Lévy femtoscopy

- a brief story of my struggles with precision data

DAY OF FEMTOSCOPY 2019

DÁNIEL KINCSES

EÖTVÖS UNIVERSITY, BUDAPEST

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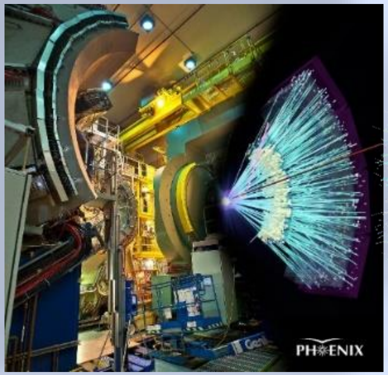
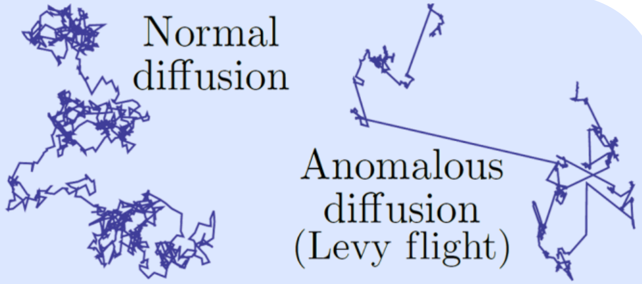


Part I. – Experimental analyses at RHIC

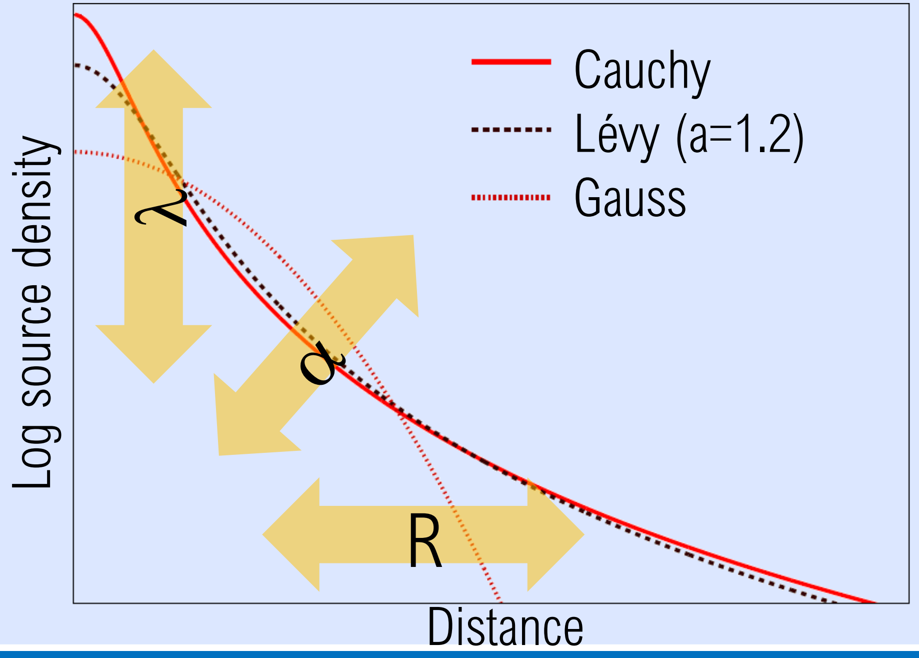
- THE HISTORY OF PHENIX PPG194 (NOW PHYS.REV. C.97.064911)
- PRELIMINARY RESULTS FROM PHENIX AND STAR
- CURRENT STATUS, STILL OPEN QUESTIONS

Lévy distributions in heavy ion physics

- Possible (competing) reasons for the appearance of Lévy-type sources:
  1. **Proximity of the critical endpoint** *Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67*
  2. **Anomalous diffusion** *(Metzler, Klafter, Physics Reports 339 (2000) 1-77, Csanad, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002)*
  3. **Jet fragmentation**



- **Symmetric Lévy-stable distribution:**
- $\mathcal{L}(\alpha,R;r) = \frac{1}{(2\pi)^3}\int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
- **Lévy-type corr. func.:** $C(Q) = 1 + \lambda \cdot e^{-(RQ)^\alpha}$
- No tail if $\alpha = 2$, power law if $\alpha < 2$; correlation between α and R, λ

