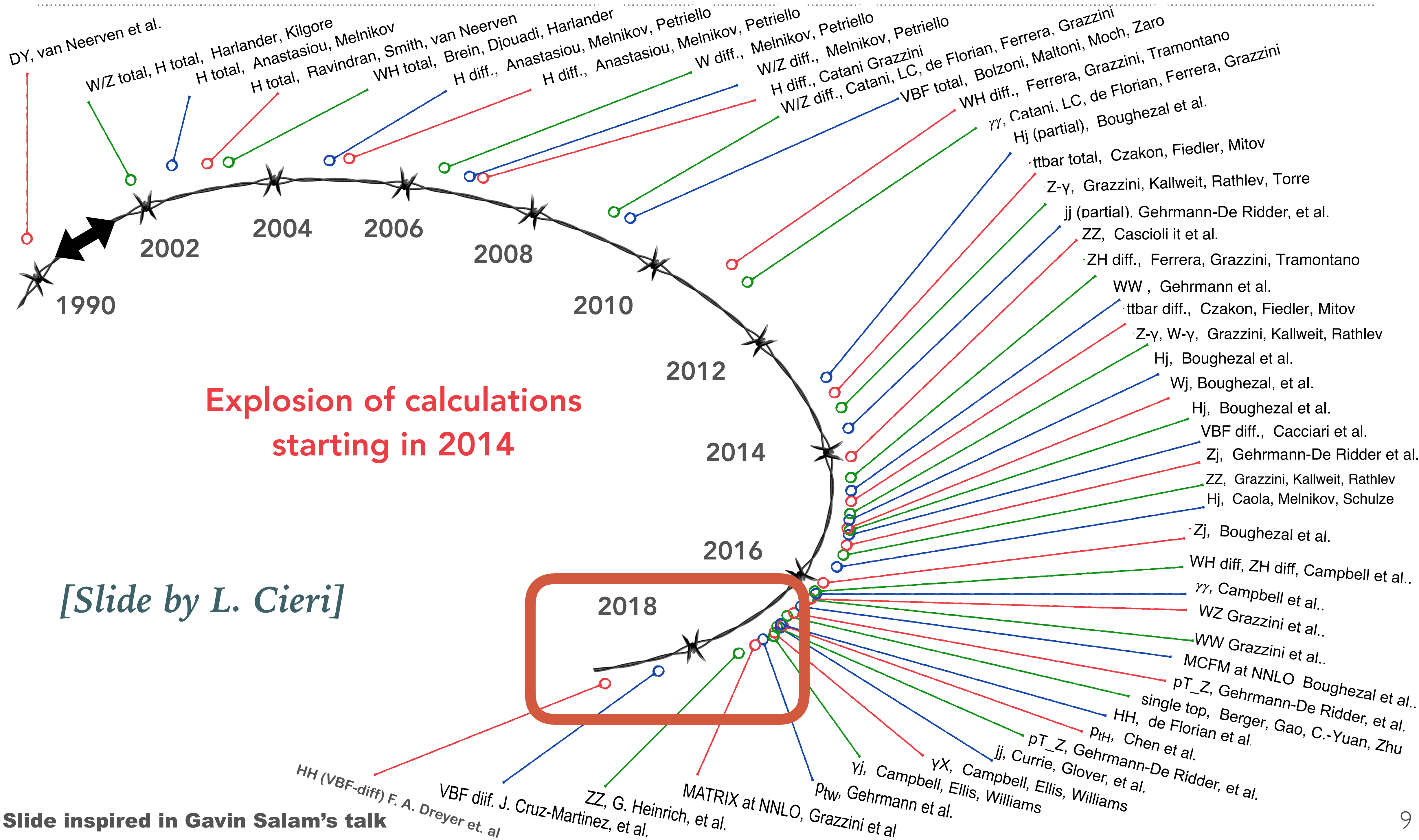
NNLO HADRON-COLLIDER CALCULATIONS VS. TIME



Slide inspired in Gavin Salam's talk

NOT REALLY...

NNLO HADRON-COLLIDER CALCULATIONS VS. TIME

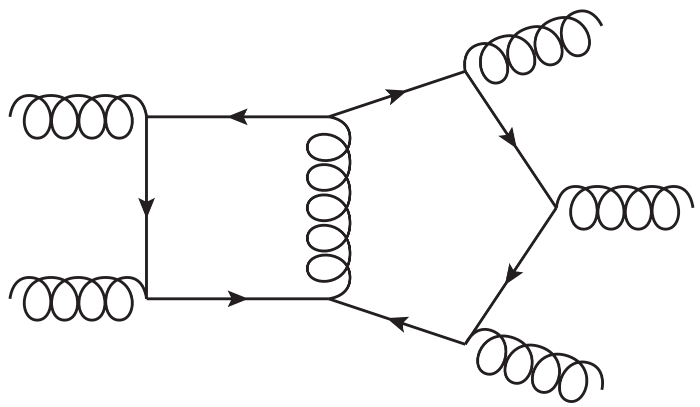


[Slide by L. Cieri]

