Theory issues on top quark mass measurement

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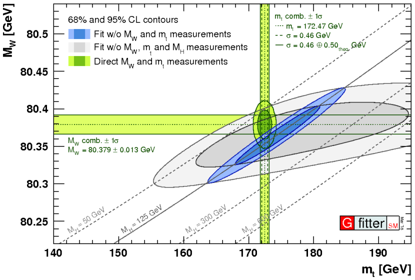
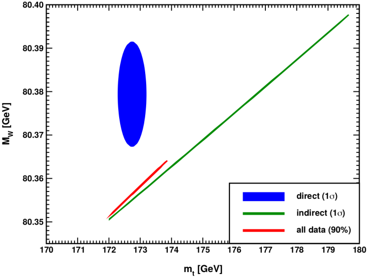
Outline

- Why the top mass
- $\overline{\rm MS}$ versus Pole mass
- Brief reminder of current status
- Problems in reaching precision below typical hadronic scales.
- Top mass from boosted top jets
- Linear Power corrections from renormalons
- Prospects and conclusions

Top and precision physics



From PDG:



$$\Delta G_\mu / G_\mu = 5 \cdot 10^{-7}; \quad \Delta M_Z / M_Z = 2 \cdot 10^{-5};$$

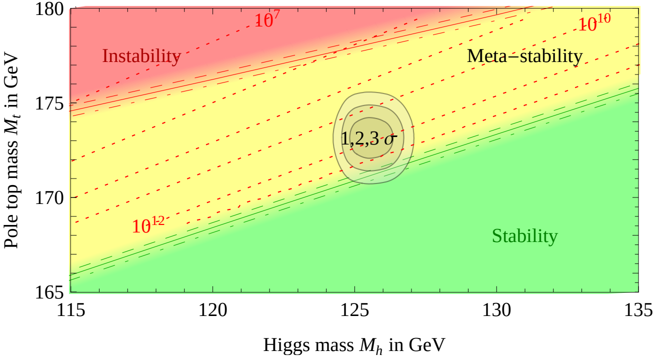
$$\Delta\alpha(M_Z)/\alpha(M_Z) = \begin{cases} 1 \cdot 10^{-4} (\text{Davier et al.; PDG}) \\ 3.3 \cdot 10^{-4} (\text{Burkhardt, Pietrzyk}) \end{cases}$$

Now that M_H is known, tight constraint on M_W-m_t, (depending on how aggressive is the error on $\alpha(M_Z)$).

But: precision on M_W is more important now ...

Top and vacuum stability

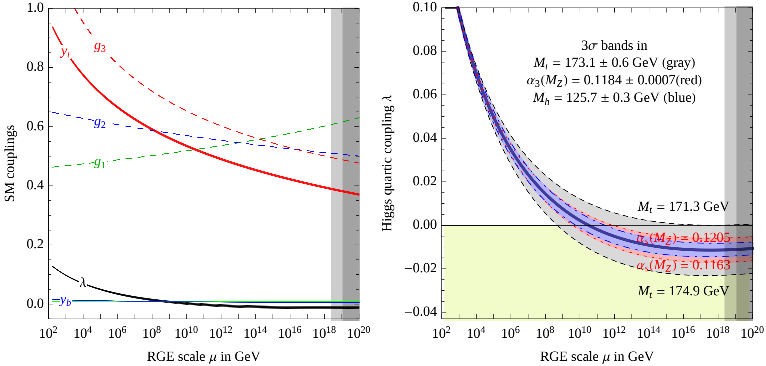
Degrassi et al. 2012



With current value of M_t and M_H the vacuum is metastable.
No indication of new physics up to the Plank scale from this.

Top and vacuum stability

Degrassi et al. 2012



The quartic coupling λ_H becomes tiny at very high field values, and may turn negative, leading to vacuum instability.
M_t as low as 171 GeV leads to $\lambda_H \to 0$ at the Plank scale.

Why the Top mass

Better to keep in mind that:

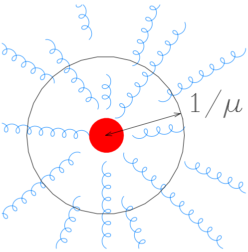
- The vacuum stability issue assumes no new physics up to the Plank scale (very strong assumption)
- In EW fits the bottleneck seems to be now the W mass.

Yet:

- We get the feeling that top mass is an important parameter.
- Its measurement at hadron colliders is quite challenging, and thus interesting.

What exactly do we wish to measure?

The mass of a heavy quark is also carried by its gluon field.



We can decide to include all the field accompanying the quark down to infinite distance.
This is the POLE MASS.

Or we can cut it off, keeping only contributions at distance below some scale $1/\mu$ (i.e., keeping only momenta above μ.
These are the SHORT DISTANCE MASSES.