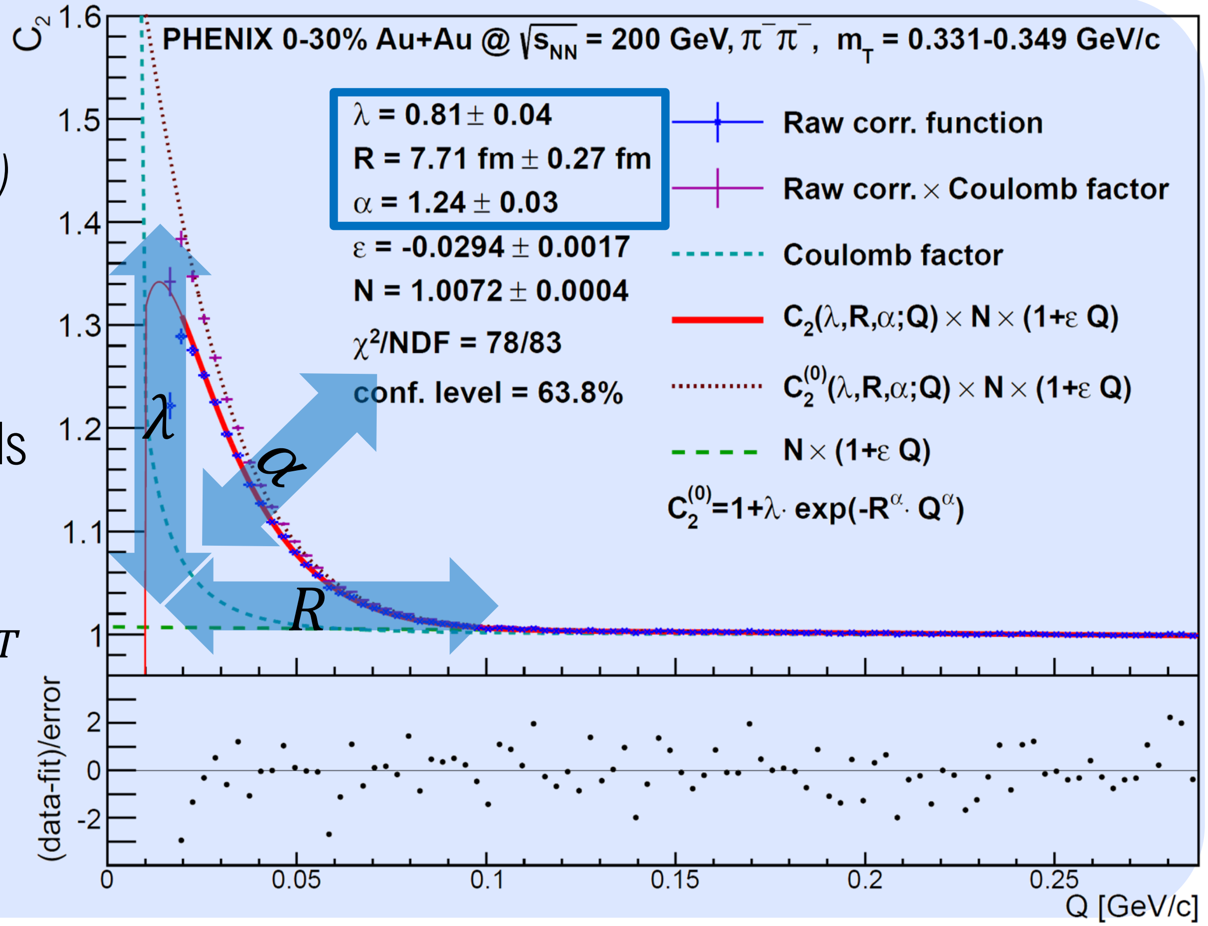


PHENIX - Example $C_2(Q_{LCMS})$ with a Lévy fit

- Measured in m_T bins

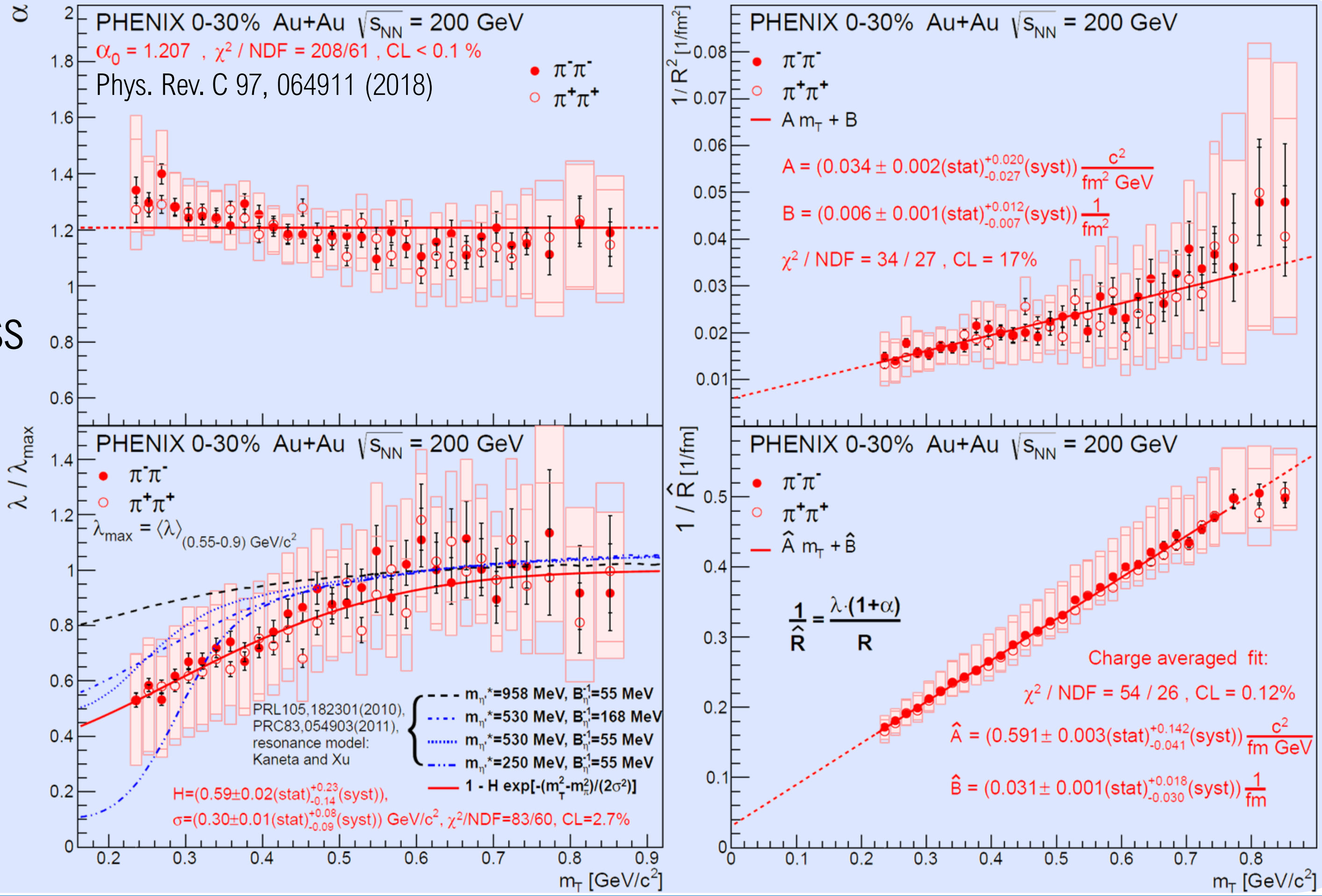
  (<u>the number of bins is important</u>! Stay tuned)
- **Fitted with Coulomb-incorporated function**
- Coulomb-factor displayed separately
- All fits converged, good confidence levels
- χ values scatter around 0 properly
- Physical parameters: R, λ, α vs. pair m_T
- Recall α: Lévy index, 0.5 at CEP

