

Update on the η' mass-modification effect in exact solutions of hydrodynamics

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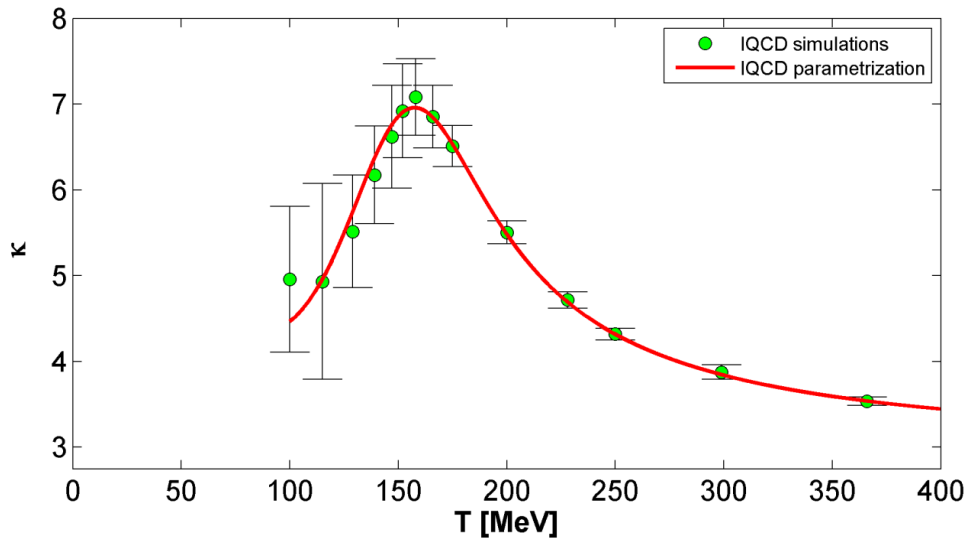
Day of Femtoscopy, Gyöngyös
31st of October, 2019

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Outlook

- **Lattice QCD EoS parametrization from hydro**
- **Non-relativistic solutions with $c_s(T)$**
- **Quark-hadron transitions**
 - **Crossover**
 - **2nd order PT**
- **Fine tuning:**
 - **Multi-component hadronic solution**
 - **Temperature dependent mass**
- **Observables**
 - **Scaling behaviour is explained by MC-solution**

Motivation



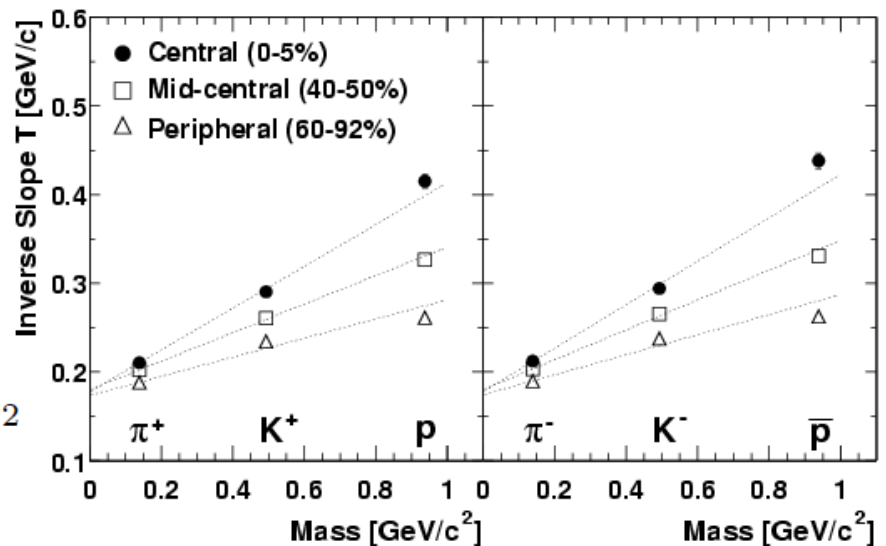
$\kappa(T) = \varepsilon/p$
 from lattice QCD
[arXiv:1007.2580](https://arxiv.org/abs/1007.2580)

***Can they be understood
 in a consistent picture?***

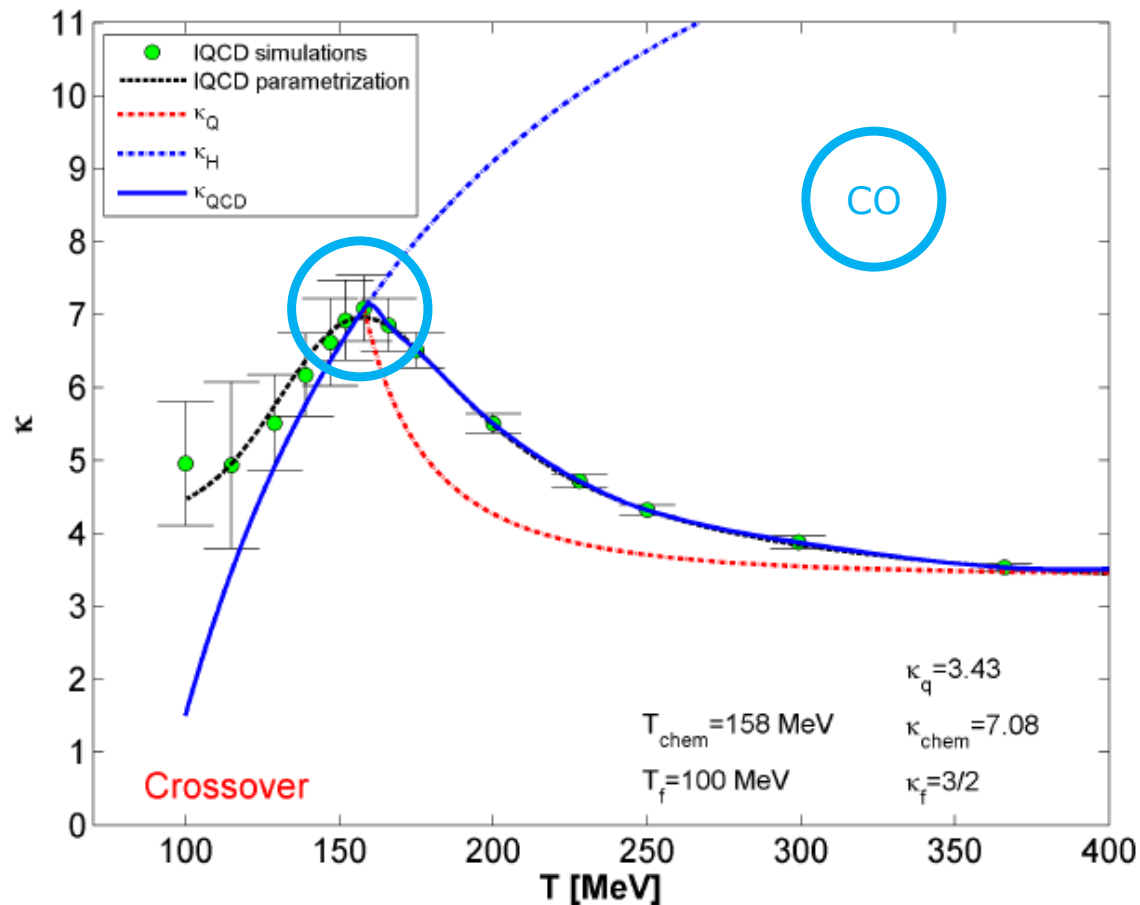
Scaling behaviour of single
 particle spectra

[nucl-ex/0307022](https://arxiv.org/abs/nucl-ex/0307022)

$$T = T_f + m \langle u_t \rangle^2 \implies T_i = T_f + m_i \langle u_t \rangle^2$$



Crossover (CO), lattice QCD EoS



- EoS of the whole $[T_f, T_0]$ interval:

$$\kappa(T) = \Theta(T_{chem} - T)\kappa_H(T) + \Theta(T - T_{chem})\kappa_{QCD}(T)$$

