

Parametrization of light baryons

Yannick Wunderlich

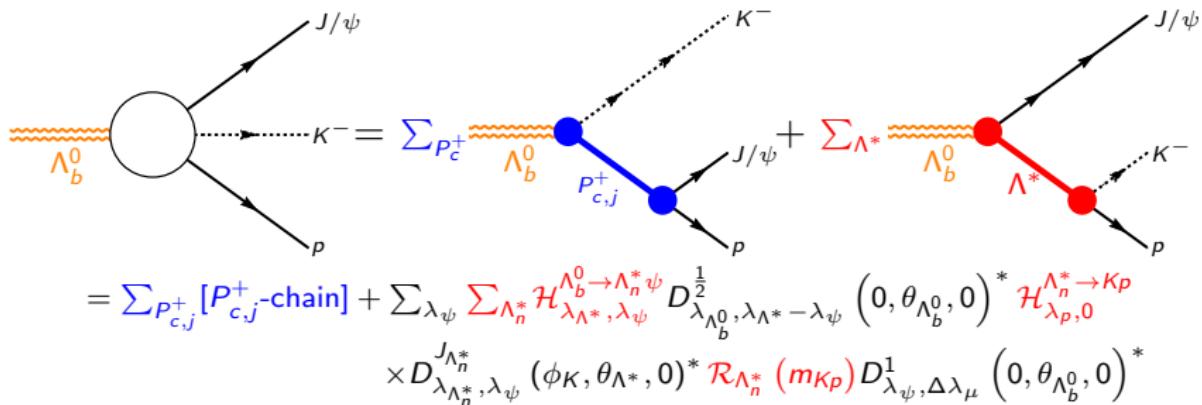
HISKP, University of Bonn

October 29, 2020



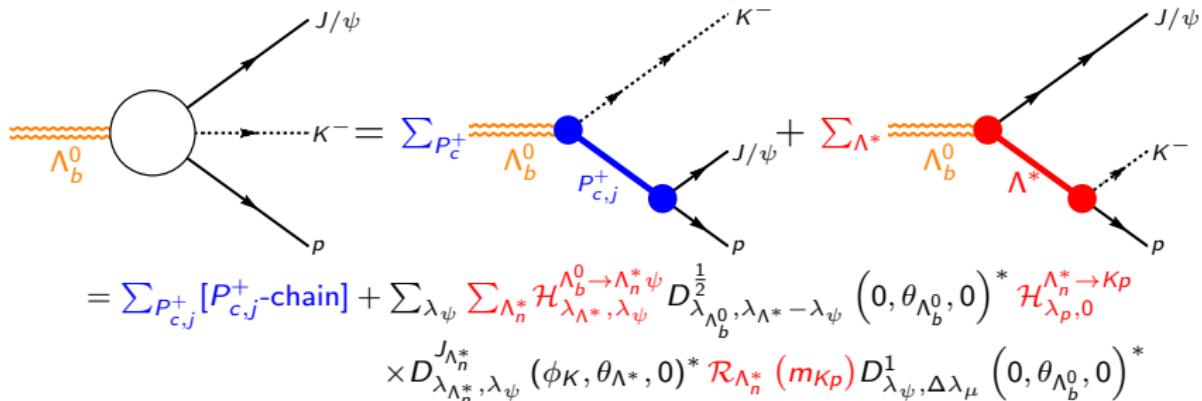
Introduction & Motivation

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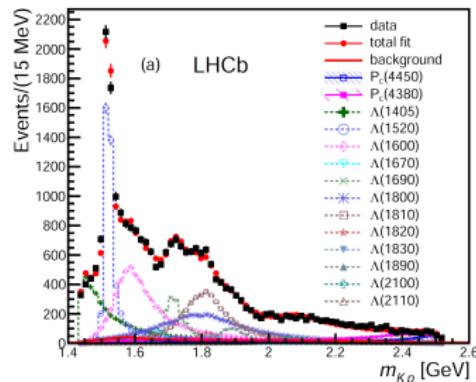


*) Isobar-lineshapes: Breit-Wigner amplitudes

$$\mathcal{R}_{\Lambda_n^*} (m_{Kp}) = \left[\frac{p}{M_{\Lambda_b^0}} \right]^{L_1} \frac{B'_{L_1} (p, p_0, d) B'_{L_2} (q, q_0, d)}{M_{0, \Lambda_n^*}^2 - m_{Kp}^2 - i M_{0, \Lambda_n^*} \Gamma (m_{Kp})} \left[\frac{q}{M_{0, \Lambda_n^*}} \right]^{L_2} .$$

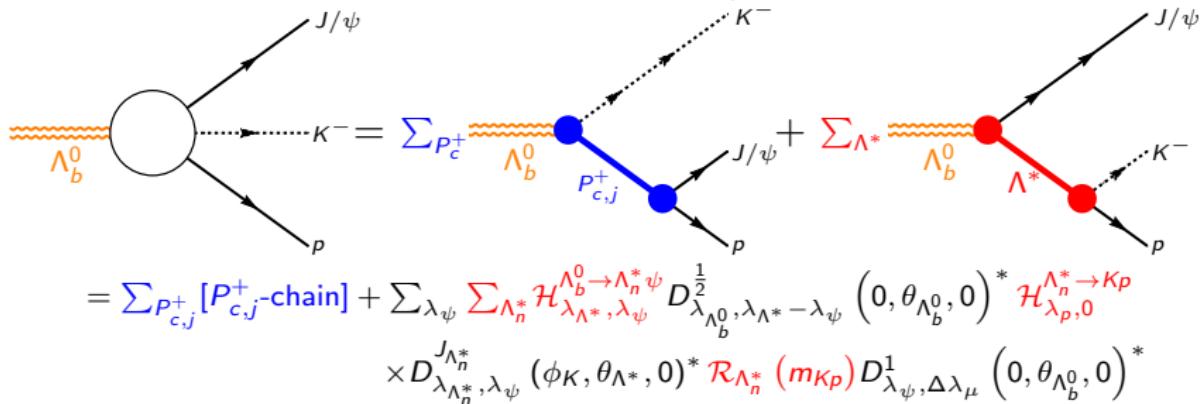
Blatt-Weisskopf factors: $B'_L (p, p_0, d)$,

Mass-dependent width: $\Gamma (m)$.



