Self-similar description of heavy-ion and p+p collisions

GÁBOR KASZA

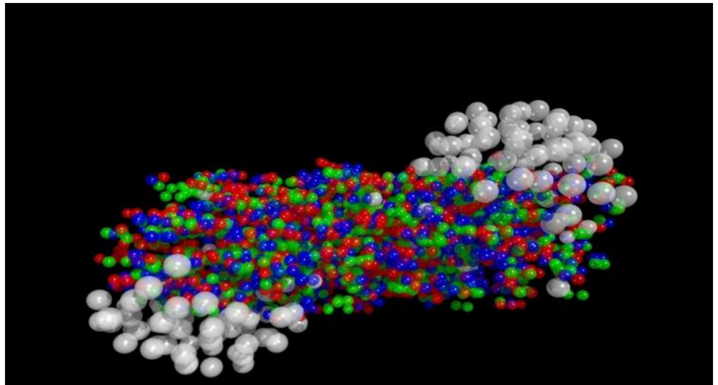
DAY OF FEMTOSCOPY, GYÖNGYÖS

31ST OF OCTOBER, 2019

NKIFH No. FK-123842 & FK-123959
EFOP 3.6.1-16-2016-00001

Various application of hydrodynamics

- Fluid dynamics: flow of liquid and gases
- Examples:
  - *Calculating forces and moments in aircrafts*
  - *Weather forecast*
  - *Describing nebulae*
  - *Inner structure of stars (magnetohydrodynamics)*
  - *Modelling fission weapon detonation*
  - *Describing the Quark Gluon Plazma (QGP)*
- Different systems in many aspects: however, hydro works well in all cases
- Why is hydrodynamics so effective?



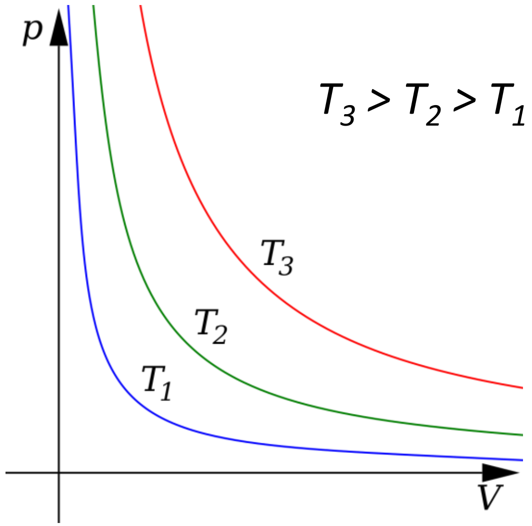
Heavy-ion collision: in the overlap area, the sQGP is created

Scaling behaviour

- Ideal gas: isothermal process → constant value of *T* constrains the possible states
- *p* and *V* are arbitrary, but their product is not:

$$pV = Nk_BT = konstans$$

- In this case <u>*the system scales with the temperature:*</u>



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- For adiabatic expansions:

$$\frac{pV}{T} = Nk_B = konstans$$

- Applicable to any *N*, *<u>the system scales with the particle number</u>*
- It is also a general feature of hydro:

„*<u>Hydrodynamics has no internal scale</u>*”

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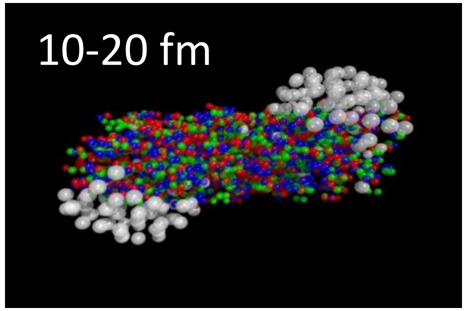

Analogy in heavy-ion physics?
This is the topic of this presentation!

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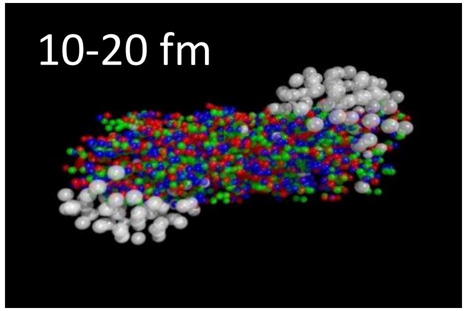


Csörgő-Kasza-Csanád-Jiang solution

- Rindler coordinates, velocity field: $(\tau,\eta_x) = \left(\sqrt{t^2 - r_z^2},\ \frac{1}{2}\ln\left[\frac{t+r_z}{t-r_z}\right]\right),\quad u^\mu = \left(\cosh\left(\Omega\right), \sinh\left(\Omega\right)\right)$
- 1+1 dimensional, parametric, almost self-similar solution:

Csörgő T., Kasza G., Csanád M., Jiang Z.: arXiv:1805.01427, arXiv:1806.06794

$$\eta_x(H) = \Omega(H) - H,$$

$$\Omega(H) = \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}}\arctan\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh\left(H\right)\right)$$

$$\sigma(\tau,H) = \sigma_0\left(\frac{\tau_0}{\tau}\right)^\lambda \mathcal{V}_\sigma(s)\left[1+\frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2}},$$

$$T(\tau,H) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s)\left[1+\frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2\kappa}},$$

$$\mathcal{T}(s) = \frac{1}{\mathcal{V}_\sigma(s)},$$

$$s(\tau,H) = \left(\frac{\tau_0}{\tau}\right)^{\lambda-1}\sinh(H)\left[1+\frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\lambda/2}$$