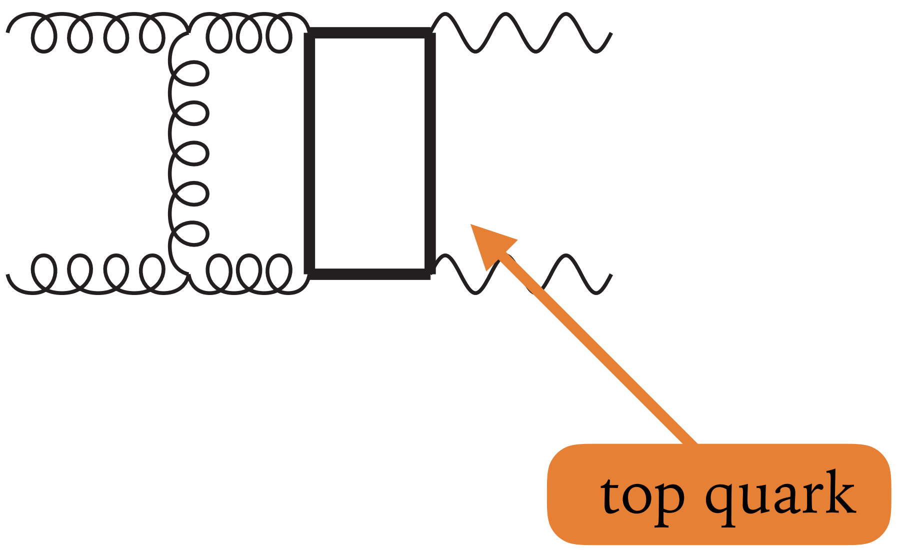
Slide inspired in Gavin Salam's talk

WHAT WE CANNOT DO (YET…)

Properly modelling LHC processes with high precision requires ***more external particles*** and ***massive internal states***



2 → 3 processes



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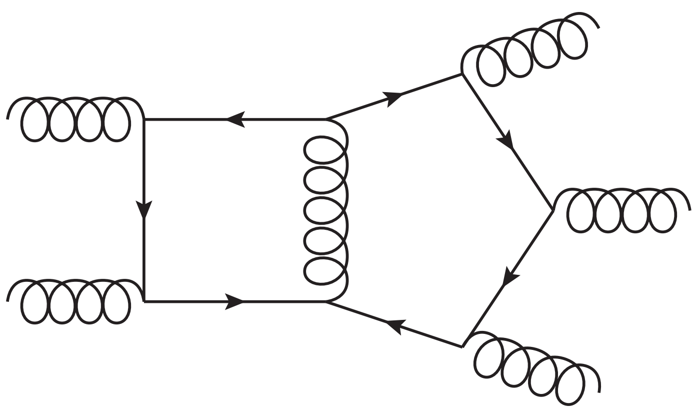
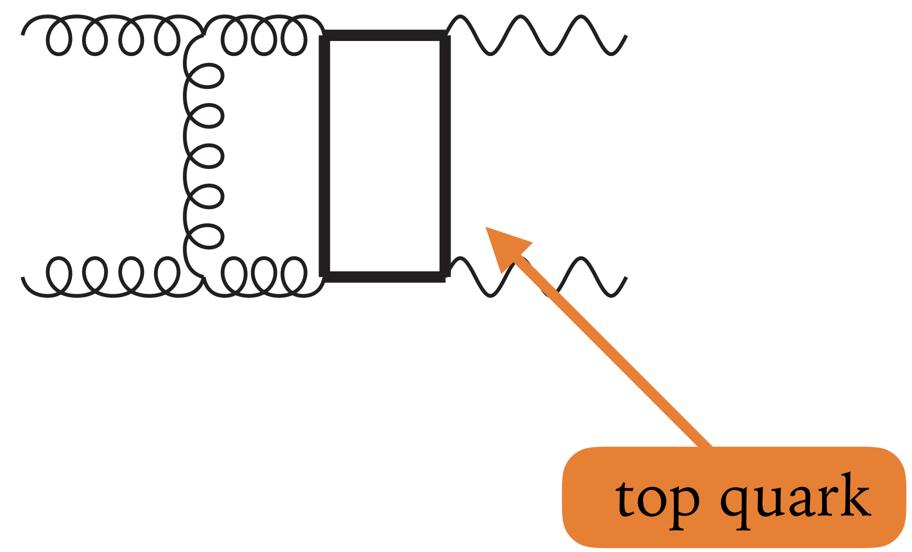
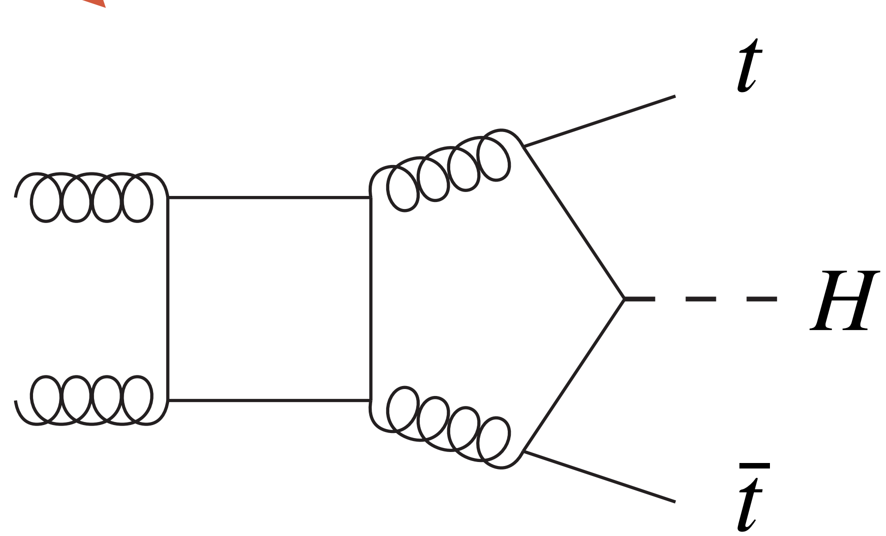


$2 \to 3$ processes





$t\bar{t} + X$ production…?!?!?
A lot of physics potential in particular in $t\bar{t}H$!!!