PHENIX, Phys.Rev. C97 (2018) no.6, 064911, arXiv:1709.05649



200 GeV 1D Lévy HBT results

- **α: not 0.5 and not 2.0**
- R: hydro scaling
- λ: „hole”,
  not incompatible with mass modification
- $\hat{R}$: new scaling variable



A story of precision

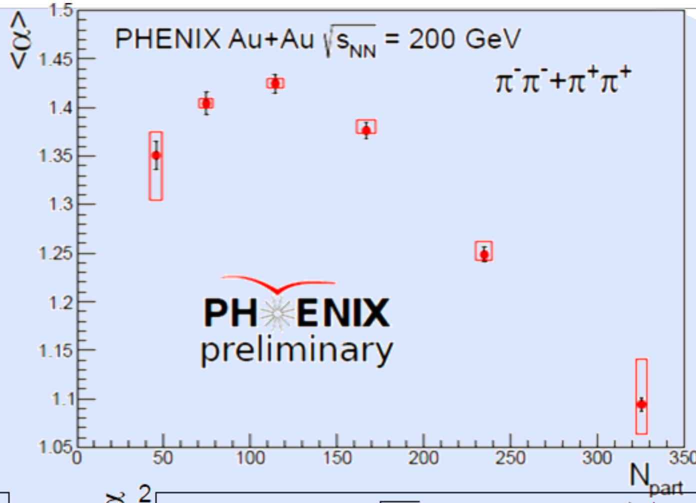
- Early femtoscopy analyses – low statistics (+ fit quality not checked carefully) – Gaussian is ok
- **Precision data** – heavy tails appear – Gaussian does not provide good description anymore

