

T -dependence of η' and η masses and the restorations of the $U_A(1)$ and the $SU_A(3)$ chiral symmetry

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The issue of the (effective) restoration of the $U_A(1)$ symmetry

- In QCD, $U_A(1)$ and $SU_A(3)$ chiral symmetry are **explicitly** broken by current quark masses: only slightly by m_u and m_d & not too badly by s -quark mass m_s \rightarrow **chiral limit(s)** make sense [with 3 (or 2) $m_q \rightarrow 0$].
- **But approximate chiral $SU_A(3)$ symmetry = absent due to DChSB, signaled by $\langle \bar{q}q \rangle$ condensates and by the octet of very light (almost) Goldstone bosons: $\pi^{0,\pm}, K^{0,\pm}, \bar{K}^0, \eta$.**
... But as lattice now agrees, chiral symmetry should be restored as a crossover (for $\mu \sim 0$) around $T_{\text{Ch}} \sim 155$ MeV: $\langle \bar{q}q \rangle(T) \rightarrow 0$.
- η' **very massive**, as even in chiral limit, $m_q \rightarrow 0$, $U_A(1)$ is broken explicitly on the quantum level by nonabelian ("gluon") axial anomaly:

$$\partial_\alpha \bar{\psi}(x) \gamma^\alpha \gamma_5 \frac{\lambda^0}{2} \psi(x) \propto F^a(x) \cdot \tilde{F}^a(x) \equiv F_{\mu\nu}^a(x) \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^a(x) \neq 0,$$

which holds at any E and $T \Rightarrow ?$ Would **only** $T \rightarrow \infty$ restore $U_A(1)$!?!?

NO, since **DChSB** and $U_A(1)$ anomaly are tied through quark bilinears such as $\langle \bar{q}q \rangle$ and QCD topological susceptibility $\chi \Rightarrow$ **Expect an effective restoration signaled by vanishing or diminishing of $U_A(1)$ -violating quantities** (e.g., large $M_{\eta'}$, difference π - $a_0(980)$, ...) **over the chiral symmetry crossover ... BUT ...**

... still debatable what happens with $U_A(1)$ symmetry restoration!

- Presently, no consensus within lattice community whether $U_A(1)$ is badly broken or effectively restored at the chiral crossover critical temper. $T = T_{Ch}$

[Sharma for HotQCD collaboration, e-Print: arXiv:1801.08500]

- Already older works found sizable $U_A(1)$ breaking above T_{Ch} [Bernard+al, PRL78 (1997)598, Chanrasekharan+al, PRL82(1999)2463, Ohno+al, PoS LATTICE 2011(2011)210 arXiv:1111.1939]

... and, this is confirmed by some recent works: notably by HotQCD collab.

[Bazavov+al, PRD86(2012)0904503] and by Karsch & collaborators [Buchhoff+al, PRD89(2014) 054514, Sharma+al, NPA956(2016)793, Dick+al, PRD91(2015)095504] as high as $T = 1.5 T_{Ch}$.

- BUT, some recent works claim that $U_A(1)$ breaking above T_{Ch} is overestimated in the continuum limit (blaming lattice artifacts near ChLim). Some then conclude that $U_A(1)$ anomaly is consistent with zero above T_{Ch} , including also Graz group Rohrhofer+al, Phys.Rev.D96(2017)094501 arXiv 1707.01881, but most vocal were researchers around JLQCD collaboration [Cossu+al,

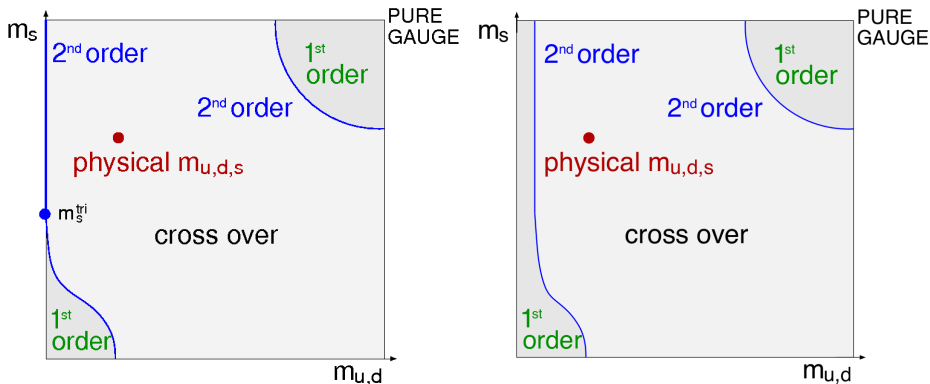
PRD93(2016)034507 arXiv:1510.07395, PRD87&88 (2013)114514&019901

These disappearances of $U_A(1)$ anomaly seem to be associated with the chiral limit - see, e.g., Tomiya+al, PRD96(2017)034509.