A NNLO REVOLUTION (WHAT WOULD IT REQUIRE)

From the virtual amplitude side, we need a calculation technique that is:

1. **fast** *(provides numerical results fast)*
2. **reliable** *(provides them reliably over the full interesting phase space)*
3. **flexible** *(easily adaptable to different processes)*

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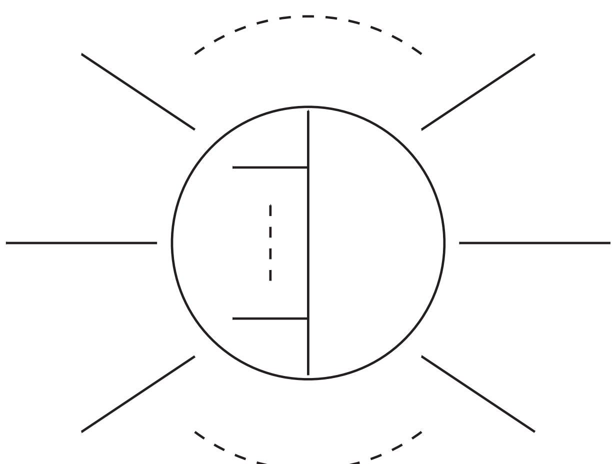
One way to go about it: **standard approach *(divide et impera)***



$$= \sum_{i=1}^{N} R_i(x_1, \ldots, x_r)\, \mathcal{I}_i(x_1, \ldots, x_n)$$

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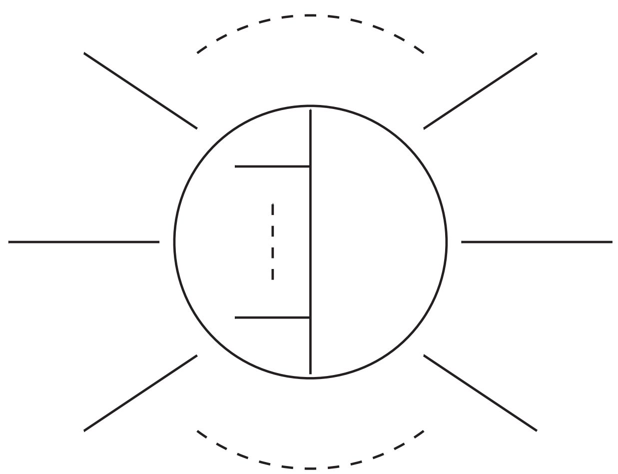

$$= \sum_{i=1}^{N} R_i(x_1,\ldots,x_r)\,\mathcal{I}_i(x_1,\ldots,x_n)$$

Coefficients depend on the **process considered**

Fundamental process-independent building blocks: **Master Integrals**

Very complicated rational functions, hundreds of **Mbs** for complicated processes:

Algebraic Complexity

Very involved **special functions** with complicated mathematical properties:

Analytic complexity

ALGEBRAIC COMPLEXITY

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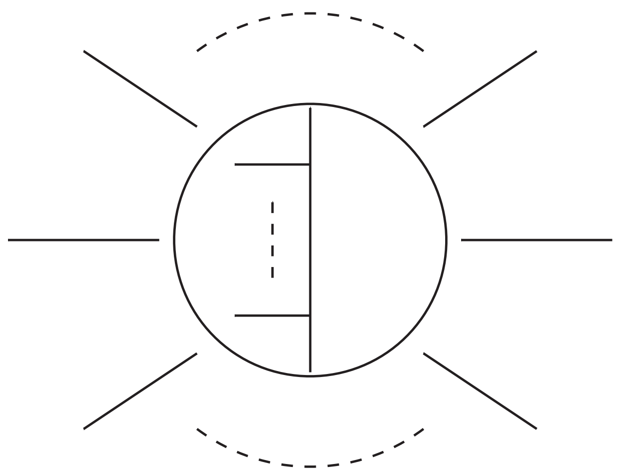
First problem is ***"getting the integrand"***:

(i.e. whatever expression we need to integrate over the loop momenta)



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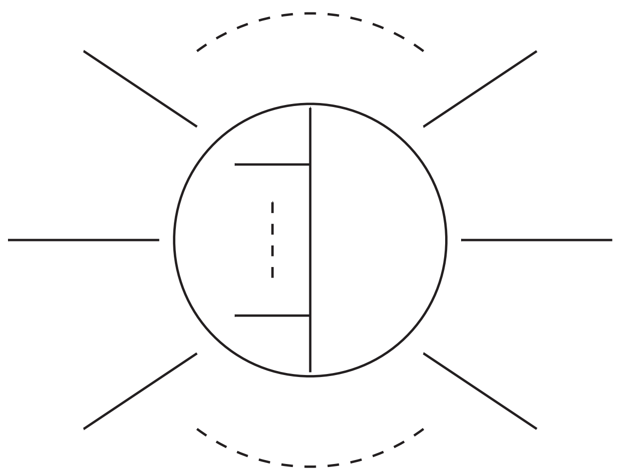

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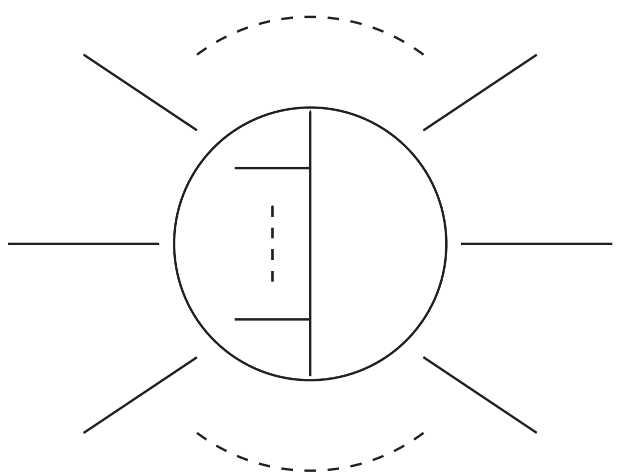

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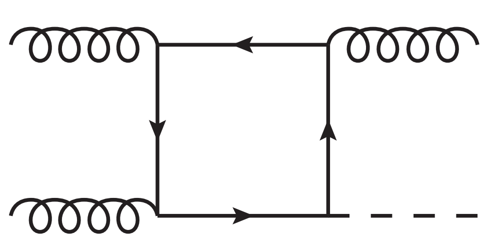

$$\sim \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k-p_2)^2 (k-p_2-p_3)^2 (k-p_1-p_2-p_3)^2}$$