Multi-component (MC) HM solutions

<p>QM</p> <p>$T > T_{chem}$</p>	$\sigma(\vec{r}, t) = \sigma_0 \frac{V_0}{V} e^{-s/2}$ $(1 + \kappa) \left[\frac{d}{dT} \left(\frac{\kappa T}{1 + \kappa} \right) \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X \left(\ddot{X} - R\omega^2 \right) = Y\ddot{Y} = Z \left(\ddot{Z} - R\omega^2 \right) = \frac{1}{1 + \kappa}$
<p>MC HM</p> <p>$T < T_{chem}$</p>	$n_i(\vec{r}, t) = n_{i,0} \frac{V_0}{V} e^{-s/2}$ $\left[\frac{d}{dT} (\kappa T) \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X \left(\ddot{X} - R\omega^2 \right) = Y\ddot{Y} = Z \left(\ddot{Z} - R\omega^2 \right) = \frac{T}{\langle m \rangle}$
<p>SC HM</p> <p>$T < T_{chem}$</p>	$n(\vec{r}, t) = n_0 \frac{V_0}{V} e^{-s/2}$ $\left[\frac{d}{dT} (\kappa T) \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X \left(\ddot{X} - R\omega^2 \right) = Y\ddot{Y} = Z \left(\ddot{Z} - R\omega^2 \right) = \frac{T}{m}$

$$m \Leftrightarrow \langle m \rangle = \frac{\sum_i m_i n_i}{\sum_i n_i}$$

Multi-component hadronic matter (HM):
Expands and rotates together

Dynamics at a QCD Crossover

- Temperature equation at a crossover

$$(1 + \kappa) \left[\frac{d}{dT} \left(\frac{\kappa T}{1 + \kappa} \right) \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$$

- Equation of state

$$\varepsilon = \sum_i m_i n_i + \kappa p$$

- Infinitesimal relations

$$dp = \sigma dT$$

$$d\varepsilon = \sum_i m_i dn_i + \kappa \sigma dT + p d\kappa$$

Dynamics at a QCD Crossover

- Temperature equation at a crossover

$$(1 + \kappa) \left[\frac{d}{dT} \left(\frac{\kappa T}{1 + \kappa} \right) + \frac{f_H}{1 + \kappa} \left\langle \frac{dm}{dT} \right\rangle \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$$

- Equation of state

$$\varepsilon = \sum_i m_i(T) n_i + \kappa p$$

- Infinitesimal relations

$$dp = \sigma dT + \sum n_i dm_i$$

$$d\varepsilon = \sum_i m_i dn_i + \kappa \left(\sigma + \sum_i n_i \frac{dm_i}{dT} \right) dT + p d\kappa$$

if m is T dependent...

Dynamics at a QCD Crossover

- Equations of motion:

$$X \left(\ddot{X} - R\omega^2 \right) = Y\ddot{Y} = Z \left(\ddot{Z} - R\omega^2 \right) = \frac{1}{1 + \kappa + f_H \frac{\langle m \rangle}{T}}.$$

- Low temperature limit with $\kappa(T < T_f) = \kappa_f$ condition:

$$\lim_{T \rightarrow 0} \frac{1}{1 + \kappa + f_H \frac{\langle m \rangle}{T}} = \frac{T}{\langle m \rangle}$$

- High temperature limit:

$$\lim_{T \rightarrow \infty} \frac{1}{1 + \kappa + f_H \frac{\langle m \rangle}{T}} = \frac{1}{1 + \kappa_Q}$$

Dynamics at a QCD Crossover

- Equations of motion:

$$X \left(\ddot{X} - R\omega^2 \right) = Y\ddot{Y} = Z \left(\ddot{Z} - R\omega^2 \right) = \frac{1}{1 + \kappa + f_H \frac{\langle m(T) \rangle}{T}}.$$

- Low temperature limit with $\kappa(T < T_f) = \kappa_f$ condition:

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- High temperature limit:

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if m is T dependent...

Slopes: linear mass dependence

	SC HM	MC HM
$\omega_0 = 0$	$T_x = T_f + m_f \dot{X}_f^2$ $T_y = T_f + m_f \dot{Y}_f^2$ $T_z = T_f + m_f \dot{Z}_f^2$	$T_{x,i} = T_f + m_{i,f} \dot{X}_f^2$ $T_{y,i} = T_f + m_{i,f} \dot{Y}_f^2$ $T_{z,i} = T_f + m_{i,f} \dot{Z}_f^2$
$\omega_0 \neq 0$ (K' frame)	$T'_{xx} = T_f + m_f (\dot{X}_f^2 + \omega_f^2 R_f^2)$ $T'_{yy} = T_f + m_f \dot{Y}_f^2$ $T'_{zz} = T_f + m_f (\dot{Z}_f^2 + \omega_f^2 R_f^2)$	$T'_{xx,i} = T_f + m_{i,f} (\dot{X}_f^2 + \omega_f^2 R_f^2)$ $T'_{yy,i} = T_f + m_{i,f} \dot{Y}_f^2$ $T'_{zz,i} = T_f + m_{i,f} (\dot{Z}_f^2 + \omega_f^2 R_f^2)$

- Transform to the lab. frame ($\omega_0 \neq 0$) with \mathbf{M} rotation matrix:

$$\mathbf{T}_i^{-1} = \mathbf{M}^{-1} \mathbf{T}'_i^{-1} \mathbf{M}$$

- Linear mass dependence
- Scaling behaviour

Hydro scaling: $T = T_f + m \langle u_t \rangle^2$

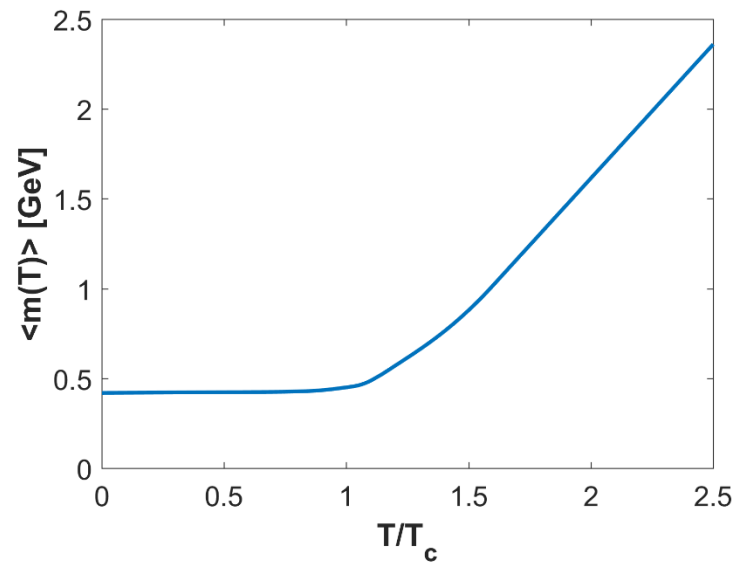
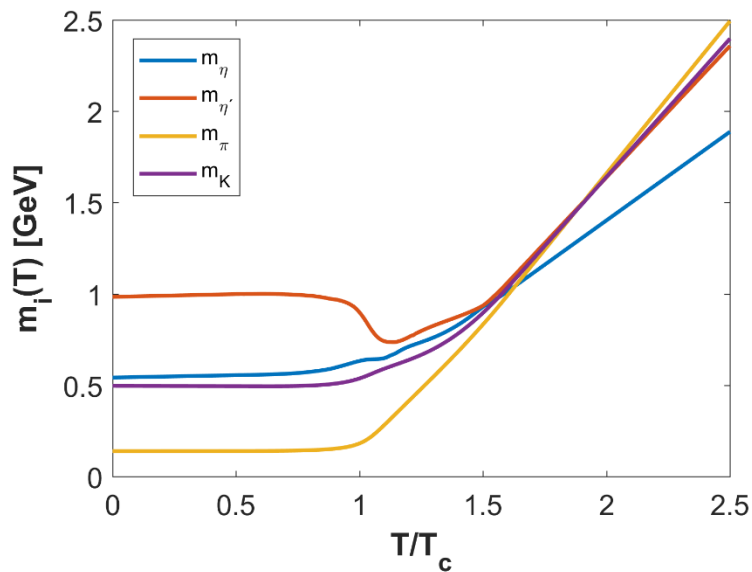
HBT results: Gaussian radii