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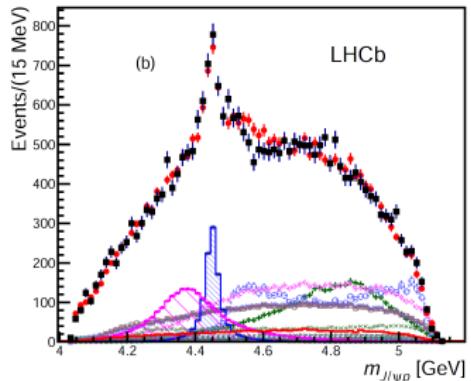


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BnGa-approach for analysis of hyperon resonances

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- *) Coupled-channels approach using K -matrix (actually: D -matrix):

$$K_{ij} = \sum_{\alpha} \frac{g_i^{\alpha} g_j^{\alpha}}{m_{\alpha}^2 - m^2} + c_{ij}.$$

- *) Many (i.e. 15) channels: $i, j = K^- p, \pi^0 \Lambda, \pi^0 \Sigma^0, \dots, \pi^0 \pi^0 \Lambda, \dots$
- *) Database (23000 points): σ_0 for 7 reactions; asymmetry P for 5 reactions; data-'points' for 6 three-body final states (event-sets for 2 final states)

BnGa-approach for analysis of hyperon resonances

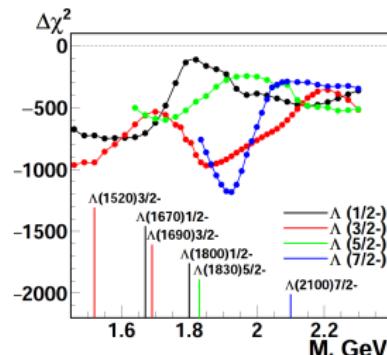
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- *) Database (23000 points): σ_0 for 7 reactions; asymmetry P for 5 reactions; data-'points' for 6 three-body final states (event-sets for 2 final states)
- *) Fit-strategy:
 - (i) Primary fit: fit only hyperons listed with *** and **** by the PDG, using BW's
 - (ii) Many exploratory fits: include candidate-resonance with fixed mass (and J^P) → fit remaining parameters → vary mass → re-fit ...
⇒ 'mass-scans'

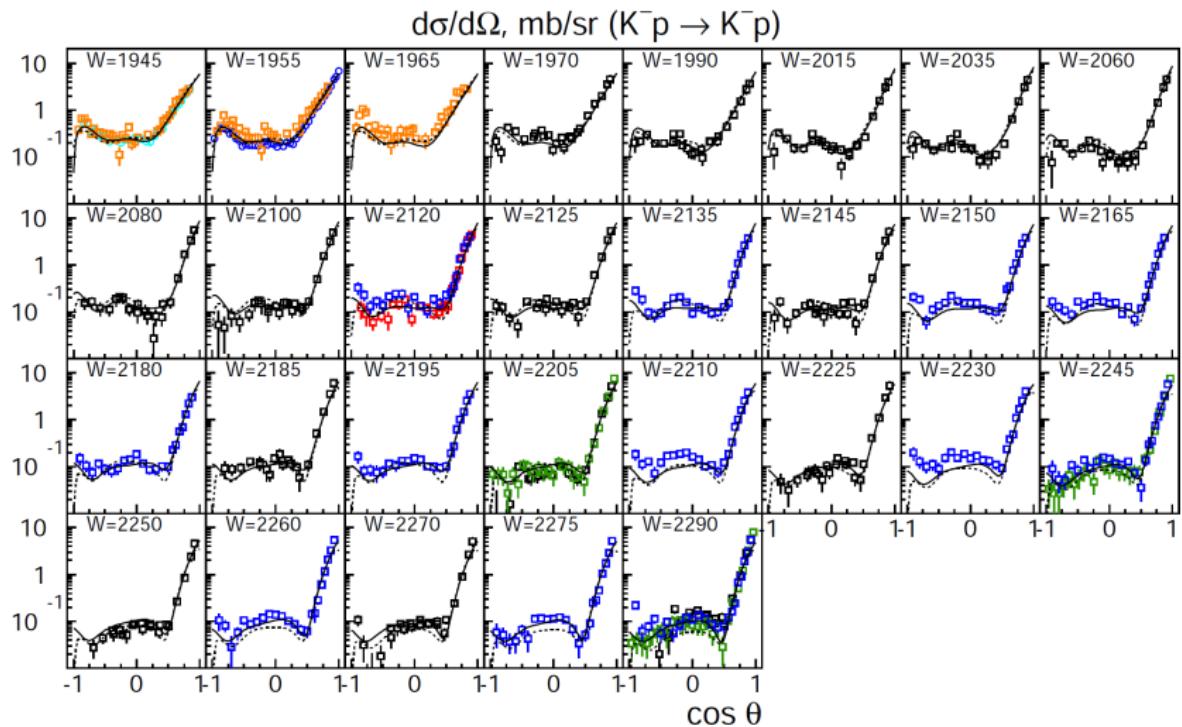


[Matveev et al., arXiv:1907.03645]