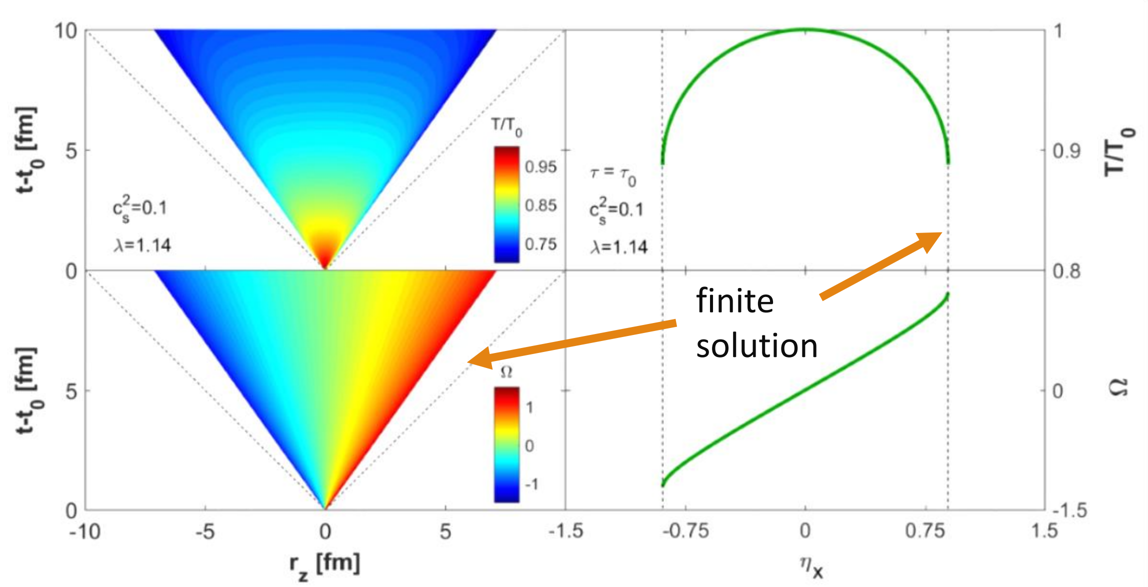
λ: acceleration parameter
(Hwa-Bjorken: *λ=1*)

↓

accelerating solution

↓

realistic *dN/dη*



Scaling behaviour in high-energy physics

- *dN/dy* is obtained from the CKCJ solution (in self-similar approximation):

$$\frac{dN}{dy} \approx \left.\frac{dN}{dy}\right|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa,\lambda)-1}\left(\frac{y}{\alpha(1,\lambda)}\right) \exp\left(-\frac{m}{T_{\rm eff}}\left[\cosh^{\alpha(\kappa,\lambda)}\left(\frac{y}{\alpha(1,\lambda)}\right)-1\right]\right)$$

Physical parameters:
λ: acceleration parameter
κ: inverse square of c_s
T_{eff}: effective temperature
m: particle mass

- If $|y| << 2+(\lambda-1)^{-1}$, Gaussian rapidity-density:

$$\frac{dN}{dy} \approx \frac{\langle N \rangle}{(2\pi\Delta^2 y)^{1/2}} \exp\left(-\frac{y^2}{2\Delta^2 y}\right) \longrightarrow \frac{1}{\Delta^2 y} = (\lambda-1)^2\left[1+\left(1-\frac{1}{\kappa}\right)\left(\frac{1}{2}+\frac{m}{T_{\rm eff}}\right)\right]$$

- Depends on the combination of the physical parameters through the width (*Δy*)
- *λ, m, T_{eff} and κ can be arbitrary, but their combination is not: Δy is determined by fits*
- Physical differences are only apparent in the width of the distribution

Csörgő, Kasza, Csanád, Jiang solution:
arXiv:1805.01427

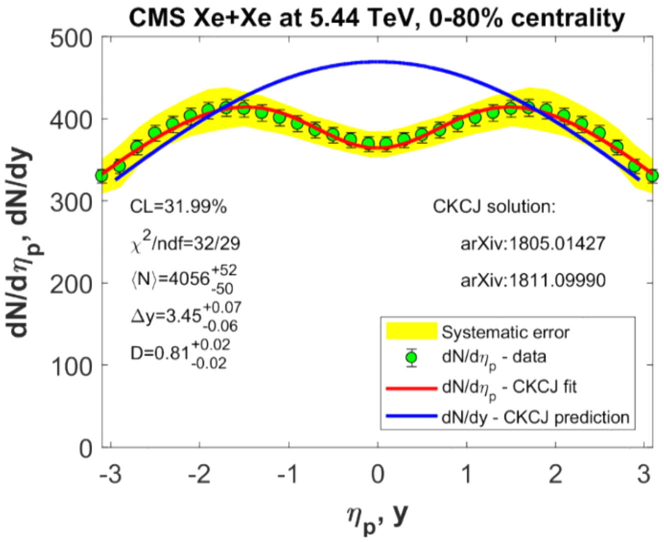
and several applications:
arXiv:1806.06794
arXiv:1811.09990

Scaling behaviour in high-energy physics

- The pseudorapity distribution is the product of *dN/dy* and the Jakobian:

$$\frac{dN}{d\eta_p} \approx \frac{\langle N \rangle}{(2\pi\Delta^2 y)^{1/2}} \frac{\cosh(\eta_p)}{(D^2 + \cosh^2(\eta_p))^{1/2}} \exp\left(-\frac{y^2}{2\Delta^2 y}\right)\Bigg|_{y=y(\eta_p)}$$