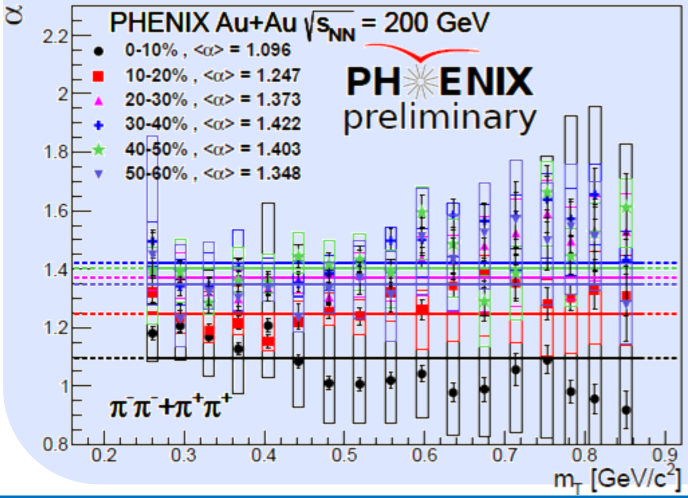
- PHENIX, *Phys.Rev. C97 (2018) no.6, 064911* - first published Lévy analysis for heavy-ion collisions, **proves that the source shape substantially differs from a Gaussian distribution**

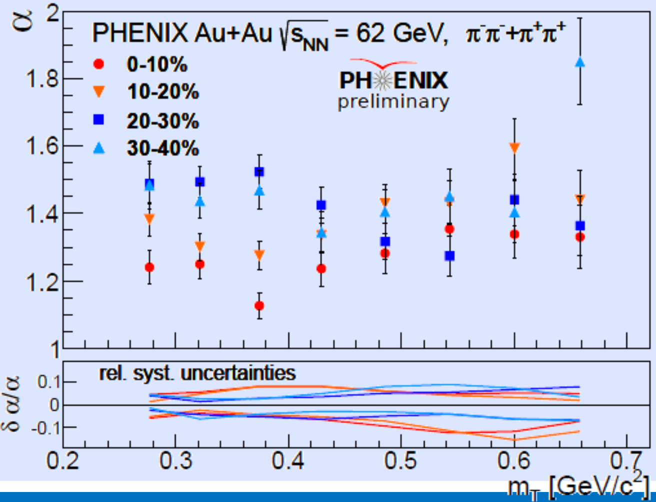
- Another new thing is the **detailed m_T dependence** – 31 m_T bins are used!
- What is the reason for using this many bins? (besides being interested in detailed trends)
  - Somewhat confidential information: because of fit quality!
    - The extracted parameters remain stable, but **fit quality gets worse with less m_T bins**
    - Less bins – more statistics in a given bin – higher precision
    - **Is it worse because of higher precision, or because of averaging over a wide range m_T ?**

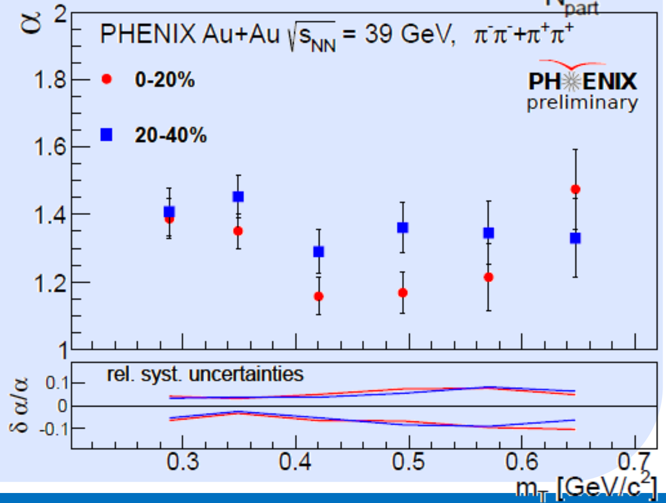
PHENIX preliminary – centrality and collision energy dependence

- Results shown at QM17, QM18, WPCF17, WPCF18
- Again we had to play with precision and find the thin line:
  too many bins result in inadequate statistics,
  too few bins result in bad fit quality
  - 200 GeV – 6 cent. bins, 17 m_T bins
  - 62 GeV – 4 cent. bins, 8 m_T bins
  - 39 GeV – 2 cent. bins, 6 m_T bins