- ↪ Final fit: use resonance-configuration with best $\Delta\chi^2$ as initial condition for the final big K - (or D -) matrix fit

Description of data in the BnGa-approach

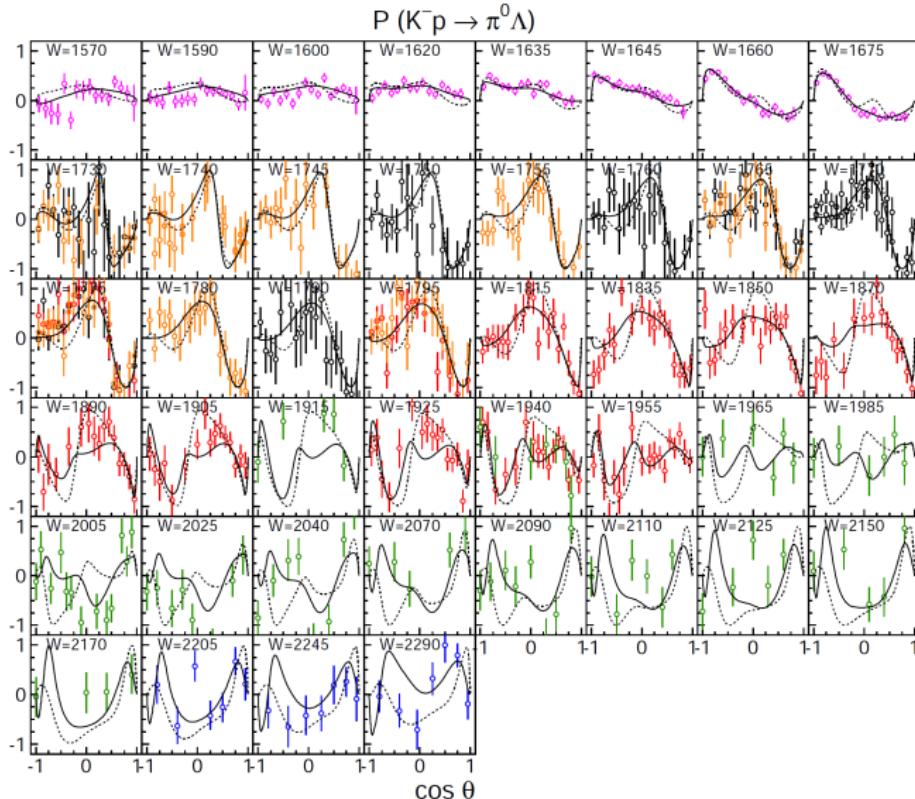
- *) Cross sections (mostly) very well-described for all energies:



[Matveev et al., arXiv:1907.03645]

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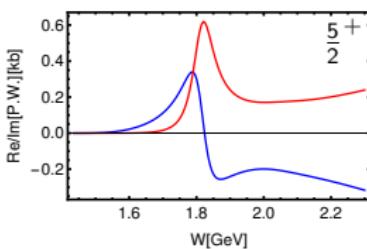
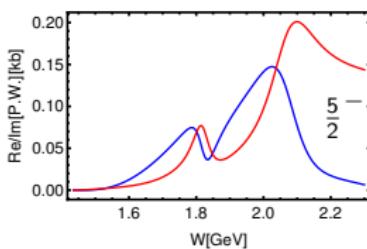
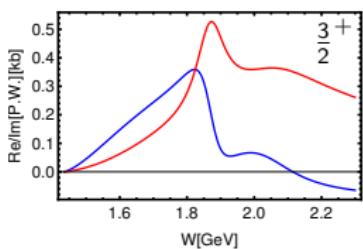
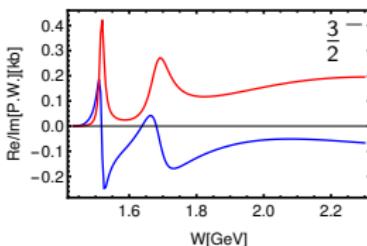
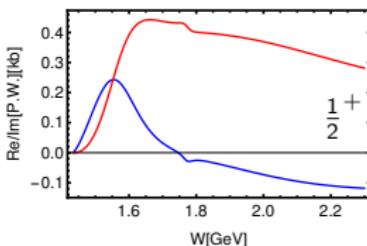
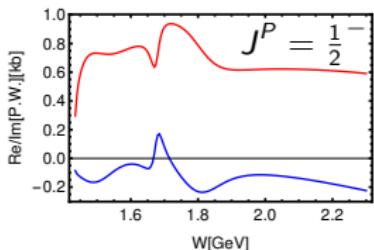
*) P -asymmetry data sometimes scarce, leave room for improvement:



[Matveev et al., arXiv:1907.03645]

