Update on longitudinal impedance requirements

Ivan Karpov

Acknowledgements:

Theodoros Argyropoulos, Elena Shaposhnikova, Alexey Burov, Rama Calaga, James A. Mitchell, Sergey Antipov, Helga Timko

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Longitudinal single-bunch stability

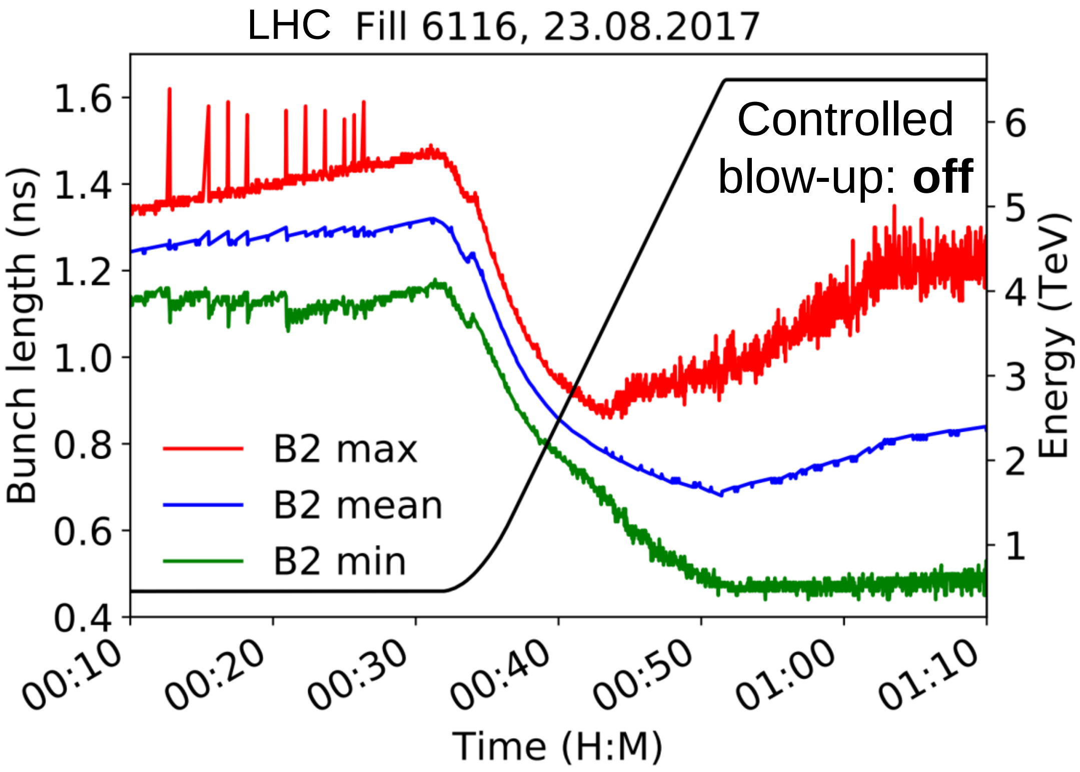
Loss of Landau damping (LD) in longitudinal plane is important intensity limitation in LHC.

Based on Sacherer formalism the intensity threshold is *(F. Sacherer, 1971)*

rf voltage bunch length*

$$N_b^{\mathrm{thr}} \propto \frac{V_{\mathrm{rf}}\tau^5}{(\mathrm{Im}Z/n)_{\mathrm{eff}}}$$

effective impedance



Bunch length shrinks during acceleration → Controlled blow-up must be applied to keep beam stable

*$\tau = \tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$ is scaled from full-width half-maximum (FWHM) bunch length

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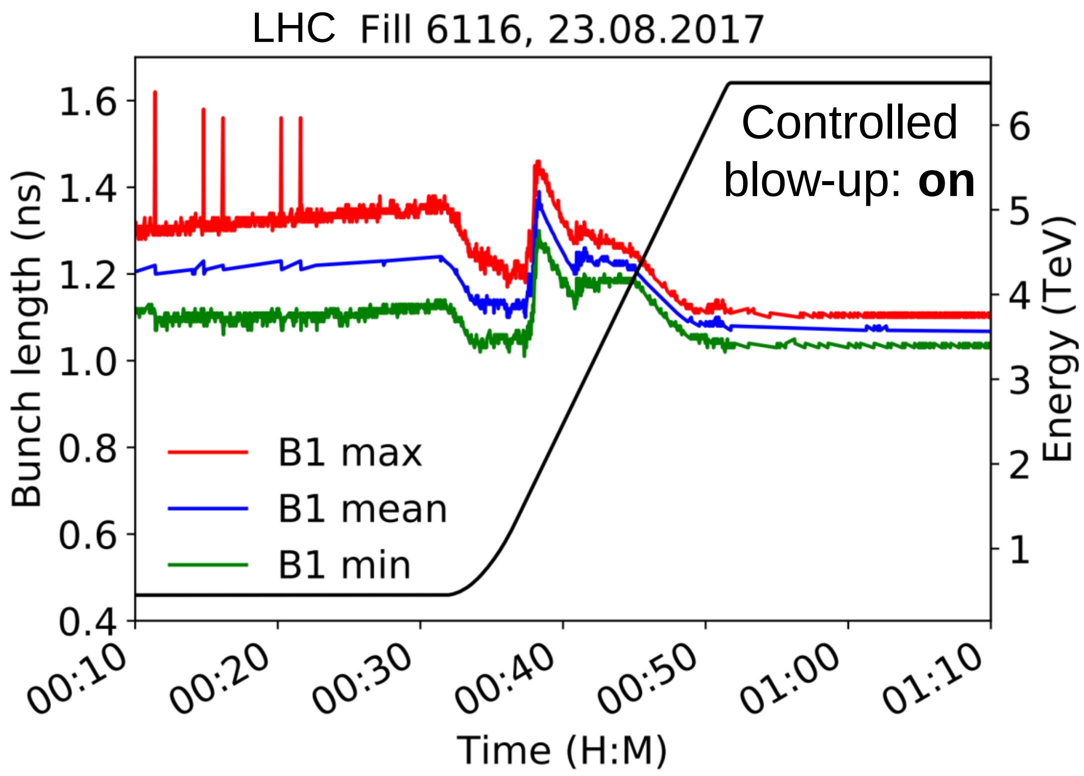

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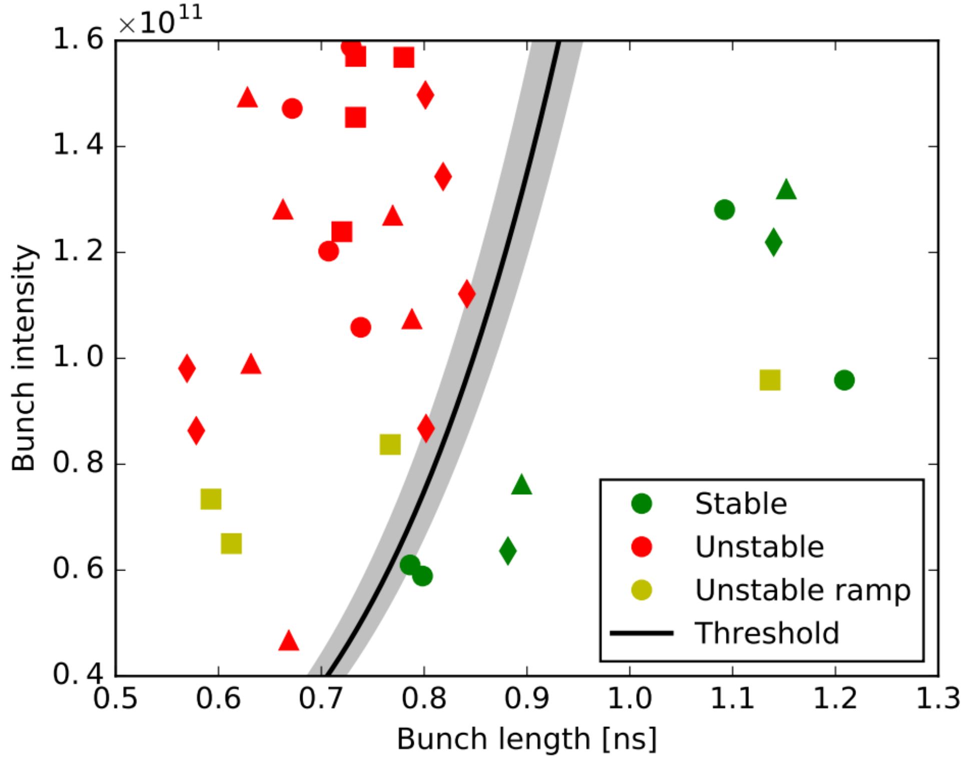
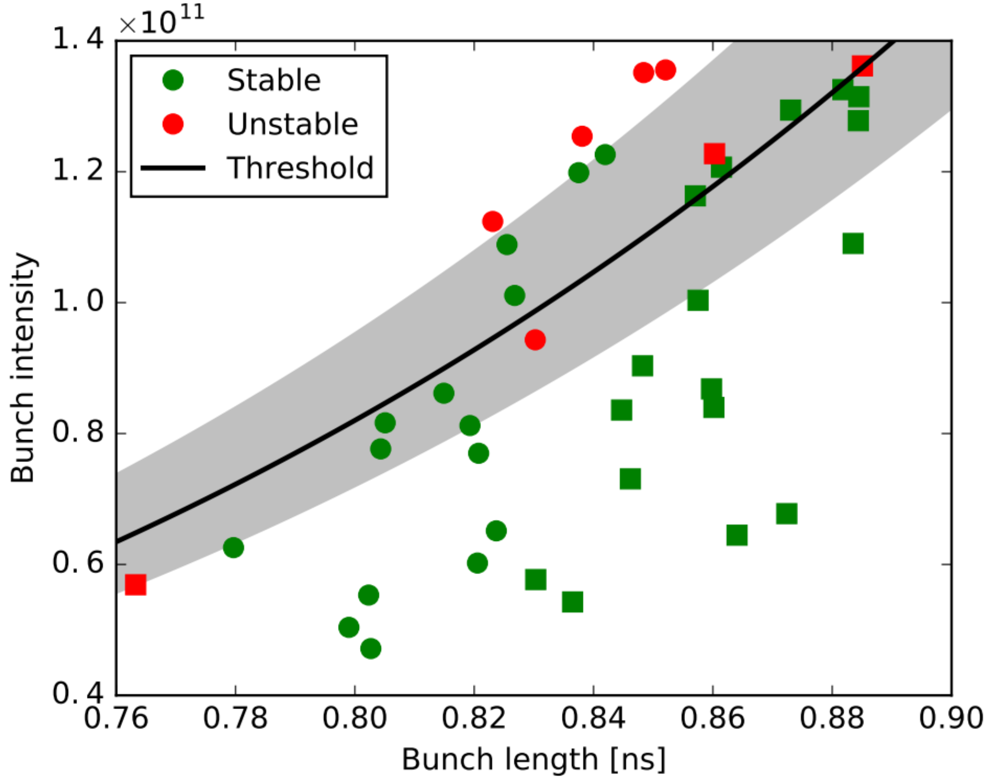
Loss of Landau damping in LHC: measurements

MDs in 2015, V_{rf} = 12 MV, beam energy 6.5 TeV

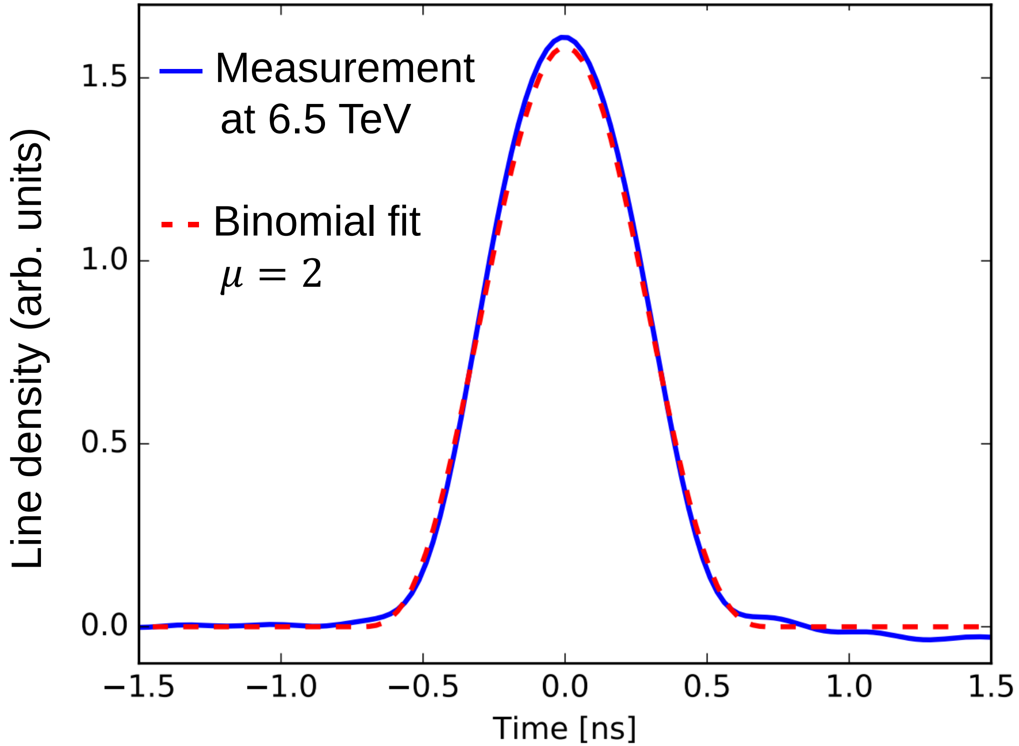


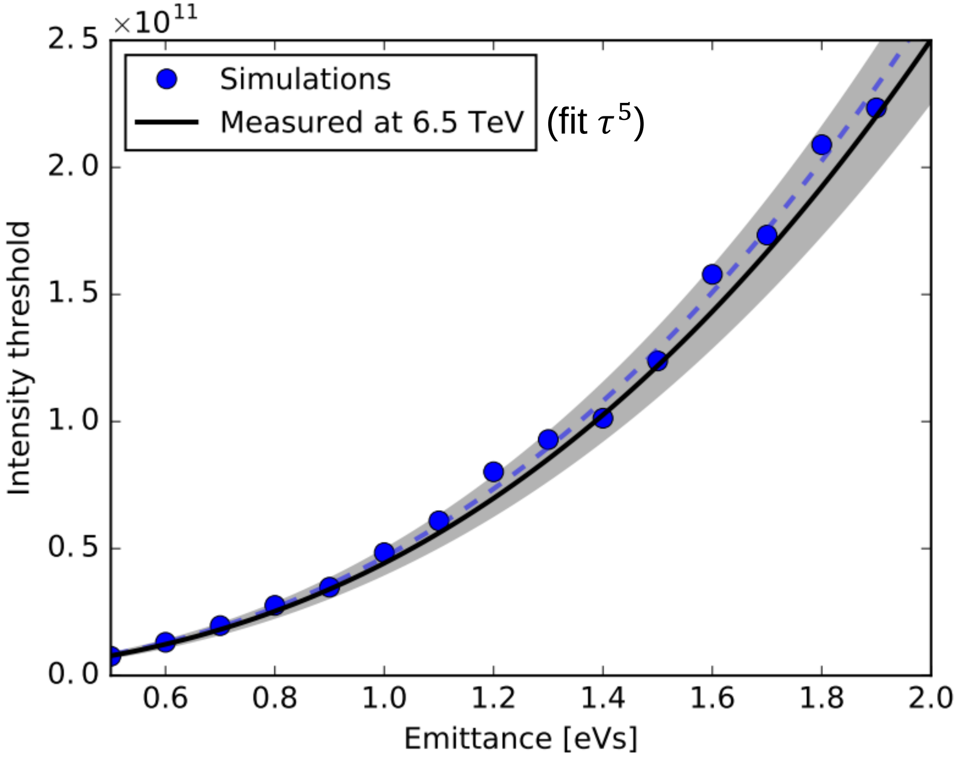


Measurements performed at different conditions and stability parameter was calculated for all bunches $\xi = V_{\mathrm{rf}}\tau^5/N_b$ *(PhD thesis J. E. Muller, 2016)*