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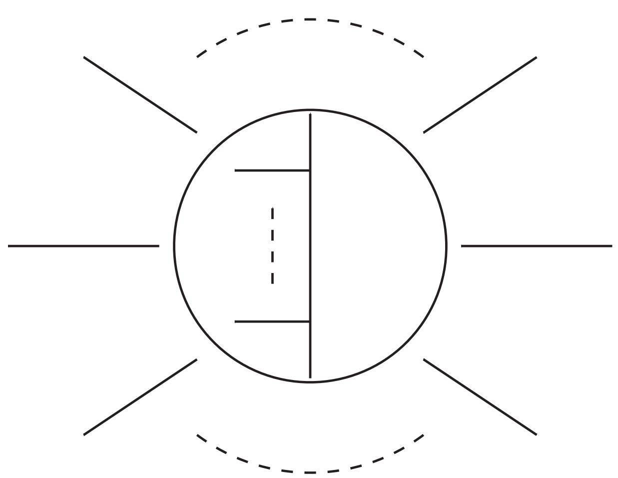

$= \sum$ *Feynman Diagrams* $\rightarrow$?

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Interesting problem: can we exploit simplifications from **4-dimensional external states**? -> [Chen '19] [Peraro, **Tancredi** '19](*)

(*) Simplify way to extract form factors using 't Hooft-Veltman scheme

ALGEBRAIC COMPLEXITY

Once we have the integrand: $\sim \mathcal{O}(10^4)$ for a typical two-loop $2 \to 2$ process @ LHC

$$\mathcal{M}^{\ell\text{-loops}} \sim \int \prod_l \frac{d^d k_l}{(2\pi)^d} \frac{S_1^{a_1} \ldots S_\sigma^{a_\sigma}}{D_1^{b_1} \ldots D_t^{b_t}} \quad \textbf{with} \quad \begin{cases} S_r = k_i \cdot p_j \\ D_r = q_r^2 - m_r^2 \end{cases}$$

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Luckily, not all independent ⟶ Integration by parts identities **(IBPs)** [Chetyrkin, Tkachov, 1981] [Laporta, 2001]

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Allow to reduce all integrals to **Master Integrals**

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Allow to reduce all integrals to **Master Integrals**

Conceptually "simple", requires ***enormous computational resources***

improvement in algorithms

KIRA, [Maierhöfer, Usovitsch, Uwer]
Block-dig system [Guan, Liu et al]

Unitarity-compatible IBPs

[Ita, Abreu, Page, Bosma, Kosower, Georgoudis, Larsen, Zhang, Zeng, …]

Finite-fields based techniques

[von Manteuffel, Schabinger]
FiniteFlow [Peraro]

A (VERY) NEW POINT OF VIEW: INTERSECTION THEORY

Master integrals are a **basis:**

can we define a __scalar product__ among Feynman integrals? If so, can we use it to __project any integral on a basis of master integrals__? [Mastrolia, Mizera '19, Frellesvig et al '19; Caron-Huot et al '19]

$\nu =$ dimension of the vector space

- **Vector decomposition** $$I = \sum_{i=1}^{\nu} c_i \, J_i$$
- **Projections** $$c_i = \begin{cases} I \cdot J_i \; , & J_i \cdot J_j = \delta_{ij} \\ \sum_{j=1}^{\nu} I \cdot J_j (C^{-1})_{ji} \; , & J_i \cdot J_j = C_{ij} \neq \delta_{ij} \end{cases}$$
- **Completeness** $$\sum_{i,j} J_j \, (C^{-1})_{ji} \, J_i = \mathbb{I}_{\nu \times \nu}$$

The two questions:

1) what is the vector space dimension ν ?

2) what is the *scalar product* "$\cdot$" between integrals ?

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Intersection numbers provide the coefficients! Obtained "just" computing residues of the relevant integrands

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ν = dimension of the vector space

- **Vector decomposition** $$I = \sum_{i=1}^{\nu} c_i \, J_i$$
- **Projections**
- **Completeness**

Preliminary interesting results for decompositions into masters "without IBPs"

—> *generalisation of 1-loop unitarity?*

Keep an eye on it!

The two questions:

1) what is the vector space dimension ν ?

2) what is the *scalar product* "$\cdot$" between integrals ?

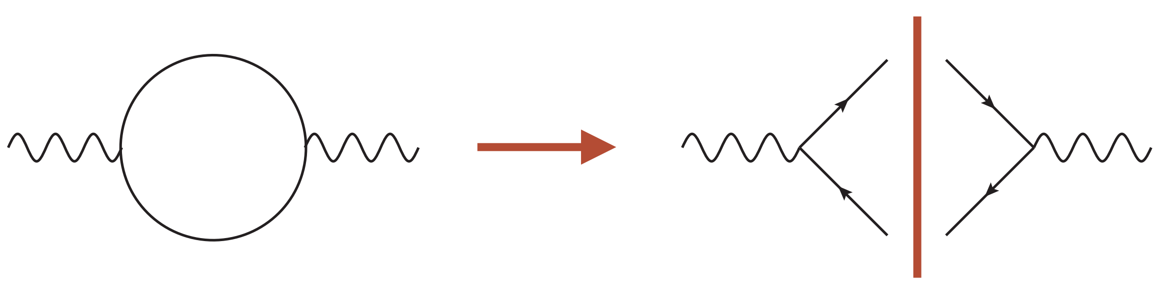
ANALYTIC COMPLEXITY

(A.K.A COMPUTING THE INTEGRALS…)

ANALYTIC COMPLEXITY

Feynman integrals are source of **analytic structure** of the scattering amplitudes

Multivalued functions. **Branch-cut structure dictated by causality / unitarity!**



$$\sim \int_{4m^2}^{\infty} \frac{ds'}{s'-s-i\epsilon} \frac{1}{\sqrt{s'(s'-4m^2)}}$$

$$\sim \frac{1}{\sqrt{s(s-4m^2)}} \ln\left(\frac{\sqrt{s-4m^2}+\sqrt{s}}{\sqrt{s-4m^2}-\sqrt{s}}\right)$$

Multivalued functions, language of Riemann surfaces…

ANALYTIC COMPLEXITY

Numerical or analytical?

pro and cons?