- The rapidity density is depressed at midrapidity by $\frac{1}{\sqrt{1+D^2}}$



$$D = \frac{m}{\bar{p}_T}$$

„*D*epression” or „*D*epth” parameter

K. G. , Csörgő T.:
arXiv:1811.09990
arXiv:1910.03428

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- Main question: Does this scaling behaviour appear in the data?
- We fitted the pseudorapidity density data with the CKCJ solution …

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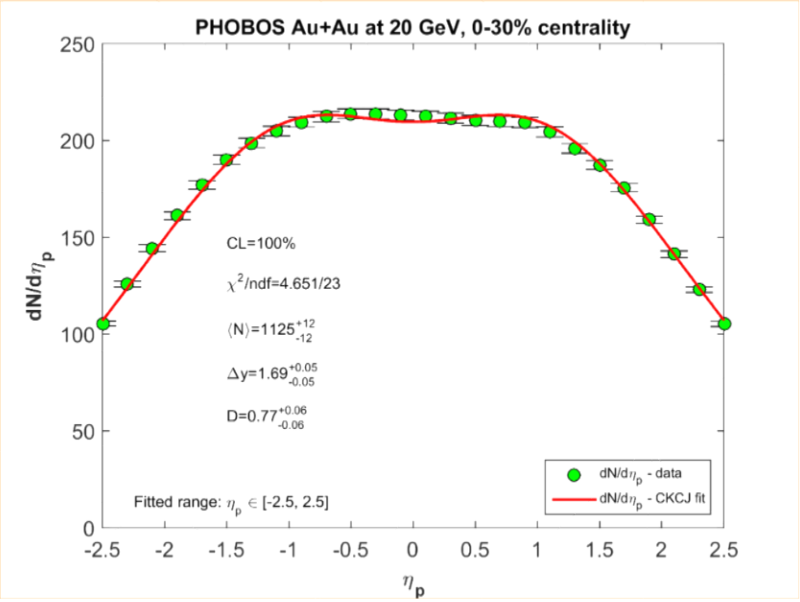

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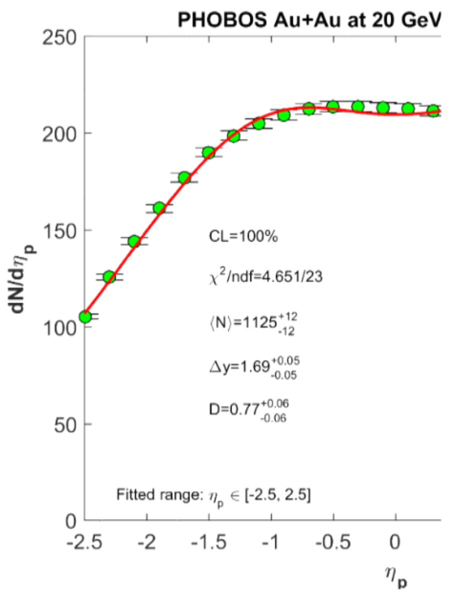