→ The threshold $\xi_{\mathrm{th}} = 0.5 \times (\max \xi_{\mathrm{unst}} + \min \xi_{\mathrm{st}}) = \ (5.0 \pm 0.5) \times 10^{-5}\,(\mathrm{ns})^5\mathrm{V}$

Comparison of measurements and simulations

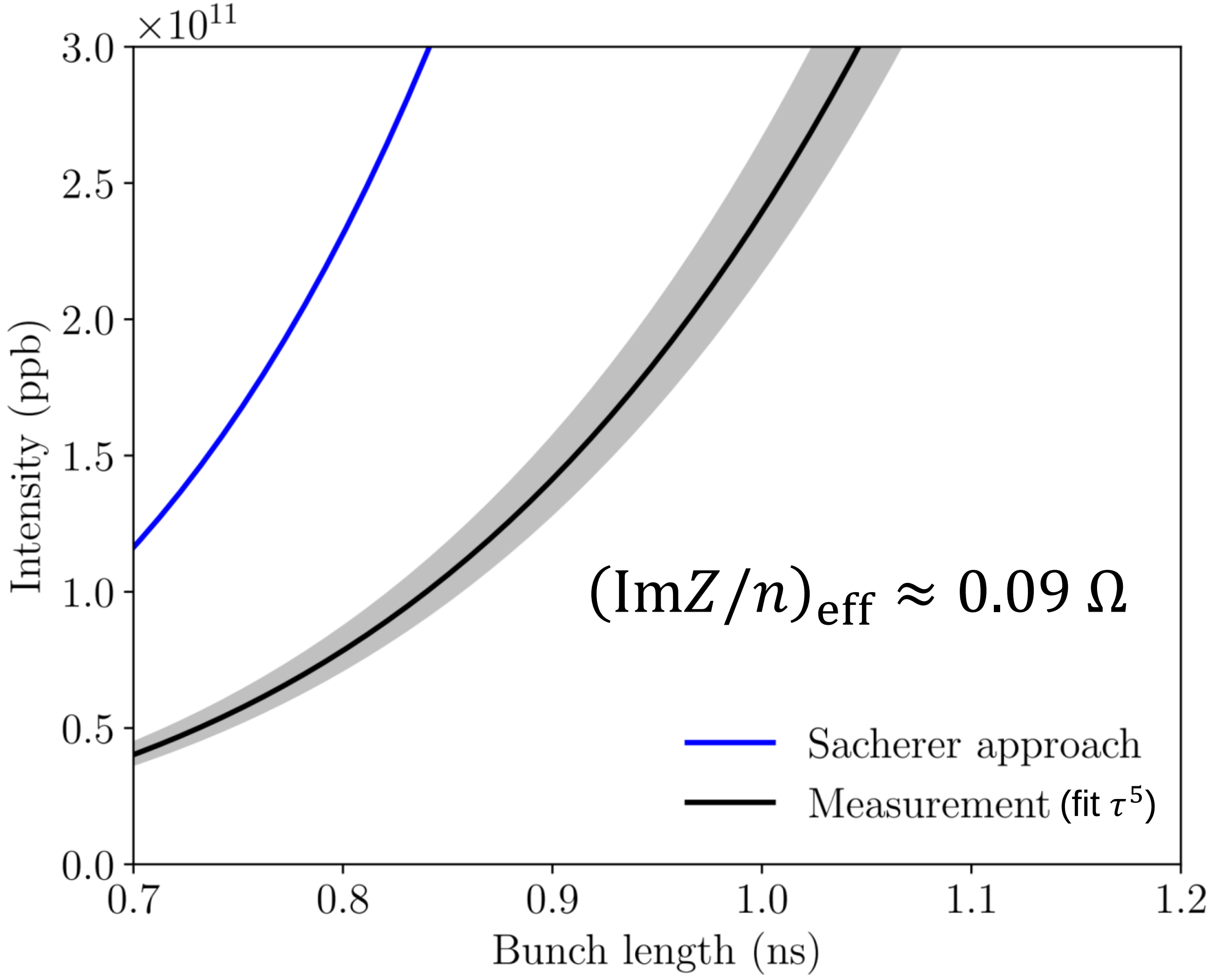




Simulation with full LHC longitudinal impedance model *(B. Salvant et al., HB2012)* & binomial bunch distribution $\lambda(t) = \lambda_0[1 - 4t^2/\tau^2]^{\mu+0.5}$

→ Very good agreement between measurements and BLonD particle tracking simulations (*PhD thesis J. E. Muller, 2016*).

Comparison of measurements and analytic calculations



→ The analytically calculated thresholds using Sacherer approach are 3 – 4 times higher.

→ We need a different approach to evaluate stability for future machines (for example FCC-hh).

Analytic approaches

Vlasov equation for a small perturbation of stationary distribution can be converted in matrix equations by:

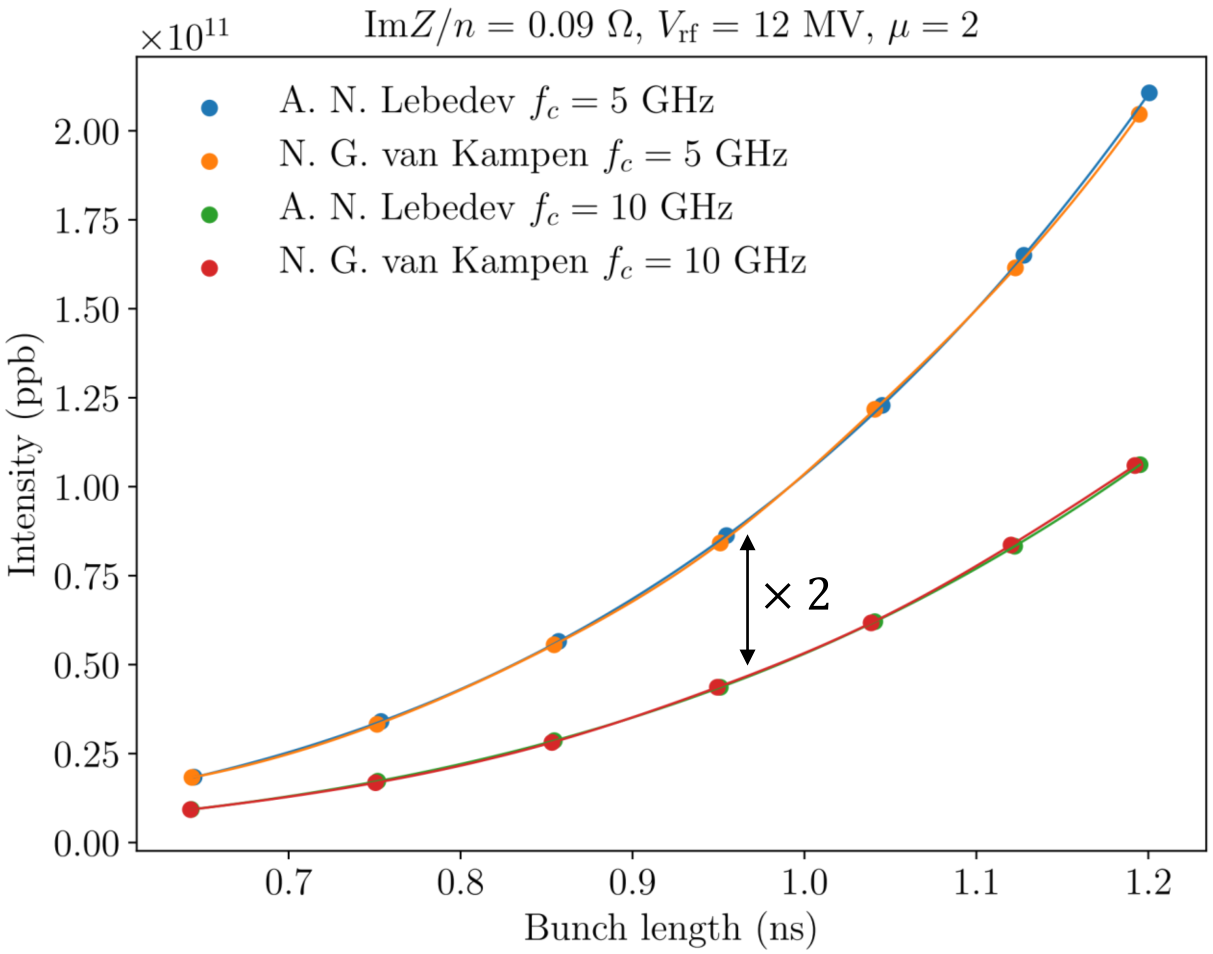
1. Discretization in action variable *(K. Oide & K. Yokoya, 1990)*.
   Loss of LD is interpreted as emerged van Kampen coherent modes (semi-analytic code by *A. Burov, 2010,* recently translated in Python by *T. Argyropoulos, 2019*).

2. Expanding in modes of ring azimuth *(A. N. Lebedev, 1967)*.

Both approaches were implemented in a new semi-analytic code MELODY (Matrix Equations for LOngituDinal stabilitY evaluation, 2019)

Comparison of analytic approaches

LHC at 6.5 TeV, $\mathrm{Im}Z/n = 0.09\ \Omega$ = const

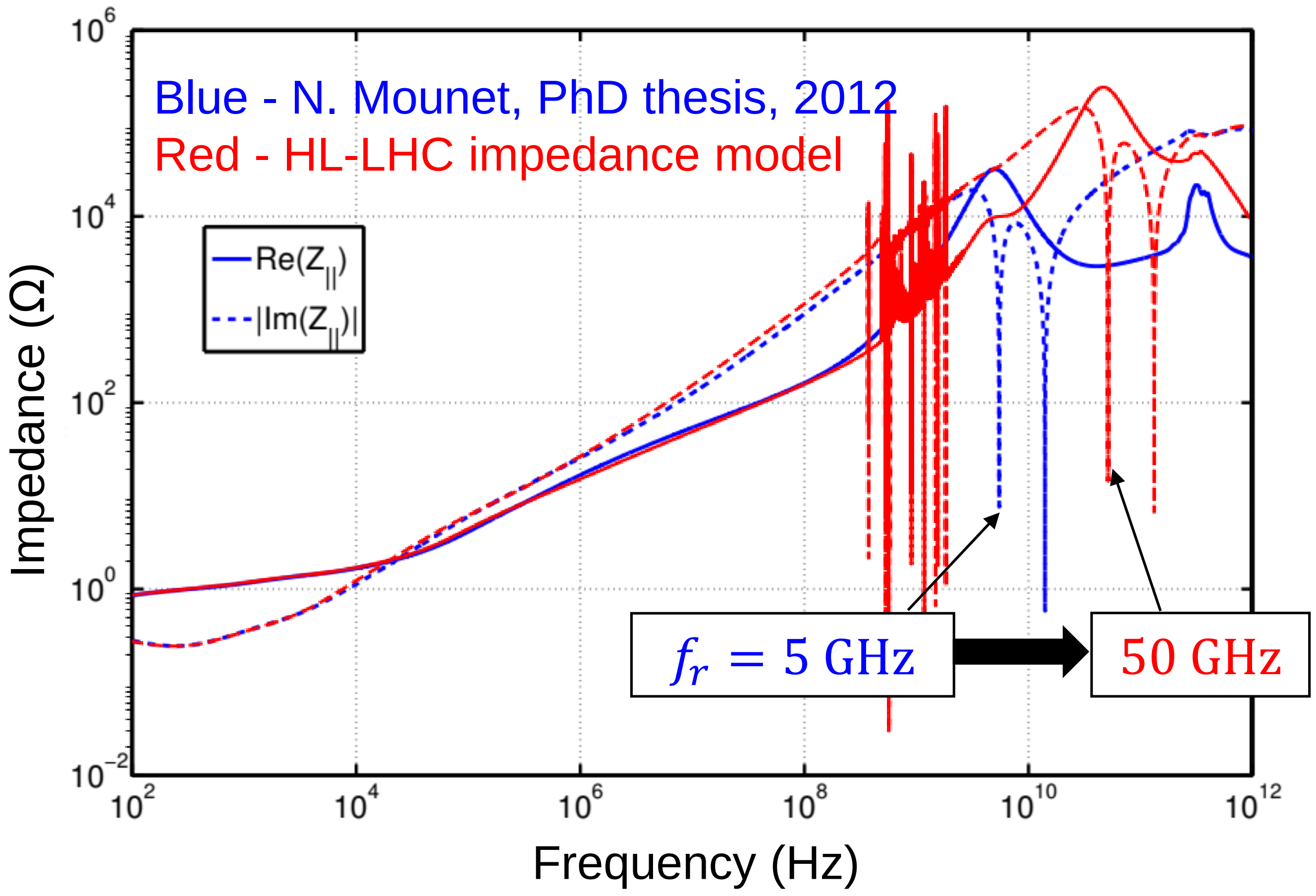


→ Very good agreement between both approaches

→ Often used for estimations assumption $\mathrm{Im}Z/n$ = const, turns out to be not physical (results don't converge, threshold depends on cut-off frequency f_c)