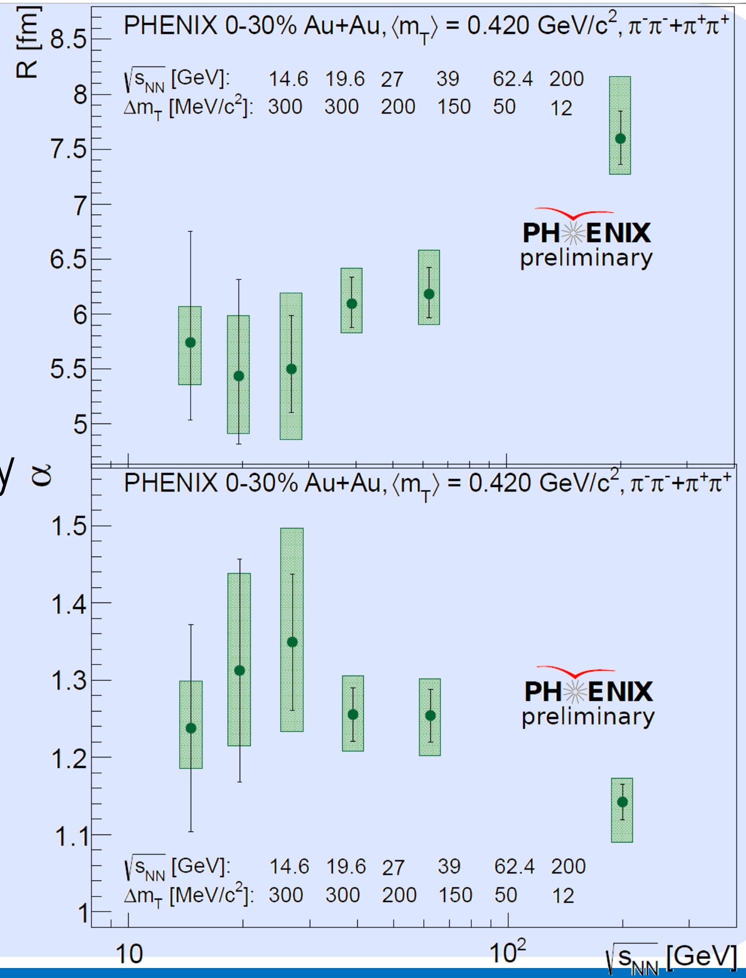




Collision energy dependence

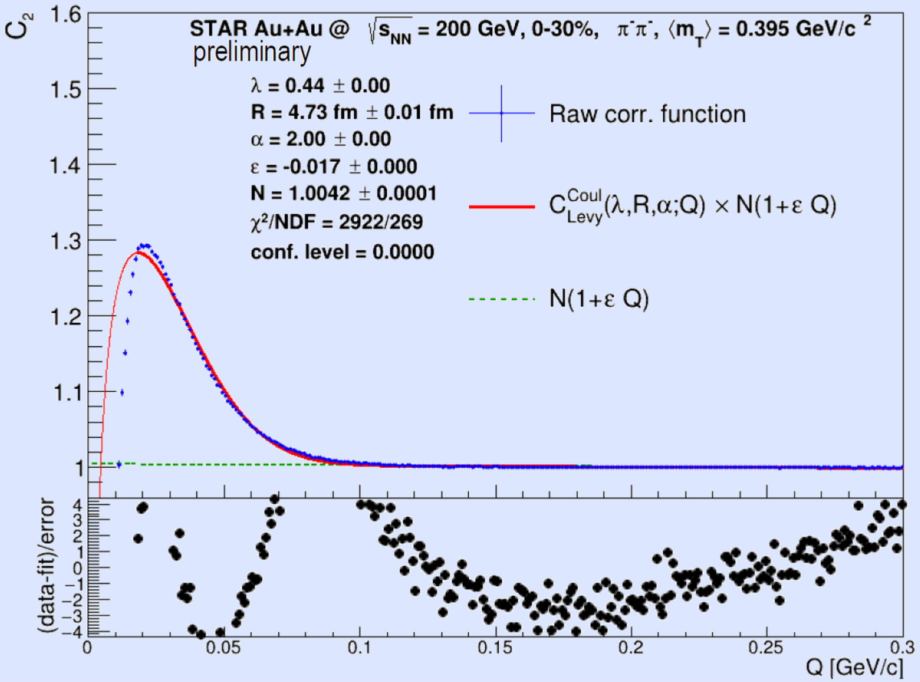
QM18, WPCF18, CPOD18

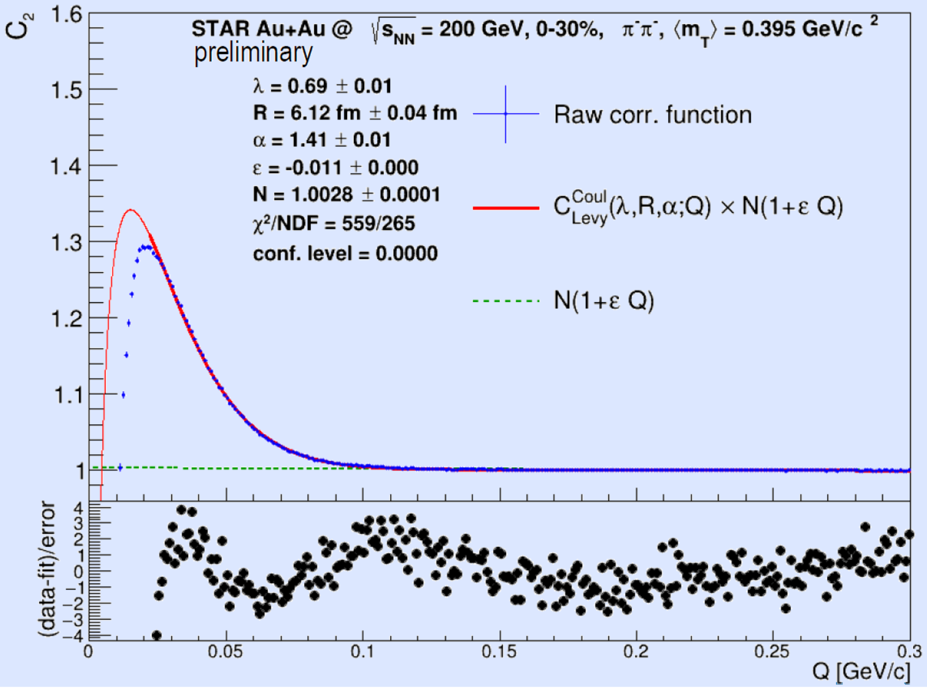
- **The struggle with precision continues**
- Low energy = low statistics
  - We must use only one wide bin in cent. and m_T
- Can we use the same bin width at different energies?
  - No! Again, at high energies, too precise data – bad fit quality
- Can we use arbitrarily wide m_T bins at low energies?
  - No! The **results are only stable up to $\Delta m_T \approx$ 100 MeV** (discovered after the preliminary)
  - **With 100 MeV wide bins we cannot use the 15, 19, 27 GeV**
  - We're still trying to find a way out of this