- Then our model* approach to η - η' may show that these two kinds of results can be reconciled, since it is consistent with both - depending whether one uses "massless" $\langle \bar{q}q \rangle_0$ or "massive" $q\bar{q}$ condensates: Horvatić, Kekez & D.K., Phys.Rev. D99 (2019) 014007, and spinoff for axions in Universe 5 (2019) 208.

What happens with $U_A(1)$ symmetry restoration matters a lot - see Columbia plot!

Left: $U_A(1)$ broken by anomaly, right: $U_A(1)$ restored (C.Fischer arXiv1810.12938)



General renorm-group arguments (Pisarski:1983ms) \Rightarrow QCD with 3 degenerate light flavors has a 1st order phase transition in chiral limit, whereas in QCD with (2+1) flavors (*i.e.*, s -quark significantly more massive), a 2nd order chiral-limit transition is also possible and even more likely (*e.g.*, Ejiri:2009ac, Ding:2019fzc). A 2nd order chiral-limit transition is exhibited by most DSE models – *e.g.*, clearly through the characteristic drop of their “massless” condensates $\langle \bar{q}q \rangle_0$.

Quantum-level breaking of $U_A(1)$ causes anomalously high $\eta' \approx \eta_0$ mass

QCD chiral behavior (reproduced by, e.g., DS approach) **of the non-anomalous parts** of masses of light $q\bar{q}'$ pseudoscalars: $M_{q\bar{q}'}^2 = \text{const} (m_q + m_{q'})$.

\Rightarrow non-anomalous parts of the masses cancel in Witten-Veneziano rel. (WVR):

$$M_{\eta'}^2 + M_{\eta}^2 - 2 M_K^2 = \frac{2N_f}{f_\pi^2} \chi_{\text{YM}} = \text{anomalous mass}^2 \equiv M_{U_A(1)}^2 \approx \Delta M_{\eta_0}^2,$$

$$\chi = \int d^4x \langle 0|Q(x)Q(0)|0 \rangle, \quad Q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x)$$

QCD topological susceptibility χ = a direct measure of $U_A(1)$ breaking \Rightarrow (partial) $U_A(1)$ restoration is indicated by vanishing or reduction of χ and related quantities, like $M_{U_A(1)} \approx \Delta M_{\eta_0} \approx \Delta M_{\eta'}$.

- $Q(x)$ = topological charge density operator
- In WVR, χ is pure-gauge, YM one, $\chi_{\text{YM}} \leftrightarrow \chi_{\text{quench}}$, obtained long ago by lattice - harder for χ of light-flavorQCD, but can use DiVecchia-Veneziano

relation:
$$\chi = \frac{-\langle \bar{q}q \rangle_0}{\sum_{q=u,d,s} \frac{1}{m_q}} + C_m(\text{unknown corrections, higher } \mathcal{O} \text{ in small } m_q)$$

$= 1^{\text{st}}$ & simplest example of $U_A(1)$ breaking given by **chiral symmetry breaking**

The 2nd example of tied breaking of $U_A(1)$ and chiral symmetries:

Leutwyler-Smilga relation (LS), also connecting YM and full QCD quantities (like WVR), “making” χ_{YM} out of much smaller χ :

$$\text{At } T = 0 \quad \chi_{\text{YM}} = \frac{\chi}{1 + \frac{\chi}{\langle \bar{q}q \rangle_0} \sum_{q=u,d,s} \frac{1}{m_q}} \equiv \tilde{\chi} \rightarrow \tilde{\chi}(T)$$

where for the light quarks

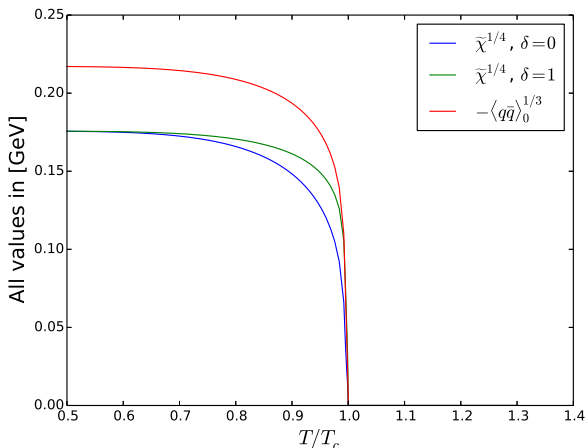
$$\chi = \frac{-1}{\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0}} + C_m$$