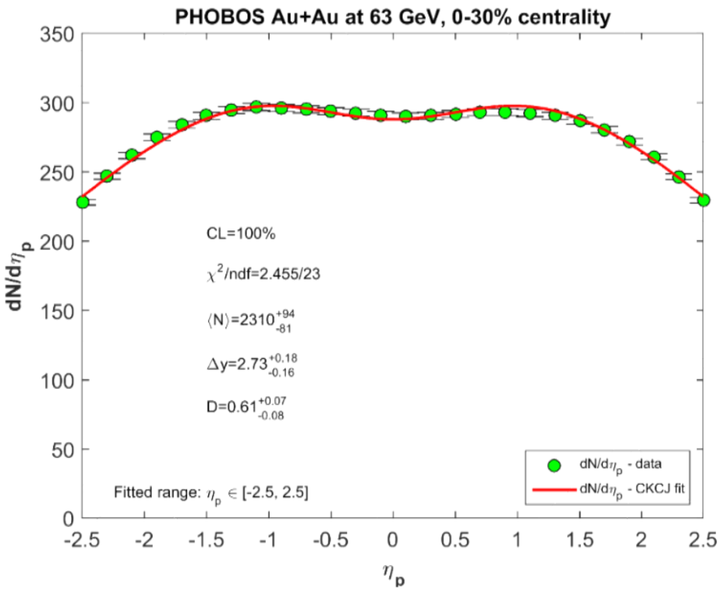

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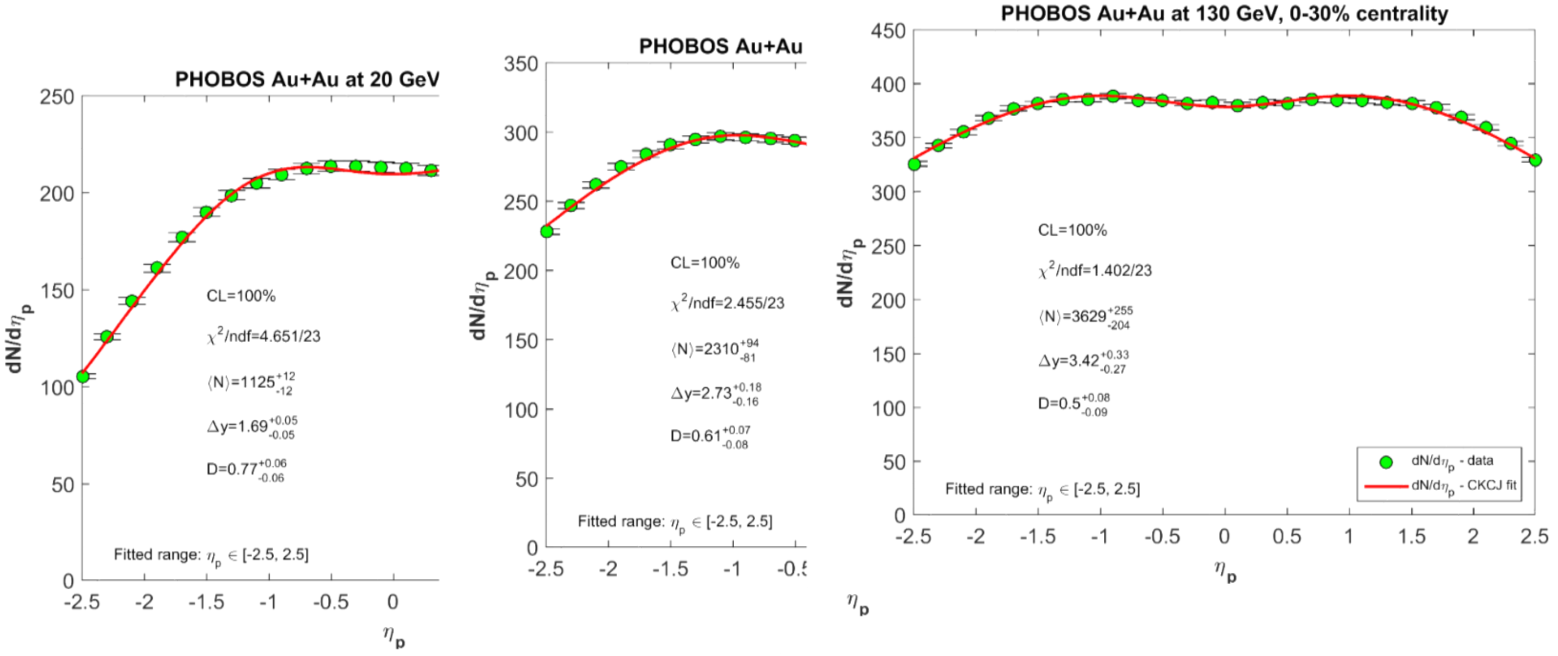

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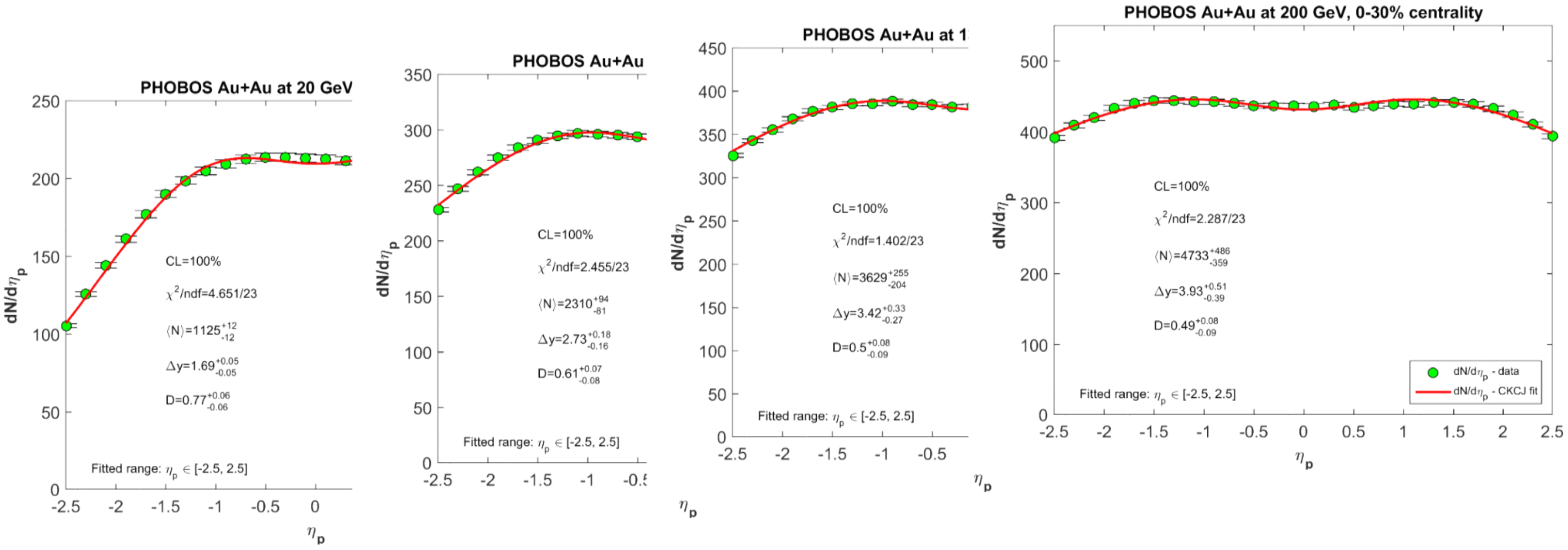