Comparing BnGa partial-waves to Breit-Wigner amplitudes

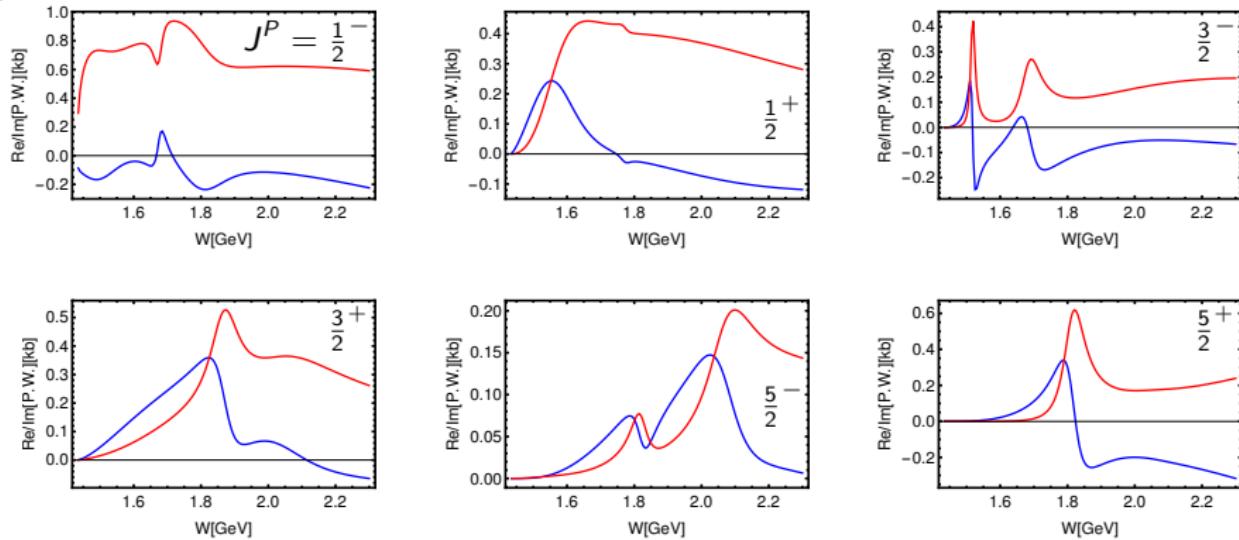
*) Consider BnGa $\bar{K}N \rightarrow \bar{K}N$ partial waves for isospin $I = 0$: [\[pwa.hiskp.uni-bonn.de\]](http://pwa.hiskp.uni-bonn.de)



—: real part BnGa, —: imaginary part BnGa

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Idea: Try to reproduce BnGa partial-waves using:

$$\mathcal{M}(W) = c_1 * \text{BW}_1(M_0^1, \Gamma_0^1; W) + c_2 * \text{BW}_2(M_0^2, \Gamma_0^2; W) + \dots,$$

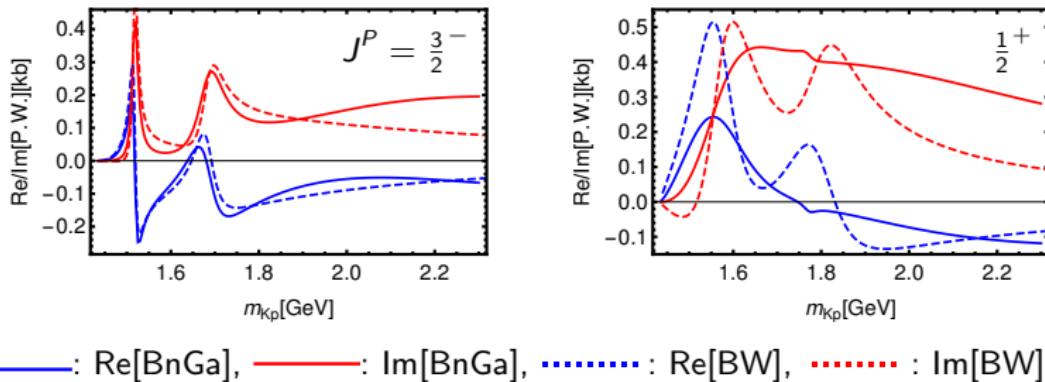
with $c_i \in \mathbb{C}$ ('helicity-couplings') and $\{M_0^i, \Gamma_0^i\}$ fixed to LHCb-values.

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*) $J^P = \frac{3}{2}^-$: sum of $\Lambda(1520)$ and $\Lambda(1690)$.

*) $J^P = \frac{1}{2}^+$: sum of $\Lambda(1600)$ and $\Lambda(1810)$.

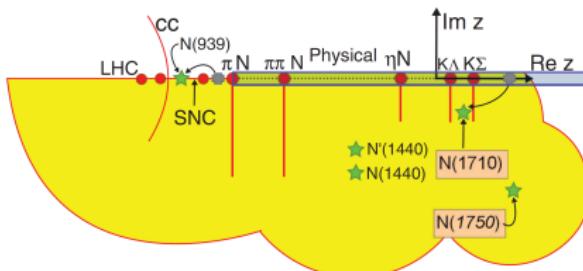
The L+P parametrization

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- *) Illustration of (complicated) analytic structure of the P_{11} partial wave from the Jülich-Bonn model:

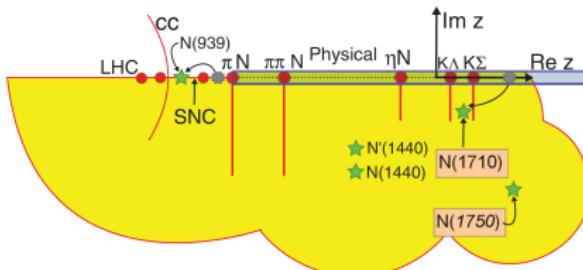
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→ Try to model analytic structure using L+P- ('Laurent+Pietarinen') Ansatz:

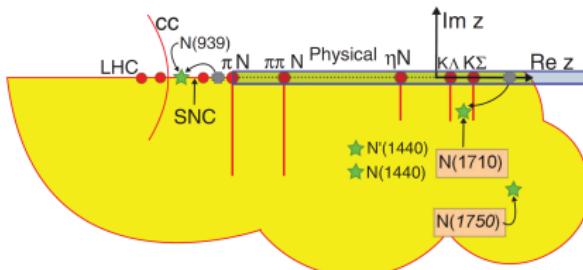
$$\mathcal{M}(W) = \underbrace{\sum_j^{N_{\text{pole}}} \frac{x_j + iy_j}{W_j - W}}_{\text{'Laurent'}} + \underbrace{\sum_{k=0}^{N_1} \mathbf{c}_k X(\alpha, x_P; W)^k + \sum_{l=0}^{N_2} \mathbf{d}_l Y(\beta, x_Q; W)^l}_{\text{'Pietarinen'}} + \dots$$

- Pole-position: $W_j \in \mathbb{C}$; Residue: $a_{-1}^{(j)} = x_j + iy_j$
- Pietarinen-functions: $X(\alpha, x_P; W) := \frac{\alpha - \sqrt{x_P - W}}{\alpha + \sqrt{x_P - W}}$, with:
 - *) 'shape-parameter': $\alpha \in \mathbb{R}$; branch-point coordinate: $x_P \in \mathbb{C}$.

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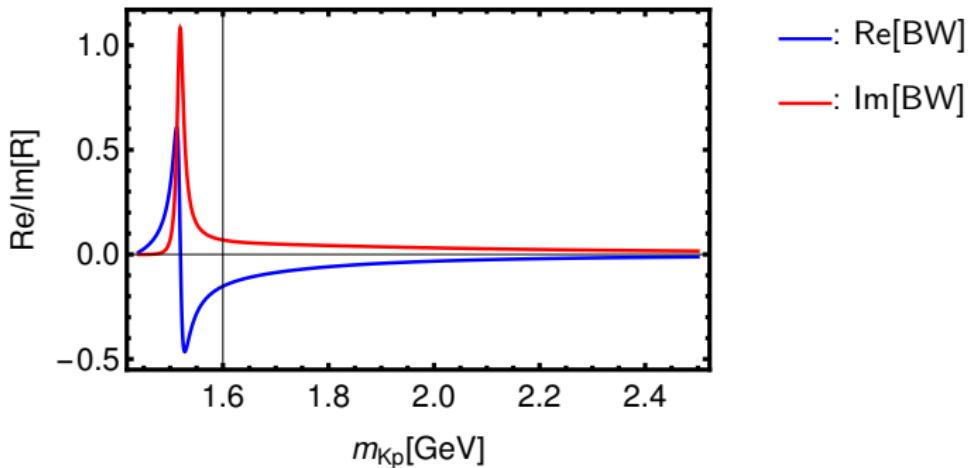
→ Could something like this work for the LHCb lineshape-functions?

Comparing LHCb Breit-Wigner's with L+P

*) Try 1 Pole + 1 Pietarinen-function:

$$\mathcal{R}(m_{Kp}) = \frac{a_{-1}}{\left(\text{Re}m_{Kp}^{\text{pole}} - i\text{Im}m_{Kp}^{\text{pole}}\right) - m_{Kp}} + \sum_{k=0}^4 c_k X(\alpha, \nu; m_{Kp})^k.$$