→ Realistic impedance model needs to be used

LHC/HL-LHC impedance model



Broad-band impedance model
(LHC Design Report, 2004)

$$R = Q\ \mathrm{Im}Z/n \frac{f_r}{f_0}$$

$$Q = 1, f_r = 5\ \mathrm{GHz}$$

$\mathrm{Im}Z/n = 0.07\ \Omega$ for injection optics

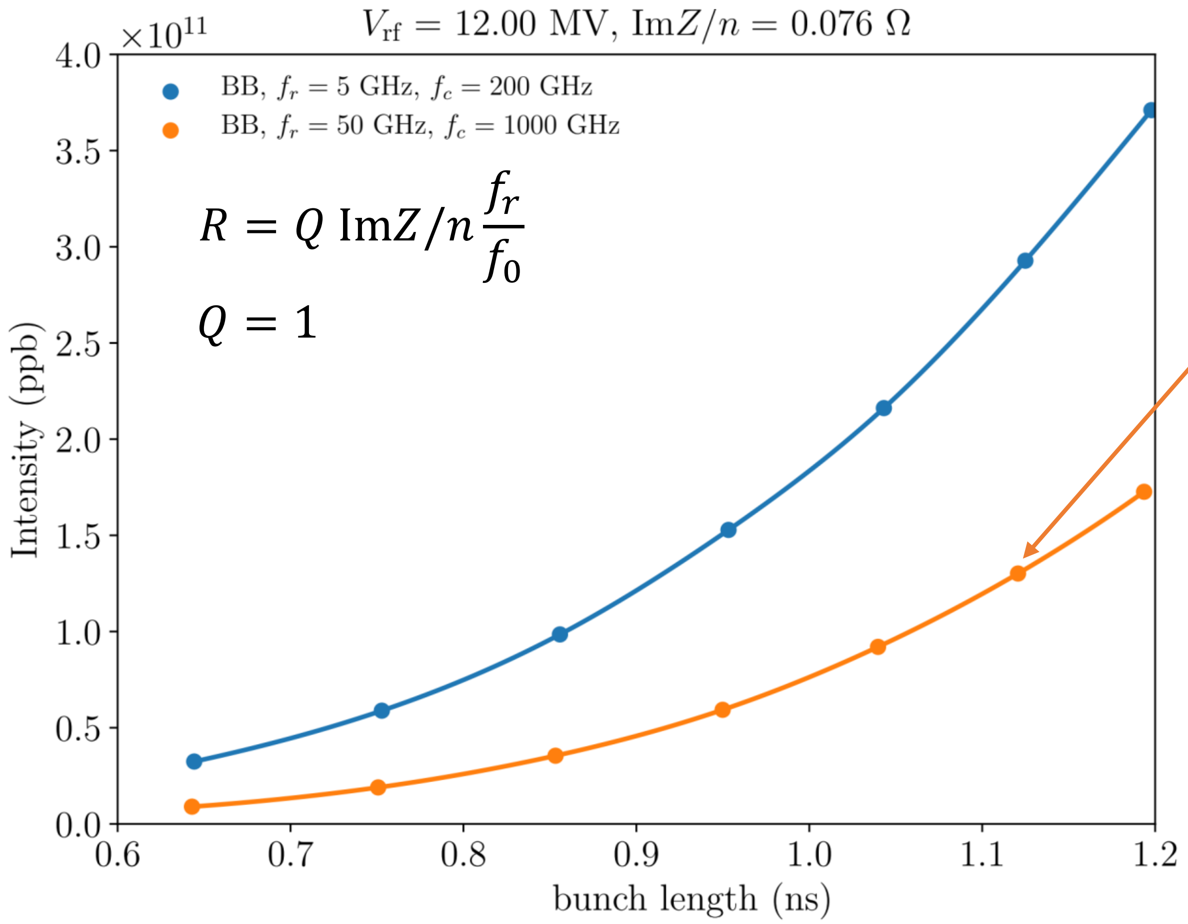
$\mathrm{Im}Z/n = 0.076\ \Omega$ for squeezed optics

→ The resonant frequency of the broadband model was changed from 5 GHz to 50 GHz (*D. Amorim, 2018*).

→ This change has a significant impact on longitudinal single-bunch stability

Stability threshold for broad-band impedance

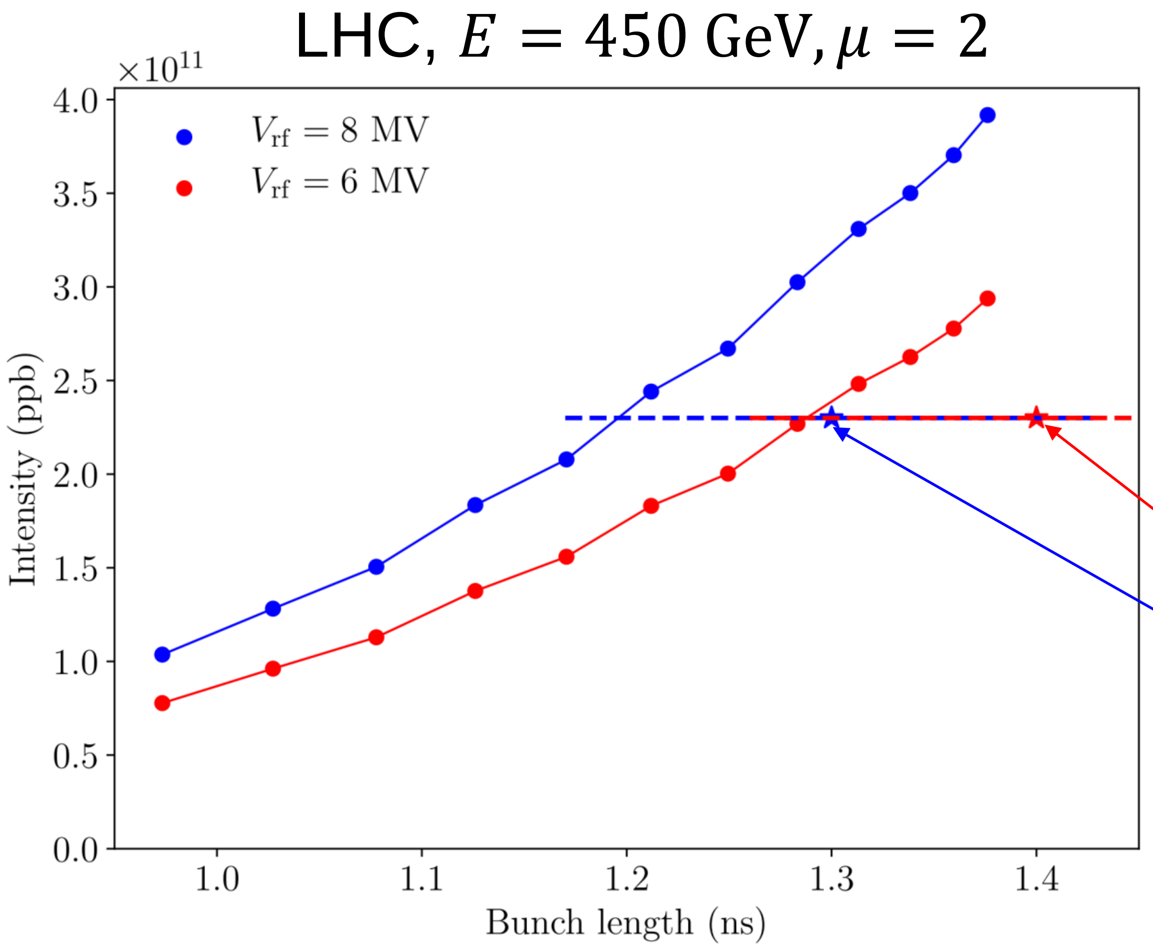
MELODY results



→ Changing resonant frequency from 5 GHz to 50 GHz results in reduction of the threshold by factor of 3.

→ The LHC/HL-LHC broad-band impedance model needs to be revised.

Single-bunch stability at 450 GeV



Results using MELODY for smoothed impedance (resistive wall + broad-band model at 5 GHz)

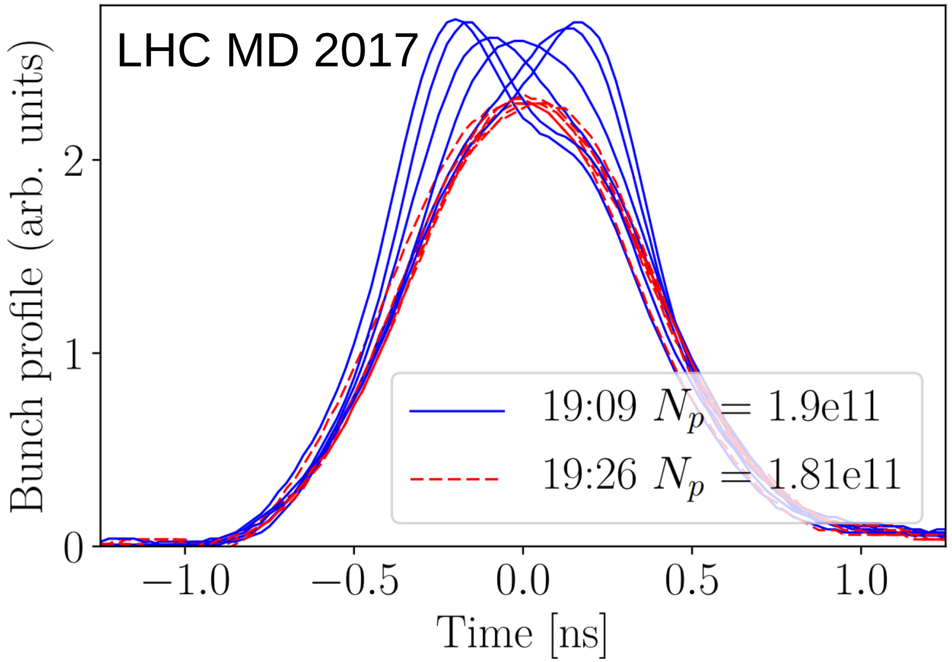
For LIU bunch from SPS (1.65 ns, 10MV@200MHz + 1.6 MV@800 MHz), bunch length in LHC (in absence of injection errors):

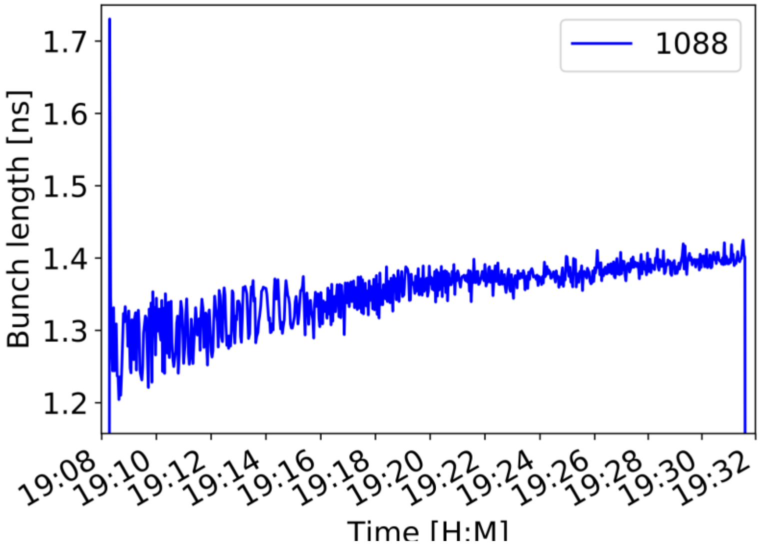
1.4 ns for 6 MV (LHC nominal 2017)

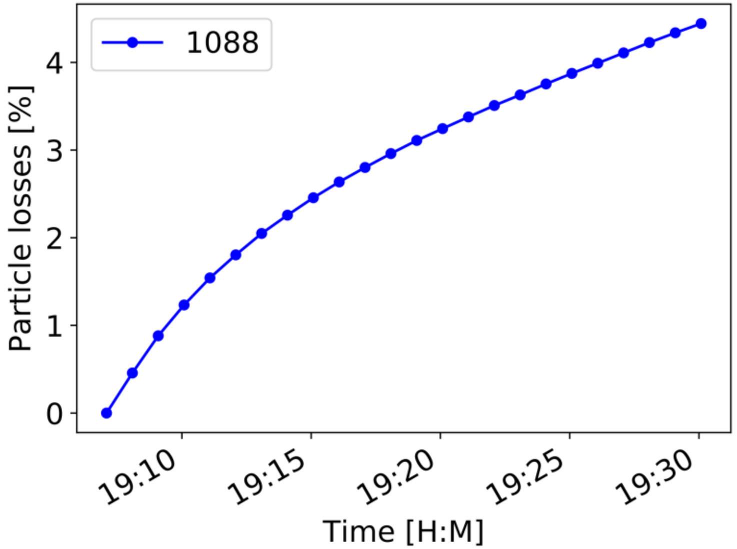
1.3 ns for 8 MV (HL-LHC design report)