Lévy Femtoscopy at STAR (first 1D preliminary results)

- Precision problem → same structure on the χ distribution as at PHENIX when data is „too precise”
- Since then: using better cuts, better event mixing – the problem is still there
- A new challenge: Low Q behavior still unclear





Summary of experimental results

- Gaussian fits do not describe the heavy-ion data
- **Lévy fits provide a better description up to a certain precision**
- **PHENIX** – We always had to „play" with precision and find the thin line where we have enough data but not too much so the fit quality is acceptable
- **STAR** – we cannot really avoid the precision problem anymore, **have to find a description beyond the simple Lévy fits**

- One possibility to go beyond simple Lévy fits – **Lévy expansion technique**, which is being tested on STAR data (with not much success unfortunately)
- Another thing we could check is the effect of **including the strong interaction** as well

Part II. – Exploring final state interactions

- LÉVY FITS AND THE FINAL-STATE INTERACTIONS
- INCLUDING THE STRONG INTERACTION

Lévy fits and the final-state interactions

- **Single particle distribution:** $N_1(p) = \int dx\, S(x,p)$
- **Pair momentum distribution:** $N_2(p_1,p_2) = \int dx_1 dx_2 S(x_1,p_1)S(x_2,p_2)|\psi(x_1,x_2)|^2$
- **Correlation function:** $C(p_1,p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_2(p_2)}$
- **Pair source/spatial correlation:**

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

relative coordinate

relative pair momentum

average pair momentum

Pair wave function

$$C(Q,K) = \frac{\int D(r,K)\left|\psi_Q(\mathrm{r})\right|^2 dr}{\int D(r,K) dr}$$

- **Core-Halo model:** $S = \sqrt{\lambda}\, S_C + (1-\sqrt{\lambda})S_H \xRightarrow{R_H\ large} C(Q) = 1 - \lambda + \lambda \cdot \frac{\int D_C(r)\left|\psi_Q(\mathrm{r})\right|^2 dr}{\int D_C(r) dr}$

Lévy fits and the final-state interactions

- **Lévy parametrization without final state effects**: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- **Bowler-Sinyukov procedure:**

Intercept parameter (correlation strength)

Lévy exponent

Possible linear background (usually negligible)

$$C(Q) = \left(1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q)$$

FSI correction

Lévy scale parameter

- **FSI-correction:**

Two-particle wave function (with FSI)