	SC HM	MC HM
$\omega_0 = 0$	$R_x^{-2} = X_f^{-2} \frac{T_x}{T_f}$ $R_y^{-2} = Y_f^{-2} \frac{T_y}{T_f}$ $R_z^{-2} = Z_f^{-2} \frac{T_z}{T_f}$	$R_{x,i}^{-2} = X_f^{-2} \frac{T_{x,i}}{T_f}$ $R_{y,i}^{-2} = Y_f^{-2} \frac{T_{y,i}}{T_f}$ $R_{z,i}^{-2} = Z_f^{-2} \frac{T_{z,i}}{T_f}$
$\omega_0 \neq 0$ (K' frame)	$R'_{xx}{}^{-2} = X_f^{-2} \frac{T'_{xx}}{T_f} \left[1 - \frac{T'^2_{xz}}{T'_{xx} T'_{zz}} \right]$ $R'_{yy}{}^{-2} = Y_f^{-2} \frac{T'_{yy}}{T_f}$ $R'_{zz}{}^{-2} = Z_f^{-2} \frac{T'_{zz}}{T_f} \left[1 - \frac{T'^2_{xz}}{T'_{xx} T'_{zz}} \right]$	$R'_{xx,i}{}^{-2} = X_f^{-2} \frac{T'_{xx,i}}{T_f} \left[1 - \frac{T'^2_{xz,i}}{T'_{xx,i} T'_{zz,i}} \right]$ $R'_{yy,i}{}^{-2} = Y_f^{-2} \frac{T'_{yy,i}}{T_f}$ $R'_{zz,i}{}^{-2} = Z_f^{-2} \frac{T'_{zz,i}}{T_f} \left[1 - \frac{T'^2_{xz,i}}{T'_{xx,i} T'_{zz,i}} \right]$

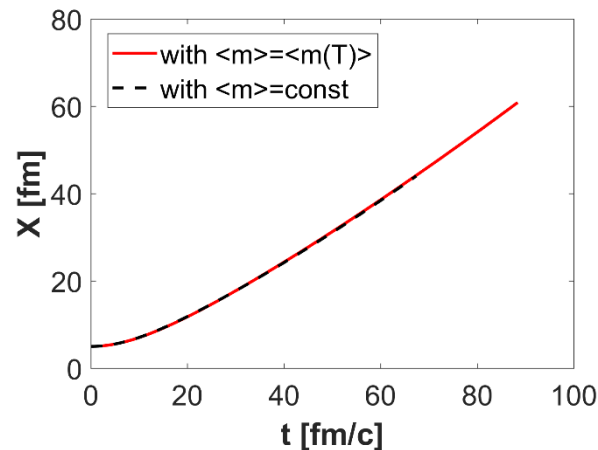
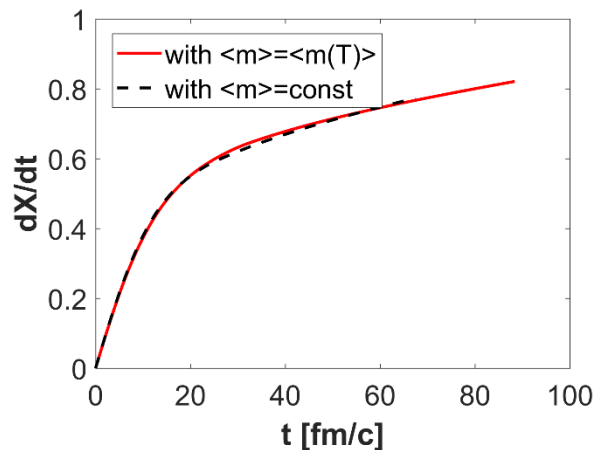
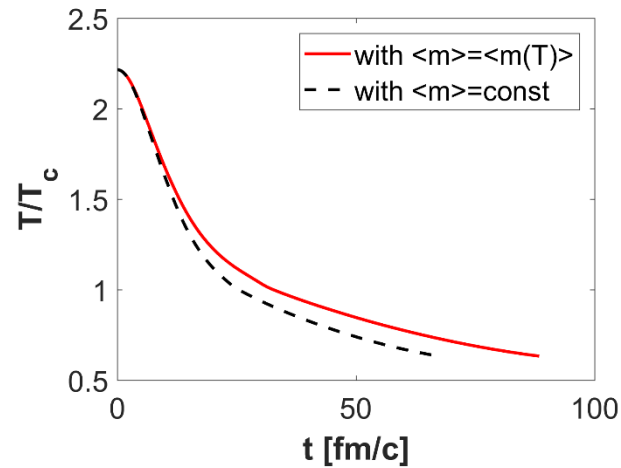
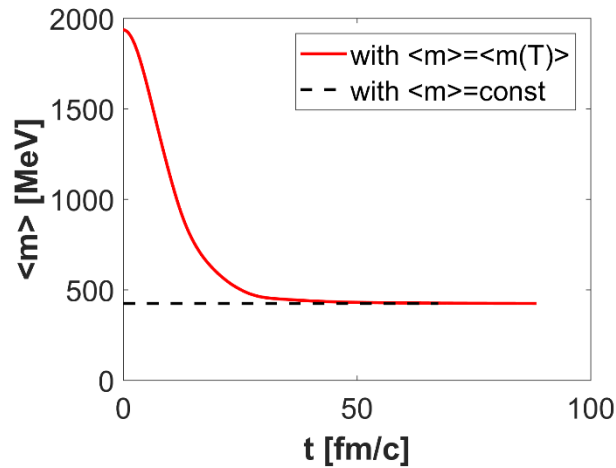
- Transform to the lab. frame ($\omega_0 \neq 0$): $\mathbf{R} = \mathbf{M}^{-1} \mathbf{R} \mathbf{M}$
- Linear mass dependence
- Scaling behaviour

$m_{\eta'}(T)$ and $m_{\eta'} = \text{const}$ comparison

- Temperature dependence of masses: D. Klabucar's calculations (Rank 2)
- Calculation of $\langle m(T) \rangle$ was based on Kaneta model