Two voltages V_{rf} provide similar single-bunch stability

Persistent oscillations after injection



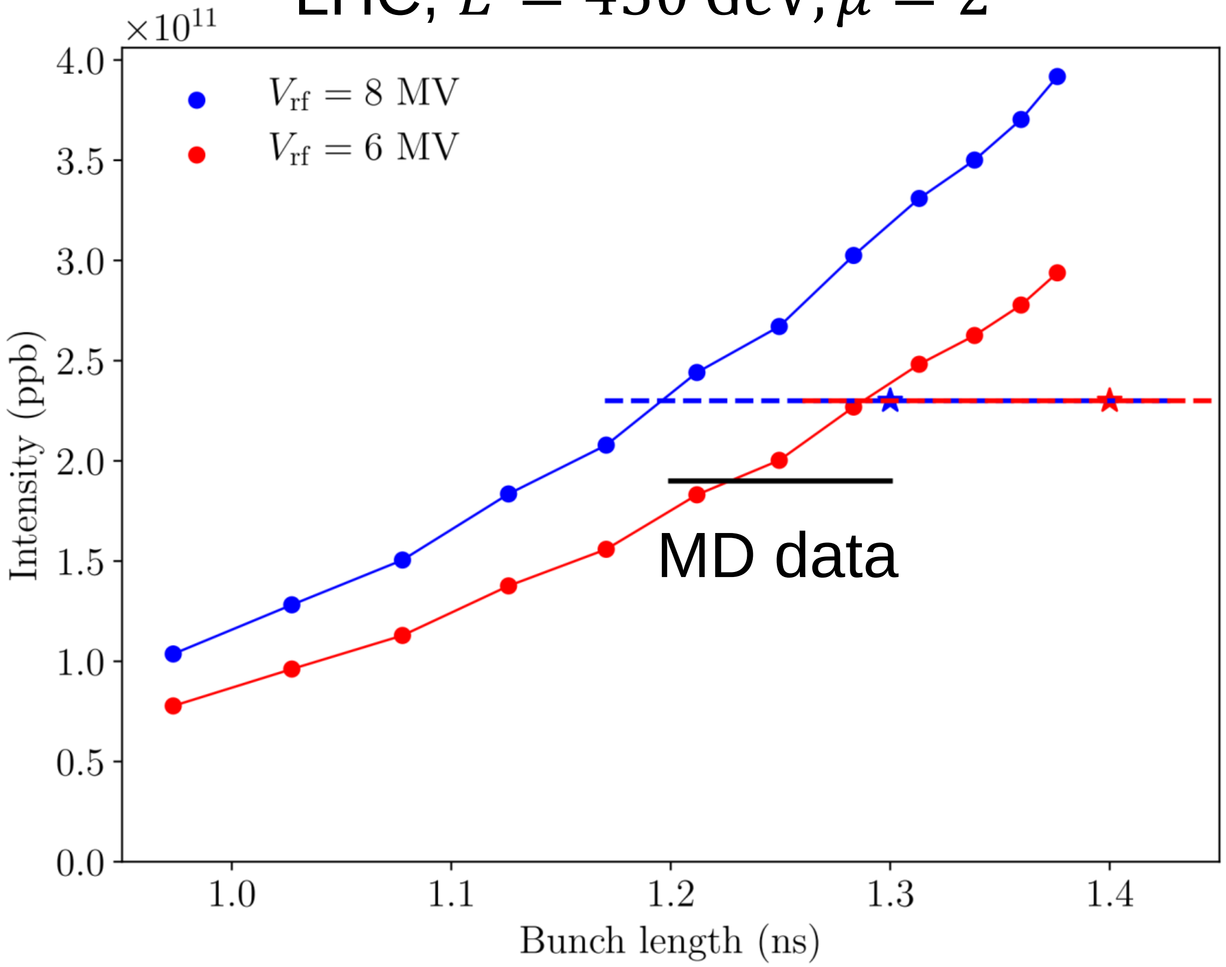


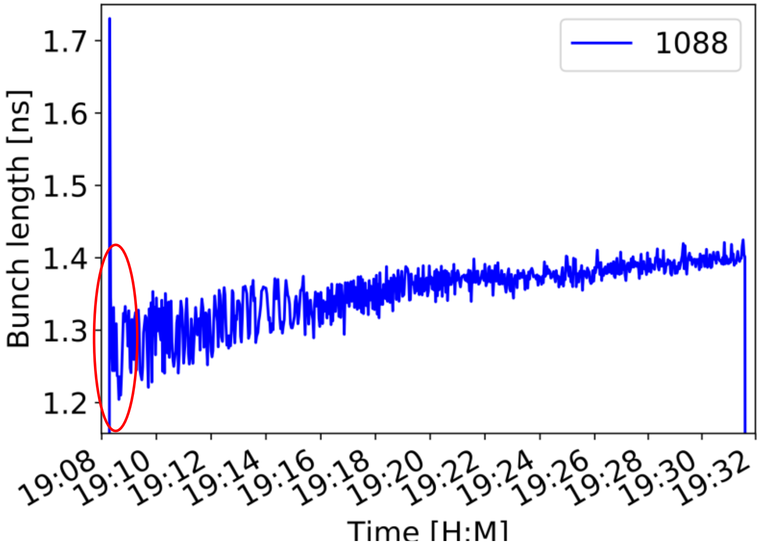


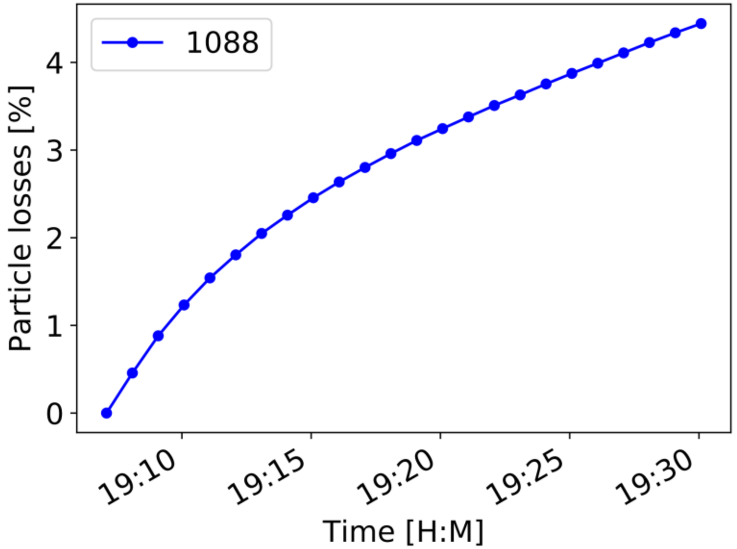
During 20 min oscillations lead to ~10 % bunch lengthening and ~5% particle loss *(H. Timko et al., HB2018)*
Similar oscillations were observed in Tevatron (*R. Moore, PAC2003*)

Persistent oscillations after injection

LHC, $E = 450$ GeV, $\mu = 2$

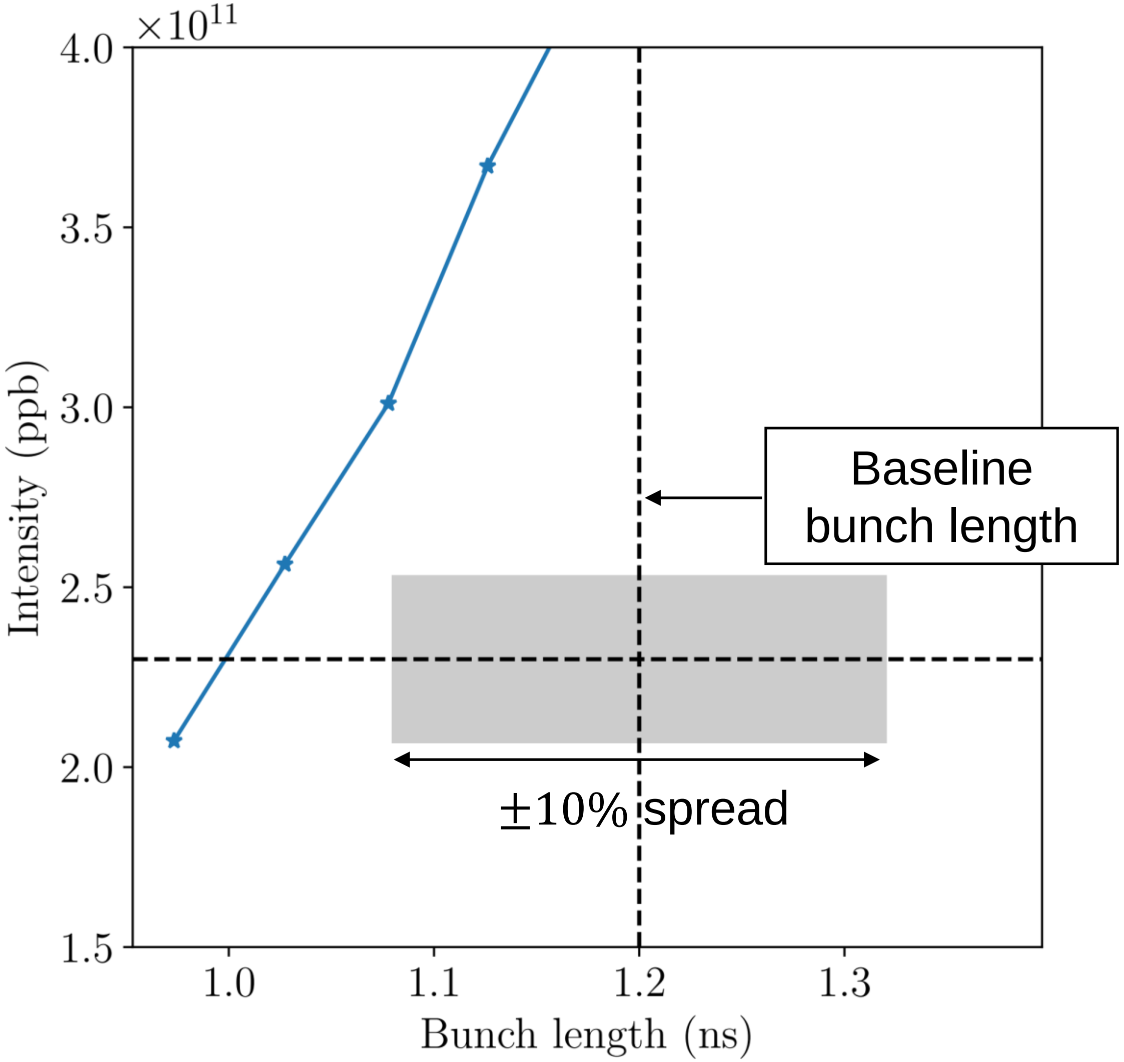






Single-bunch stability at 7 TeV

LHC, $V_{\mathrm{rf}} = 16\ \mathrm{MV}, E = 7\ \mathrm{TeV}, \mu = 2$



Results using MELODY for smoothed impedance (resistive wall + broad-band model at 5 GHz)

→ Sufficient stability for $\tau = 1.2$ ns with margin for $\pm 10\%$ bunch length (and intensity) spread

Next steps:
→ To repeat calculations with revised broad-band impedance model
→ To study effect of High-Order-Modes (HOM) on single-bunch stability

Stability of multi-bunch beam

Multi-bunch instabilities were not observed so far in LHC

HL-LHC: higher intensity & HOMs of crab cavities (CC)

For ≈ 3000 bunches macro-particle simulations are computationally expensive

$\rightarrow$ Analytical approaches are used to define requirements for HOM damping

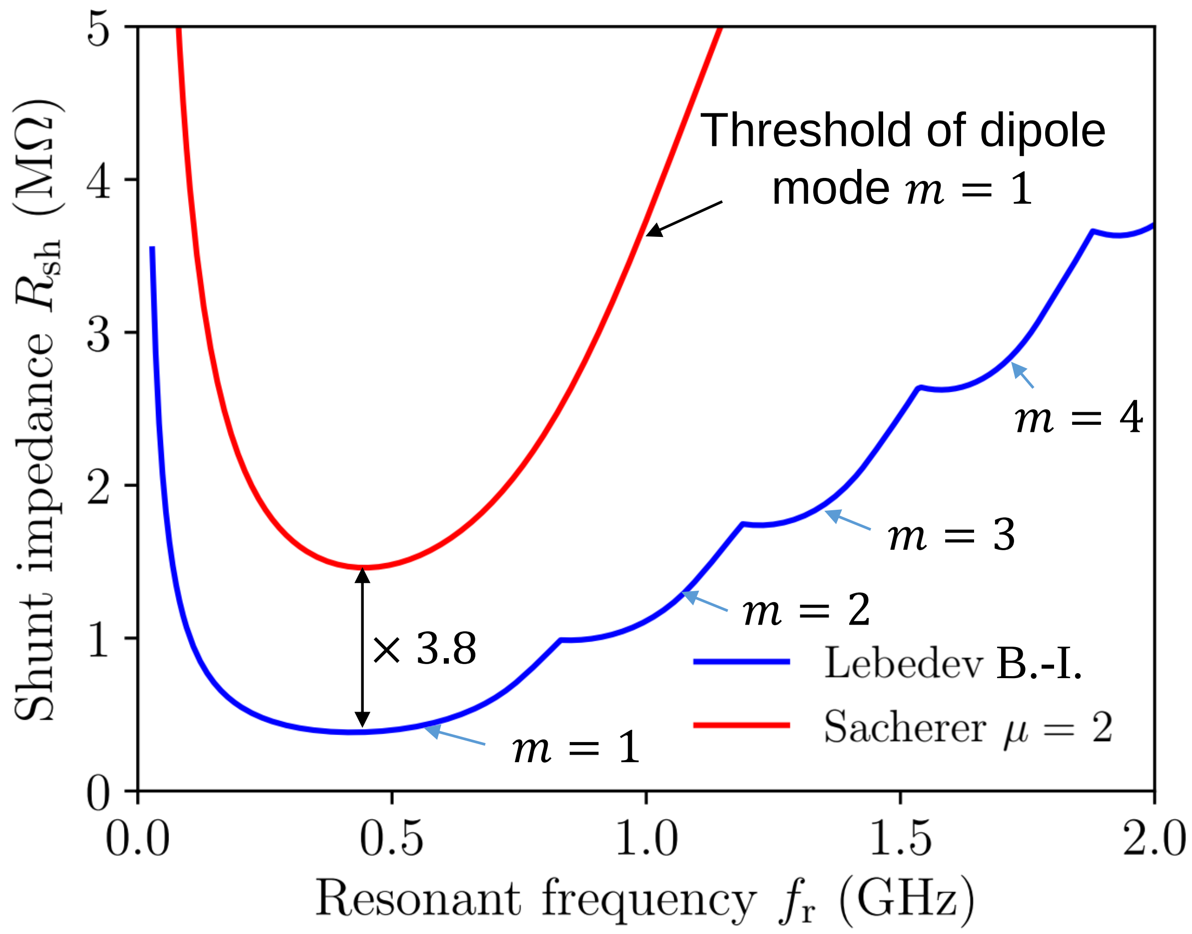
Analytical stability evaluation can be based on:

- Sacherer stability diagram *(F. Sacherer, 1971)*
- Lebedev equation (stability diagram by *V. Balbekov, S. Ivanov, 1987)*

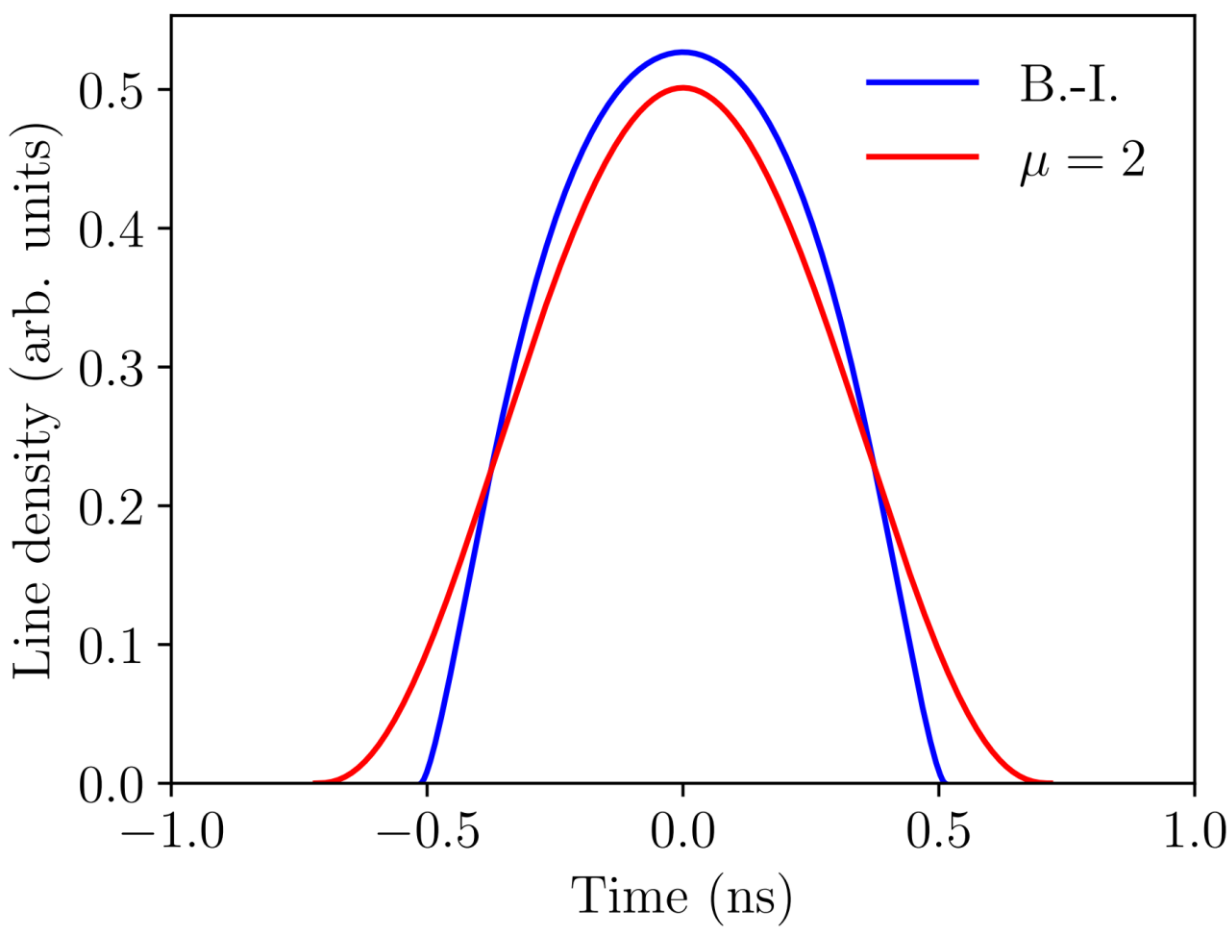
There was a significant discrepancy between the results of two approaches (*E. Shaposhnikova, LHC-CC'10* and *A. Burov, LHC-CC'11*)

Lebedev vs Sacherer approach

$V_{\mathrm{rf}} = 16\ \mathrm{MV}, \tau = 1.2\ \mathrm{ns}, E = 7\ \mathrm{TeV}$