ANALYTIC COMPLEXITY

Numerical or analytical?

pro and cons?

There exist ***numerical techniques*** that attempt to define general and flexible strategies for the evaluation of Feynman integrals

- Sector decomposition **(see S. Jones' talk)** HH, $H + jet$ @ **NLO through top loop**
- **Local subtraction** of multiloop amplitudes **[Anastasiou, Sterman '18]**
- Application of **loop-tree duality** **[Capatti, Hirschi et al '19]** [Interesting, keep an eye!]
- …

ANALYTIC COMPLEXITY

Numerical or analytical?

pro and cons?

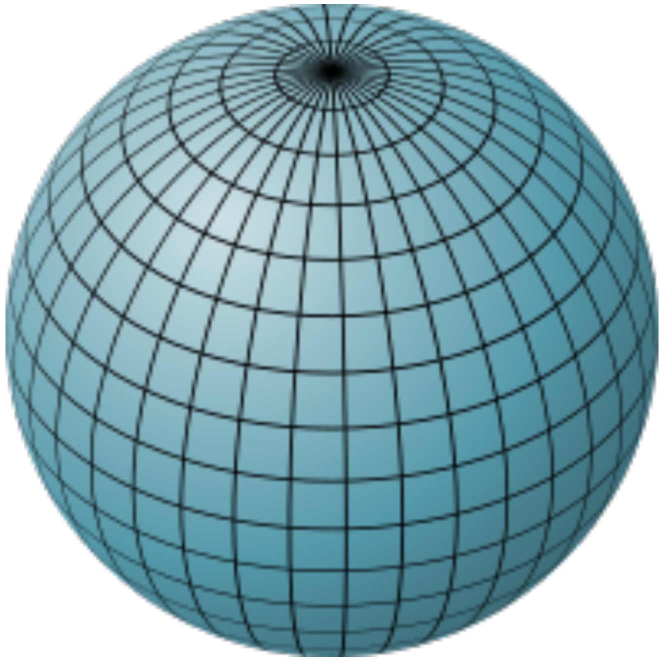
We are discovering how <u>geometry</u> can help ***simplify our approach to calculation of Feynman integrals***

Feynman integrals naturally live on (complex) **Riemann surfaces**

$$\mathcal{I} \sim \int_0^\infty \prod_j dx_j \frac{\mathcal{U}^n}{\mathcal{F}^m}, \quad \mathcal{U}\,,\,\mathcal{F} \quad pol. \quad \rightarrow \quad y = \mathcal{F}(x_1, \ldots, x_n)^k$$

EASY CASE: GENUS 0

For the simplest kind of surface: **Riemann Sphere —> genus 0**



Multiple Polylogarithms are born!

Integrate Rational Functions

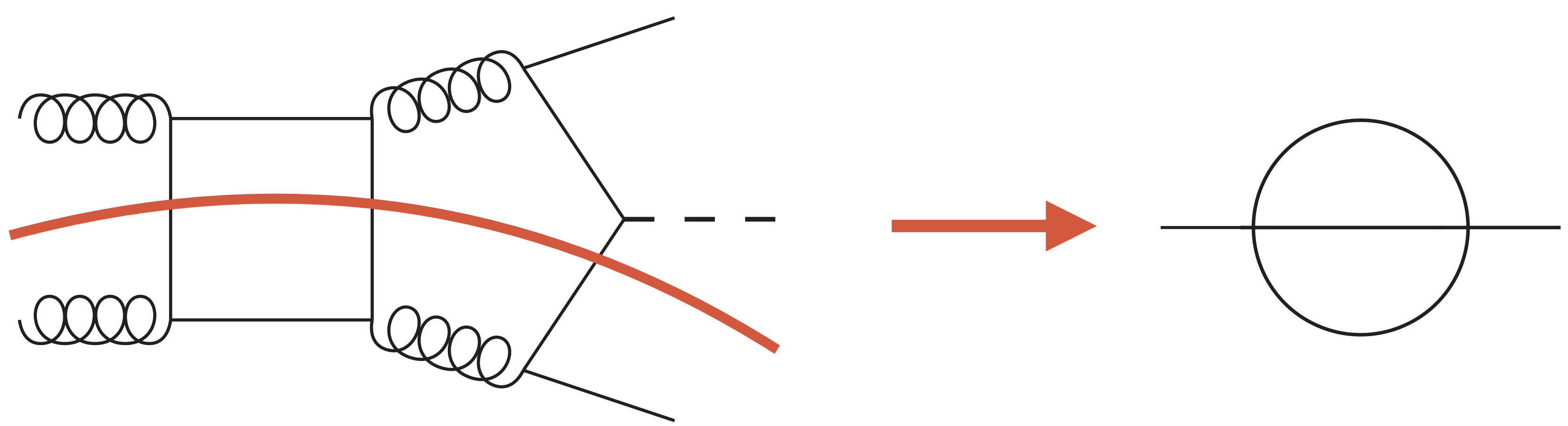
With complex poles on the Riemann sphere

$$G(c_1, c_2, \ldots, c_n, x) = \int_0^x \frac{dt_1}{t_1 - c_1} G(c_2, \ldots, c_n, t_1)$$

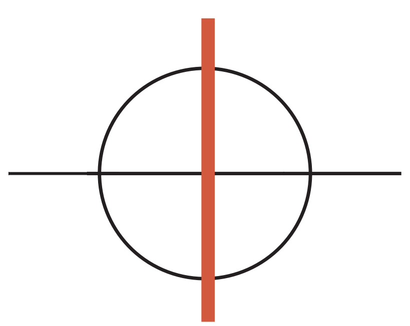
$$= \int_0^x \frac{dt_1}{t_1 - c_1} \int_0^{t_1} \frac{dt_2}{t_2 - c_2} \ldots \int_0^{t_{n-1}} \frac{dt_n}{t_n - c_n}$$