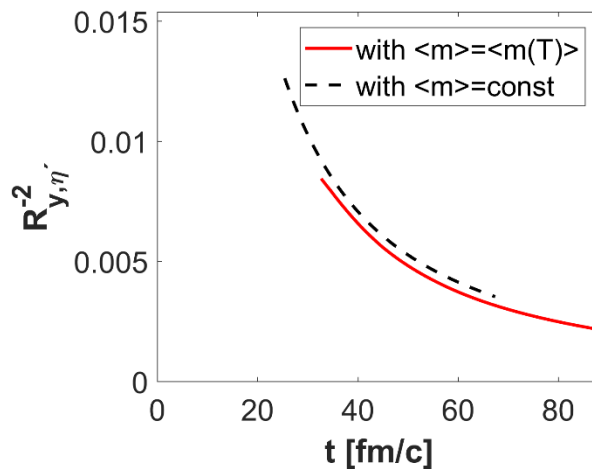
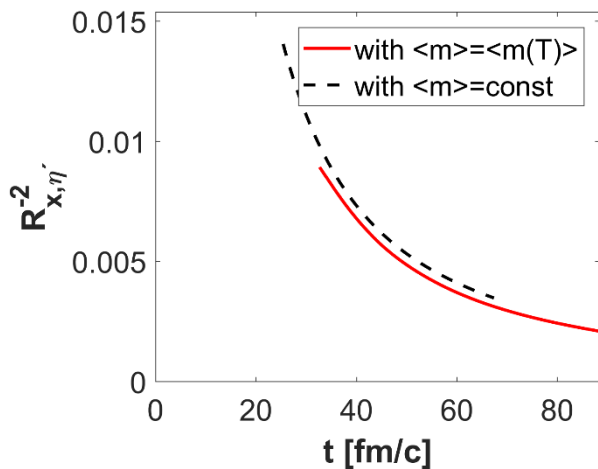
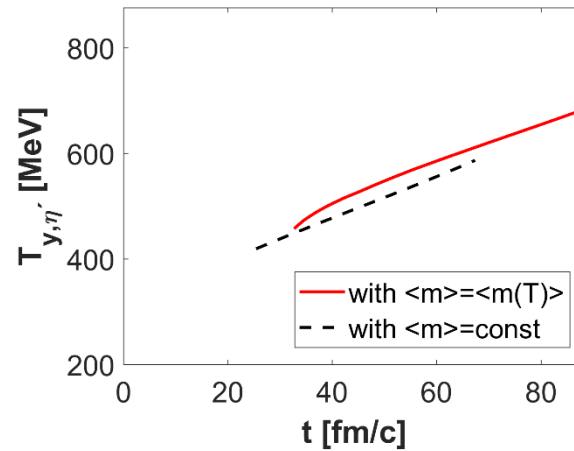
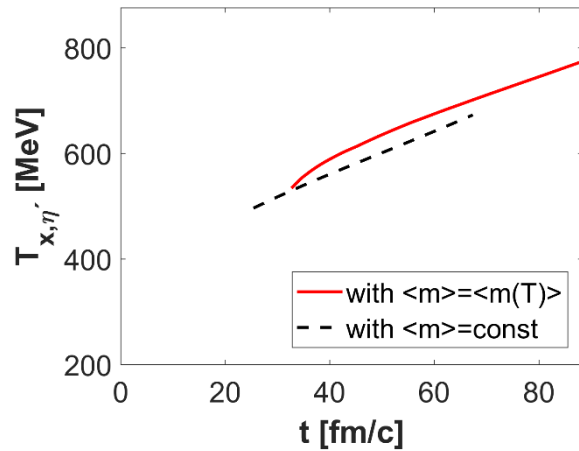


$m_{\eta'}(T)$ and $m_{\eta'} = \text{const}$ comparison (dynamics)



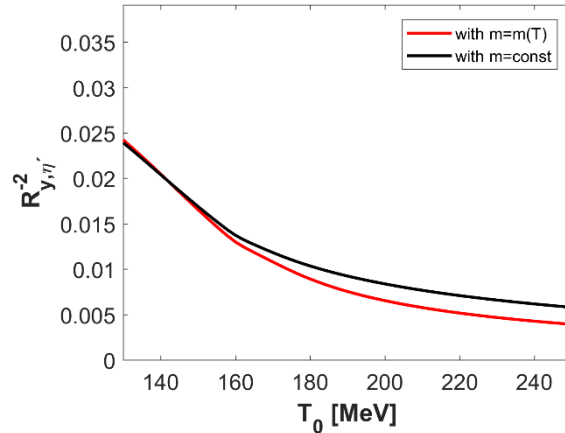
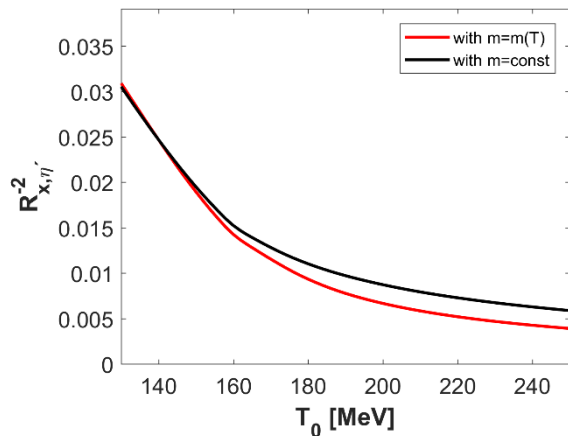
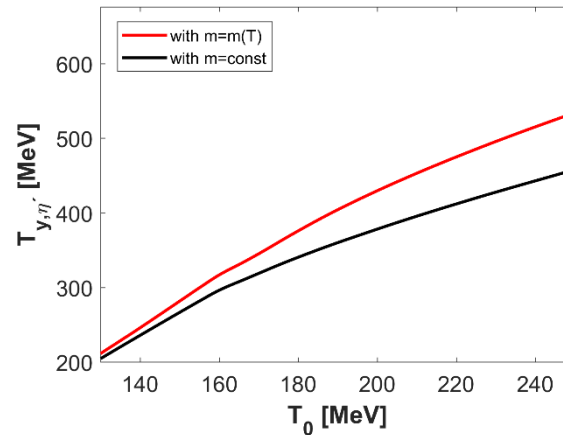
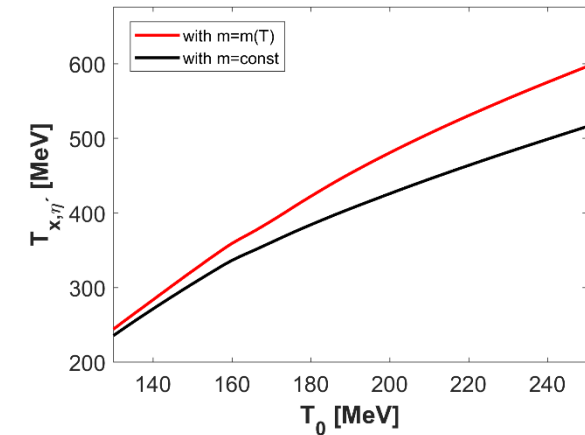
$T_0 = 350$ MeV
 $X_0 = 5$ fm
 $Y_0 = 6$ fm
 $Z_0 = 4$ fm
 $dX/dt|_{t_0} = 0$
 $dY/dt|_{t_0} = 0$
 $dZ/dt|_{t_0} = 0$
 $\omega_0 = 0.02$ c/fm
 $T_{\text{chem}} = 158$ MeV
 $T_f = 100$ MeV

$m_{\eta'}(T)$ and $m_{\eta'} = \text{const}$ comparison (observables)



$X_0 = 5$ fm
 $Y_0 = 6$ fm
 $Z_0 = 4$ fm
 $dX/dt|_{t_0} = 0$
 $dY/dt|_{t_0} = 0$
 $dZ/dt|_{t_0} = 0$
 $\omega_0 = 0.02$ c/fm
 $T_{\text{chem}} = 158$ MeV
 $T_f = 100$ MeV

$m_{\eta'}(T)$ and $m_{\eta'} = \text{const}$ comparison (observables)



For every T_0 :
 $X_0 = 5$ fm
 $Y_0 = 6$ fm
 $Z_0 = 4$ fm
 $dX/dt|_{t_0} = 0$
 $dY/dt|_{t_0} = 0$
 $dZ/dt|_{t_0} = 0$
 $\omega_0 = 0$ c/fm
 $T_{\text{chem}} = 158$ MeV
 $T_f = 100$ MeV

Summary

- IQCD equation of state is parametrized by considerations based on hydro
- This parametrization is ready to apply in the $c_s \rightarrow c_s(T)$ generalization of CKCJ solution (future plan)
- $m \rightarrow m(T)$ generalization: hardly effect the dynamics and the observables
- Hydro does not really sensitive to η' mass drop