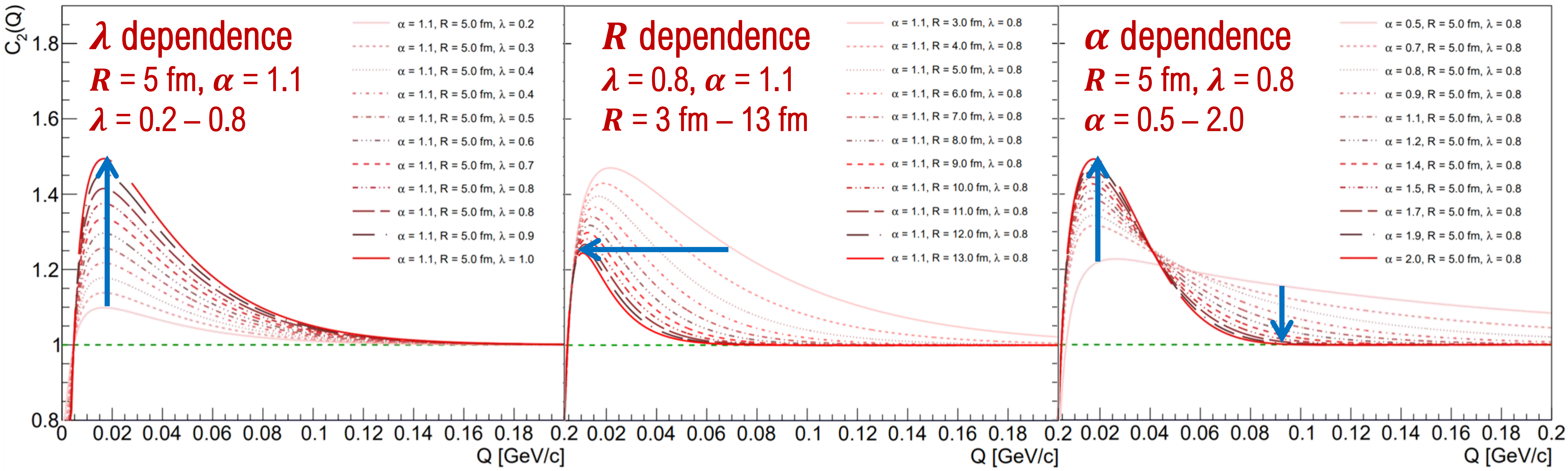
$$K(Q;\alpha,R) = \frac{\int D(r)\left|\psi^{FSI}(r)\right|^2 dr}{\int D(r)\left|\psi^{(0)}(r)\right|^2 dr}$$ → **calculated numerically**

Spatial correlations

Two-particle wave function (plane wave)

Shape of the correlation functions with Coulomb effect included

$$C(Q) = 1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)$$



Including the strong interaction

- **FSI-correction (calculated numerically):**

$$D(r) = \mathcal{L}\left(\alpha, 2^{\frac{1}{\alpha}}R, r\right)$$

$$K(Q;\alpha,R) = \frac{\int D(r)|\psi^{FSI}(r)|^2 dr}{\int D(r)|\psi^{(0)}(r)|^2 dr}$$

Plane wave

Confluent hypergeometric function

$$\psi^{Coul}(\boldsymbol{r}) = \frac{1}{\sqrt{2}}\frac{\Gamma(1+i\eta)}{e^{\pi\eta/2}}\{e^{i\boldsymbol{qr}}F(-i\eta, 1, i(kr - \boldsymbol{qr})) + [\boldsymbol{r} \leftrightarrow -\boldsymbol{r}]\}$$

$$\eta = \frac{\alpha_{EM} m_\pi c^2}{2\hbar q c}$$

3-dim. mom. diff. in pair rest frame ($\boldsymbol{q_{PCMS}}$)

$$q = Q/2$$

- **Final state interactions appear in the two-particle wave function**
- How to include strong interaction? Take partial wave expansion of the known Coulomb-scattering wave-function, subtract the $l = 0$ term, and add this term back with strong phase-shift included:

With the strong phase shift Δ_k we can write the full wave function as

$$\Psi_{\mathbf{k}}^{(\mathrm{CS})}(\mathbf{r}) = \psi_{\mathbf{k}}^{(-)}(\mathbf{r}) - \frac{e^{-i\delta_0^c}}{2k}\mathcal{F}_{k,0}(r) + \frac{e^{-i(\delta_0^c+\Delta_k)}}{2k}\mathcal{F}_{k,0}^{\Delta_k}(r) = \psi_{\mathbf{k}}^{(-)}(\mathbf{r}) - \frac{i}{2k}e^{-i(\delta_0^c+\Delta_k)}\sin\Delta_k\left(\mathcal{F}_{k,0} + i\mathcal{G}_{k,0}\right).$$

Including the strong interaction

Substituting the formulas for the respective wave functions encountered here, we get

$$\Psi_{\mathbf{k}}^{(\mathrm{CS})}(\mathbf{r}) = e^{-ikr}\Big\{\mathcal{N}^{*}\mathbf{F}\big(1-i\eta, 1, i(kr+\mathbf{kr})\big) + 2i\sin\Delta_k e^{-i\Delta_k} e^{\pi\eta/2} e^{-2i\delta_0^{\mathrm{c}}} U\big(1-i\eta, 2, 2ikr\big)\Big\}.$$

$$\mathbf{F}(a,b,z) = \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(a)\Gamma(b+n)}\frac{z^n}{n!}$$