They are related in perturbation theory by a power expansion in α_s with well defined coefficients:

$$M_{\rm pole} = M(\mu)\left(1 + \sum_{i=1}^{\infty} c_i \alpha_s^i(\mu)\right)$$

What exactly do we wish to measure?

The pole mass includes gluon field contributions at distances near and above the confinement scales:

$$\Delta m|_{\mu>\Lambda} \approx \int_{1/\Lambda}^{\infty} \left(\frac{g^2}{r^2}\right)^2 \mathrm{d}^3 r \approx g^2(\Lambda)\Lambda \approx \Lambda$$

Intuitive reasoning tells us that this must imply an ambiguity in the pole mass of order Λ.

After all: uncalculable confinement effects may cut off this integral at a scale near Λ, leading to an ambiguity of order Λ.

Pole mass ambiguity

The relation of the pole mass m_p to the $\overline{\mathrm{MS}}$ mass m is (Marquard,A.V.Smirnov,V.A.Smirnov,Steinhauser, 2015)

$$m_p = m(1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.375\alpha_s^3 + (8.49 \pm 0.25)\alpha_s^4)$$

The asymptotic behaviour of the coefficients (Beneke,Braun+Beneke,1994)

$$N\ m_t(2b_0)^n\Gamma(n+1+b)\left(1+\sum_{k=1}^{\infty}\frac{s_k}{n^k}\right), \qquad b = \frac{b_1}{b_0^2},$$

yields a good fit to the exact result, so that higher order terms can be estimated, yielding a very accurate conversion formula, with typical size

$$m_p = m + \underbrace{7.557}_{\mathtt{NLO}} + \underbrace{1.617}_{\mathtt{N^2LO}} + \underbrace{0.501}_{\mathtt{N^3LO}} + \underbrace{0.195}_{\mathtt{N^4LO}} + \underbrace{0.300}_{\mathtt{N^{5,6,..}LO}}\ \ \mathrm{GeV}$$

(Pineda etal, 2001,2014 for bottom;
Beneke,Marquard,Steinhause,PN, 2016 and
Hoang,Lepenik,Preisser, 2017 for top)

Mass renormalon

The $\mathcal{O}(\Lambda)$ ambiguity appears due to the factorial growth (the $\Gamma(n+1)$ factor) of the coefficients of the mass relation (Pole mass renormalon), leading to an asymptotic expansion.

Some authors have quoted an ambiguity of 1 GeV.

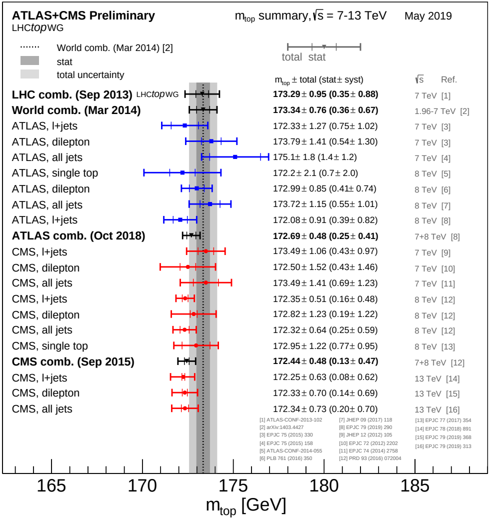
More refined estimates give much smaller results:

- Beneke, Marquard, Steinhauser, PN 2016: 110 MeV.
- Hoang, Lepenik, Preisser, 26 Jun 2017: 250 MeV.
- Pineda etal, 2001,2013,2014 190,200 MeV.
  (in a bottom physics context, but valid also for top.)

What do we wish to measure?

- The top impact on precision physics involves short distance phenomena, with power corrections (controlled by a short-distance OPE) of size $\leq \Lambda^2/M_{\rm EW}^2$.
- An ambiguity of the top mass of order Λ would translate into corrections of order $\Lambda/M_{\rm EW}$.
- If we aim at precisions better than Λ, we should target a short distance mass like the $\overline{\rm MS}$ mass.

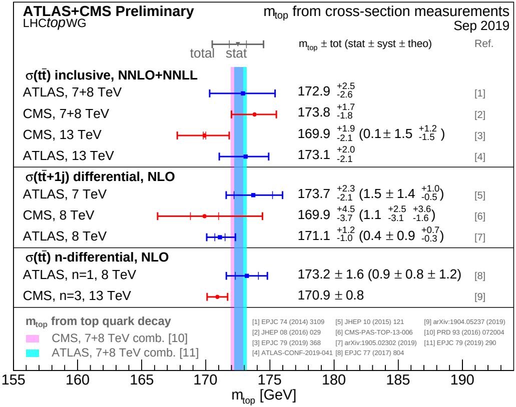
Current Measurements: “Direct Measurements”



Direct measurements use as top mass sensitive observable a reconstructed top mass from top decay products.

The claimed precision is around 500 MeV.