Backup slides

Non-relativistic hydrodynamics

- Nonrelativistic approximation:

$$|\vec{v}|^2 \ll c^2$$

QM ($T_i \geq T \geq T_{chem}$)	HM ($T_{chem} > T \geq T_f$)
$\partial_t \sigma + \nabla(\sigma \mathbf{v}) = 0$	$\partial_t n_i + \nabla(n_i \mathbf{v}) = 0, \quad \forall i$
$T\sigma(\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$	$\sum_i m_i n_i (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$
$\frac{1+\kappa}{T} \left[\frac{d}{dT} \frac{\kappa T}{1+\kappa} \right] (\partial_t + \mathbf{v}\nabla)T + \nabla \mathbf{v} = 0$	$\frac{1}{T} \left[\frac{d(\kappa T)}{dT} \right] (\partial_t + \mathbf{v}\nabla)T + \nabla \mathbf{v} = 0$
$p = \sigma T / (1 + \kappa)$	$p = \sum_i p_i = T \sum_i n_i$

$$\varepsilon + p = \sum_i \mu_i n_i + T\sigma,$$

$$\varepsilon + p \approx T\sigma, \quad (T_i \geq T \geq T_{chem}),$$

$$\varepsilon + p \approx \sum_i m_i n_i \quad (T_{chem} > T \geq T_f).$$

Relativistic hydrodynamics

QM ($T_i \geq T \geq T_{chem}$)	HM ($T_{chem} > T \geq T_f$)
$\partial_\mu \sigma u^\mu = 0$ $T \sigma u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu) p$ $\frac{1+\kappa}{T} \left[\frac{d}{dT} \frac{\kappa T}{1+\kappa} \right] u^\mu \partial_\mu T + \partial_\mu u^\mu = 0$ $p = \sigma T / (1 + \kappa)$	$\partial_\mu n_i u^\mu = 0, \quad \forall i$ $\left(\sum_i m_i n_i \right) u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu) p$ $\frac{1}{T} \left[\frac{d(\kappa T)}{dT} \right] u^\mu \partial_\mu T + \partial_\mu u^\mu = 0$ $p = \sum_i p_i = T \sum_i n_i$

$$\varepsilon + p = \sum_i \mu_i n_i + T \sigma,$$

$$\varepsilon + p \approx T \sigma, \quad (T_i \geq T \geq T_{chem}),$$

$$\varepsilon + p \approx \sum_i m_i n_i \quad (T_{chem} > T \geq T_f).$$

Rotation angles, p and q space

- Rotation angle of the momentum space (K' frame):

$$\theta'_p = \frac{1}{2} \arctan \left(\frac{2T'_{xz}}{T'_{xx} - T'_{zz}} \right) = \frac{1}{2} \arctan \left(\frac{2\omega R}{\dot{X} + \dot{Z}} \right)$$

In laboratory frame: $\theta_p = \vartheta_f + \theta'_p$

- Rotation angle of the HBT space (K' frame):

$$\begin{aligned} \theta'_{q,i} &= \frac{1}{2} \arctan \left(\frac{2XZT'_{xz,i}}{Z^2T'_{xx,i} - X^2T'_{zz,i}} \right) = \\ &= \frac{1}{2} \arctan \left(\frac{2m_i XZ\omega R (\dot{X} - \dot{Z})}{(T + m_i\omega^2 R^2)(Z^2 - X^2) + m_i (Z^2\dot{X}^2 - X^2\dot{Z}^2)} \right) \end{aligned}$$

In laboratory frame: $\theta_{q,i} = \vartheta_f + \theta'_{q,i}$

- $\theta_{q,i}, \theta_p$ correlation: it can be a new possibility to measure the type of the quark-hadron transition

Observables from the new solutions - triaxial, rotating and expanding

Coordinate-space ellipsoid at the beginning of time evolution

Final coordinate-space ellipsoid at freeze-out

“Momentum-space ellipsoid” (eigenframe of single-particle spectrum)

“HBT-space ellipsoid” (eigenframe of HBT correlations)

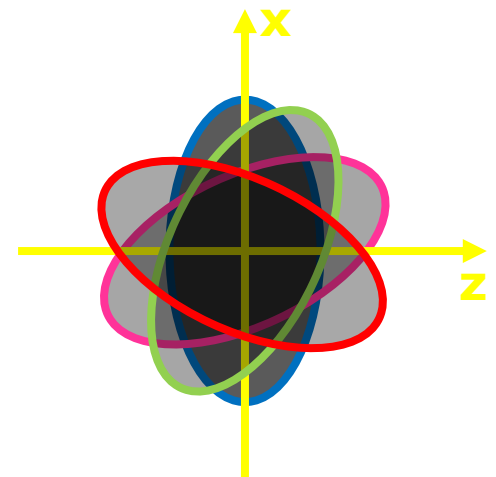
ϑ_f

θ_p

θ_q

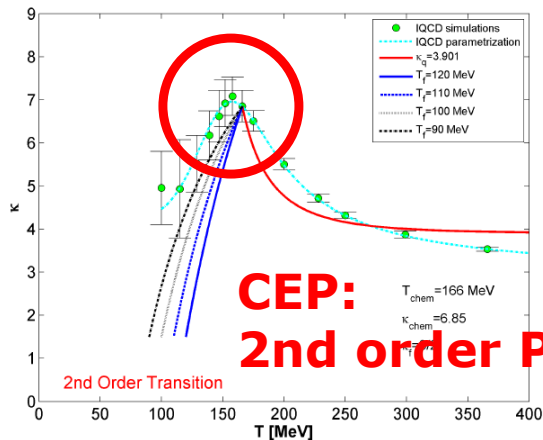
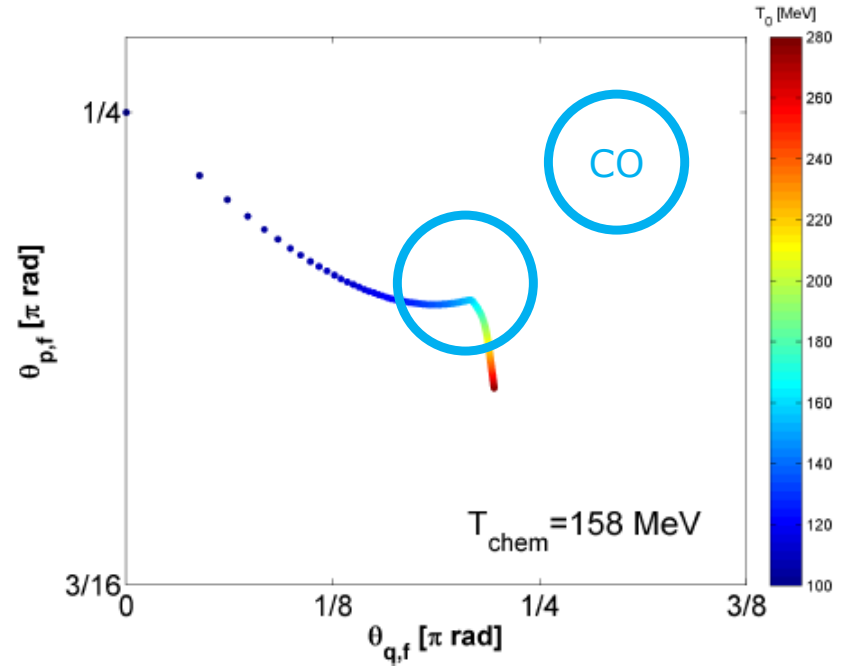
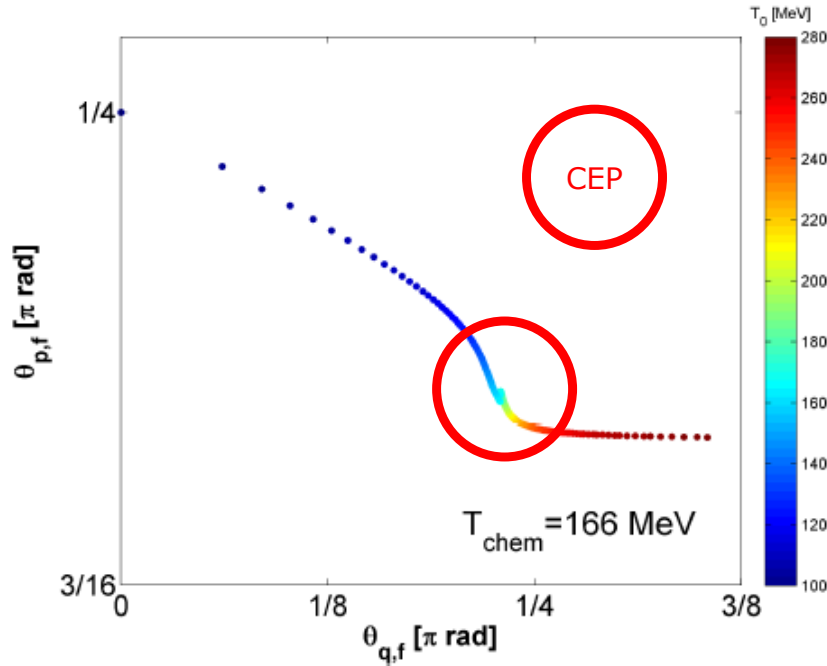
$$\theta'_p = \frac{1}{2} \arctan \left(\frac{2T'_{xz}}{T'_{xx} - T'_{zz}} \right) = \frac{1}{2} \arctan \left(\frac{2\omega R}{\dot{X} + \dot{Z}} \right)$$

$$\theta'_{q,i} = \frac{1}{2} \arctan \left(\frac{2XZT'_{xz,i}}{Z^2T'_{xx,i} - X^2T'_{zz,i}} \right)$$