- C_m = small corrections of higher orders in small m_q .
However, neglecting it, *i.e.*, $C_m = 0$, would imply $\chi_{\text{YM}} = \infty$.
Conversely, $\chi_{\text{YM}} = \infty$ in LS returns the leading term of χ .
For axions, χ_{YM} is not needed \Rightarrow the leading term of χ will suffice.
- LS relation fixes the value of the correction at $T = 0$:

$$\frac{1}{C_m} = \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} - \chi_{\text{YM}}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} \right)^2.$$

- The conjecture on $\tilde{\chi}(T)$ supported by Shore's generalization of WV relation.

Chiral condensate $\langle q\bar{q} \rangle_0(T)$ and resulting $\tilde{\chi}(T)$



This sharp chiral transition enforces at $T = T_c \equiv T_{Ch}$ the abrupt transition to the NS-S asymptotic regime of vanishing $U_A(1)$ anomaly influence:

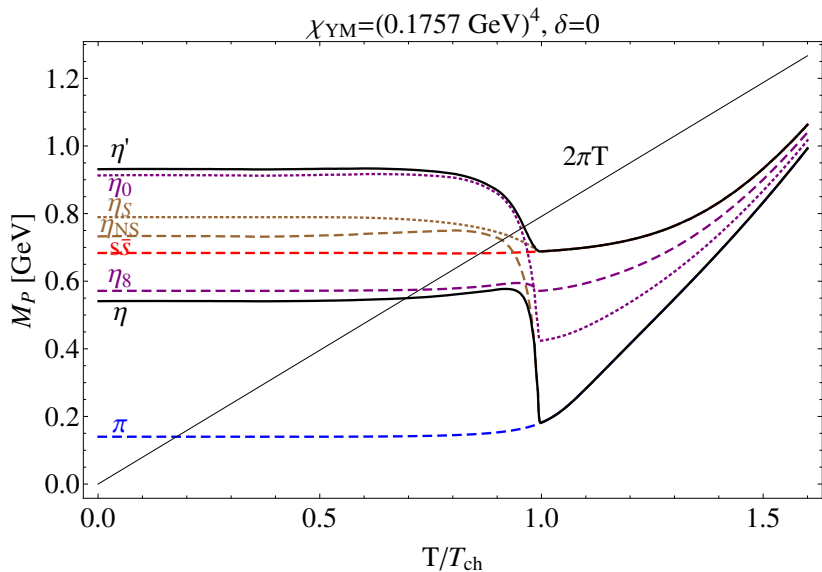
$M_{\eta'}(T) \rightarrow M_{s\bar{s}}(T)$, and $M_{\eta}(T) \rightarrow M_{NS}(T) \rightarrow M_{\pi}(T)$, and $\phi(T) \rightarrow 0$.

Acceptable or even good for η' , but η would be in conflict with experiment.

Prediction good for η' , but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]:

Anomalous contribution from WVR:



Shore's generalized $WV = 3^{\text{rd}}$ example of tying $U_A(1)$ and $SU_A(3)$

$$(f_{\eta'}^0)^2 M_{\eta'}^2 + (f_{\eta}^0)^2 M_{\eta}^2 = \frac{1}{3}(f_{\pi}^2 M_{\pi}^2 + 2f_K^2 M_K^2) + 2N_f A \quad (1)$$

$$f_{\eta'}^0 f_{\eta'}^8 M_{\eta'}^2 + f_{\eta}^0 f_{\eta}^8 M_{\eta}^2 = \frac{2\sqrt{2}}{3}(f_{\pi}^2 M_{\pi}^2 - f_K^2 M_K^2) \quad (2)$$

$$(f_{\eta'}^8)^2 M_{\eta'}^2 + (f_{\eta}^8)^2 M_{\eta}^2 = -\frac{1}{3}(f_{\pi}^2 M_{\pi}^2 - 4f_K^2 M_K^2) \quad (3)$$