Current Measurements: “Indirect Measurements”



The so called “Indirect methods” make use of various cross sections or distributions as top mass sensitive observables.

- Direct measurements are certainty affected by linear power corrections, i.e. corrections of order Λ, that may be estimated as variations in the results due to changes in the Monte Carlo model as far as its low energy component is concerned (hadronization, shower-hadronization matching, colour reconnection, etc.).
- Among the indirect measurements, only the one using the total cross section may be considered a candidate for a measurement free from linear power corrections; but not a very useful one, considering its large error.

On current measurements

- A longstanding tradition leaves the indirect mass determination without an appropriate mass definition (the summary plot does not say anything like "Pole Mass" or "$\overline{\mathrm{MS}}$ mass", contrary to the plot of indirect measurements.)
- There is now some consensus among theorists that direct measurements can be considered as Pole Mass measurements, as long as one keeps in mind that they are affected by corrections of order Λ that need to be estimated in some way (which is also the case for the "indirect" measurements).
  See the contribution on "Top Mass: Theoretical issues", G. Corcella, P. Nason, A. Hoang and H. Yokoya,
  from: Report from Working Group 1:
  Standard Model Physics at the HL-LHC and HE-LHC, 2019, CERN Yellow Rep.Monogr. 7 (2019) 1-220, arXiv:1902.04070.

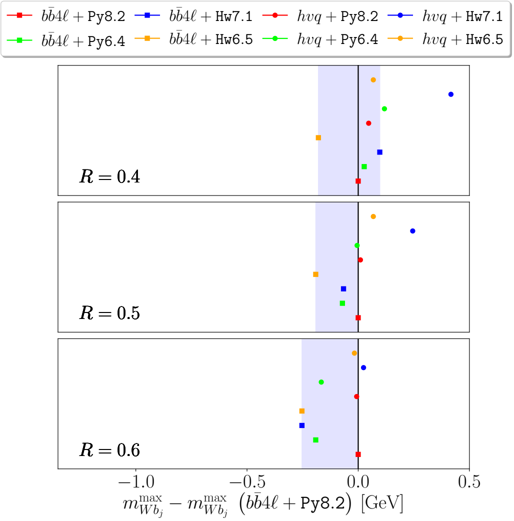
On current measurements

Keep in mind that as long as we are not at the level of reaching precision near typical hadronic scales:

- Direct measurements should be (and are!) considered pole mass measurements.
- Scheme differences (between the pole and $\overline{\mathrm{MS}}$ mass, for example) are of order $\alpha_s m_t$. So, a mass scheme for the direct measurements MUST BE SPECIFIED.

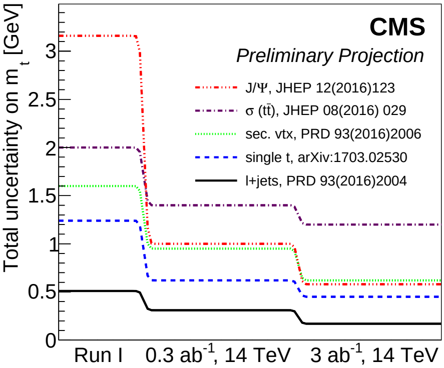
All problems and subtleties arise if we want to reach precision at or below typical hadronic scales. In this framework, many things are debatable, and much more work from theorists is needed.

An example: $\mathcal{O}(\Lambda)$ effects in direct measurements



(Ježo, Oleari, Ferrario Ravasio, P.N. 2019) Focus upon the groups of squares (our best generator) at fixed R. They span a range not larger than 250 MeV. This yields a lower bound on the intrinsic theoretical error associated with $\mathcal{O}(\Lambda)$ ambiguities.

Prospecs: HL LHC



We will have the opportunity to increase precision. But in order to reach the 100 MeV accuracy, the problem of linear power corrections must be addressed and solved.

Precision below Λ: boosted top jets.

Hoang and collaborators, in a long sequel of publications have addressed the problem considering as top mass sensitive observable the mass of an ultrarelativistic top jet.

This observable is affected by linear power corrections. However, via SCET techniques, they argue that these power corrections can be parametrized in terms of those that arise for light jets.
(see Hoang,Lepenik,Stahlhofen 2019 and references therein.)

Although it may be difficult to reach the desired precision with boosted top jets, this works gives an example of a relation between the short distance top mass and a hadronic observables that can be controlled in a sound theoretical framework.

Precision below Λ: boosted top jets.

Connection with Shower generators:

- The availability of precise calculations of the jet mass for highly boosted top jets also allow for a comparison with shower Monte Carlo describing the same regime, in an attempt to relate the mass parameter in the Shower model with a well-defined, theoretical short distance mass in the SCET calculation (see Hoang,Plätzer,Samitz, 2018).
- At the moment, far from application at hadronic colliders.

Linear Power Corrections: the Renormalon perspective

Renormalons provide a context where we can study power corrections in their relation to the asymptotic behaviour of the perturbative expansion.