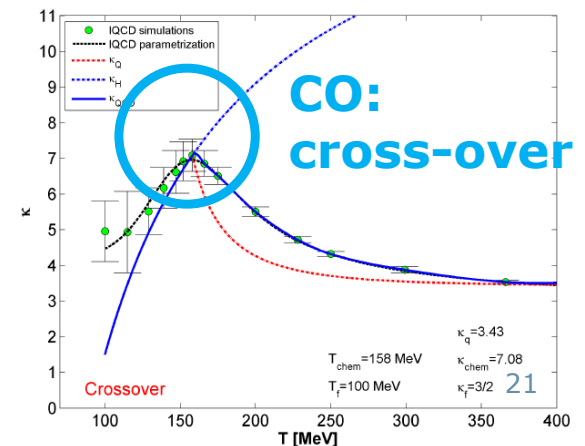


M.I. Nagy and T. Csörgő: [arXiv:1606.09160](https://arxiv.org/abs/1606.09160)

Correlation of p and q space difference of dynamics at CEP and CO QCD transition

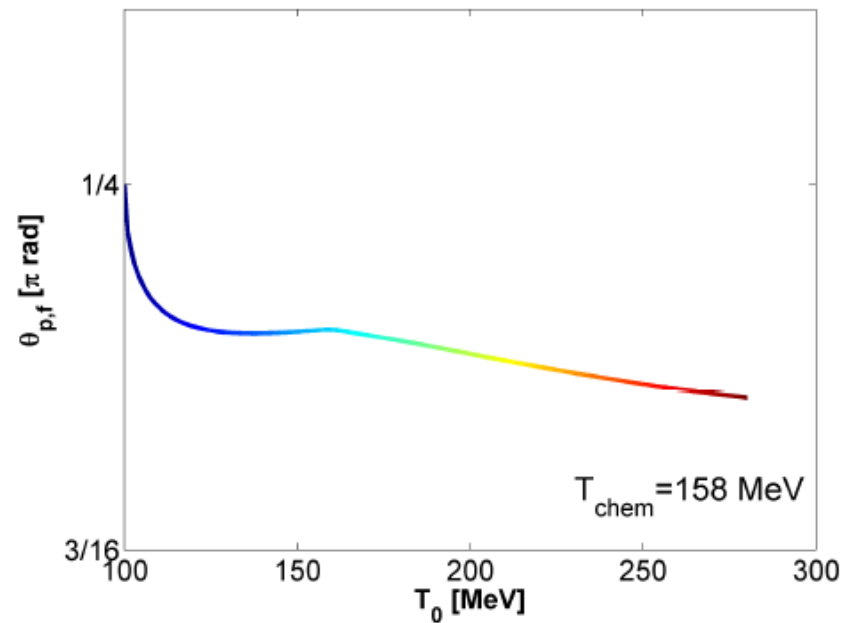
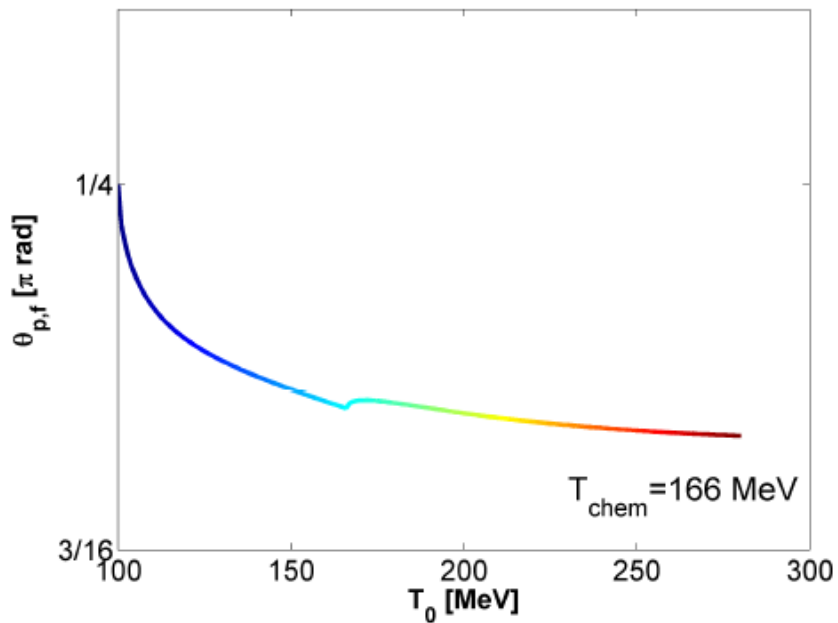


Correlation of angles in momentum and in HBT space: sensitive to CEP, CO, in general to QCD EoS



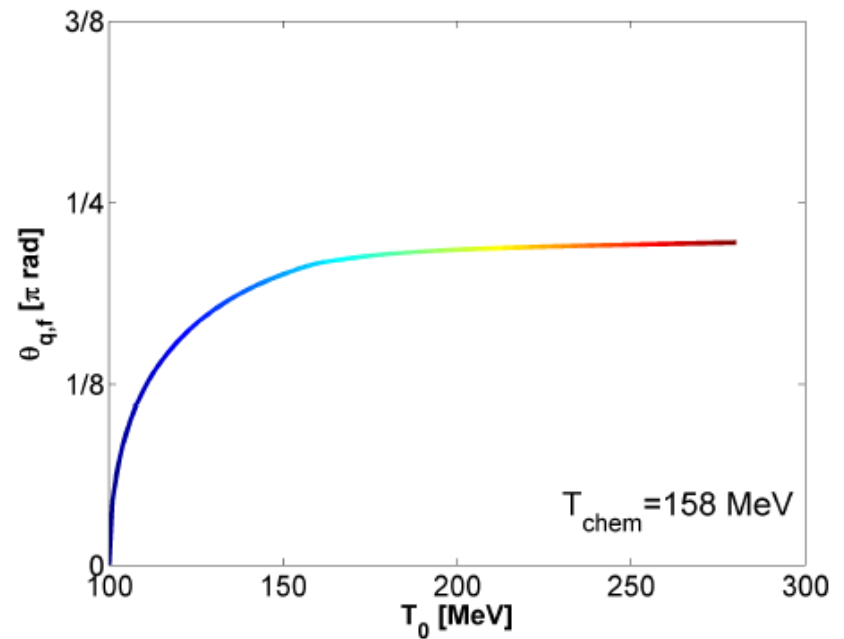
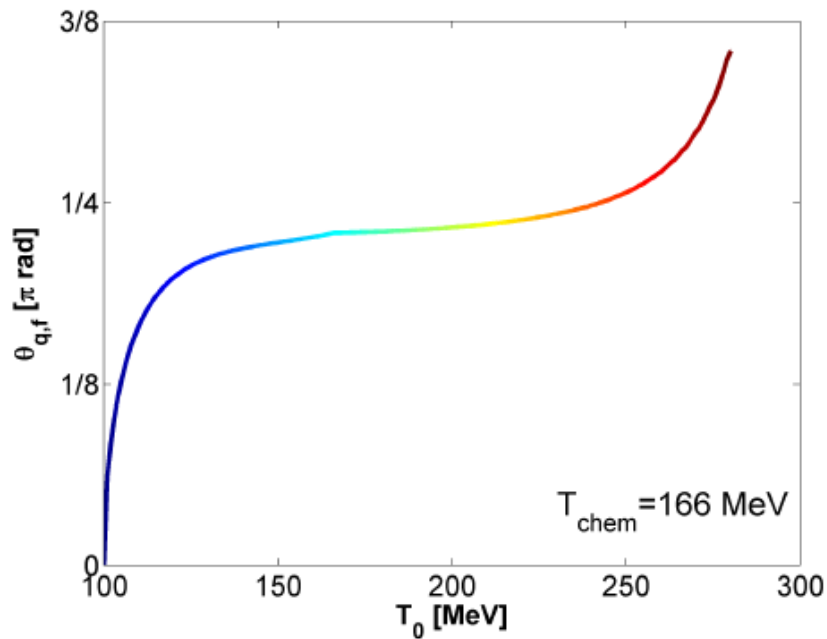
Rotation angles, p space