The role of χ_{YM} taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi \left(\frac{1}{\langle \bar{u}u \rangle m_u} + \frac{1}{\langle \bar{d}d \rangle m_d} + \frac{1}{\langle \bar{s}s \rangle m_s} \right)} \quad (4)$$

A behaves with T as a full QCD quantity, **but**, at $T = 0$, $A = \chi_{\text{YM}} + \mathcal{O}(\frac{1}{N_c})$

Again, $A = \infty$ returns the leading term of

$$\chi = \frac{-1}{\frac{1}{m_u \langle \bar{u}u \rangle} + \frac{1}{m_d \langle \bar{d}d \rangle} + \frac{1}{m_s \langle \bar{s}s \rangle}} + C'_m \quad (5)$$

Massive-quark condensates employed \Rightarrow crossover around $T \sim T_{\text{Ch}}$

(Large N_c limit & approximating 3 condensates by $\langle \bar{q}q \rangle_0$, returns the LS relation.)

QCD topological susceptibility $\sqrt{\chi(T)}$ gives axion mass $\times f_a$

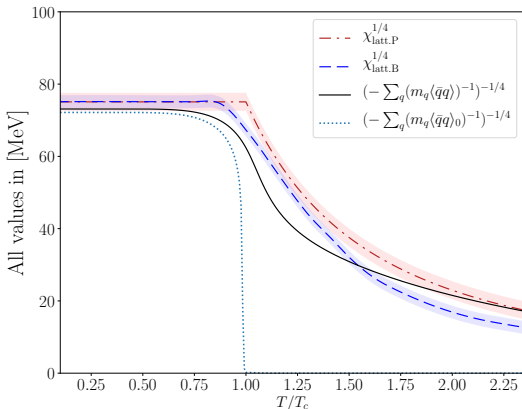
- For all temperatures: $m_a^2(T) f_a^2 = \chi(T) =$ **full** QCD topological susceptibility

- At $T = 0$,
$$m_a^2 f_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} f_\pi^2 M_\pi^2 \rightarrow \frac{\text{isospin}}{\text{limit}} \rightarrow (78.9 \text{ MeV})^4$$

- This agrees well with results, including $\chi(T)$, from “our” DS-BSE chirally well-behaved model (separable: simplified, but phenomenologically successful)

- Agrees well with $\chi(T)$ from lattice studies of axion mass: [Petreczky & al. PLB \(2016\)](#) and [Borsany & al. Nature \(2016\)](#)

- $\chi(T)$ from our usual DS-BSE model: successful at $T = 0$, no additional fitting for $T > 0$: condensates $\langle \bar{q}q \rangle(T)$ of **massive** $q = u, d, s$ essential to yield **good crossover** T -dependence of $\chi(T)$ for good T -dependence of η and η' masses.



Briefly on axions as solutions for Strong CP problem

- QCD has the Strong CP problem: **no experimental evidence of any CP-symmetry violation in strong interactions, in spite of its θ -term:**

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{CPsymmetric}}^{\text{QCD}} + \bar{\theta} \frac{g^2}{32\pi^2} F_{\mu\nu}^b \tilde{F}^{b\mu\nu} \quad (\tilde{F}^{b\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^b)$$

- The θ -term is a **total divergence, but it cannot be discarded. It contributes anyway** (unlike in QED) due to **nontrivial topological structures in QCD** – e.g., **instantons** (probably yielding, e.g., anomalously large $M_{\eta'}$ \Rightarrow important for solving the $U_A(1)$ problem).
 - **The experimental bound is mysteriously small:** $|\bar{\theta}| < 10^{-10}$. **Why?!?**
- $\bar{\theta} = \theta_{\text{QCD}} + \text{Arg Det } \hat{M}_q \Rightarrow$ setting $\bar{\theta} = 0$ just “by hand” is **fine-tuning**.

How to get $\bar{\theta} \approx 0$!!?