Confluent hypergeometric function

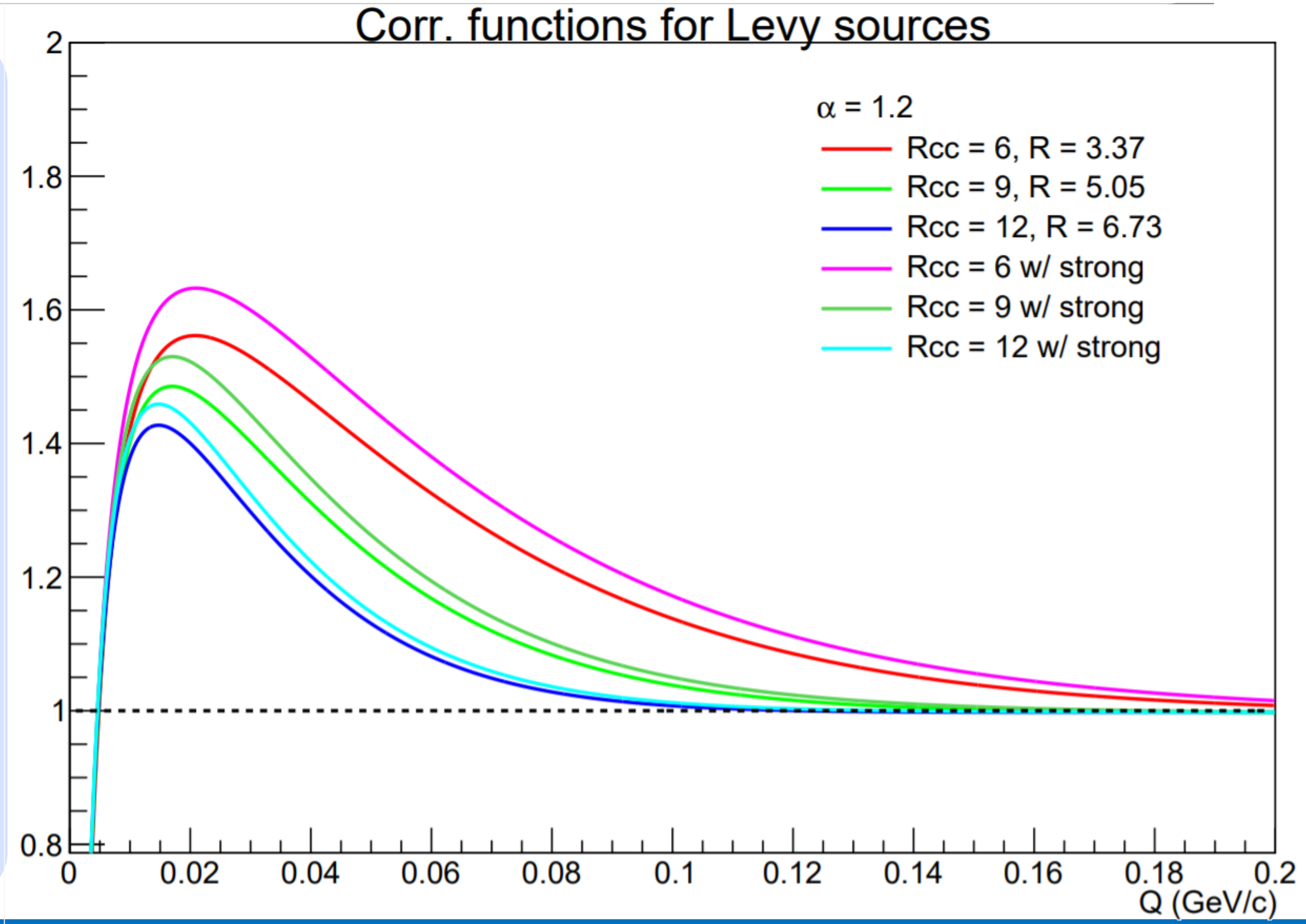
$$U(a,b,z) = \frac{\pi}{\sin(\pi b)}\left\{\frac{\mathbf{F}(a,b,z)}{\Gamma(a+1-b)} - z^{1-b}\frac{\mathbf{F}(a+1-b, 2-b, z)}{\Gamma(a)}\right\}$$

Kummer's function (in case of integer b, l'Hospital's rule to be used…)

- The effect of the strong interaction appears in the **strong phase-shift** Δ_k
- The phase-shift can be characterized by two parameters, the **scattering length** f_0, and **effective range** d_0
- The values for the aforementioned parameters can be found in literature (these are not new fit parameters!)
- **The C(Q) functional form containing the Coulomb+Strong interactions can be calculated numerically**

C(Q) containing the Coulomb+Strong interactions

- It seems that the strong interaction has a **non-negligible effect**!
- By eye it seems that it affects mostly the strength (λ), maybe R and α as well
- **Many cross-checks needed**, we already found some typos which changed the results drastically, but **it seems we are converging**
- Detailed studies are ongoing, **stay tuned!**



Summary

- **Lévy fits provide a good description** of experimental heavy-ion data only **up to a certain precision**
- If the data is „too precise”, Lévy fits may fail to describe the data,
  although the values of the extracted parameters are stable
- **To face this issue, we can explore** the Lévy expansion technique,
  as well as **the effect of final state interactions** in more details
- Both the PHENIX and STAR analyses are facing the same problem of precision, but **there is hope**
- We are currently finalizing and writing up the calculations and results of the SI investigation
  much more details to be shown at Zimányi School this year

Thank you for your attention!