

Motivation

Basic Problem in QFT: UV (given) $\xrightarrow{\text{RG}}$ IR ?

Develop general tools for attacking this problem

Typical Examples are gauge theories in 3d or 4d

4d SU(N) Yang Mills (no matter)

Loose: IR is gapped and confining

$$\text{Action } S \supseteq \frac{\Theta}{8\pi^2} \int \underbrace{\text{Tr}(F \wedge F)}_{\text{instanton density}} \quad \Theta \in [0, 2\pi) \quad \text{periodic}$$

A circle's worth of QFTs. How does IR depend on Θ ?

Strongly coupled at dynamical scale. Λ_{QCD} believe all particles have masses $m \sim \Lambda_{\text{QCD}} > 0$

What new can be said? Organize IRs of QFTs:

- gapless: described by conformal field theory (free or interacting)
- gapped: particle masses $m > 0$. Subcases:
 - trivially gapped IR empty no interesting correlators
 - TQFT topological correlation functions (eg 3d CS theories)

conjecture: SU(N) YM at $\Theta=0$ trivially gapped

Can IR be boring for all Θ ? NO!

Result: IR at $\Theta=\pi$ cannot be trivially gapped!

(GJKS+CLS) (Goal: understand argument)

Even more can be said! $\Theta=0, \pi$ special

time reversal T $\Rightarrow T(\Theta) = -\Theta$

flips sign of E_{top}

$\Theta=0, \pi$ fixed pts with T sym

Possibilities for $\Theta=\pi$:

- gapless (very novel and exotic)

- gapped, non-trivial TQFT subcases:

- confinement: do large wilson loops have area law or perimeter law?

- T invariance: does the vacuum preserve or spontaneously break T sym?

Result: SU(N) YM at $\Theta=\pi$ cannot be simultaneously gapped, confining, T inv.

Loew: T spontaneously broken. \Rightarrow two vacua + stable domain walls

Techniques: Symmetry + Anomalies

Develop theory of RG invariants ('t Hooft anomalies)

Symmetry is RG invariant!

UV theory has \Rightarrow local operators in reps of G
global sym G

\Downarrow RG

IR theory has " "

sym G

Note: IR Hilbert space in infinite spatial volume
may spontaneously break G , but operators remain G inv

Useful, but in IR symmetry may act trivially

simple example: complex massive scalar has $U(1)$ sym
but IR is trivial. Unit operator in trivial rep of $U(1)$.

Under what circumstances can we ensure that sym acts in IR?

Source Gauge Fields

Probe theory with global sym by coupling to a background gauge field A

$$S' \supseteq \int d^d x A^\nu J_\nu ; \quad \partial^\nu J_\nu = 0 \quad \text{conserved current}$$

A is a classical field. Not summed over.

integral over fluctuating fields defines $Z[A]$ partition function

encodes correlators $\langle J_1(x_1) \dots J_n(x_n) \rangle$.

In correlation functions J conserved at separated pts but what about contact terms?

Equivalently, is $Z[A+d\lambda] = Z[A]$? (gauge invariant)

in general:

$$Z[A+d\lambda] = Z[A] \exp(i \int d^d x \omega(\lambda, A))$$

correction local since J conserved at separated pts

Try to adjust $Z[A]$ to restore gauge invariance

$$Z[A] \mapsto Z[A] \exp(i \int d^d x f(A))$$

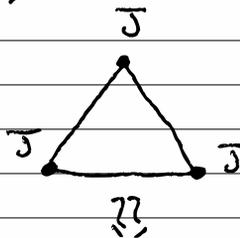
local counterterms

such transformations modify contact terms in $\langle \dots \rangle$.

Definition = If ω cannot be eliminated by any f we say there is an 't Hooft anomaly

(classified by a cohomology problem)

Basic Example



Weyl fermions Ψ of charge Q_I
enter chiral $U(1)$ sym

$$d^d J_\mu \sim \left(\sum_I Q_I^3 \right) F_A \wedge F_A.$$

Anomalies are RG Invariant

Consider $Z[A, g]$ g background metric

By investigating $g \rightarrow c^{\sigma} g$ we probe long distances

Wess-Zumino consistency

$$\delta_{\sigma} \delta_{\lambda} \log Z[A, g] = \delta_{\lambda} \delta_{\sigma} \log Z[A, g]$$

$$\underbrace{\text{scale dependence of anomaly}} = \delta_{\lambda} \int \langle T_{\mu}^{\mu} \rangle = 0$$

↑
should vanish
since T gauge inv.

connect to previous discussion:

$Z[A]$ typically non-local. Encodes separated pts $\langle \dots \rangle$

if theory trivially gapped all correlators reduce to contact terms

$$\Rightarrow Z[A] = \exp(i \int d^d x f(A)) \quad \text{if trivially gapped}$$

sometimes also called: invertible theory / SPT

Note: A trivially gapped thy has no anomaly

\Rightarrow A theory with non-zero anomaly cannot flow to a trivially gapped thy.

fundamental conclusion