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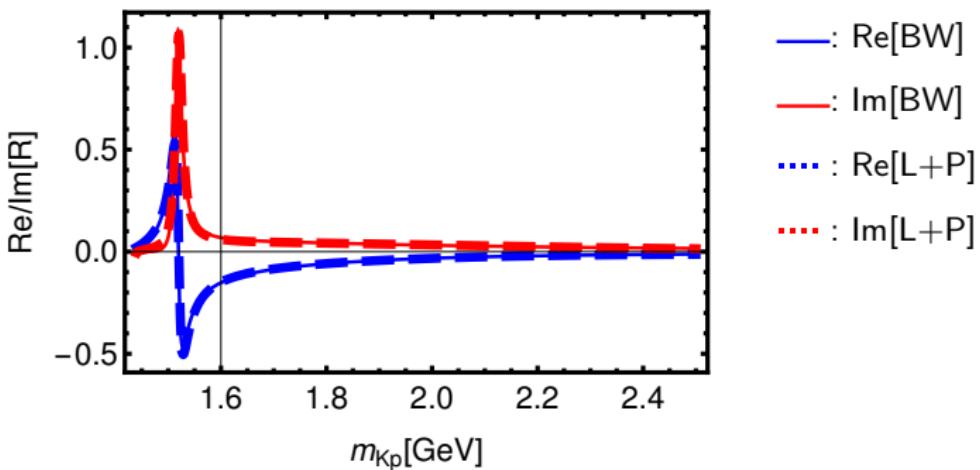
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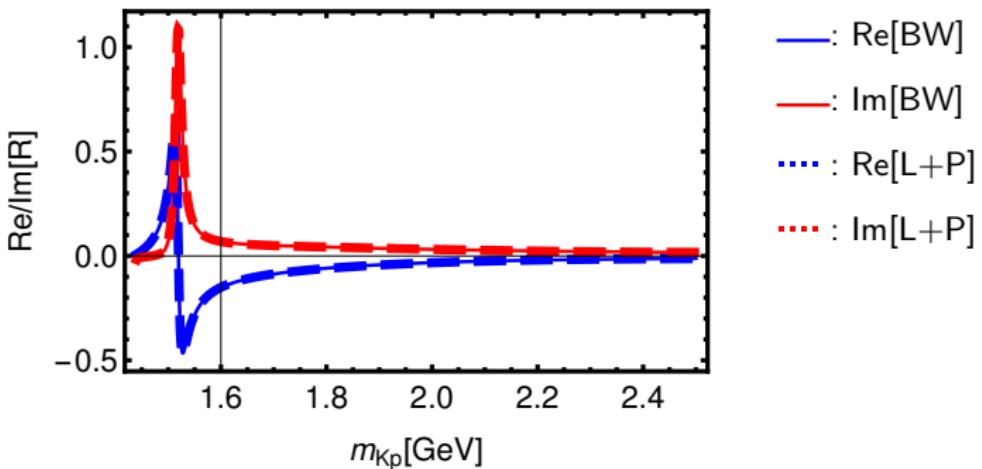
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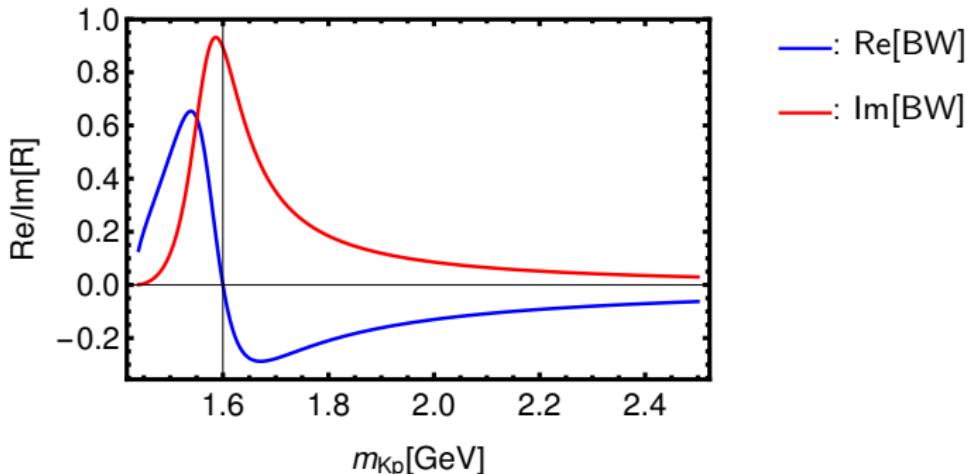
$$\hookrightarrow \text{Rem}_{Kp}^{\text{pole}} = (1.518 \pm 0.201) \text{ GeV}, \text{Imm}_{Kp}^{\text{pole}} = (0.0074 \pm 0.1920) \text{ GeV}.$$

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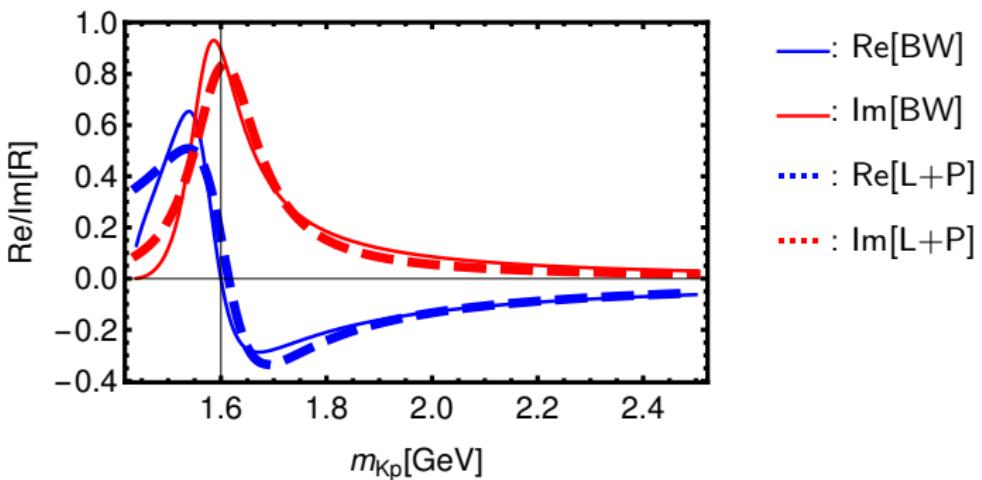
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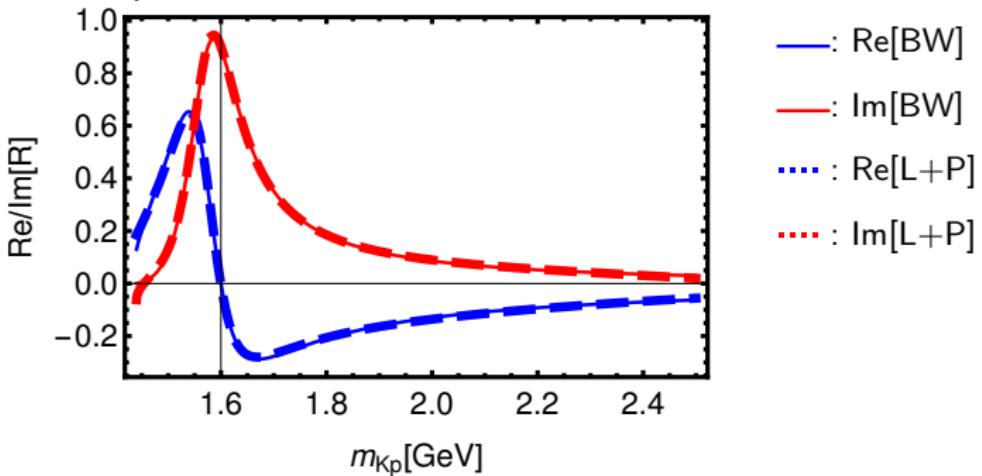
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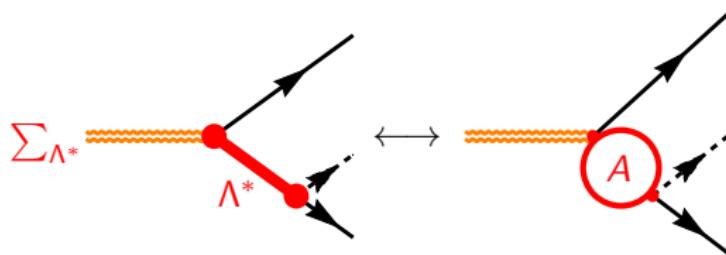
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↪ $\text{Rem}_{Kp}^{\text{pole}} = (1.5705 \pm 0.7930)$ GeV, $\text{Imm}_{Kp}^{\text{pole}} = (0.0554 \pm 0.6844)$ GeV.
cf.: PDG-BW par.: mass $\in [1.560, 1.700]$ GeV, width/2 $\in [0.025, 0.125]$ GeV.