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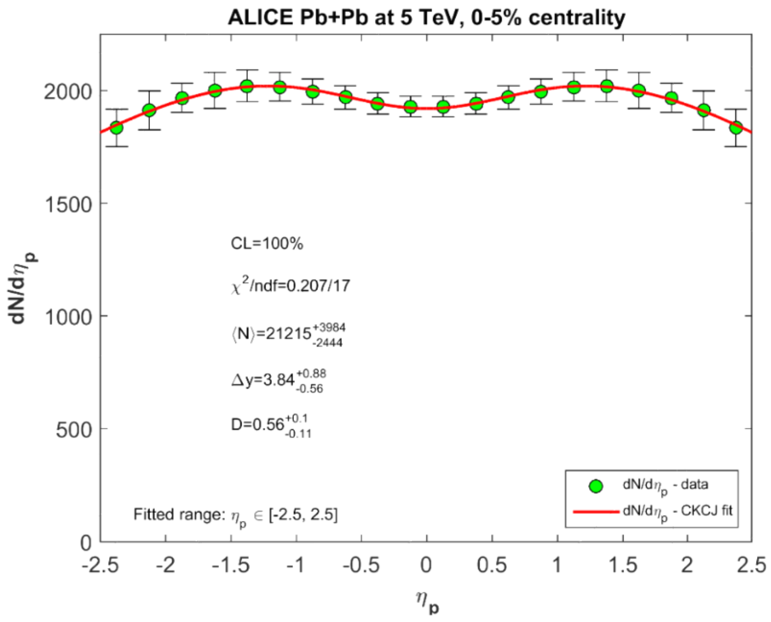

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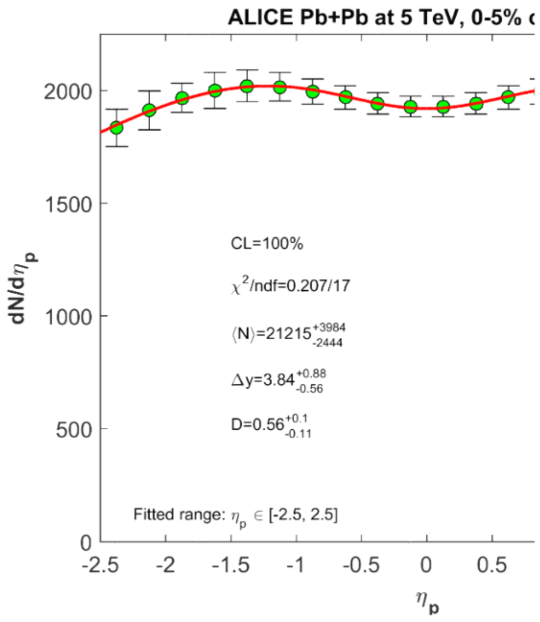

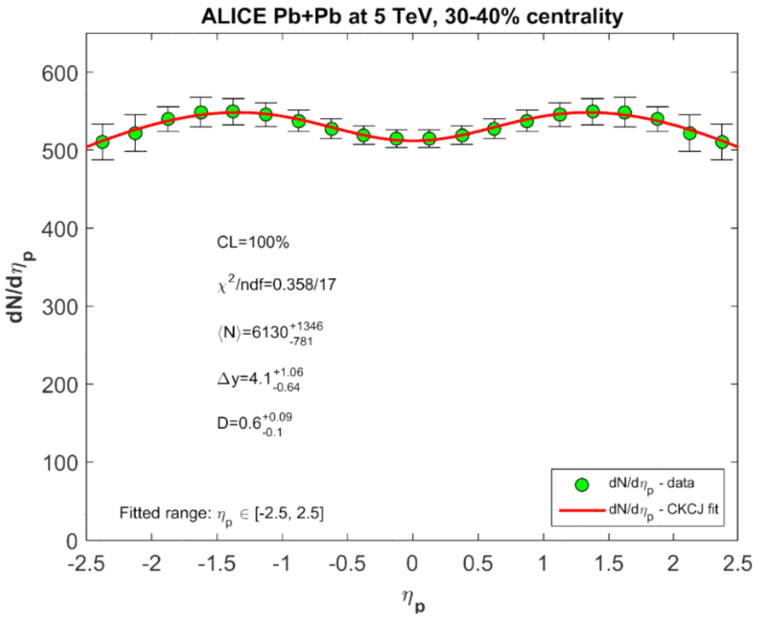

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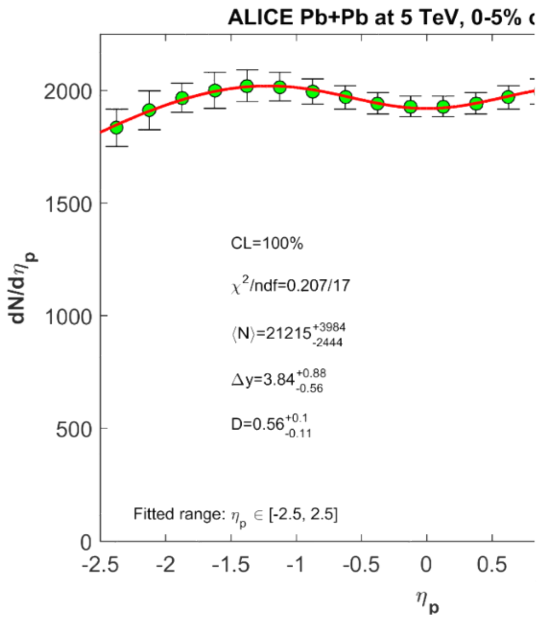

