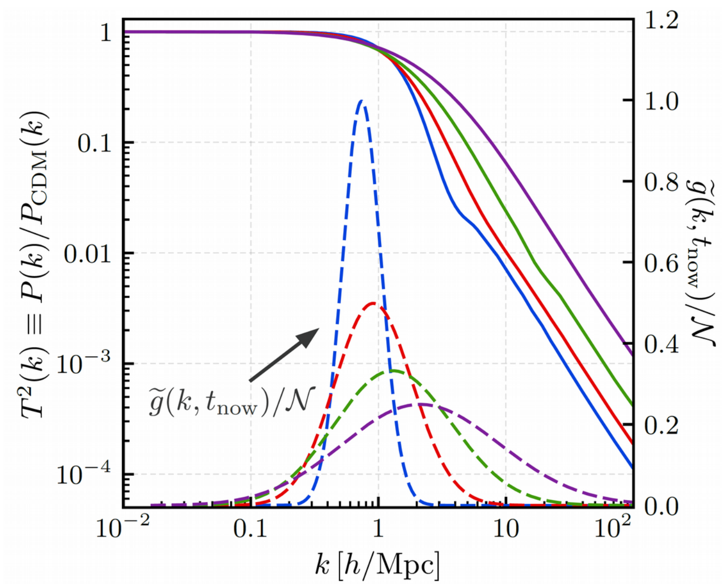
A Reconstruction Conjecture: Deciphering the Structure of the Dark Sector from the Matter Power Spectrum



Brooks Thomas

LAFAYETTE COLLEGE

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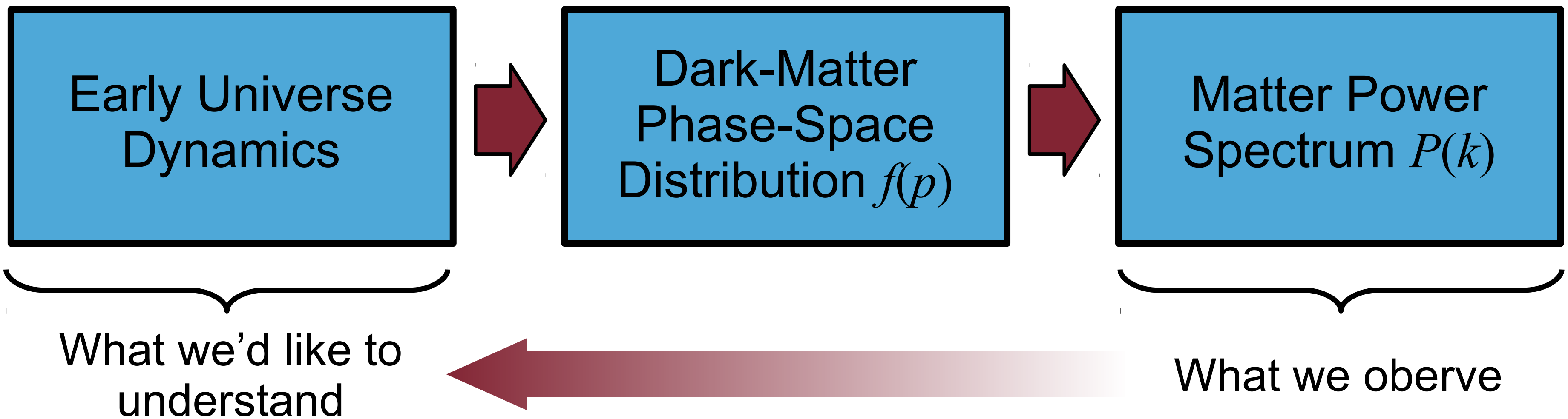
Based on work done in collaboration with:

- **K. R. Dienes, F. Huang, J. Kost, and S. Su [arXiv:2001.02193] (Accepted for publication in PRD).**

The Basic Question

- The early-universe dynamics which produces the dark matter gives rise to a particular dark-matter phase-space distribution $f(p)$. This, in turn, affects the shape of the matter power spectrum $P(k)$.



Q To what extent can we work backwards and ***reconstruct*** the properties of $f(p)$ – and the dynamics that gave rise to it – from information encoded in $P(k)$?

- While the maps from the underlying physics to $f(p)$, and from $f(p)$ to $P(k)$ are clearly not invertible, it is nevertheless possible to "work backwards" and obtain substantial information about the dark sector from information contained in the matter power spectrum.