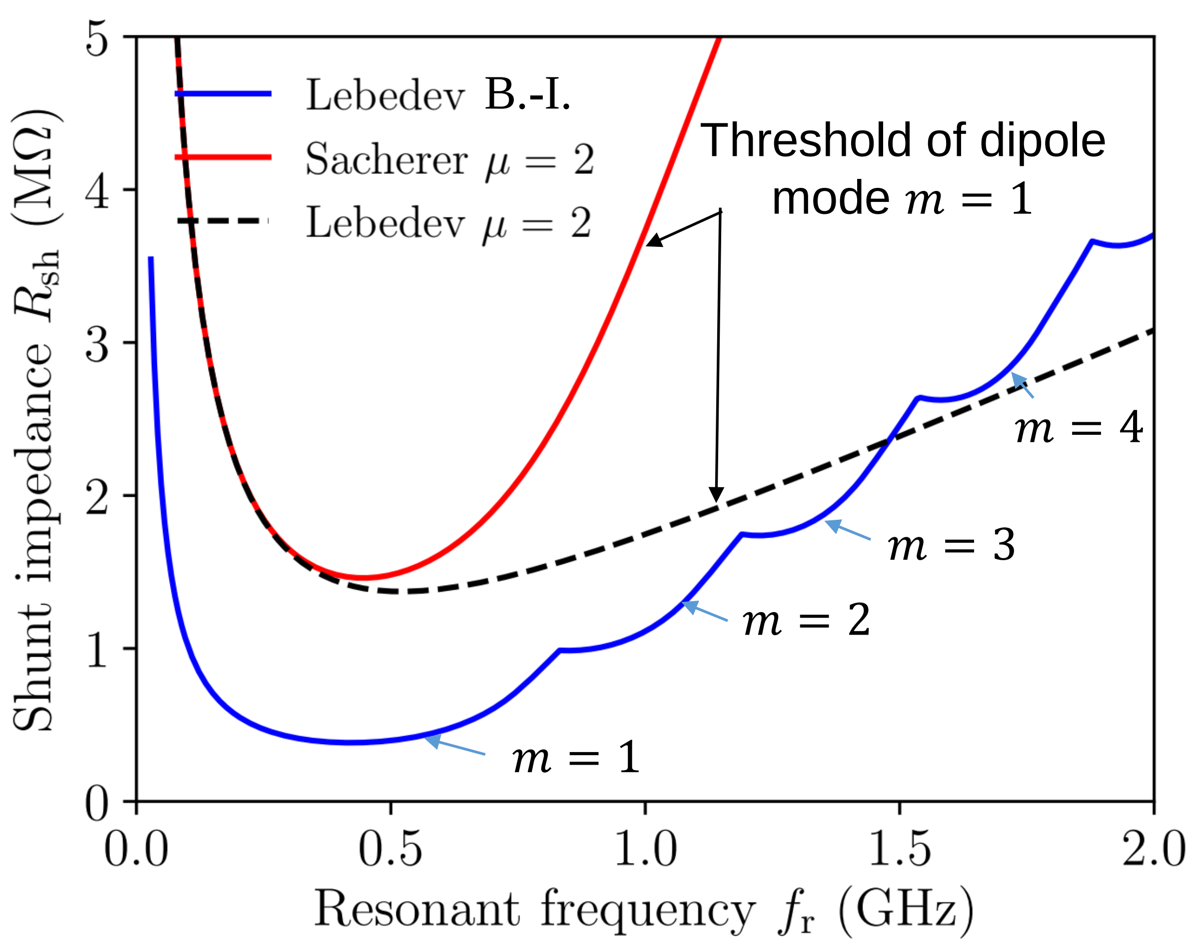


Binomial vs Balbekov-Ivanov distribution



Lebedev vs Sacherer approach

$V_{\mathrm{rf}} = 16\ \mathrm{MV}, \tau = 1.2\ \mathrm{ns}, E = 7\ \mathrm{TeV}$



→ Factor of 4 difference is due to different distribution function.

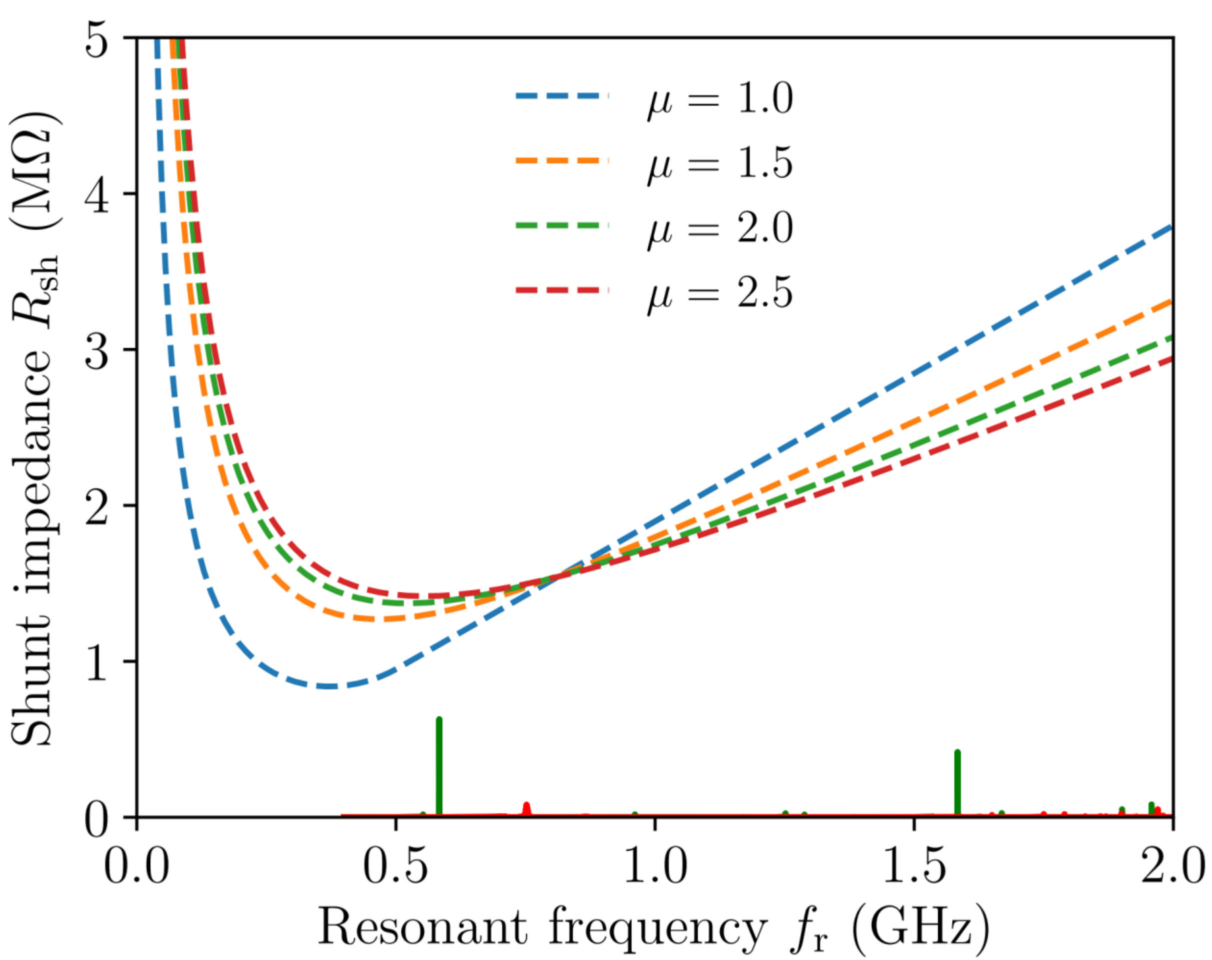
→ Stability diagram approach based on Lebedev equation was extended to binomial distribution.

→ For $\mu = 2$, the minimum thresholds are similar, but Sacherer approach underestimates threshold at higher frequencies

→ Sacherer approach can be obtained as a low frequency expansion of Lebedev equation (*E. Shaposhnikova et al., MCBI19*)

Results for HL-LHC flat top

$V_{\mathrm{rf}} = 16\ \mathrm{MV}, \tau = 1.2\ \mathrm{ns}, E = 7\ \mathrm{TeV}$



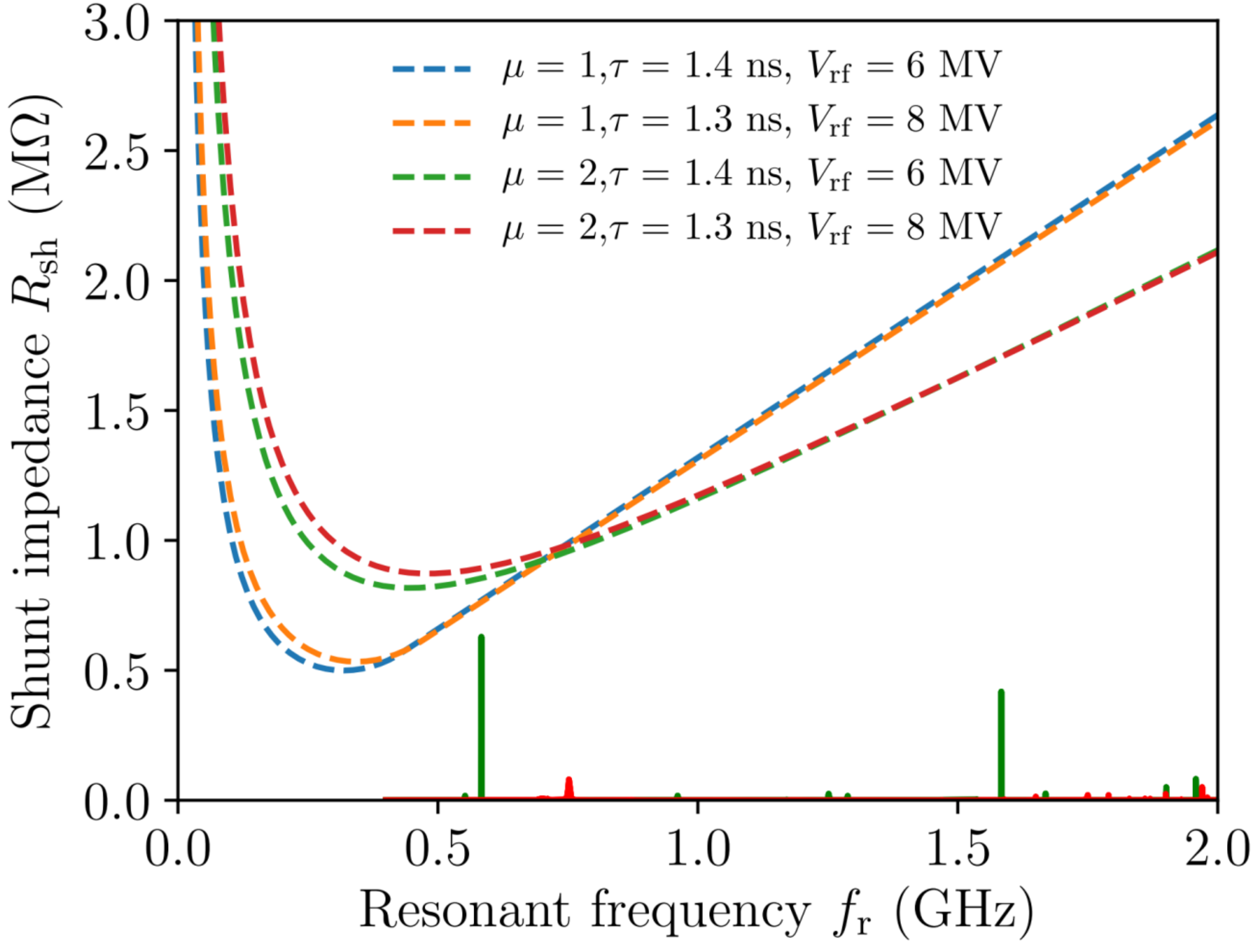
Crab cavity HOMs:
HL-LHC Double Quarter Wave (DQW) × 4
HL-LHC RF-Dipole (RFD) × 4

→ Thresholds for distributions with different μ and the same FWHM bunch length are similar (except $\mu = 1$)

→ Only one HOM is close to the stability limit for the worst-case scenario without frequency spread between CC.

Results for HL-LHC flat bottom

$E = 450$ GeV



Crab cavity HOMs:
HL-LHC Double Quarter Wave (DQW) × 4
HL-LHC RF-Dipole (RFD) × 4

→ Thresholds are similar for 6 MV and 8 MV of rf voltage for the same bunch parameters at the SPS extraction.

→ Recommendation: further damping of the first high Q mode of DQW CC could be addressed for margin in machine operation.

Summary

Single-bunch stability:

- Bunch parameters are affected by the loss of Landau damping
- Sacherer stability diagram in longitudinal plane should be used with caution. More complete formalisms (van-Kampen modes and Lebedev equation) are available for accurate semi-analytical threshold estimations.
- LHC/HL-LHC impedance model needs to be revised for longitudinal stability evaluation.

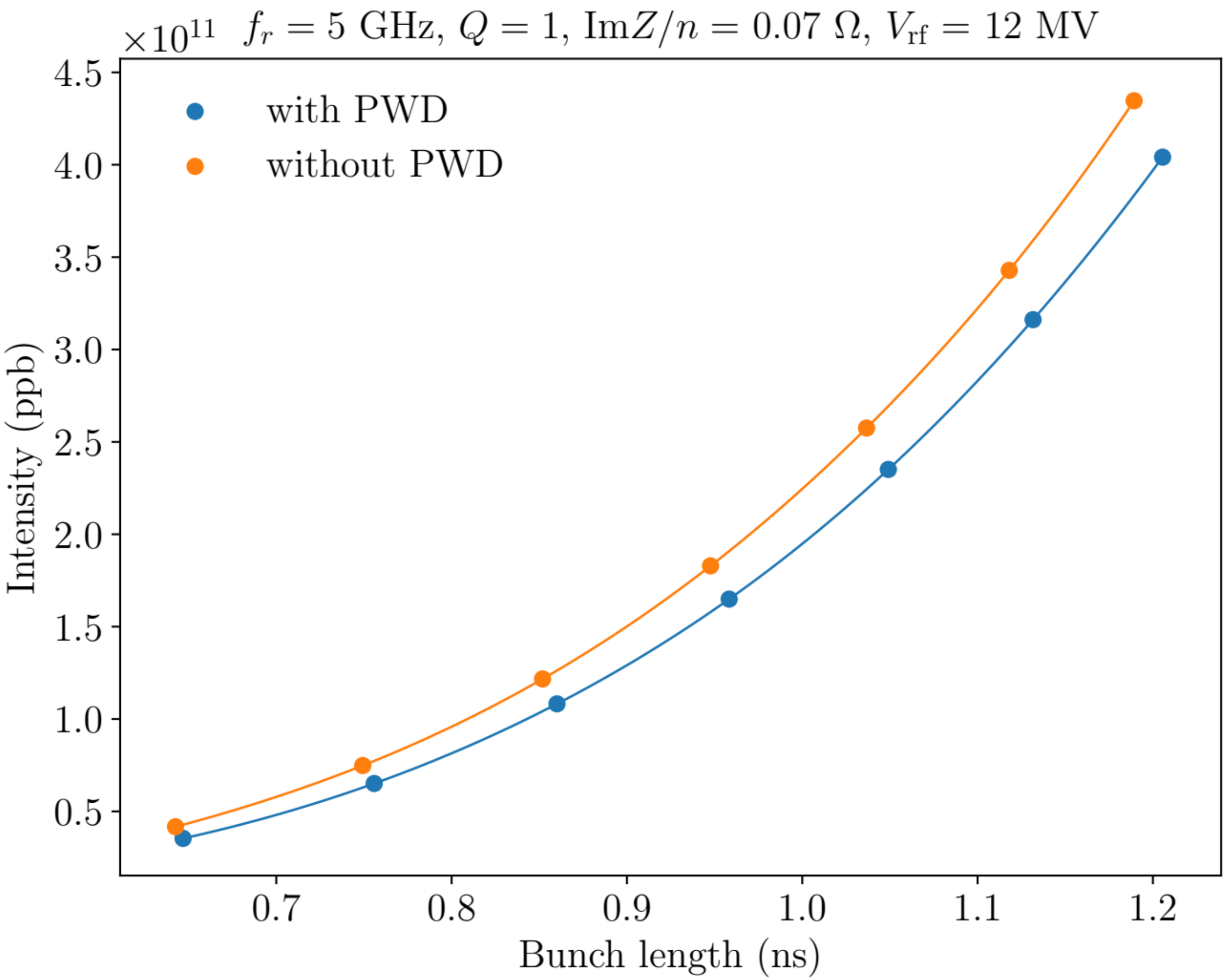
Multi-bunch stability:

- Thresholds of coupled-bunch instability depend on distribution function but are similar for the same FWHM bunch length (binominal distribution).
- To increase stability margin, a spread of HOM frequencies between crab cavities needs to be introduced and further damping of the first high Q mode of DQW CC is recommended.