- **Nowadays preferred solution:** a new particle beyond SM: **axion** a
- **Axions are very interesting also for cosmology as candidates for dark matter.**

Axions as quasi-Goldstone bosons

- Peccei & Quinn introduced a new axial global symmetry $U(1)_{PQ}$ which is **broken spontaneously at some scale f_a** (f_a = free parameter of axion theories, determines absolute value of the axion mass m_a , but cancels from combinations such as $m_a(T)/m_a(0)$.)
- the pseudoscalar axion field $a(x)$ is the (would-be massless) Goldstone boson of this spontaneous breaking. Then,

$$\mathcal{L}_{\text{axion}}^{\text{QCD}+} = \mathcal{L}_{\text{CPsymmetric}}^{\text{QCD}} + \left(\bar{\theta} + \frac{a}{f_a} \right) \frac{g^2}{32\pi^2} F_{\mu\nu}^b \tilde{F}^{b\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a + \mathcal{L}_{\text{int}}^{a\psi}$$

- But, the $U(1)_{PQ}$ symmetry is **also broken explicitly** by the gluon axial anomaly through axion's coupling with gluons $\Rightarrow m_a \neq 0$.
- Gluons generate an effective axion potential, which leads to the **axion expectation value $\langle a \rangle$** such that $(\bar{\theta} + \langle a \rangle / f_a) = 0$, minimizing the potential \Rightarrow **strong CP problem solved, irrespective of the initial $\bar{\theta}$** .

("Misalignment production" is relaxation from any value in the early Universe towards the effective potential minimum at $\bar{\theta} = -\langle a \rangle / f_a$. The resulting axion oscillation energy is a "cold dark matter" candidate.)

Evaluation of $q\bar{q}$ condensates from propagators

Solving the gap SD equation \Rightarrow dressed propagators $S_q(p)$

The usual expression for the condensate of the flavor q for $T > 0$ becomes

$$\langle \bar{q}q \rangle = -N_c \int_p \text{Tr} [S_q(p)] \equiv -N_c T \sum_{n_q \in \mathbb{Z}} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \text{Tr} [S_q(p)]$$

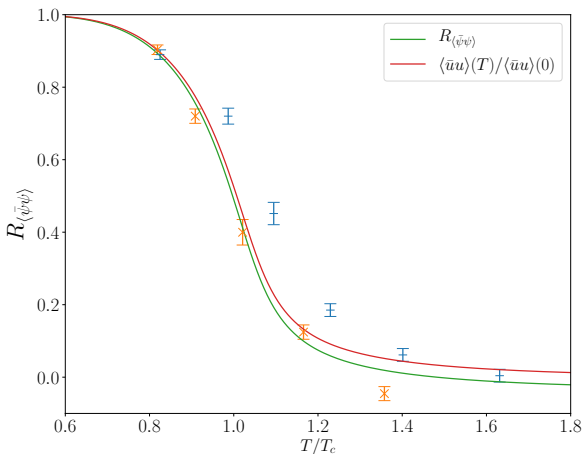
Tr = trace in Dirac space. The combined integral-sum symbol says: when $T > 0$, the 4-momentum integration \rightarrow 3-momentum integr. & sum over Matsubaras $\omega_q = (2n_q + 1)\pi T$, $n_q \in \mathbb{Z}$.

- Well known: $q\bar{q}$ condensates are finite only for massless quarks, $m_q = 0$.
"Massive" condensates must be subtracted of their divergences.
- The arbitrariness of sensible procedures is in practice slight, *i.e.*, only small differences between the results of various sensible subtractions.
- First consider the (normalized) subtraction proposed on lattice by Burger+al (2011), here applied to our condensates of light (u - and d -) quarks:

$$R_{\langle \bar{\psi}\psi \rangle}(T) = R_{\langle \bar{u}u \rangle}(T) = \frac{\langle \bar{u}u \rangle(T) - \langle \bar{u}u \rangle(0) + \langle \bar{q}q \rangle_0(0)}{\langle \bar{q}q \rangle_0(0)}.$$

Comparison of subtracted & normalized lattice- and DS-condensates

Relative T -dependence of the subtracted (and normalized) condensate $R_{\langle\bar{\psi}\psi\rangle}$. The lattice data points are from Fig. 6 of Kotov, Lombardo & Trunin, PLB794 (2019), scaled for the critical temperatures T_χ from their Table 2, which are different for the “crosses” (lattice data for $m_\pi \approx 370$ MeV) and “bars” (lattice data for $m_\pi \approx 210$ MeV).