THE ELLIPTIC WORLD: GENUS 1

At **two loops**, MPLs are not enough -> there is more than rational functions



The sunrise integral



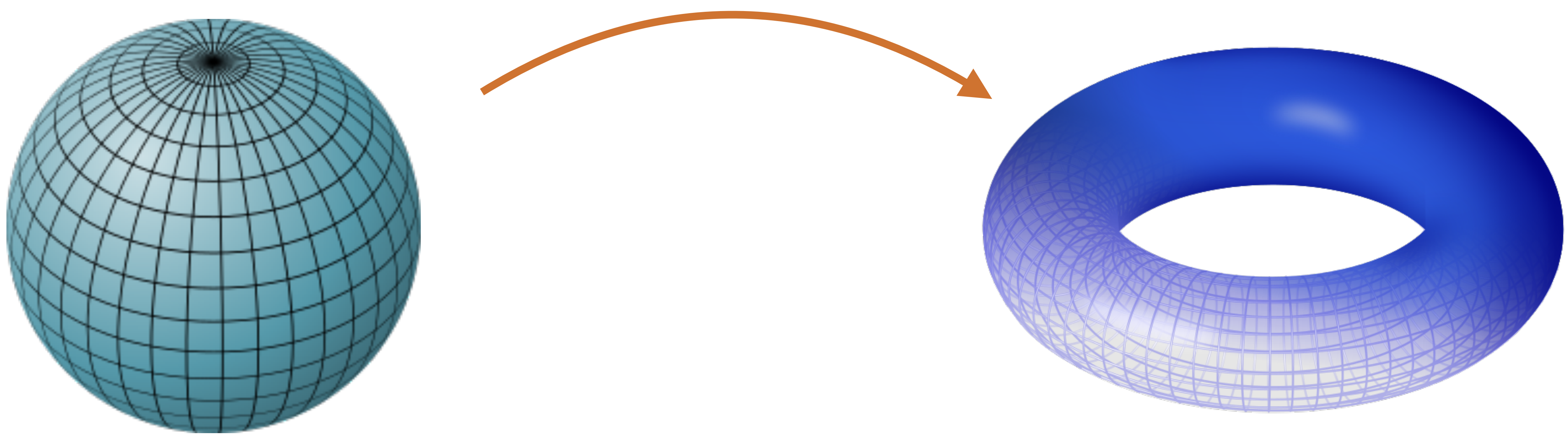
$$= \frac{1}{\sqrt{(3m-\sqrt{s})(\sqrt{s}+m)^3}}\,\mathrm{K}\left(\frac{16m^3\sqrt{s}}{(3m-\sqrt{s})(\sqrt{s}+m)^3}\right)$$

$$\mathrm{K}(x) = \int_0^1 \frac{dz}{\sqrt{(1-z^2)(1-x\,z^2)}}$$

Complete elliptic integral of 1st kind

THE ELLIPTIC WORLD: GENUS 1

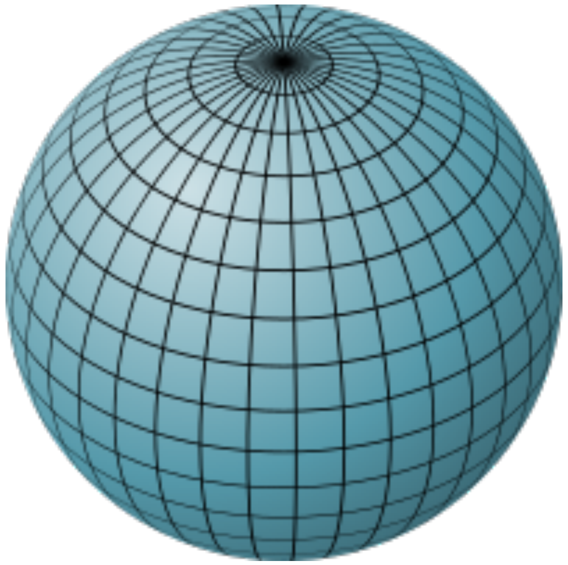
It turns out that the **geometry** associated to an elliptic curve is a **complex torus**!



From Genus 0 to **Genus 1**!

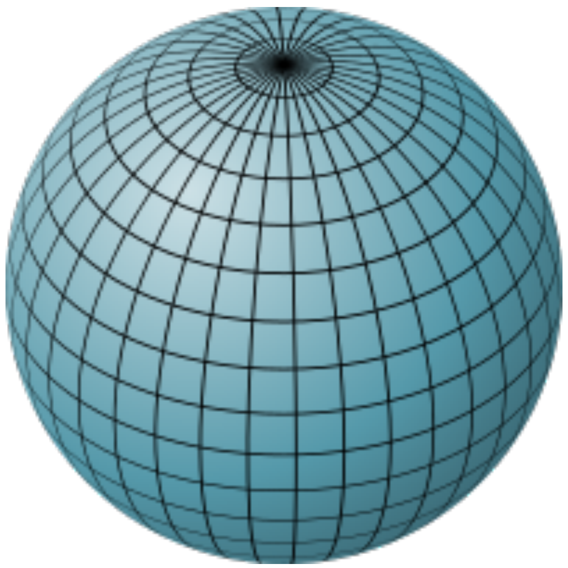
But apart from the geometry, not much changes conceptually!