Thank you for your attention!

Spare slides

Impact of potential well distortion



Difference is about 10 %

Landau damping for multi-bunch beam

For narrow band impedance with ω_r, only one resonant harmonic $k_r = \omega_r/\omega_0 = lM + n$ can be kept (M - number of equidistant bunches) in Lebedev' equation:

$$\frac{k}{Z_k} = -\frac{i I_0\, h M}{V\cos\phi_s} G_{kk}(\Omega)$$

From stability diagram *(V. Balbekov, S. Ivanov, 1987):*

$$\frac{k}{Z_k} > \frac{I_0\, h M}{V\cos\phi_s}\,\mathrm{Im}G_{kk}^{max}$$

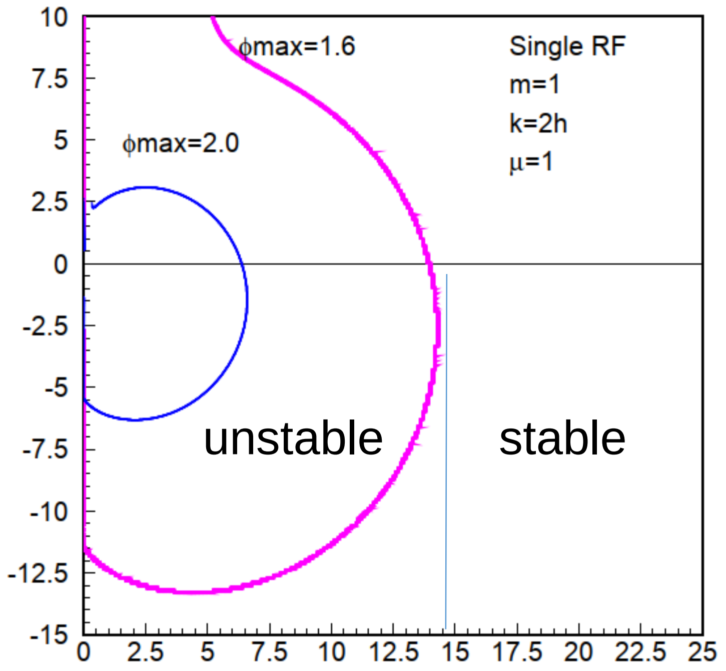
with

$$\mathrm{Im}G_{kk}(\Omega) = \frac{\pi}{A_N}\sum_{m=1}^{\infty}\frac{F'(\mathcal{E}_m) I_{mk}^2(\mathcal{E}_m)}{\omega_s'(\mathcal{E}_m)}$$

→ Beam is stable if vertical line $1/R_{sh}$ is inside stability region

Stability diagram for

$$F(\mathcal{E}) = F_0(1-\mathcal{E}/\mathcal{E}_{max})^{\mu}$$



$$Z_k^{-1} = 1/R_{sh} + i\,Q(\omega/\omega_r - \omega_r/\omega)/R_{sh}$$

Multi-bunch threshold

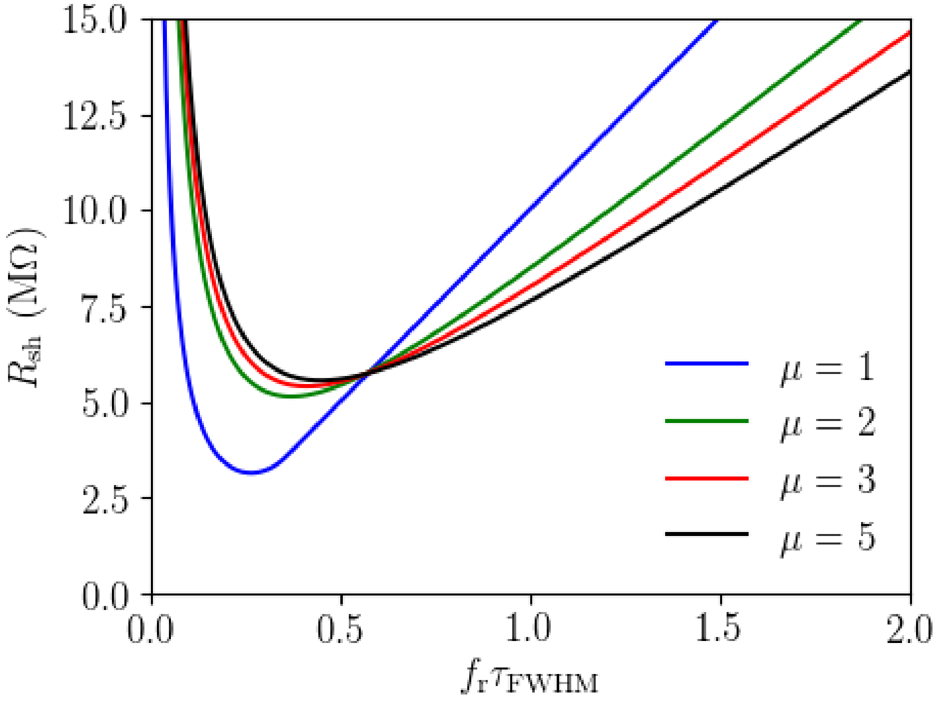
In single RF system the threshold (no acceleration) for binomial distribution

$$F(\mathcal{E}) = F_0(1-\mathcal{E}/\mathcal{E}_{max})^{\mu}$$

$$R_{sh} < \frac{V\ (\pi f_{rf}\tau_b)^3}{32\, I_0}\, G_{\mu}(f_r\tau_b)$$ where

$$G_{\mu}(x) = \frac{x}{\mu(\mu+1)} \min_{y\in[0,1]} \left[\sum_{m=1}^{\infty}(1-y^2)^{\mu-1} J_m^2(\pi x y)\right]^{-1}$$

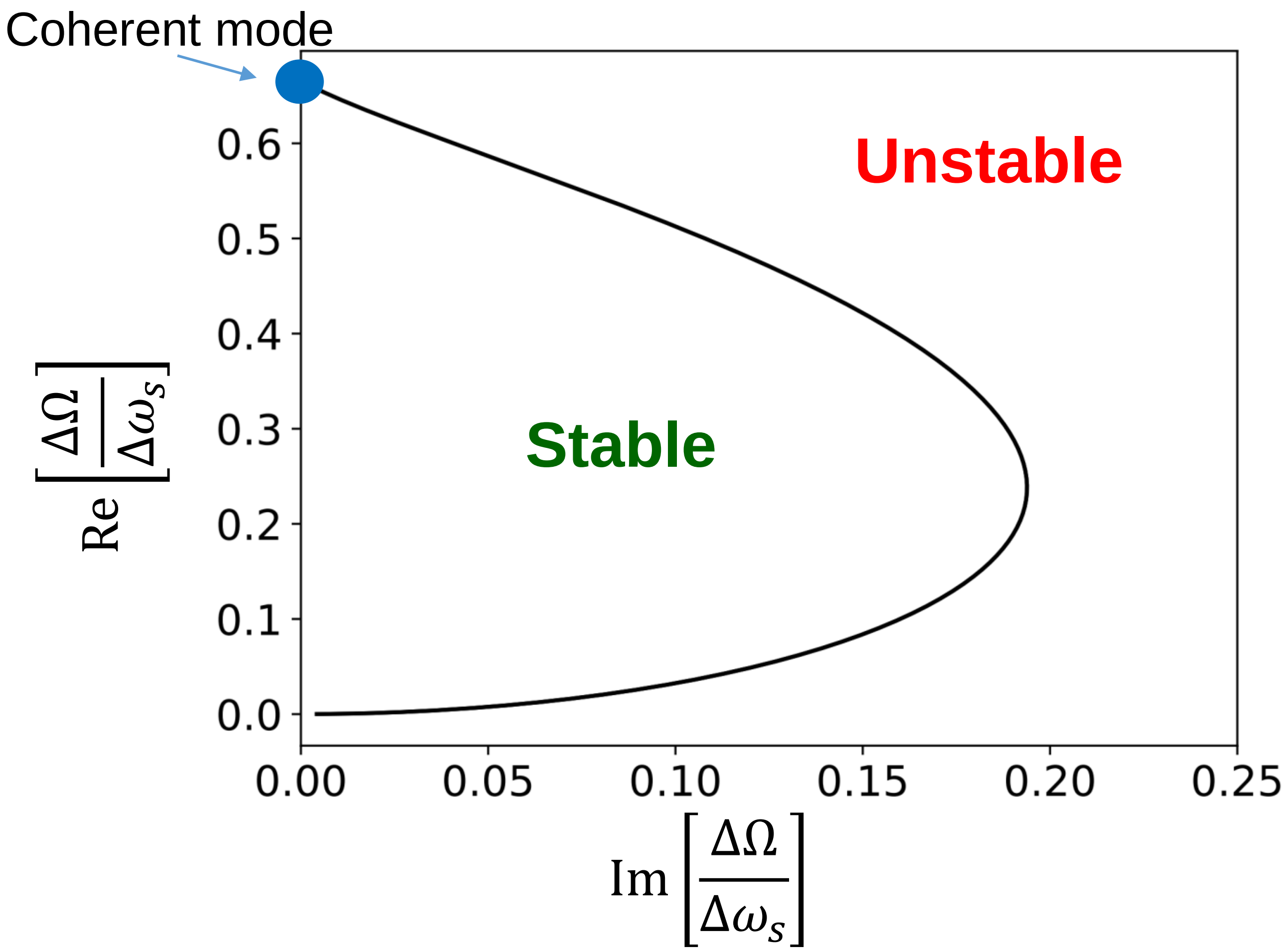
→ The FWHM bunch length is important



Threshold R_{sh} for coupled-bunch instabilities in FCC-hh at 50 TeV for nominal intensity N_b=10^{11}, V_{rf} = 38 MV, γ_t =99.3 *(I. K, E.Shaposhnikova, IPAC'19)*

Sacherer's formalism

Stability diagram for parabolic line density



Landau damping is lost if coherent mode shift $\Delta\Omega$ normalized by incoherent spread $\Delta\omega_s$ lies outside of stability diagram (F. Sacherer, 1971)

Simplified threshold $N_b = V_{\mathrm{rf}}\tau^5/\xi$

Stability parameter $\xi \propto (Z/n)_{\mathrm{eff}}$

Effective impedance

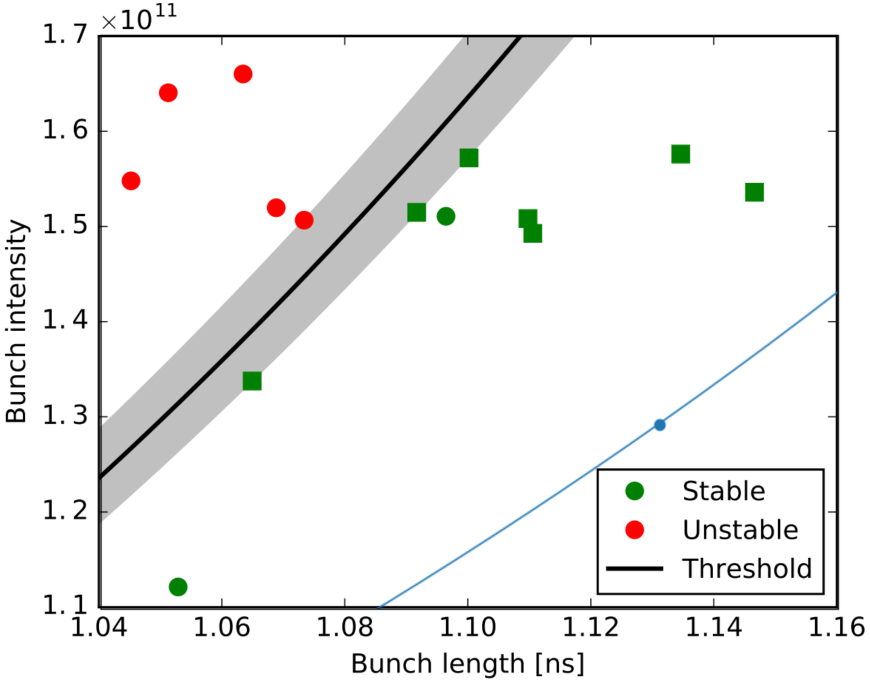
$$(Z/n)_{\mathrm{eff}} = \omega_0 \frac{\sum_q \frac{Z^{\parallel}(\omega_q)}{\omega_q}\frac{J_{3/2}^2(\omega_q\tau)}{|\omega_q\tau|}}{\sum_q \frac{J_{3/2}^2(\omega_q\tau)}{|\omega_q\tau|}}$$

$\omega_q = q\omega_0 + \omega_s$

→ For the case of the LHC impedance model $\xi \approx 1.4 \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$, ($(Z/n)_{\mathrm{eff}} \approx 0.09\ \Omega$)

Measurements of the loss of Landau damping threshold in LHC

MD 2011,
V_{rf} = 5 MV, E = 450 GeV



Measurement were performed at different conditions, but with all efforts only a limited parameter space was available during each of MDs (*PhD thesis J. E. Muller, 2016*). Threshold curves correspond to a fit $N_b = V\tau^5/\xi$.

→ As the result, $\xi \ = \ (5.0 \pm 0.5) \times 10^{-5}\ (\mathrm{ns})^5\mathrm{V}$ was obtained.

→ The thresholds predicted from Sacherer' stability diagrams are 3 – 4 times higher than measured thresholds.