- θ_p as a function of the initial temperature
- Left: 2nd Order PT, right: crossover



Rotation angles, q space

- $\theta_{q,i}$ as a function of the initial temperature
- Left: 2nd Order PT, right: crossover



Triaxial, rotating exact hydro solutions

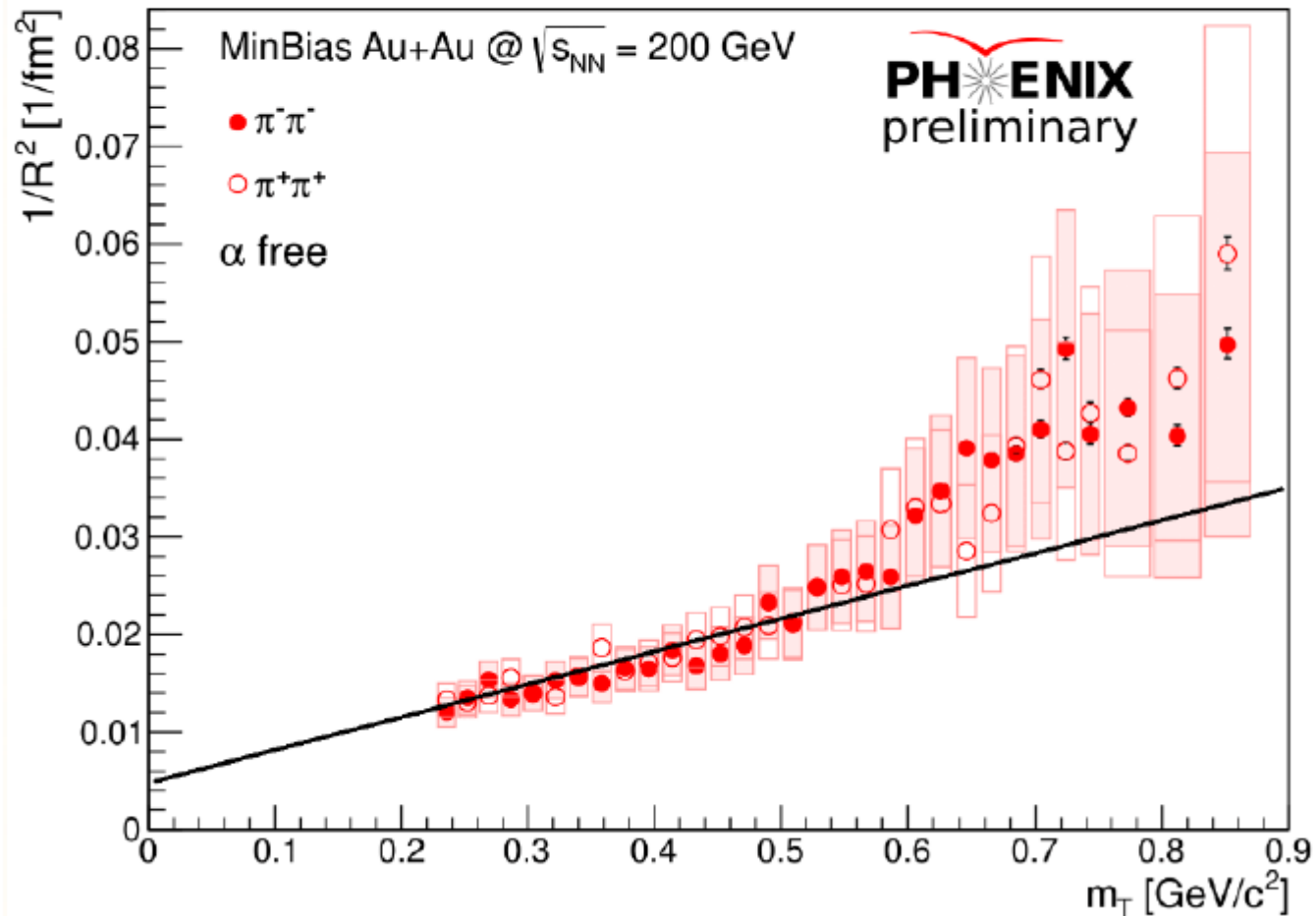
In both frames:	
$H_x = \frac{\dot{X}}{X}, \quad H_y = \frac{\dot{Y}}{Y}, \quad H_z = \frac{\dot{Z}}{Z},$ $V = (2\pi)^{3/2} XYZ, \quad n = n_0 \frac{V_0}{V} \exp(-s/2),$ $\dot{\vartheta} \equiv \frac{\omega}{2}, \quad \omega = \omega_0 \frac{R_0^2}{R^2}, \quad R = \frac{X+Z}{2},$	$\frac{d[T\kappa(T)]}{dT} \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0 \quad \text{if } \kappa(T) \neq \text{const},$ $T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \quad \text{if } \kappa(T) = \text{const}$ $X(\ddot{X} - \omega^2 R) = Y\ddot{Y} = Z(\ddot{Z} - \omega^2 R) = \frac{T}{m_0},$
in laboratory frame K :	in the co-rotating frame K' :
$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} + \left(\frac{1}{Z^2} - \frac{1}{X^2} \right) [(r_x^2 - r_z^2) \sin^2 \vartheta + r_x r_z \sin(2\vartheta)]$ $\mathbf{v}(\mathbf{r}, t) = \mathbf{v}_H(\mathbf{r}, t) + \mathbf{v}_R(\mathbf{r}, t)$ $\mathbf{v}_H(\mathbf{r}, t) = \begin{pmatrix} (H_x \cos^2 \vartheta + H_z \sin^2 \vartheta) r_x \\ H_y r_y \\ (H_x \sin^2 \vartheta + H_z \cos^2 \vartheta) r_z \end{pmatrix} + (H_z - H_x) \frac{\sin(2\vartheta)}{2} \begin{pmatrix} r_z \\ 0 \\ r_x \end{pmatrix}$ $\mathbf{v}_R(\mathbf{r}, t) = \dot{\vartheta} \begin{pmatrix} r_z \\ 0 \\ -r_x \end{pmatrix} + \dot{\vartheta} \begin{pmatrix} \left(\frac{X}{Z} \cos^2 \vartheta + \frac{Z}{X} \sin^2 \vartheta \right) r_z \\ 0 \\ -\left(\frac{X}{Z} \sin^2 \vartheta + \frac{Z}{X} \cos^2 \vartheta \right) r_x \end{pmatrix} + \dot{\vartheta} \left(\frac{X}{Z} - \frac{Z}{X} \right) \frac{\sin(2\vartheta)}{2} \begin{pmatrix} r_x \\ 0 \\ -r_z \end{pmatrix}$	$s = \frac{r'_x{}^2}{X^2} + \frac{r'_y{}^2}{Y^2} + \frac{r'_z{}^2}{Z^2}$ $\mathbf{v}'(\mathbf{r}', t) = \mathbf{v}'_H(\mathbf{r}', t) + \mathbf{v}'_R(\mathbf{r}', t)$ $\mathbf{v}'_H(\mathbf{r}', t) = \begin{pmatrix} H_x r'_x \\ H_y r'_y \\ H_z r'_z \end{pmatrix}$ $\mathbf{v}'_R(\mathbf{r}', t) = \dot{\vartheta} \begin{pmatrix} \frac{X}{Z} r'_z \\ 0 \\ -\frac{Z}{X} r'_x \end{pmatrix}$

TABLE II: Summary of the new rotating solution of the hydrodynamical equations, written up both in the inertial, laboratory frame K and in the K' frame, where the coordinate axes rotate together with the (X, Z) axes of a triaxial ellipsoid.