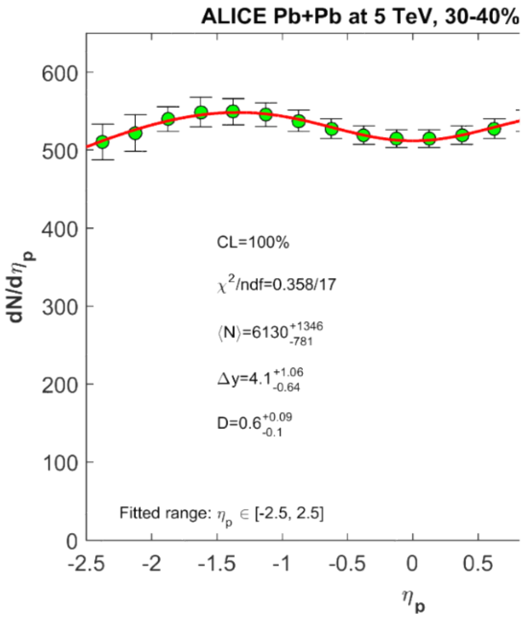

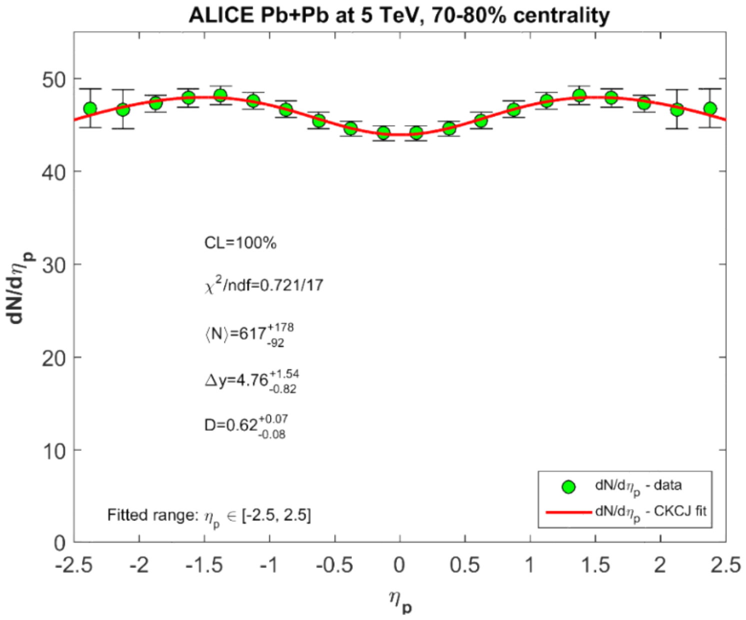

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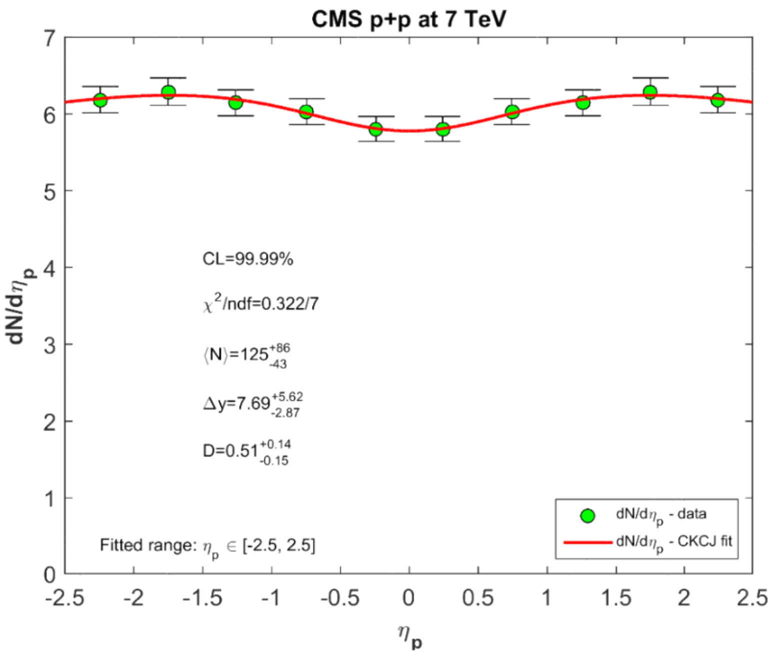

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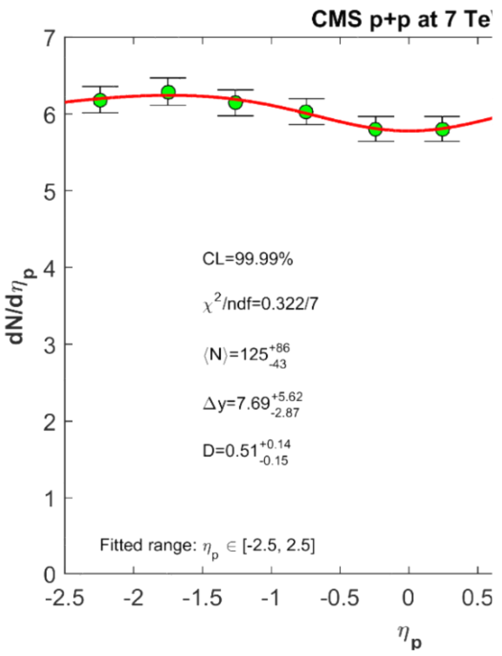