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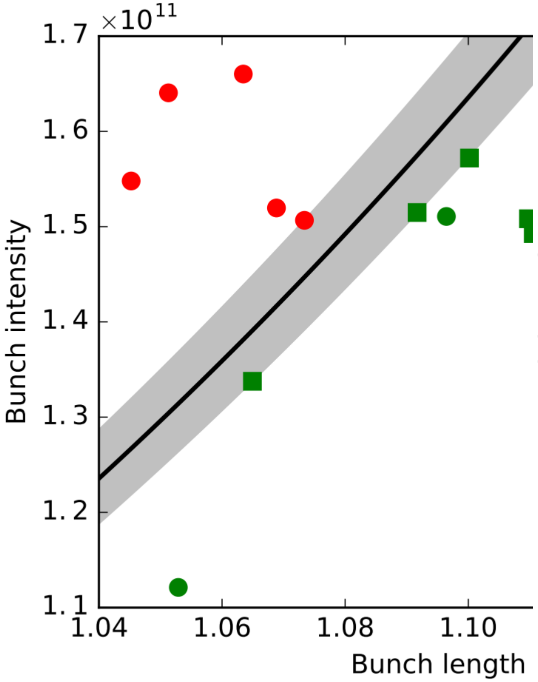
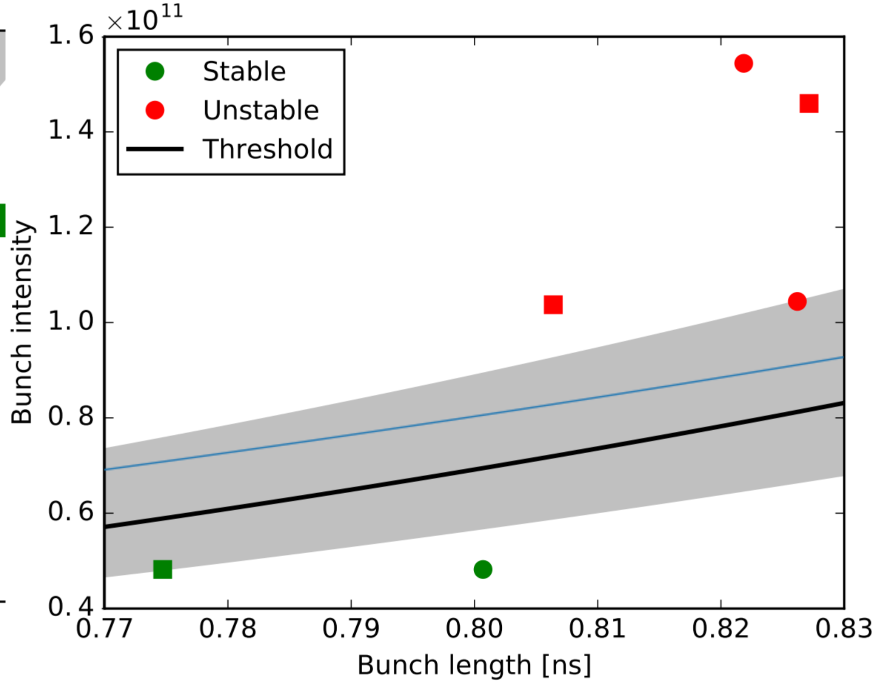


MD 2011,
V_{rf} = 12 MV, E = 4 TeV



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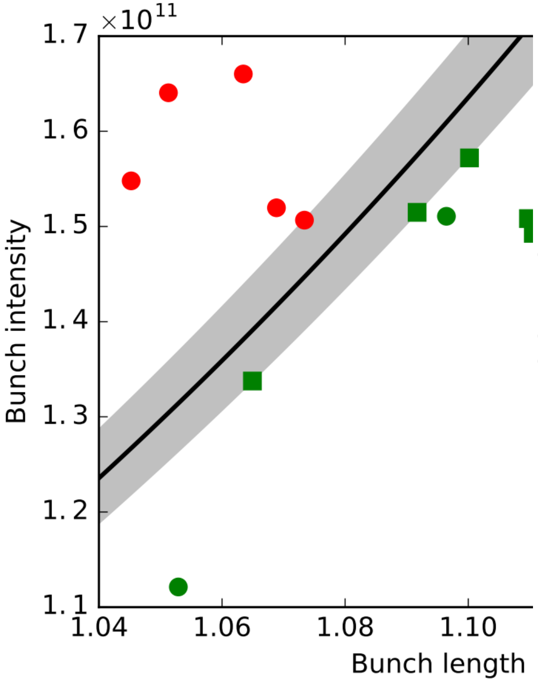

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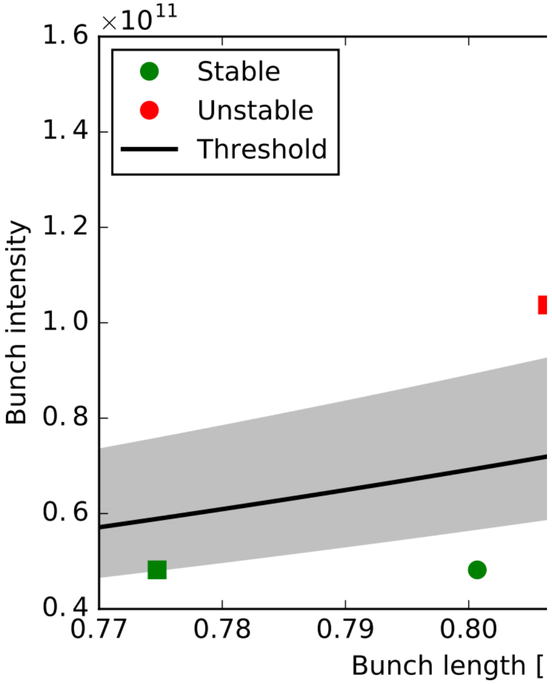
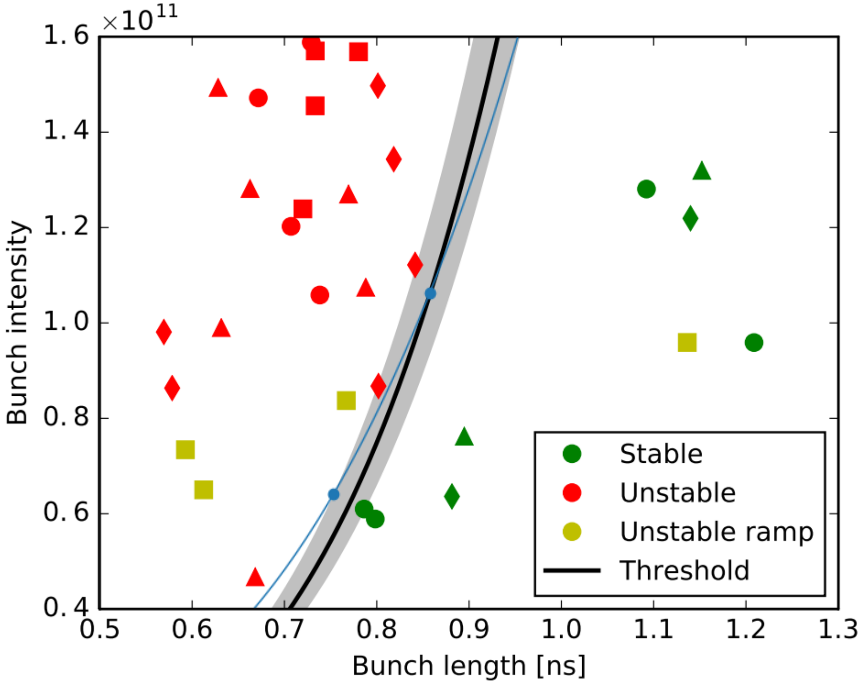


MD 2015,
V_{rf} = 12 MV, E = 6.5 TeV



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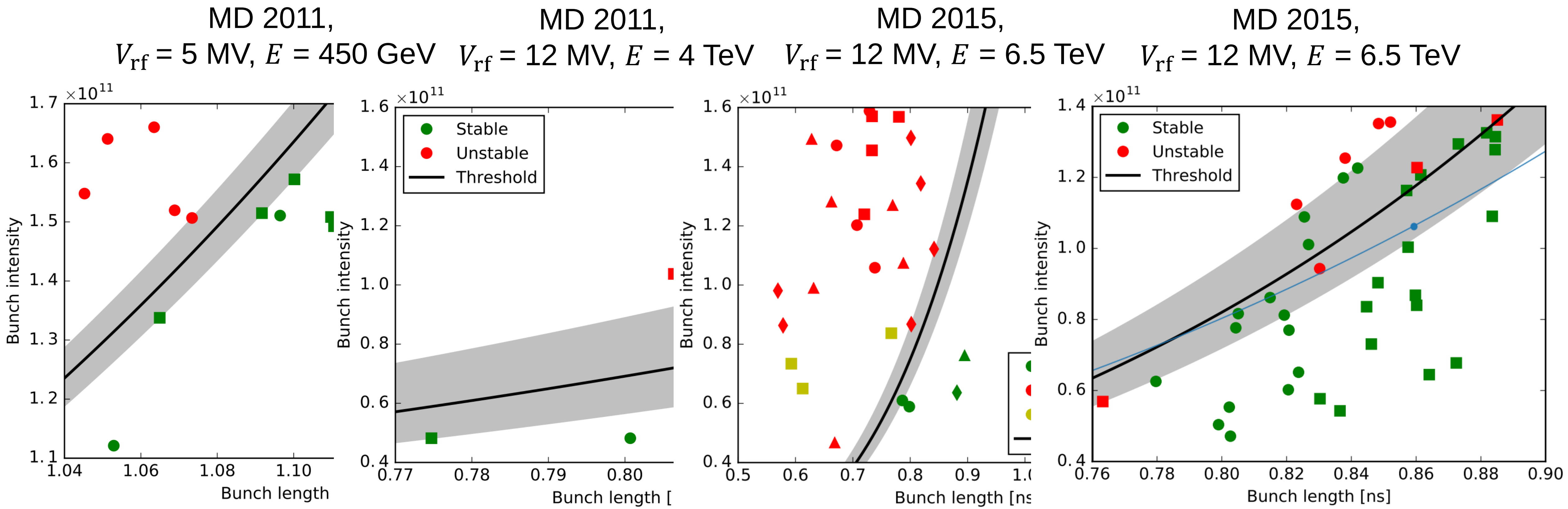

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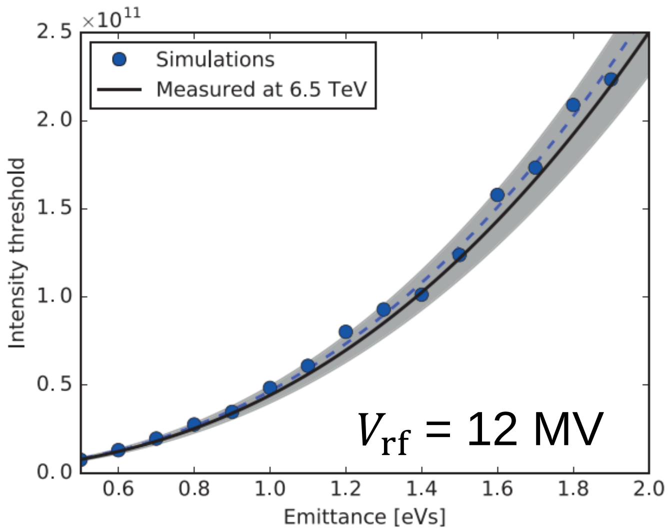
Comparisons with simulations

Simulation setup (*PhD thesis J. E. Muller, 2016*):

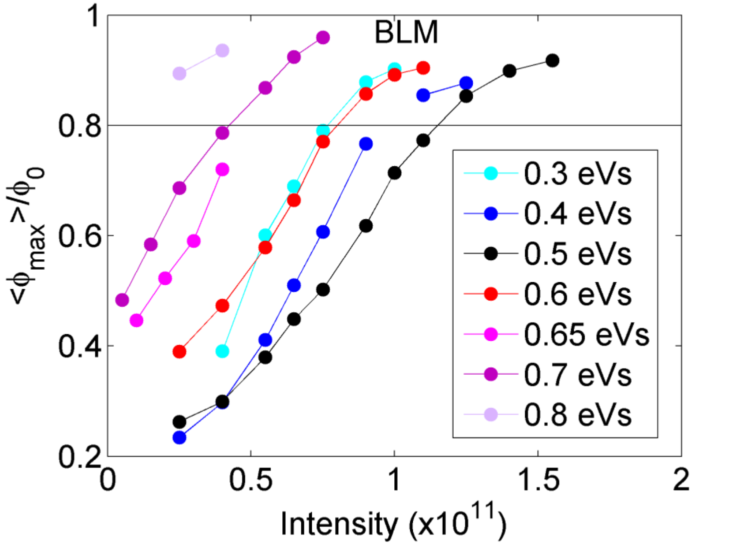
- Number of macro-particles 5×10^5
- 50 slices per bucket (f_c = 10 GHz) for induced voltage calculation using full impedance model
- Initially matched bunched is kicked by 1 degree



Stability criterion:
Bunch is stable if oscillation amplitude is reduced below 0.2 degrees

BLonD simulations using full LHC impedance model agree very well with measurements

Simulations for double rf system**



However, the stability threshold can significantly vary depending on the chosen criteria based on the final oscillation amplitude (PhD thesis T. Argyropoulos, 2015)

→ Different method to find absolute threshold is needed

Lebedev' approach

Matrix equation (A. N. Lebedev, 1967) in $(\mathcal{E},\psi)$ variables with $\mathcal{E} = \dot{\phi}^2/(2\omega_{s0}^2) + U(\phi)/(V\cos\phi_s)$

$$\boxed{j_p(\Omega) = -\frac{iI_0 h}{V\cos\phi_s}\sum_{k=-\infty}^{\infty} G_{pk}(\Omega)\frac{Z_k(\Omega)}{k} j_k(\Omega)}$$

where $G_{pk}(\Omega) = \frac{1}{A_N}\sum_{m=-\infty}^{\infty}\int_0^{\infty} F'(\mathcal{E})\,\frac{I_{mp}(\mathcal{E})\,I^*_{mk}(\mathcal{E})}{\Omega/m - \omega_s(\mathcal{E})}d\mathcal{E}$

where $I_{mk}(\mathcal{E}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{e}^{k\phi(\mathcal{E},\psi)/h - im\psi}\,d\psi$ and normalization $A_N = \int_0^{\infty}\frac{F(\mathcal{E})}{\omega_s(\mathcal{E})}d\mathcal{E}$

It allows to evaluate both single- and multi-bunch stability
Was extensively used to evaluate coupled-bunch instability thresholds due to narrow-band impedance *(V. Balbekov, S. Ivanov, 1987)* and for combination of narrow-band resonator and Im*Z/n* = const (*M. Blaskiewicz, 2009)*

→ For single-bunch case it was considered to be not numerically tractable

Landau damping: Van-Kampen modes

Method (A. Burov, 2010): find Van-Kampen modes solving Vlasov equation for perturbation *f(J,ψ,t)* of stationary distribution function *F(J)* expanded (Oide & Yokoya, 1990) as

$$f(J,\psi,t) = \mathrm{e}^{-i\Omega t} \sum \left[f_m(J)\cos m\psi + g_m(J)\sin m\psi \right]$$

