

Symmetry and its Generalizations

Goal: Search for new concepts of sym + anomalies

Topological Operators

Viewpoint on symmetry that does not require currents

initial definition $Q \equiv \int_{\text{space}} J_0$ conserv charge

More generally in d -dimensional Euclidean QFT look at:

charge $Q(M^{d-1}) \equiv \int_{M^{d-1}} *J$ ($\partial^\mu J_\mu = 0 \Leftrightarrow d *J = 0$)

any manifold

Can also consider exponentiated operators:

$$U_\alpha(M^{d-1}) \equiv \exp(i\alpha Q(M^{d-1}))$$

α now
group label

U 's preference: Discrete sym has no charges!

Key Properties:

- $U_\alpha(M^{d-1})$ depends topologically on M^{d-1}

$$\int_{M^{d-1}} *J - \int_{\tilde{M}^{d-1}} *J = \int_{\substack{\tilde{M}^{d-1} \\ \text{stokes}}} d *J = 0 \quad \left(\begin{array}{l} \text{Man small} \\ \text{deformation of } M^{d-1} \end{array} \right)$$

- $U_\alpha(M^{d-1})$ implements symmetry transformation on ops

$$e^{i\alpha q_0} \cdot \mathcal{O}(o) =$$

charge local of $\mathcal{O}(o)$
 $\leftarrow \mathcal{U}_a(g^{d-1})$ sym defect

true from Ward identity $d^* J \sim q_0 \delta^{(d)}(o)$

In general in QFT we define global sym by the existence of topological operators $\mathcal{U}_g(M^{d-1})$

$$\mathcal{U}_{g_1}(M^{d-1}) \mathcal{U}_{g_2}(M^{d-1}) = \mathcal{U}_{g_1 g_2}(M^{d-1}) \quad \text{group law}$$

Example : Ising model

$$\begin{array}{c} \downarrow \downarrow \downarrow \\ \downarrow \downarrow \downarrow \end{array} \quad \boxed{\quad} \quad \begin{array}{c} \uparrow \uparrow \uparrow \\ \uparrow \uparrow \uparrow \end{array}$$

spacetime cut in two
by \mathbb{Z}_2 symmetry defect

- since this is a symmetry we only detect the insertion at the transition lines, defect
- in general this operator is not an integral of something local

Higher Form Global Symmetry

Def : A q -form global sym characterized by topological operators of codimension $q+1$ $\mathcal{U}_g(M^{d-1-q})$

Charged operators have dimension q :

$$\begin{array}{c} L \text{ extended} \\ \text{op dim } q \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} = e^{i\alpha q_L} \begin{array}{c} L \\ | \\ | \\ | \\ | \\ | \end{array}$$

$\uparrow \mathcal{U}_a(M^{d-1-q})$

In Hilbert space picture space $\equiv Y^{d-1}$. A q -form sym gives one charge for each non-trivial $d-1-q$ cycle in Y^{d-1}

One-form Symmetry in Gauge Theory

Continuous 1-form symmetry \leftrightarrow conserved current with 2 indices

$$\partial^\mu J_{\mu\nu} = 0 \quad \leftrightarrow \quad d(*\bar{J})_{d-2} = 0$$

4d Maxwell theory (free U(1) gauge theory) has two:

$$\cdot \text{U(1)}_{\text{elec}}^{(1)} \xleftarrow{\text{1-form}} \partial^\mu F_{\mu\nu} = 0 \quad \text{by EOM}$$

$$\cdot \text{U(1)}_{\text{mag}}^{(0)} \quad \partial^\mu (*F)_{\mu\nu} = 0 \quad \text{by Bianchi}$$

charged objects are line operators. Say L extended along time
at origin of space

$$\frac{1}{2\pi} \int_S^* F \sim \text{electric charge of } L \\ (\text{e.g. Wilson line charged})$$

$$\frac{1}{2\pi} \int_S^* F \sim \text{magnetic charge of } L \\ (\text{'tHooft line charged})$$