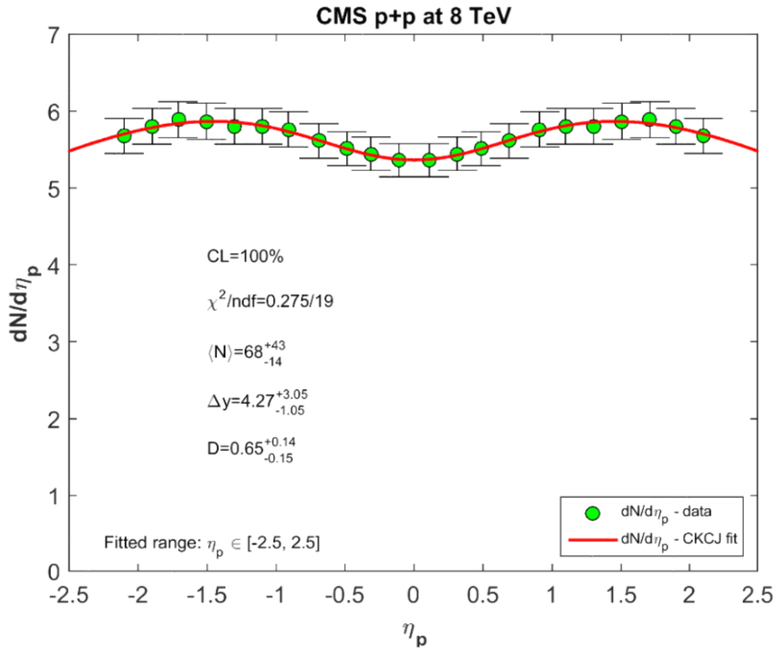

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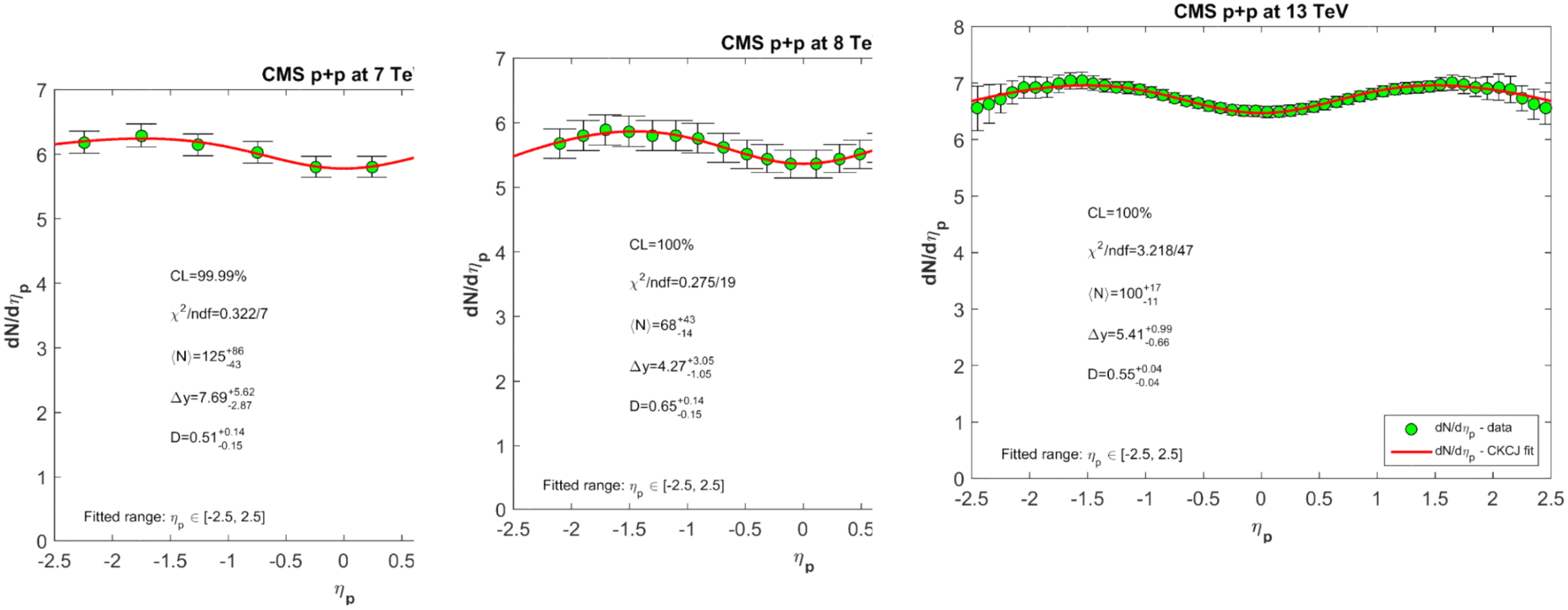

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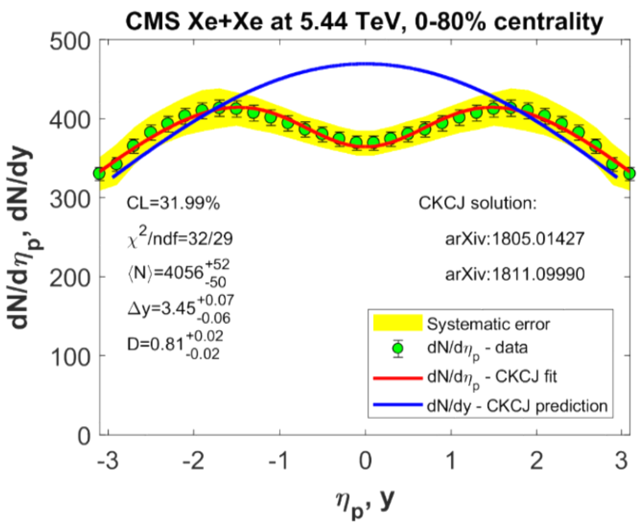