ELLIPTIC MULTIPLE POLYLOGARITHMS



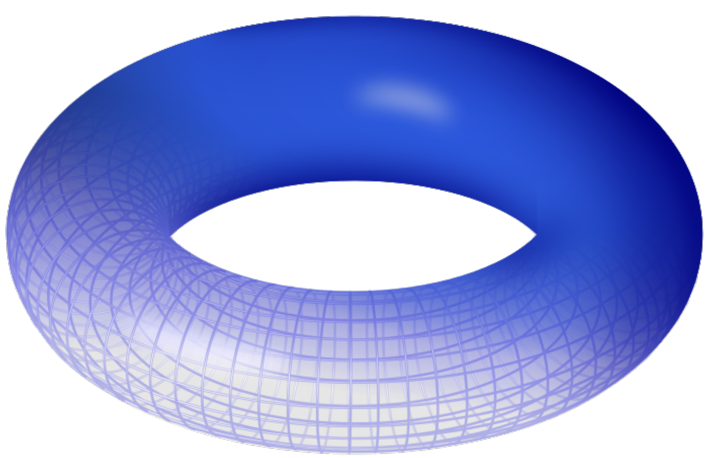
$$G(c_1, ..., c_k; x) = \int_0^x dt\, r(c_1, t) G(c_2, ..., c_k; t)$$

ELLIPTIC MULTIPLE POLYLOGARITHMS



$$G(c_1, \ldots, c_k; x) = \int_0^x dt\, r(c_1, t) G(c_2, \ldots, c_k; t)$$

Repeating MPLs construction on a more complicated geometry



$$\mathcal{E}_4\left(\begin{smallmatrix} n_1 & \cdots & n_k \\ c_1 & \cdots & c_k \end{smallmatrix}; x, \vec{a}\right) = \int_0^x dt\, \Psi_{n_1}(c_1, t, \vec{a})\, \mathcal{E}_4\left(\begin{smallmatrix} n_2 & \cdots & n_k \\ c_2 & \cdots & c_k \end{smallmatrix}; t, \vec{a}\right)$$

[Brown, Levin ’11; Adams, Weinzierl ’13,’15; Broedel, Duhr, Dulat, Penante, Tancredi ’17,’18,’19; Broedel, Mafra, Matthes, Schlotterer ’15,’16]

A lot of progress in understanding these functions, very exciting!

MODULAR FORMS AND ALGORITHMS FOR THEIR EVALUATION

A special subset, iterated integrals of <u>**modular forms**</u>, ***are now under control !***

[Zagier; Brown;… Adams, Bogner, Weinzierl, …]

[Duhr, Tancredi ’19]

$$f\,(\gamma\cdot\tau) = (c\tau+d)^n f(\tau)\,, \qquad \gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma\,.$$

$$I(f_1,\ldots,f_k;\tau) = \int_{i\infty}^{\tau} \frac{d\tau'}{2\pi i} f_1(\tau') I(f_2,\ldots,f_k;\tau')\,,$$

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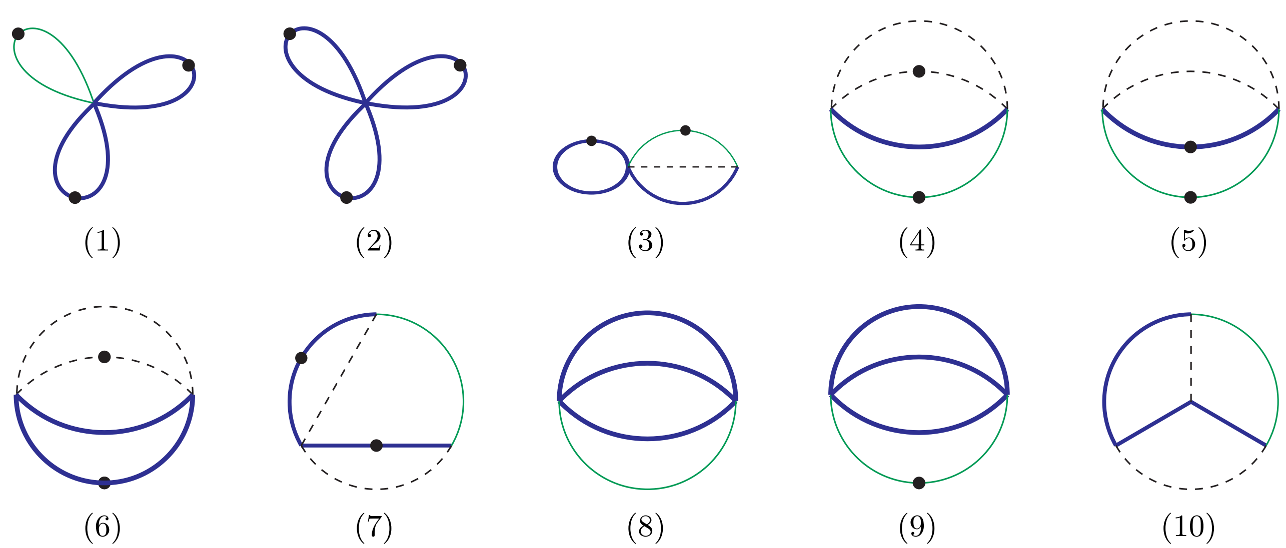
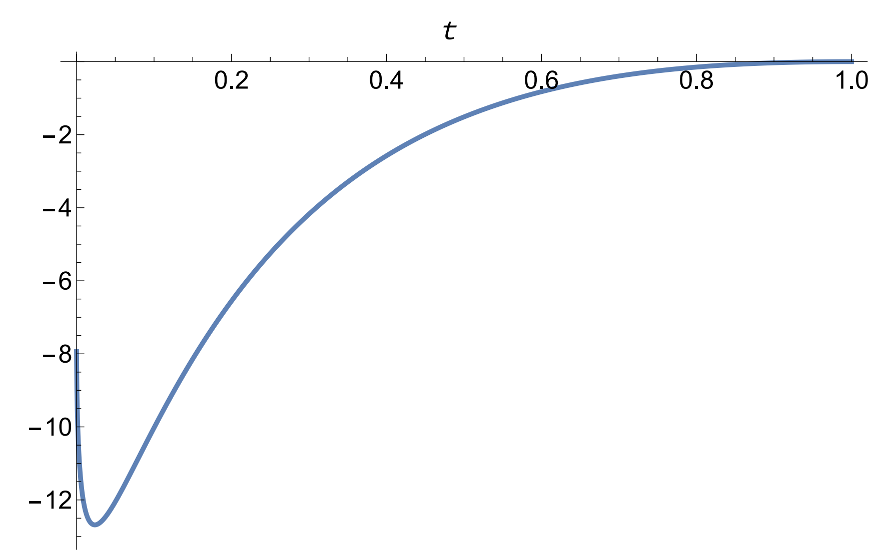
Analytic results for the **rho-parameter** in the Standard Model