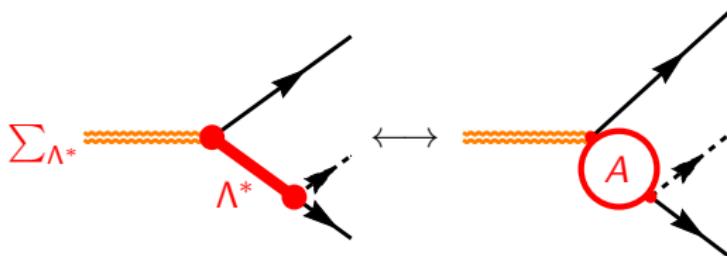
P-vector approach

) Consider replacement of the sum over Breit-Wigner's for the Λ^ 's:

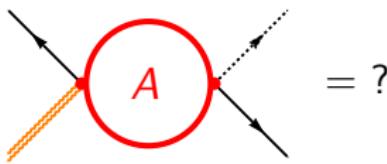


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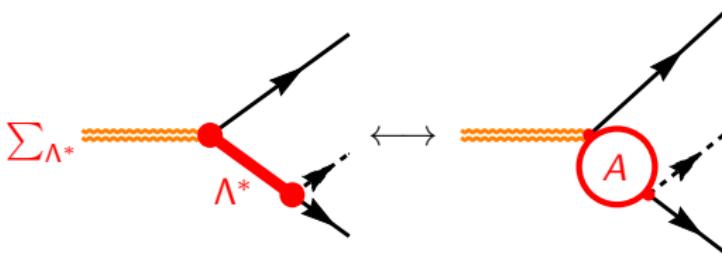


- *) Which Ansatz to choose for A ?



P -vector approach

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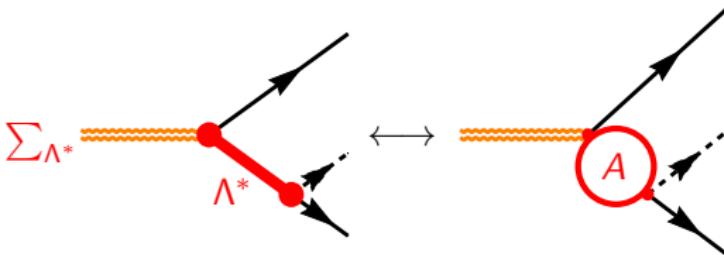
- *) Possibility: introduce K -matrix from PWA-model (e.g. BnGa) using the P -vector approach $\hat{A} = \hat{P} (\mathbb{1} - i\rho \hat{K})^{-1}$

[Aitchison Nucl. Phys. A 189, 417 (1972)]

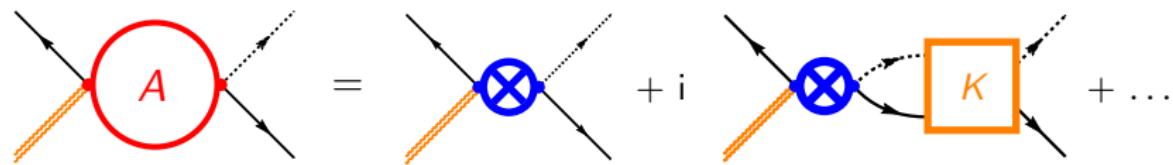
$$\begin{aligned}
 & \text{Diagram showing the P-vector approach:} \\
 & \quad \text{Top row: } \text{Diagram with a red circle } A \text{ (labeled with a red circle)} = \text{Diagram with a blue circle } P \text{ (labeled with a blue circle)} \left[\dots \right]^{-1} \text{ (labeled with an orange square)} K \text{ (labeled with an orange square)} \\
 & \quad \text{Bottom row: } \text{Diagram with a blue circle } P \text{ (labeled with a blue circle)} = \text{Diagram with a blue circle } P \text{ (labeled with a blue circle)} + i \text{ (labeled with an orange square)} K \text{ (labeled with an orange square)} + i^2 \text{ (labeled with an orange square)} K \text{ (labeled with an orange square)} K \text{ (labeled with an orange square)} + \dots
 \end{aligned}$$