- When an OPE exists for a process, renormalons can be associated to the VEV of the operators.
- When no OPE exists (or when it is not known), renormalons can give an indication on the presence and nature of power corrections.

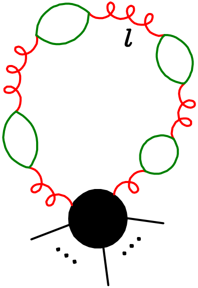
The full renormalon structure of QCD is not known. There are, however, simplified frameworks where renormalons can give useful information. They have been applied to the study of quark current correlators, structure function sum rules, heavy flavour studies and jet studies by a large number of researchers (see Beneke, 1998).

ABC of I.R. Renormalons

All-orders contributions to QCD amplitude of the form

$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p \underbrace{\frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2)\log\frac{k^2}{m^2}}}_{\text{Landau Pole}}$$

$$= \alpha_s(m^2)\sum_{n=0}^{\infty}(2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!}.$$



Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p_{\mathrm{QCD}}$.
- Typical scale dominating at order α_s^{n+1}: $m \exp(-np)$.

Renormalons and Linear Power Corrections in top decay

Ferrario Ravasio, Oleari, P.N.2019

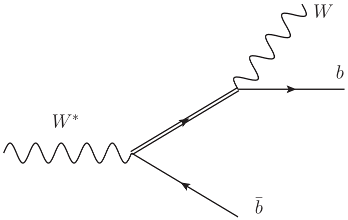
- We have investigated renormalon induced linear power corrections in top production and decays.
- We have considered a simplified production and decay process. However, we consider kinematic regions that are commonly accessible at the LHC, and we take into account top finite width.
- We rely on the so called *l*arge b_0 approximation renormalon framework.

Motivation

- Linear (i.e. $p = 1$) renormalons may affect top mass measurements at order Λ (near the present experimental accuracy).
- Previous to this work, only the top pole mass renormalon has received some attention.
- Several other sources of linear renormalons come into play in top mass measurements (for example, from jet requirements). Our aim is to understand their structure, and their interplay with the pole mass renormalon.

Our work: compute top mass sensitive observables in leading N_f one gluon correction.

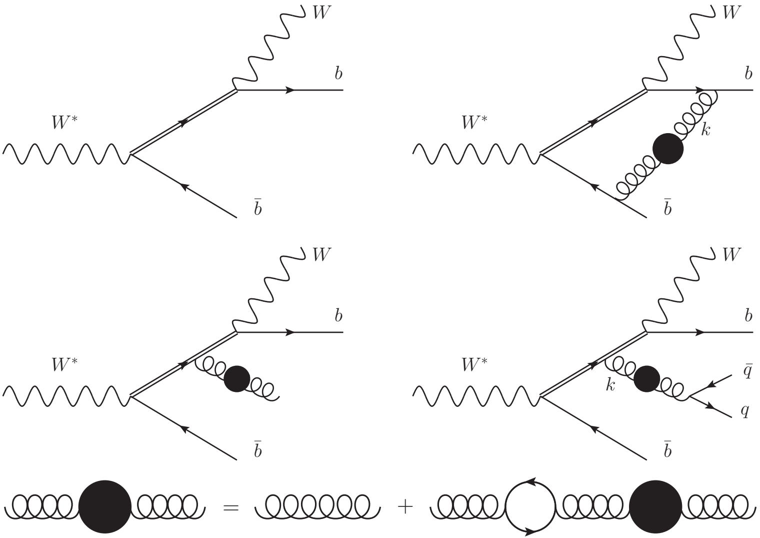
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can examine any infrared safe observable, no matter how complex.

Diagrams up to leading N_f one gluon correction



All-order result

For any (IR safe) final state observable O we compute:

$$
\begin{aligned}
\langle O\rangle_{\mathrm{b}} &= N^{(0)}\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}(\Phi_{\mathsf{b}})\,O(\Phi_{\mathsf{b}})\,, \quad \left(\text{where } N^{(0)}=\left[\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}\right]^{-1}\right)\,,\\
\widetilde{V}(\lambda) &= N^{(0)}\int \mathrm{d}\Phi_{\mathrm{b}}\,\sigma_{\mathrm{v}}^{(1)}(\lambda^2,\Phi_{\mathrm{b}})\left[O(\Phi_{\mathsf{b}})-\langle O\rangle_{\mathrm{b}}\right],\\
\widetilde{R}(\lambda) &= N^{(0)}\int \mathrm{d}\Phi_{g^*}\,\sigma_{g^*}^{(1)}(\lambda^2,\Phi_{g^*})\left[O(\Phi_{g^*})-\langle O\rangle_{\mathrm{b}}\right],\\
\widetilde{\Delta}(\lambda) &= \frac{3\pi}{\alpha_s\,T_{\mathrm{F}}}\,\lambda^2\,N^{(0)}\int \mathrm{d}\Phi_{q\bar{q}}\,\delta(k_{q\bar{q}}^2-\lambda^2)\,\sigma_{q\bar{q}}^{(2)}(\Phi_{q\bar{q}})\times\left[O(\Phi_{q\bar{q}})-O(\Phi_{g^*})\right]
\end{aligned}
$$

$\langle O\rangle_{\mathrm{b}}+\widetilde{V}(\lambda)+\widetilde{R}(\lambda)$ is the average value of O in a theory with a massive gluon with mass λ, accurate to order α_s.