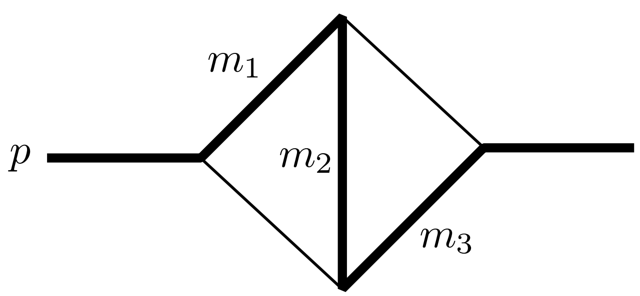
[Abreu, Becchetti, Duhr, Marzucca '19]



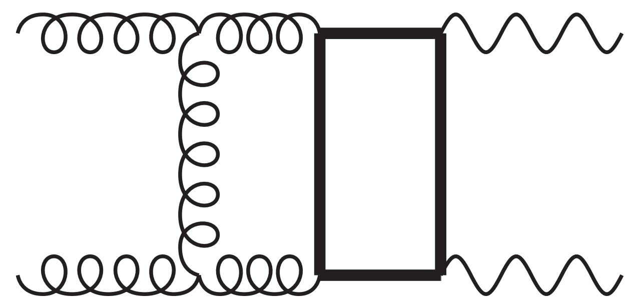


Analytic continuation and numerical evaluation **across whole phase-space**

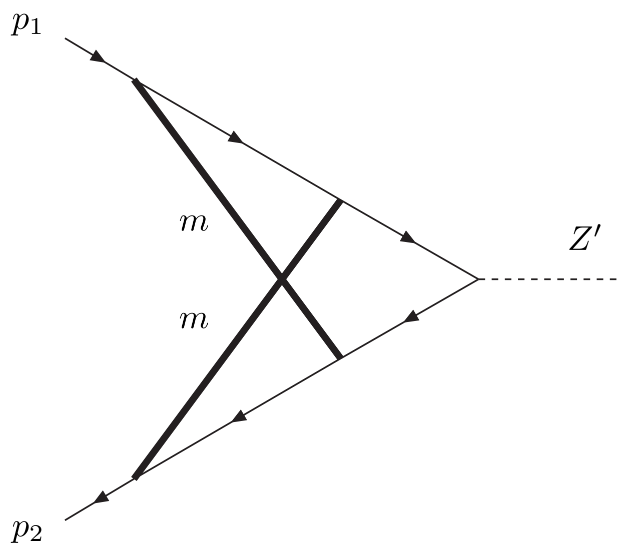
TOWARDS HIGGS AND TOPS



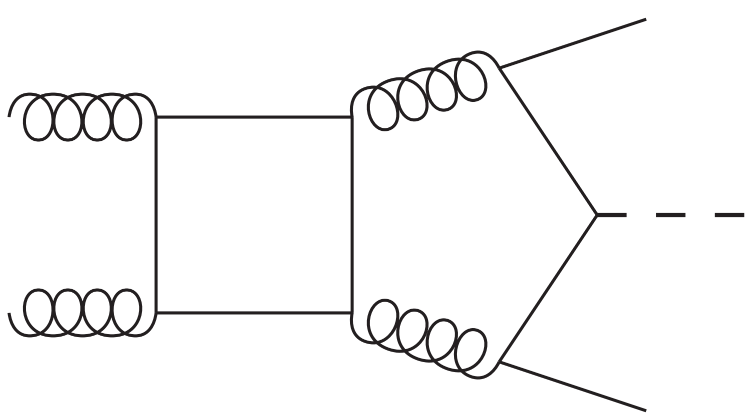
Kite integral (self-energies…)



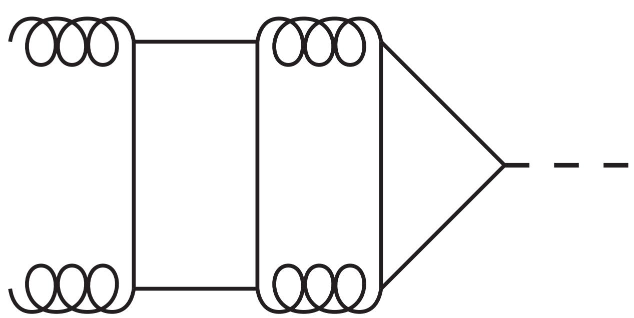
QCD with top quarks



EW form factor



ttb + X processes



H form factor at 3 loops

Iterated integrals of elliptic type are crucial for **high precision calculations** in the **Higgs and top sectors** !

CONCLUSIONS

- NNLO (and sometimes N3LO) calculations require evaluation of multiloop scattering amplitudes in the standard model
- In spite of impressive developments, most $2 \to 3$ processes remain out of reach because of the complexity of the relevant scattering amplitudes
- Complexity of two types: **Algebraic** and **Analytic**
- New techniques being developed to handle this complexity: ***Finite fields*** + ***unitarity*** for algebraic complexity and a new ***geometrical point of view*** for analytical complexity
- Pheno for $2 \to 3$ processes @ NNLO start being within reach…

Stay tuned!

THANK YOU!