M. Nagy and T. Csörgő,
[arXiv:1610.02197](https://arxiv.org/abs/1610.02197)

Hydro: mass systematics of HBT radii

■ Linear mass dependence



Crossover (CO), lattice QCD EoS

- Energydensity and pressure:

$$\varepsilon = \varepsilon_Q + \varepsilon_H = g_Q \varepsilon + g_H \varepsilon$$

$$p = p_Q + p_H = f_Q p + f_H p$$

- EoS of the crossover:

$$\kappa_{QCD} = f_Q \kappa_Q + f_H \kappa_H$$

- Weights:

$$0 \leq g_Q = \varepsilon_Q / \varepsilon \leq 1 \qquad 0 \leq g_H = \varepsilon_H / \varepsilon \leq 1$$

$$0 \leq f_Q = p_Q / p \leq 1 \qquad 0 \leq f_H = p_H / p \leq 1$$

- Connection between g_i and f_i :

$$g_Q = \frac{\kappa_Q}{\kappa_{QCD}} f_Q \qquad g_H = \frac{\kappa_H}{\kappa_{QCD}} f_H$$

Method of multi-component solutions

- Suppose we know the u^μ velocity field
- The scale variable (s) of the fireball satisfies

$$u^\mu \partial_\mu s = 0$$

- We assume that T and n are known, and a connection between n and n_i :

$$n_i = \frac{n_{i,0}}{n_0} n$$

- If $p \propto n$ then the pressure of the MC scenario is

$$p_{MC} = \sum_i p_i = p \frac{n_{i,0}}{n_0}$$

- In this way, if we know a single component solution then we get the multi component generalization with this trivial method

It works for relativistic and nonrelativistic!

Three classes of analytic solutions

Class A: $T = T(t)$ and $\kappa = \kappa(T)$

Gaussian integrals: all observables are analytic.

QM ($T_i \geq T \geq T_{chem}$)	HM ($T_{chem} > T \geq T_f$)
$\mathbf{v} = \left(\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z \right)$	$\mathbf{v} = \left(\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z \right)$
$\sigma = \sigma_0 \frac{V_0}{V} \exp \left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2} \right)$	$n_i = n_{i,c} \frac{V_c}{V} \exp \left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2} \right)$
$(1 + \kappa) \left[\frac{d}{dT} \frac{\kappa T}{1 + \kappa} \right] \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$	$\frac{d(\kappa T)}{dT} \frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$
$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{1}{1 + \kappa(T)}$	$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{\langle m \rangle}$

Class B: $T = T(t, r)$, but $\kappa \neq \kappa(T)$

Allows to handle init. temp. inhomogeneities analytically.

But class B excludes the use of lattice QCD EoS.

What about class C?

Three classes of analytic solutions

Class C: $T = T(t,r)$ AND $\kappa = \kappa(T)$ but ...

$\kappa(T)$ has to have special form! ***Is it QCD compatible?***

- Analytic solution - QM ($T > T_c$):

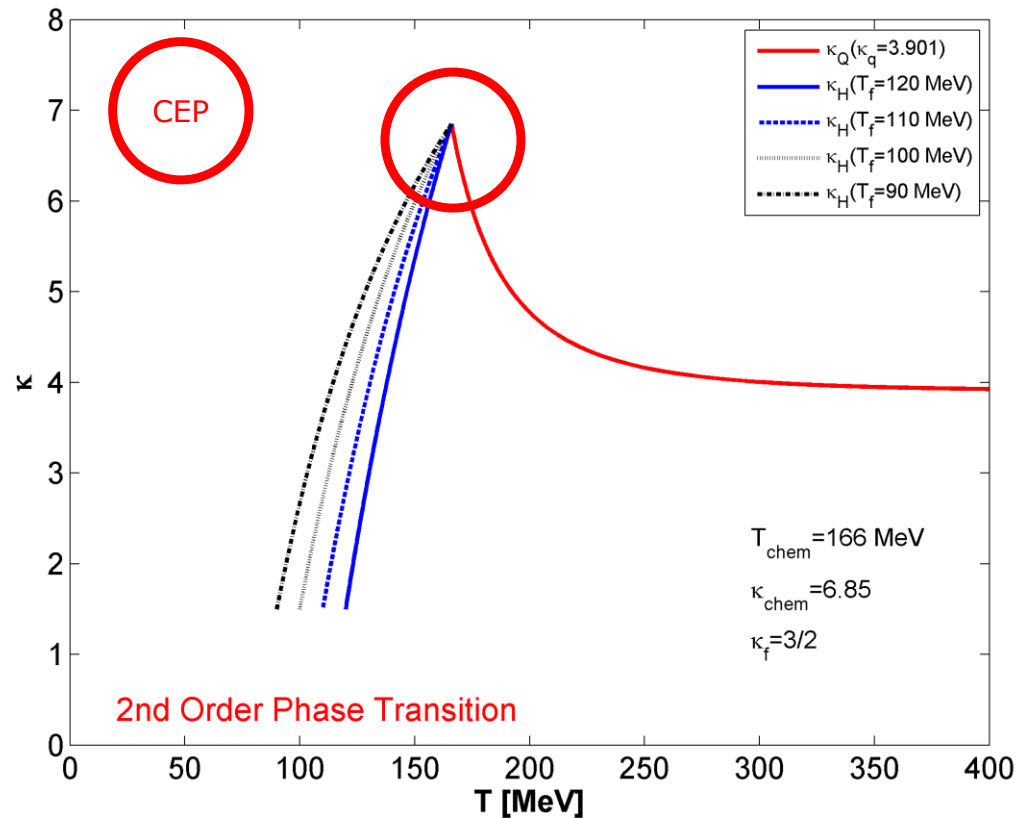
$$\kappa_Q(T) = \frac{\kappa_q \left(\frac{T}{T_{chem}} \right)^{1+\kappa_q} + \frac{\kappa_{chem} - \kappa_q}{\kappa_{chem} + 1}}{\left(\frac{T}{T_{chem}} \right)^{1+\kappa_q} - \frac{\kappa_{chem} - \kappa_q}{\kappa_{chem} + 1}}$$

- Analytic solution - HM ($T < T_c$)

$$\kappa_H(T) = \frac{\kappa_{chem} T_{chem} - \kappa_f T_f}{T_c - T_f} - \frac{\kappa_{chem} - \kappa_f}{T_{chem} - T_f} \frac{T_{chem} T_f}{T}$$

- $\kappa_Q(T)$: EoS of the quark matter
- $\kappa_{chem} = \kappa(T_{chem})$
- $\kappa_H(T)$: EoS of the hadronic matter
- $\kappa_f = \kappa(T_f)$

2nd Order Phase Transition (CEP)



- Combined EoS at T_{chem} :

$$\kappa(T) = \Theta(T_{chem} - T)\kappa_H(T) + \Theta(T - T_{chem})\kappa_Q(T)$$

Dynamics at the QCD Critical Point

- Boundary conditions:

$$\begin{aligned}T_Q(t_{chem}) &= T_H(t_{chem}) = T_{chem} \\ \vec{v}_Q(\vec{r}, t_{chem}) &= \vec{v}_H(\vec{r}, t_{chem}) \\ \kappa_Q(T_{chem}) &= \kappa_H(T_{chem})\end{aligned}$$

- From the equations of motion:

$$\frac{1}{1 + \kappa_{chem}} \approx 0.13 < \frac{T_{chem}}{\langle m \rangle} \approx 0.59$$

- The 2nd derivative of the scales jump at t_{chem}
- Second explosion: starts just after the conversion to the HM

Second explosion for $\langle m \rangle \sim 280$ MeV

Second explosion