Notice: $\widetilde{V}(\lambda)+\widetilde{R}(\lambda)$ has a finite limit for $\lambda\to 0$, while each contribution is log divergent.

defining $\widetilde{T}(\lambda) \equiv \widetilde{V}(\lambda) + \widetilde{R}(\lambda) + \widetilde{\Delta}(\lambda)$, our final result is

$$\langle O \rangle = \langle O \rangle_{\mathrm{b}} + \frac{3\pi}{\alpha_s T_{\mathrm{F}}} \int_0^\infty \frac{\mathrm{d}\lambda}{\pi} \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\widetilde{T}(\lambda) \right] \mathrm{atan} \left(\frac{\alpha_s \pi b_0}{1 + \alpha_s b_0 \log \frac{\lambda^2}{\mu^2 e^{5/3}}} \right) \tag{1}$$

If we have:

$$\widetilde{T}(\lambda) = a + b\,\lambda + \mathcal{O}\left(\lambda^2\right) \tag{2}$$

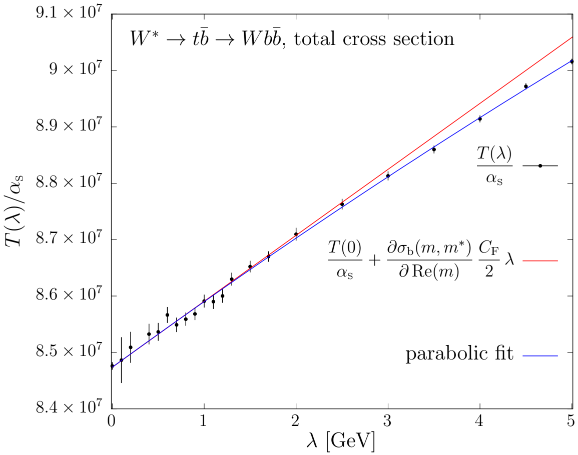
the integration has an ambiguity of order $b\Lambda_{\mathrm{QCD}}$, due to the value of λ where the denominator in the argument of the atan function vanishes (i.e. again the Landau pole).

If we expand the result in powers of α_s, we recover the factorial growth corresponding to linear power corrections.

Comments

- The need to include the Δ term has a long story:
  - Seymour,P.N. 1995, I.R. renormalons in e^+e^- event shapes.
  - Dokshitzer,Lucenti,Marchesini,Salam, 1997-1998 Milan factor
- We compute $T(\lambda)$ numerically. The $\lambda \to 0$ limit implies the cancellation of two large logs in V and R. However, the precise value at $\lambda = 0$ can also be computed directly by standard means (which we do).
- We can easily switch to the $\overline{\mathrm{MS}}$ scheme,

Total cross section



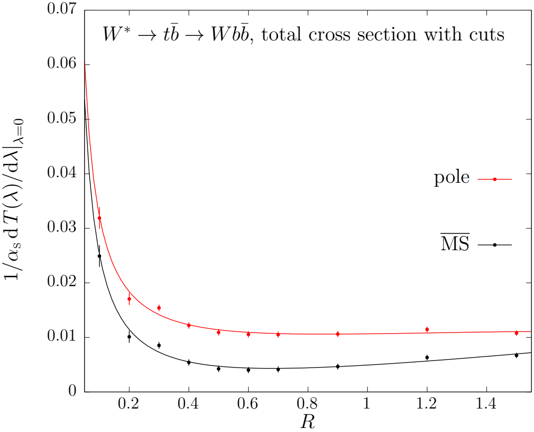
The red line is the correction to be subtracted from our result when switching to the $\overline{\mathrm{MS}}$ scheme. No linear renormalon in $\overline{\mathrm{MS}}$ scheme!

Total cross section

- For $k < \Gamma$: no renormalon in the physics! The top finite width screens the soft sensitivity of the cross section.
  The renormalon is there only if it is present in the mass counterterm; thus, it is not there in the $\overline{\mathrm{MS}}$ scheme.
- What about $k \gg \Gamma$?
  This is the narrow width limit: the cross section factorizes into a production cross section and a partial width.
  The former has no physical renormalons for obviour reasons.
  The latter does not have them for less obvious reasons.

So, the mass from the total σ is free of linear power corrections.