Transformation on fields

Examine $U_\alpha \equiv \exp(i\alpha \int_\Sigma *F)$ work locally $\Sigma = xy$ plane $t=0$

set temporal gauge $A_t = 0 \Rightarrow (*F)_{xy} \sim \partial_t A_z$

since $\partial_t A_z$ canonically conjugate to A_z we learn:

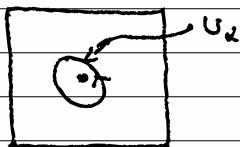
$$U_\alpha A_i U_\alpha^{-1} = \begin{cases} A_i & i \neq z \\ A_z + \alpha \delta(z) \delta(t) & \end{cases}$$

so A shifted by
a flat connection
(flat away from U_α)

Action of magnetic spin not as single.

Disorder Definition

U_a codm 2
not in place



Define insertion of U_2
in path integral st

$$\text{G} \ A = \alpha$$

small circle
surrounding U

Charged Matter

What if we include charged matter (interacting theory) ?

Wilson line charge transverse to the line

Effective charge of lone depends on distance \Rightarrow now broken!

More generally say charged particles have charge $q > 1$

Then $\cup_{\text{else}}^{(1)} \rightarrow \mathbb{Z}_q^{(1)}$ (can only fully screen mult of q)

Technically test if charges are topological

$$\exp\left(i\alpha \int_{\Sigma_1} *F\right) \exp\left(-i\alpha \int_{\tilde{\Sigma}_2} *\tilde{F}\right) = \exp\left(i\alpha \int_X J_{\text{can}}\right)$$

where $\dot{x}^* F = J_{\text{ele}}$ is the EOM

so we still have sym if $\int_x^y J_{\text{elec}} = q$; $\alpha = \frac{2\pi k}{q}$; $k \in \mathbb{Z}$

Non-Abelian Gauge Fields

$SU(N)$ YM (global form of group matters! $SU(2)$ vs $SO(3)_{\text{m}}$)

(Wilson line operators \leftrightarrow representations of $SU(N)$)

all made from
fundamental N , $\text{sym}^2 N$, $\Lambda^2 N$, adj; ...
 \square , $\square\square$, $\square\square\square$, $\square\square\square\square$

The center of $SU(N)$ is λI_N $\lambda = \exp(2\pi i k/N)$

def: N -ality of \equiv charge under center of $SU(N)$
Wilson line L \equiv # boxes mod N

In pure YM only fields are adjoints \Rightarrow screening adds N boxes

$\Rightarrow SU(N)$ YM has $\mathbb{Z}_N^{(1)}$ 1-form symmetry!

$$\left. \begin{aligned} U_L &= \exp(i\alpha N\text{-ality}(L)) \\ \alpha &= 2\pi i k/N \end{aligned} \right\}$$

Can also define via disorder or shifts by flat connections

Connection to Confinement

1-form symmetry gives a way to understand gauge theory phases

$\mathbb{Z}_N^{(1)}$ - spontaneously broken \leftrightarrow deconfined; $\mathbb{Z}_N^{(1)}$ preserved \leftrightarrow confined

Usually phrased in terms of perimeter vs area law.

$$\langle \begin{array}{c} T \\ | \\ \text{---} \\ | \\ R \end{array} \rangle \sim e$$

deconfined -perimeter
fundamental or -Area
Wilson line confined

for large R , and $T \rightarrow \infty$ view as a sum of lines

- confined, infinite wilson line $\langle f \rangle = 0$
- deconfined, " " $\langle f \rangle \neq 0$

(here we have added a counterterm to wilson line)

To see connection to symmetry. f charged under 1-form sym $U_\alpha(\Sigma)$ where Σ intersects the line ($\Sigma \approx R^2$ non-compact)

So these phases are distinguished by a charged object having ver