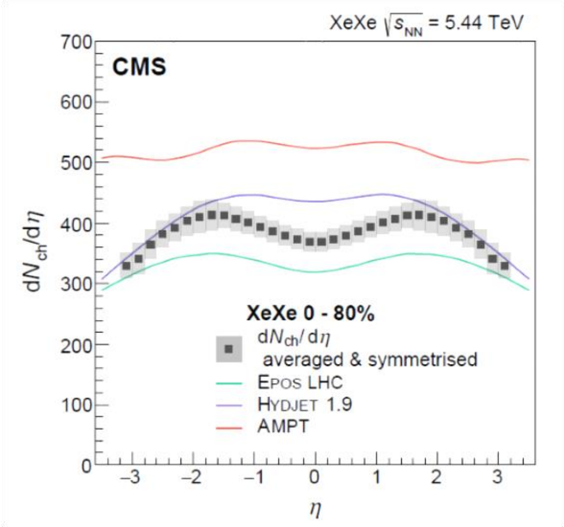

Our self-similar hydrodynamic calculations are succesful in such cases where other models fail.

CMS collab.: arXiv:1902.03603

Werner, Liu, Pierog: arXiv:hep-ph/0506232

Pierog, Karpenko, et all.: arXiv:1306.0121

Lokhtin, Snigirev: arXiv:hep-ph/0506189

Lin, Ko, et all.: arXiv:nucl-th/0411110

In conclusion…

- p+p collisions can be described as collective systems
- Our fits indicate low c_s value (≈0.35), but it can be determined only by violating the scaling behaviour
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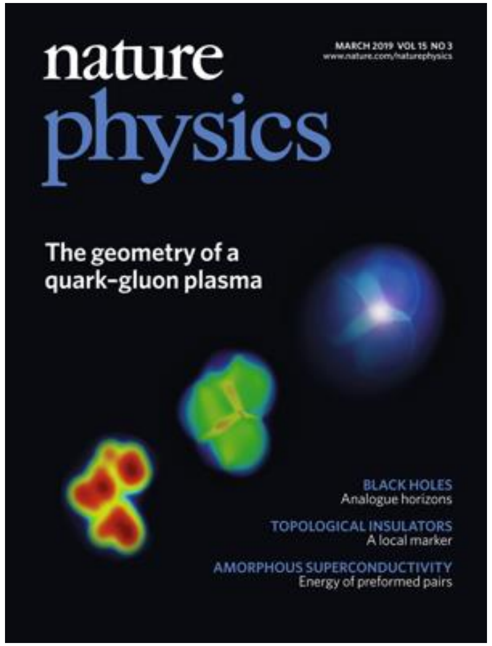

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Nijmegen preprint
HEN-405
Dec. 97

ESTIMATION OF HYDRODYNAMICAL MODEL PARAMETERS FROM THE INVARIANT SPECTRUM AND THE BOSE-EINSTEIN CORRELATIONS OF π^- MESONS PRODUCED IN (π^+/K^+)p INTERACTIONS AT 250 GeV/c

EHS/NA22 Collaboration

N.M. Agababyan[g], M.R. Atayan[g], T. Csörgő[h], E.A. De Wolf[a,1], K. Dziunikowska[b,2], A.M.F. Endler[e], Z.Sh. Garutchava[f], H.R. Gulkanyan[g], R.Sh. Hakobyan[g], J.K. Karamyan[g], D. Kisielewska[b,2], W. Kittel[d], S.S. Mehrabyan[g], Z.V. Metreveli[f], K. Olkiewicz[b,2], F.K. Rizatdinova[c], E.K. Shabalina[c], L.N. Smirnova[c], M.D. Tabidze[f], L.A. Tikhonova[c], A.V. Tkabladze[f], A.G. Tomaradze[f], F. Verbeure[a], S.A. Zotkin[c]

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[b] Institute of Physics and Nuclear Techniques of Academy of Mining and Metallurgy and Institute of Nuclear Physics, PL-30055 Krakow, Poland
[c] Nuclear Physics Institute, Moscow State University, RU-119899 Moscow, Russia
[d] High Energy Physics Institute Nijmegen (HEFIN), University of Nijmegen/NIKHEF, NL-6525 ED Nijmegen, The Netherlands
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[f] Institute for High Energy Physics of Tbilisi State University, GE-380086 Tbilisi, Georgia
[g] Institute of Physics, AM-375036 Yerevan, Armenia
[h] KFKI, Hungarian Academy of Sciences, H-1525 Budapest 114, Hungary

Abstract: The invariant spectra of π^- mesons produced in (π^+/K^+)p interactions at 250 GeV/c are analysed in the framework of the hydrodynamical model of three-dimensionally expanding cylindrically symmetric finite systems. A satisfactory description of experimental data is achieved. The data favour the pattern according to which the hadron matter undergoes predominantly longitudinal expansion and non-relativistic transverse expansion with mean transverse velocity $\langle u_t \rangle = 0.20 \pm 0.07$, and is characterized by a large temperature inhomogeneity in the transverse direction: the extracted freeze-out temperature at the center of the tube and at the transverse rms radius are 140 ± 3 MeV and 82 ± 7 MeV, respectively. The width of the (longitudinal) space-time rapidity distribution of the pion source is found to be $\Delta\eta = 1.36 \pm 0.02$. Combining this estimate with results of the Bose-Einstein correlation analysis in the same experiment, one extracts a mean freeze-out time of the source of $\langle \tau_f \rangle = 1.4 \pm 0.1$ fm/c and its transverse geometrical rms radius, $R_G(\text{rms}) = 1.2 \pm 0.2$ fm.

We hope that these results help to confirm the legitimacy of hydro in p+p collisions!

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