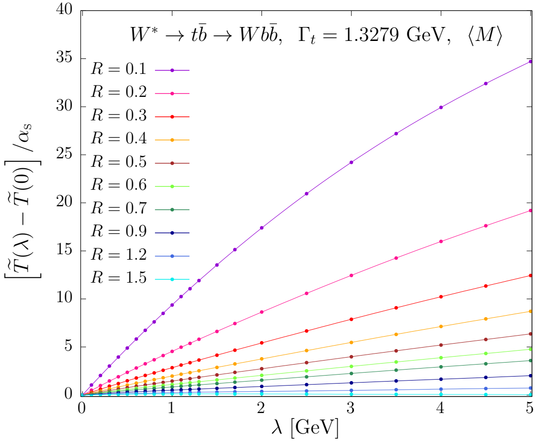
Total cross section with cuts



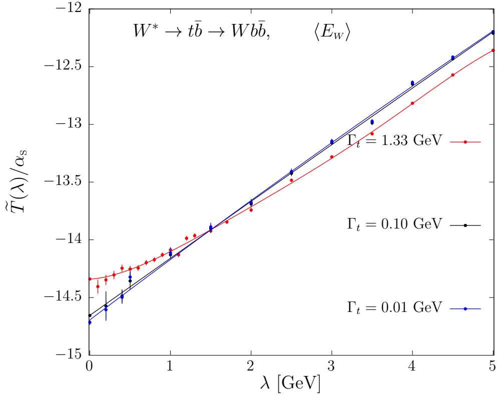
The renormalon is there also in $\overline{\mathrm{MS}}$ scheme!

The $1/R$ behaviour of the renormalon coefficient arises from jet requirements (Dasgupta,Magnea,Salam,2008)

Reconstructed top mass



Strong renormalon effects due to jets (coefficient $\propto 1/R$)



Consider $\langle E_W \rangle$.
For $k \gg \Gamma$, the slope is roughly 0.45.
The $\overline{\mathrm{MS}}$ conversion would add -0.067.
It seems that physical linear renormalons are present also in leptonic observables.

But, for $\lambda \ll \Gamma$, the slope of $T(\lambda)$ decreases, approaching 0.067!
The renormalon seems to cancel in the $\overline{\mathrm{MS}}$ scheme!

Leptonic Observables

Two questions:

- Our narrow width result seems to be in contrast with what found in heavy flavour inclusive decays, where no renormalons are present for leptonic observables if the heavy flavour mass is expressed in the $\overline{\mathrm{MS}}$ scheme (Beneke,Braun,Zakharov,1994; Bigi etal,1994).
  We have verified, however, that if $\langle E_W \rangle$ is computed in the top rest frame, no renormalons are present.
  So: no contradiction there.
- About the renormalon cancellation for finite width:
  we have also verified it for larger width values,
  and also proved it theoretically.

Prospects

- With some work, the renormalon approach can help to search for top mass observables that are free from linear renormalons.
- One may discuss calibration of jets on a theoretically sound ground.
- The fact that top CM leptonic distributions are free from linear renormalon may be exploited further.

Top CM Leptonic distributions

Kawabata,Shimizu,Sumino,Yokoya,2013,2014 have proposed a method to measure physical parameters in the decay of a massive object involving a light lepton using only the lepton spectrum, and have proposed to apply it for the measurement of the top mass.

Defining a weight function

$$W(E_\ell, m) = \int \mathrm{d}E\, D_0(E, m) \frac{1}{E\, E_l} \times \left(\text{odd function of} \quad \log \frac{E_l}{E} \right)$$

where $D_0(E, m)$ is the lepton spectrum in the top rest frame for a top of mass m. It turns out that the quantity

$$I(m') = \int \mathrm{d}E_l\, D(E_l, m') W(E_l, m),$$

where $D(E, m')$ is the lepton spectrum in the laboratory for a top of mass m', vanishes if $m = m'$.

- This observable only depends upon the lepton spectrum in the top rest frame, and is thus free from linear renormalon.
- In order for it to be useful, we must be able to compute it with sufficient accuracy.
- Assuming a NNLO calculation of the lepton spectrum, the N^3LO correction should lead to a mass error that is smaller than a typical hadronic scale.
- We can check if this is the case in our model, since we can also compute all the coefficients of the perturbative expansion.

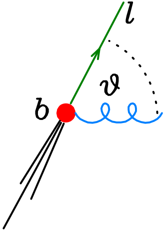
Conclusions

- Top mass measurements at hadron colliders, when the precision approaches few hundred MeV's, pose difficult and profound theoretical problems, involving our understanding of non-perturbative corrections in QCD, and of how they are implemented in shower generators.
- The traditional method: aim at an observable, measure it, extract its value from a perturbative calculation, and estimate power corrections using a shower Monte Carlo, is still a valuable strategy to follow, as long as better ways of doing it are not in sight.
- Theoretical studies on the form of linear power corrections and to what extent they can/are implemented in shower Monte Carlo are at a primitive stage, but they are promising. They can help us to understand the limitations of current measurements, and they can help to identify better observables.