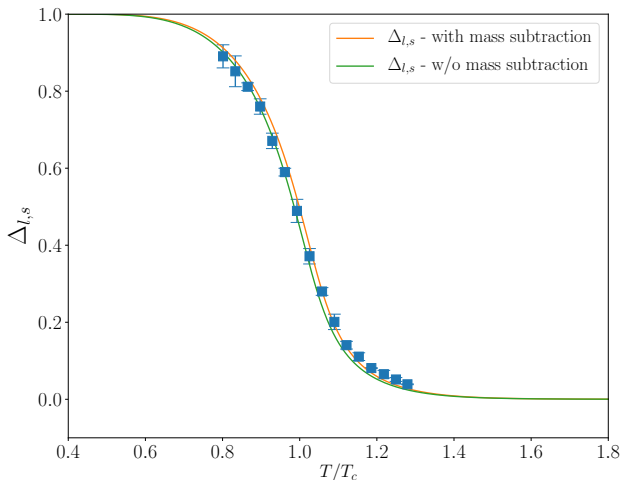


The lower, green curve results from only the $R_{\langle\bar{\psi}\psi\rangle}$ -subtraction of our u -quark condensate. The upper, red curve is $R_{\langle\bar{u}u\rangle}(T)$ when our u -quark condensate is regularized in the usual way, by subtracting the current quark mass parameter m_u from the numerator of the dressed quark propagator.

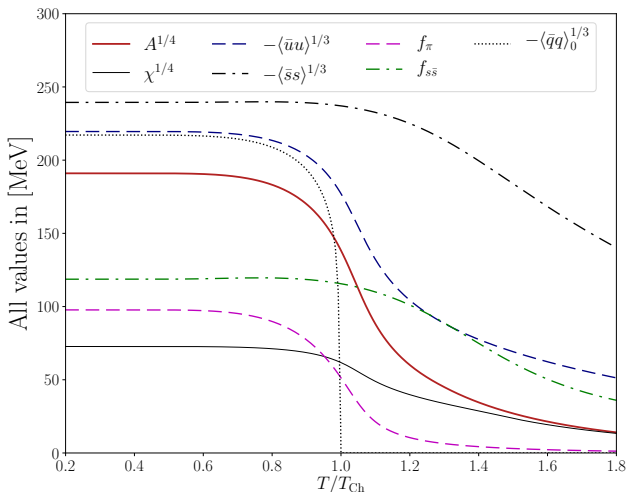
Compare $\Delta_{l,s}(T)$ regularization of the lattice and DS condensates

$$\Delta_{l,s}(T) = \frac{\langle \bar{l}l(T) \rangle - \frac{m_l}{m_s} \langle \bar{s}s(T) \rangle}{\langle \bar{l}l(0) \rangle - \frac{m_l}{m_s} \langle \bar{s}s(0) \rangle} \quad (l = u \text{ or } d \text{ in isosymmetric limit})$$

This is the most usual (normalized) subtraction on the lattice. Very good agreement with Isserstedt+al 2019 & the lattice (Borsanyi+al 2010). Red and green curve, respectively, again result from our model DS condensate $\langle \bar{u}u(T) \rangle$ with and without subtraction of m_u from the quark propagator numerator.



T -dependence of $\langle \bar{q}q \rangle$ & decay const's f_P with χ & A



FKS scheme on Shore \Rightarrow how f_P influence elements of the η - η' mass matrix:

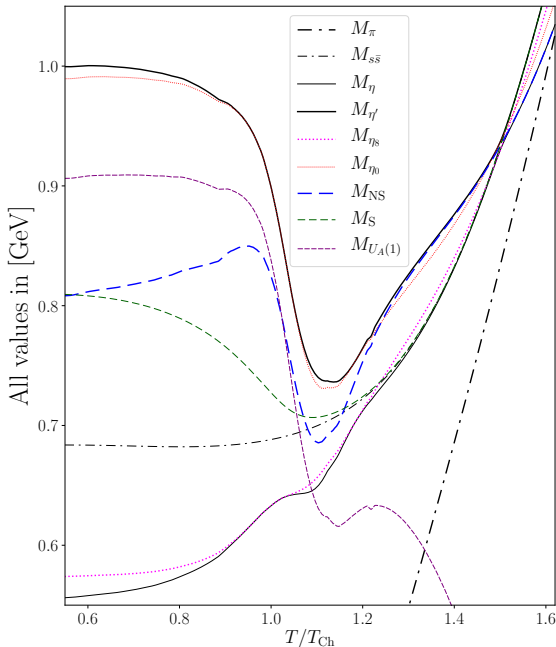
$$X = \frac{f_\pi}{f_{s\bar{s}}}, \quad M_{NS}^2 = M_\pi^2 + \frac{4A}{f_\pi^2}, \quad M_{NSS}^2 = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}, \quad M_S^2 = M_{s\bar{s}}^2 + \frac{2A}{f_{s\bar{s}}^2}$$

Zoomed η - η' complex

- $M_{\eta'}(T)$ is not changed much as condensates are changed from chiral to massive: $M_{\eta'}(T)$ falls again around T_{Ch} by 300 to 200 MeV, corresponding to melting of $\sim \frac{1}{3} M_{U_A(1)}$.