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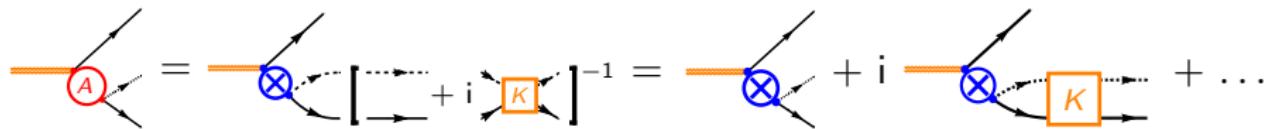
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- *) P -vector approach:



- *) Modify the Λ^* -part of the decay-amplitude accordingly:



Possible applications of the P -vector approach

*) Use K -matrix as 'background' for fits of P_c^+ -states:

$$A_k = P_j \left(\mathbb{1} - i\rho \hat{K} \right)_{jk}^{-1} = P_k + iP_j \rho_j K_{jk} + \dots$$

- K_{ij} : p.w. matrix-elements of 'frozen' K -matrix imported from the PWA-model

$$P_j = \sum_{\alpha} \frac{G_{\alpha} g_j^{\alpha}}{m_{\alpha}^2 - m^2} + \underbrace{f_j}_{=?}$$

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) Use LHCb-data in order to fit Λ^ -resonances: minimize (simplification!)

$$-2 \ln \left[\mathcal{L}_{\text{LHCb}} \left(\vec{\omega}_{P_c^+}, \vec{\omega}_{\Lambda^*} \right) \right] + \chi^2_{\text{BnGa}} (\vec{\omega}_{\Lambda^*})$$

- $\vec{\omega}_{P_c^+}$: masses, widths, hel.-couplings of P_c^+ 's

- $\vec{\omega}_{\Lambda^*}$: parameter-vector from (BnGa) light-baryon analysis

↪ Quite difficult, but more promising than using just LHCb-events for light-baryon spectroscopy ...

Conclusion and Outlook

For the example-reaction $\Lambda_b^0 \rightarrow J/\psi K^- p$, we discussed:

- *) Introducing the L+P-parametrization for the lineshape-fcts $\mathcal{R}(m_{Kp})$:
 - ✓ Introduce (or extract) pole- instead of Breit-Wigner Parameters
→ in principle less model-dependent
 - ✓ In principle possible to model complicated analytic structures using multiple Pietarinen-functions, ...
 - ✗ ... however: this would introduce too many new parameters. Instead: try at most 1 Pietarinen per lineshape-function
 - ✗ Issue: do helicity-couplings factorize around L+P-parametrization, or not?

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 - ✗ Issue: do helicity-couplings factorize around L+P-parametrization, or not?
 - *) Import the K -matrix of a reaction-model (e.g. BnGa) using the P -vector approach (or variants thereof):
 - ✓ Can improve the description of Λ^* -contributions ('background')
 - ✓ K -matrix related ideas can open window towards usage of LHCb-data for light-brayon spectroscopy
 - ✗ Implementation is probably time-consuming (i.e. not easy)
- ↪ Final suggestion: maybe use K -matrix for Λ^* 's and L+P for P_c^+ 's?

Thank You!

Additional Slides

BnGa-results for Λ^* -resonances

Negative-parity Λ -hyperons:

	Mass	Width	$\Delta\chi^2$	Status
$\Lambda(1405)1/2^-$	1420 ± 3 $1405.1^{+1.3}_{-1.0}$	46 ± 4 50.5 ± 2.0	4070	****
$\Lambda(1670)1/2^-$	1677 ± 2 1660 to 1680	33 ± 4 25 to 50	3610	****
$\Lambda(1800)1/2^-$	1811 ± 10 1720 to 1850	209 ± 18 200 to 400	1896	***
$\Lambda(2000)1/2^-$	2085 ± 14 ≈ 2060	428 ± 16 100 to 300	845	*
$\Lambda(1520)3/2^-$	1518.5 ± 0.5 1519.5 ± 1.0	15.7 ± 1.0 15.6 ± 1.0	>10 000	****
$\Lambda(1690)3/2^-$	1689 ± 3 1685 to 1695	75 ± 5 50 to 70	>10 000	****
$\Lambda(1830)5/2^-$	1821 ± 3 1810 to 1830	64 ± 7 60 to 110	1790	***
$\Lambda(2080)5/2^-$	2082 ± 13 -	181 ± 29 -	770	** new
$\Lambda(2100)7/2^-$	2090 ± 15 2090 to 2110	290 ± 30 100 to 250	5412	****

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BnGa-results for Λ^* -resonances

Positive-parity Λ -hyperons:

	Mass	Width	$\Delta\chi^2$	Status
$\Lambda(1600)1/2^+$	1605 ± 8 1560 to 1700	245 ± 15 50 to 250	$>10\,000$	****
$\Lambda(1810)1/2^+$	1773 ± 5 1750 to 1850	36 ± 6 50 to 250	46	*
$\Lambda(1890)3/2^+$	1873 ± 5 1850 to 1910	103 ± 10 60 to 200	4480	****
$\Lambda(2070)3/2^+$	2070 ± 24 -	370 ± 50 -	1144 new	**
$\Lambda(1820)5/2^+$	1822 ± 4 1815 to 1825	80 ± 8 70 to 90	$>10\,000$	****
$\Lambda(2110)5/2^+$	2086 ± 12 2090 to 2140	274 ± 25 150 to 250	1418	**

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