Backup Slides

About the absence of renormalons in top CM leptonic observables:

- A b quark in a B meson undergoes Fermi motion, i.e. it has momentum of order Λ. But its kinetic energy is of order Λ^2/m_b, because it is non-relativistic. So, no linear power corrections there.
- The decay can take place in a time fraction when the b is in a virtual state associated with the emission of a soft gluon.



The decay product are boosted with velocity $v = k/m_b$, where k is the soft gluon momentum. The corresponding change in the lepton momentum is $\delta p_l \approx vp\cos\theta$. But this effect linear in v vanish under azimuthal average.

As a result, the semileptonic spectrum has no linear power corrections *if expressed in terms of a short distance mass*

(This explanation also holds for heavy quarks produced on-shell, since their soft radiation pattern does not depend upon its spin.)

A look at the expansions

	$\sigma/\sigma_b^{\rm nocuts}(m)$			
	pole scheme		$\overline{\rm MS}$ scheme	
i	c_i	$c_i\,\alpha_S^i$	c_i	$c_i\,\alpha_S^i$
0	1.00000000	1.0000000	0.86841331	0.8684133
1	$5.003\,(0)\times 10^{-1}$	$5.411\,(0)\times 10^{-2}$	$1.480\,(0)\times 10^{0}$	$1.601\,(0)\times 10^{-1}$
2	$-6.20\,(2)\times 10^{-1}$	$-7.25\,(2)\times 10^{-3}$	$4.42\,(2)\times 10^{-1}$	$5.17\,(2)\times 10^{-3}$
3	$-3.03\,(2)\times 10^{0}$	$-3.83\,(3)\times 10^{-3}$	$6.4\,(2)\times 10^{-1}$	$8.1\,(3)\times 10^{-4}$
4	$-1.25\,(2)\times 10^{1}$	$-1.70\,(3)\times 10^{-3}$	$0\,(2)\times 10^{-2}$	$0\,(3)\times 10^{-6}$
5	$-6.4\,(2)\times 10^{1}$	$-9.4\,(3)\times 10^{-4}$	$1\,(2)\times 10^{-1}$	$1\,(3)\times 10^{-5}$
6	$-3.9\,(1)\times 10^{2}$	$-6.2\,(2)\times 10^{-4}$	$0\,(1)\times 10^{0}$	$0\,(2)\times 10^{-6}$
7	$-2.9\,(1)\times 10^{3}$	$-5.0\,(2)\times 10^{-4}$	$0\,(1)\times 10^{1}$	$0\,(2)\times 10^{-6}$
8	$-2.5\,(1)\times 10^{4}$	$-4.6\,(2)\times 10^{-4}$	$0\,(1)\times 10^{2}$	$0\,(2)\times 10^{-6}$
9	$-2.4\,(1)\times 10^{5}$	$-4.9\,(2)\times 10^{-4}$	$0\,(1)\times 10^{3}$	$0\,(2)\times 10^{-6}$
10	$-2.6\,(1)\times 10^{6}$	$-5.8\,(2)\times 10^{-4}$	$0\,(1)\times 10^{4}$	$-1\,(2)\times 10^{-6}$