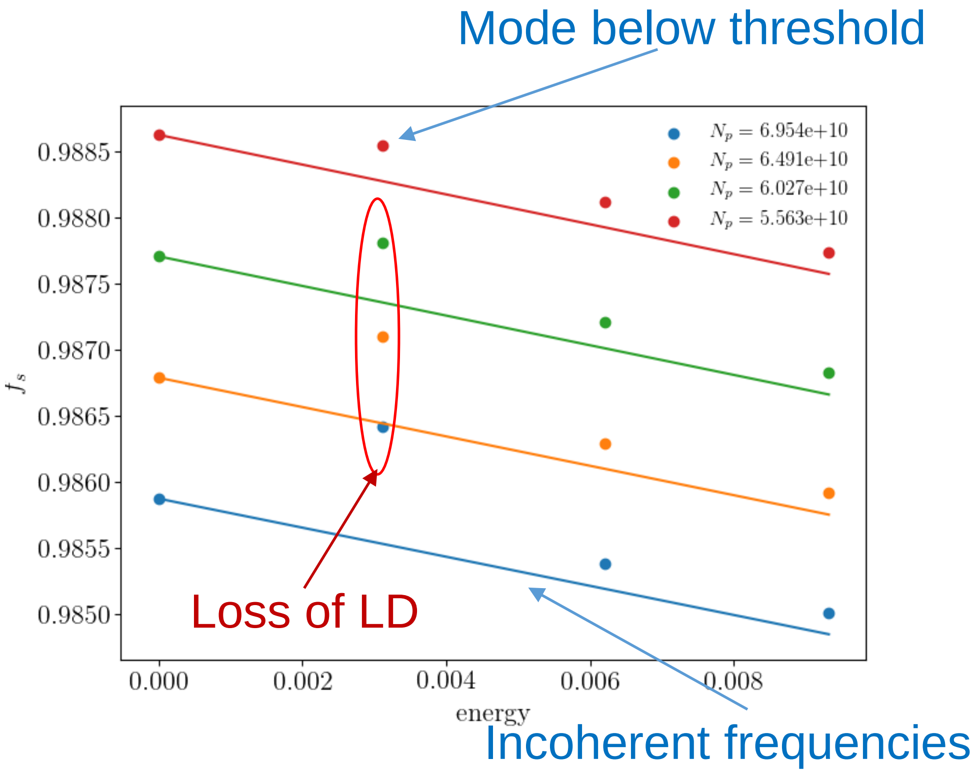
Without mode coupling the matrix equation (in action J) is

$$(\Omega^2 - m^2\omega_s^2) f_m(J) = m^2 \omega_s(J) F'(J) \int V_m(J,J') f_m(J') dJ'$$

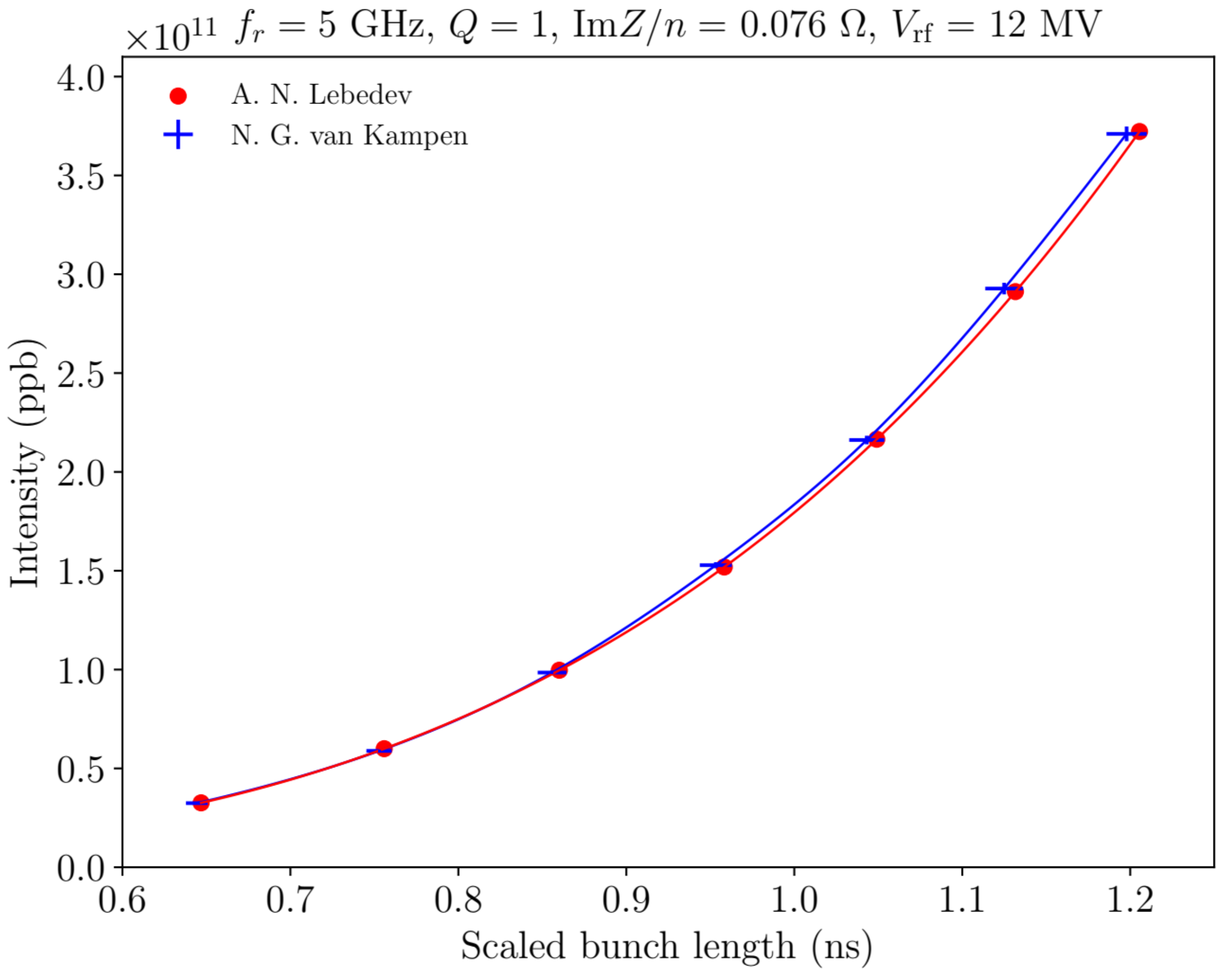
where $$V_m(J,J') = 2\,\mathrm{Im} \sum_k \frac{Z_k}{k} I_{mk}(J) I^*_{mk}(J')$$

and $$I_{mk}(J) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{k\phi(J,\psi)/h - im\psi}\, d\psi$$

Continuous spectrum - singular modes from incoherent band. Discrete modes - coherent solutions described by regular eigenfunctions → their existence outside incoherent band serve as criterion for loss of LD



Van-Kampen vs Lebedev approaches



A new numerical code was developed to solve Lebedev eigenvalue problem

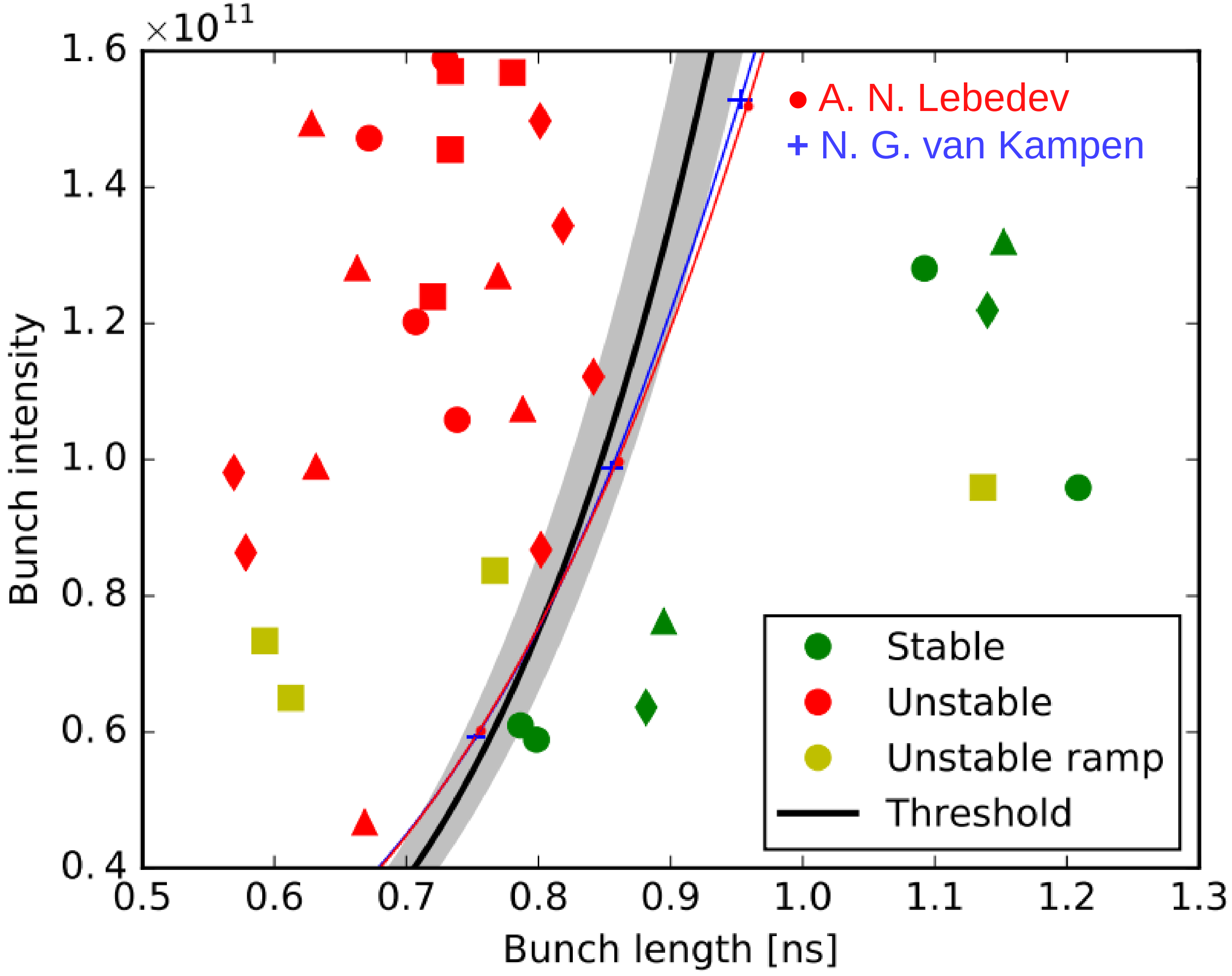
The thresholds are calculated using Lebedev and Van-Kampen and approaches for broadband impedance and following parameters:
$R = (\mathrm{Im}Z/n) f_r/f_0, f_r = 5\ \mathrm{GHz}, Q = 1,$
$f_c = 20\ \mathrm{GHz},\ \mathrm{Im}Z/n = 0.076, \mu = 2,$
$V_{\mathrm{rf}} = 12\ \mathrm{MV}$

Distribution function $F(\mathcal{E}) = F_0(1 - \mathcal{E}/\mathcal{E}_{max})^{\mu}$

→ Very good agreement between two approaches

Preliminary comparisons with measurements

Measurements at 6.5 TeV, $V_{\mathrm{rf}} = 12\ \mathrm{MV}$ (*PhD thesis J. E. Muller, 2016*)



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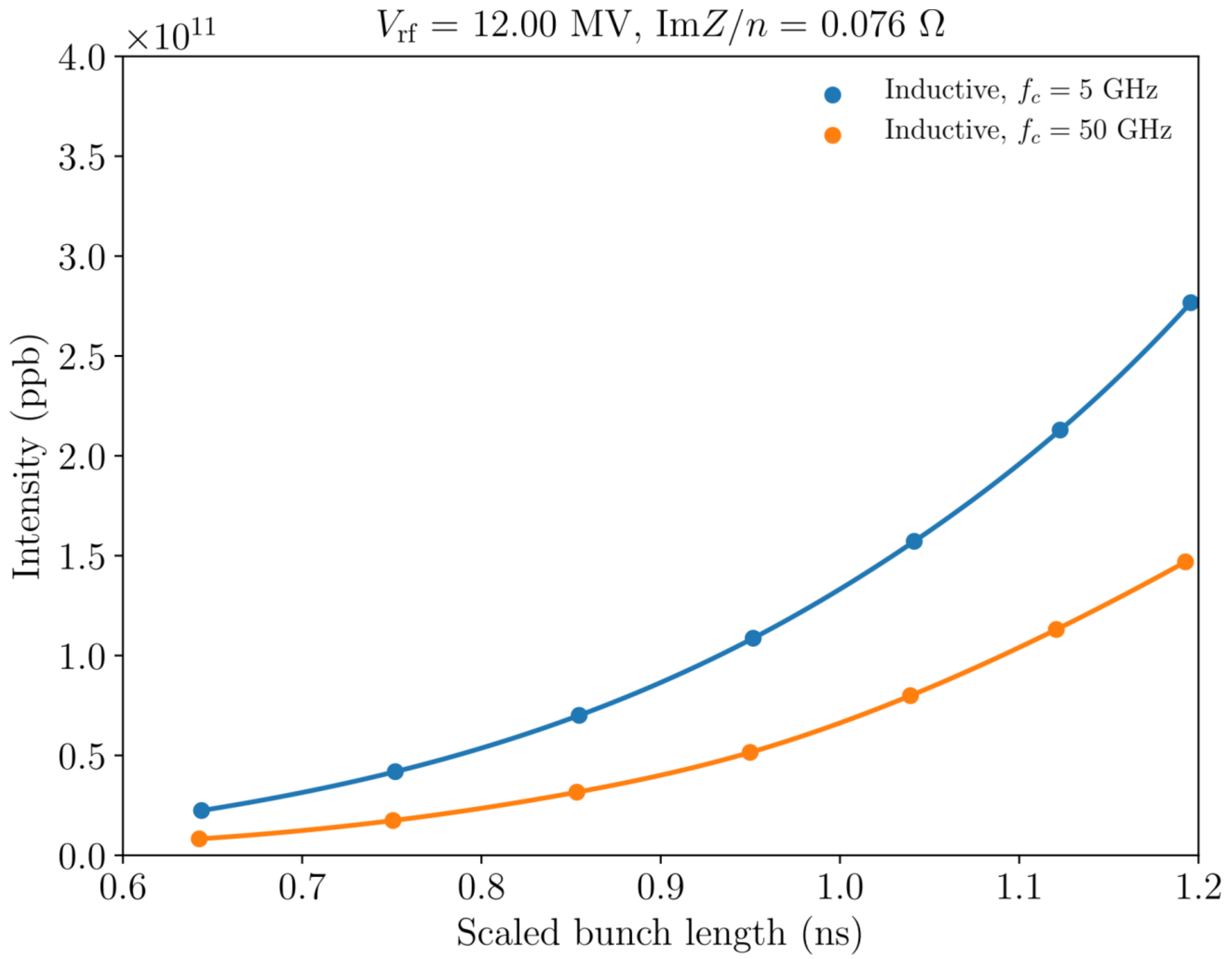
→ Reasonable agreement between measurements and semi-analytic calculations.
→ Scaling law is different in comparison to simplified Sacherer's approach

Calculation results



Case of inductive impedance $\mathrm{Im}Z/n = \mathrm{const}$.

Threshold significantly depends on the cut-off frequency f_c

For the case of symmetric potential well

$$I_{mk}(\mathcal{J}) \approx i^m J_m\left(\frac{k}{h}\sqrt{2\mathcal{J}}\right)$$

So diagonal elements of the matrix diverge

$$V_m(\mathcal{J},\mathcal{J}) = 2\mathrm{Im}Z/n \sum_k J_m^2\left(\frac{k}{h}\sqrt{2\mathcal{J}}\right) \to \infty$$

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→ Realistic impedance model needs to be used