- But η does not exhibit any mass drop at all, now. It stays predominantly η_8 till anticrossing at $\sim 1.5 T_{Ch}$.

Similarly $\eta' \sim \eta_0$ long after T_{Ch} , and only after this anti-X with η , η' tends to a pure $s\bar{s}$.



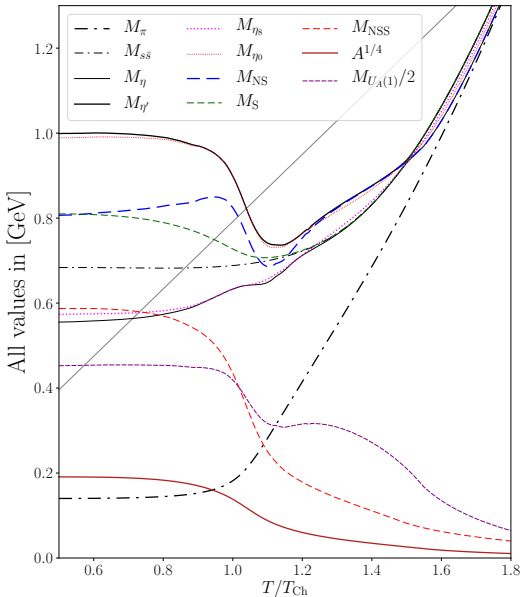
T -dependence of $M_P(T)$ up to $T = 1.8T_{\text{Ch}}$ [Horvatić & al. Phys.Rev. D99 (2019) 014007]

- $\mathcal{C}(T) \neq \text{const}$, adjusted to enable reaching arbitrary high T 's, results otherwise very similar to previous case with $\mathcal{C}(T) = \mathcal{C}(0)$.
- Other limitations of rank-2 separable model make it hard to find solutions beyond $\sim 1.8T_c$.

But it is enough to exhibit cleanly the asymptotic regime beyond anticrossing at $1.5T_{\text{Ch}}$.

Along with A , influence on anomalous masses is given by M_{NSS} and $(\frac{1}{2})M_{U_A(1)}$.

Utopistic in practice? - but in principle, accurate experimental knowledge of $M_{\eta'}(T)$ would tell us about $A(T)$ and thus about $\chi(T) \propto m_a(T)^2$.



Summary

- Our approach ties the $U_A(1)$ SB to the DChSB so closely, expressing χ & A through $m_q \langle \bar{q}q \rangle$, $q = u, d, s$, that the restoration of the chiral symmetry must lead to the restoration of the $U_A(1)$ symmetry at least partially, and surely on the level of the η' & η masses.
- **Full $U_A(1)$ restoration occurs together with the chiral one at $T = T_{Ch}$ only for the chiral-limit condensate $\langle q\bar{q} \rangle_0$. However, such an abrupt restoration is excluded by the behavior of η .**
- Condensates with **explicit ChSB fall with T much more slowly** than $\langle q\bar{q} \rangle_0$. Our “massive” condensates yield $\chi(T)$ in reasonable agreement with $\chi(T)$ from lattice studies of the T -dependence of axion mass. We now find that **η does not exhibit any mass drop at all**, while the **significant drop of the η' mass signals only a partial restoration of $U_A(1)$ symmetry, consuming only about $\frac{1}{3} M_{U_A(1)}$.**
- For realistic explicit chiral breaking, there is an intermediate region between the chiral restoration at $T = T_{Ch}$ and the η - η' anticrossing at $T = 1.5 T_{Ch}$ which marks the effective $U_A(1)$ restoration. The anomalous contributions then become sufficiently weak, and η - η' complex enters the NS-S asymptotic regime:

$$M_{\eta'}(T) \rightarrow M_{s\bar{s}}(T), \text{ \& } M_{\eta}(T) \rightarrow M_{NS}(T) \rightarrow M_{\pi}(T), \text{ \& } \phi(T) \rightarrow 0.$$