A look at the expansions

Reconstructed mass

	$R = 0.1$		$R = 0.5$		$R = 1.5$	
i	pole	$\overline{\text{MS}}$	pole	$\overline{\text{MS}}$	pole	$\overline{\text{MS}}$
0	172.8280	163.0146	172.8201	163.0040	172.7533	162.9244
1	$-7.597\,(0) \times 10^{0}$	$2.163\,(0) \times 10^{-1}$	$-2.785\,(0) \times 10^{0}$	$5.030\,(0) \times 10^{0}$	$4.446\,(0) \times 10^{-1}$	$8.268\,(0) \times 10^{0}$
2	$-4.136\,(2) \times 10^{0}$	$-2.852\,(2) \times 10^{0}$	$-1.255\,(1) \times 10^{0}$	$2.9\,(1) \times 10^{-2}$	$1.029\,(8) \times 10^{-1}$	$1.387\,(1) \times 10^{0}$
3	$-2.397\,(2) \times 10^{0}$	$-1.973\,(2) \times 10^{0}$	$-5.96\,(2) \times 10^{-1}$	$-1.72\,(2) \times 10^{-1}$	$1.4\,(1) \times 10^{-2}$	$4.38\,(1) \times 10^{-1}$
4	$-1.505\,(2) \times 10^{0}$	$-1.337\,(2) \times 10^{0}$	$-3.13\,(2) \times 10^{-1}$	$-1.44\,(2) \times 10^{-1}$	$-6\,(1) \times 10^{-3}$	$1.63\,(1) \times 10^{-1}$
5	$-1.038\,(2) \times 10^{0}$	$-9.50\,(2) \times 10^{-1}$	$-1.88\,(2) \times 10^{-1}$	$-1.00\,(2) \times 10^{-2}$	$-9.7\,(9) \times 10^{-3}$	$7.86\,(9) \times 10^{-2}$
6	$-7.94\,(2) \times 10^{-1}$	$-7.35\,(2) \times 10^{-1}$	$-1.33\,(1) \times 10^{-1}$	$-7.3\,(1) \times 10^{-2}$	$-1.05\,(8) \times 10^{-2}$	$4.89\,(8) \times 10^{-2}$
7	$-6.79\,(2) \times 10^{-1}$	$-6.33\,(2) \times 10^{-1}$	$-1.09\,(1) \times 10^{-1}$	$-6.3\,(1) \times 10^{-2}$	$-1.12\,(7) \times 10^{-2}$	$3.53\,(7) \times 10^{-2}$
8	$-6.51\,(2) \times 10^{-1}$	$-6.08\,(2) \times 10^{-1}$	$-1.04\,(1) \times 10^{-1}$	$-6.1\,(1) \times 10^{-2}$	$-1.25\,(7) \times 10^{-2}$	$3.08\,(7) \times 10^{-2}$
9	$-6.99\,(2) \times 10^{-1}$	$-6.54\,(2) \times 10^{-1}$	$-1.12\,(1) \times 10^{-1}$	$-6.7\,(1) \times 10^{-2}$	$-1.47\,(7) \times 10^{-2}$	$3.09\,(7) \times 10^{-2}$
10	$-8.37\,(2) \times 10^{-1}$	$-7.83\,(2) \times 10^{-1}$	$-1.35\,(1) \times 10^{-1}$	$-8.1\,(1) \times 10^{-2}$	$-1.85\,(9) \times 10^{-2}$	$3.57\,(9) \times 10^{-2}$

A look at the expansions

	$\langle E_W \rangle$			
	pole scheme		$\overline{\text{MS}}$ scheme	
i	c_i	$c_i\,\alpha_s^i$	c_i	$c_i\,\alpha_s^i$
0	121.5818	121.5818	120.8654	120.8654
1	$-1.435\,(0) \times 10^1$	$-1.552\,(0) \times 10^0$	$-7.192\,(0) \times 10^0$	$-7.779\,(0) \times 10^{-1}$
2	$-4.97\,(4) \times 10^1$	$-5.82\,(4) \times 10^{-1}$	$-3.88\,(4) \times 10^1$	$-4.54\,(4) \times 10^{-1}$
3	$-1.79\,(5) \times 10^2$	$-2.26\,(6) \times 10^{-1}$	$-1.45\,(5) \times 10^2$	$-1.84\,(6) \times 10^{-1}$
4	$-6.9\,(4) \times 10^2$	$-9.4\,(6) \times 10^{-2}$	$-5.7\,(4) \times 10^2$	$-7.8\,(6) \times 10^{-2}$
5	$-2.9\,(3) \times 10^3$	$-4.4\,(5) \times 10^{-2}$	$-2.4\,(3) \times 10^3$	$-3.5\,(5) \times 10^{-2}$
6	$-1.4\,(3) \times 10^4$	$-2.2\,(4) \times 10^{-2}$	$-1.0\,(3) \times 10^4$	$-1.7\,(4) \times 10^{-2}$
7	$-8\,(2) \times 10^4$	$-1.3\,(4) \times 10^{-2}$	$-5\,(2) \times 10^4$	$-8\,(4) \times 10^{-3}$
8	$-5\,(2) \times 10^5$	$-9\,(4) \times 10^{-3}$	$-2\,(2) \times 10^5$	$-4\,(4) \times 10^{-3}$
9	$-3\,(2) \times 10^6$	$-7\,(4) \times 10^{-3}$	$-1\,(2) \times 10^6$	$-2\,(4) \times 10^{-3}$
10	$-3\,(2) \times 10^7$	$-6\,(5) \times 10^{-3}$	$0\,(2) \times 10^6$	$-1\,(5) \times 10^{-4}$
11	$-3\,(3) \times 10^8$	$-7\,(6) \times 10^{-3}$	$0\,(3) \times 10^6$	$0\,(6) \times 10^{-5}$
12	$-4\,(3) \times 10^9$	$-9\,(9) \times 10^{-3}$	$0\,(3) \times 10^8$	$1\,(9) \times 10^{-3}$

- The "benefit" of the renormalon-free expansion is visible starting with the NNLO term in the total cross section: terms beyond NNLO are smaller in the $\overline{\mathrm{MS}}$ scheme.
- For the reconstructed mass both the $\overline{\mathrm{MS}}$ and pole scheme have problems. For large radii (most of the mass of the decay products is captured) the pole mass is better.
- For E_W the improved behaviour is visible starting from order 6-7, as expected, corresponding to virtualities of the order of the top width.