Describing the Phase-Space Distribution

- We're going to describe the phase-space distribution in a slightly atypical way. We'll begin with some motivation.

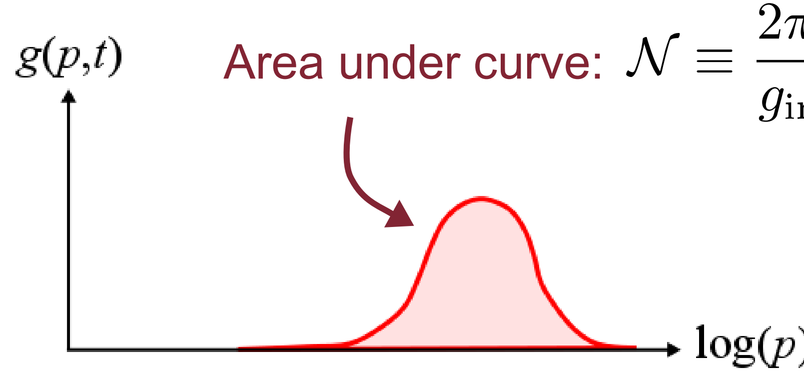
$$x(t) = x(t')\frac{a(t)}{a(t')} \qquad p(t) = p(t')\frac{a(t')}{a(t)} \quad \Rightarrow \quad \frac{d\log(p)}{dt} = -H(t)$$

Number of internal degrees of freedom

- Physical number density: $$n(t) = \frac{g_{\text{int}}}{2\pi^2}\int dp\, p^2 f(p,t)$$
- Comoving number density: $$N(t) = n(t)a^3(t) = \frac{g_{\text{int}}}{2\pi^2}\int d\log p\, \boxed{p^3 a^3 f(p,t)}$$
- This motivates us define the "log-space" DM phase-space distribution:

$$\boxed{g(p,t) = a^3(t)p^3 f(p,t)}$$

Area under curve: $\mathcal{N} \equiv \frac{2\pi^2}{g_{\text{int}}} N(t)$



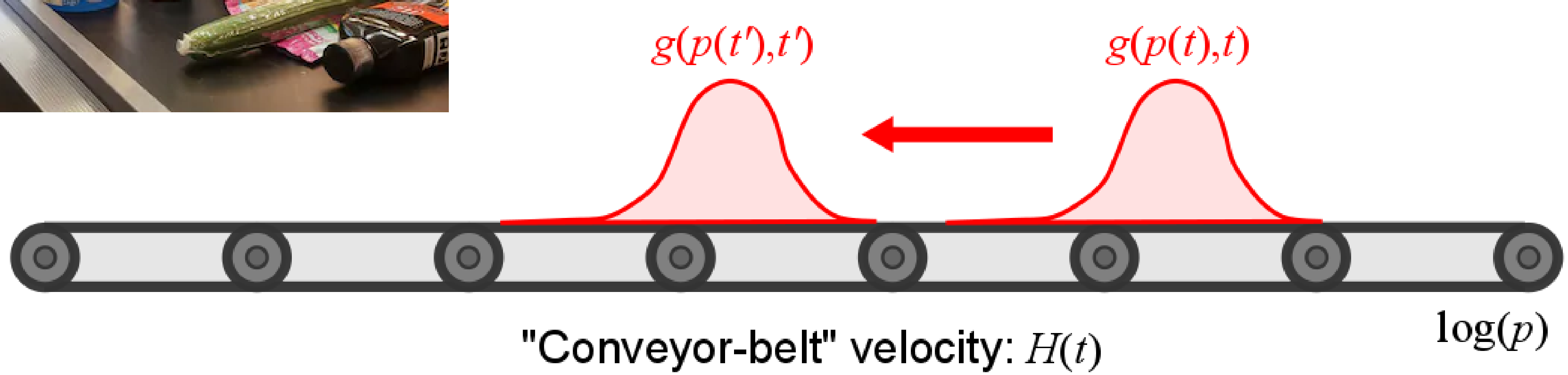
The Cosmological Conveyor Belt

- The reason $g(p,t)$ turns out to be a useful quantity is that it evolves with time in a particularly straightforward manner.
- Indeed, in the absence of sources/sinks, $N(t') = N(t)$ is conserved and $g(p,t)$ quantity evolves with time according to the relation

$$\boxed{g(p(t'),t') = g(p(t),t)}$$

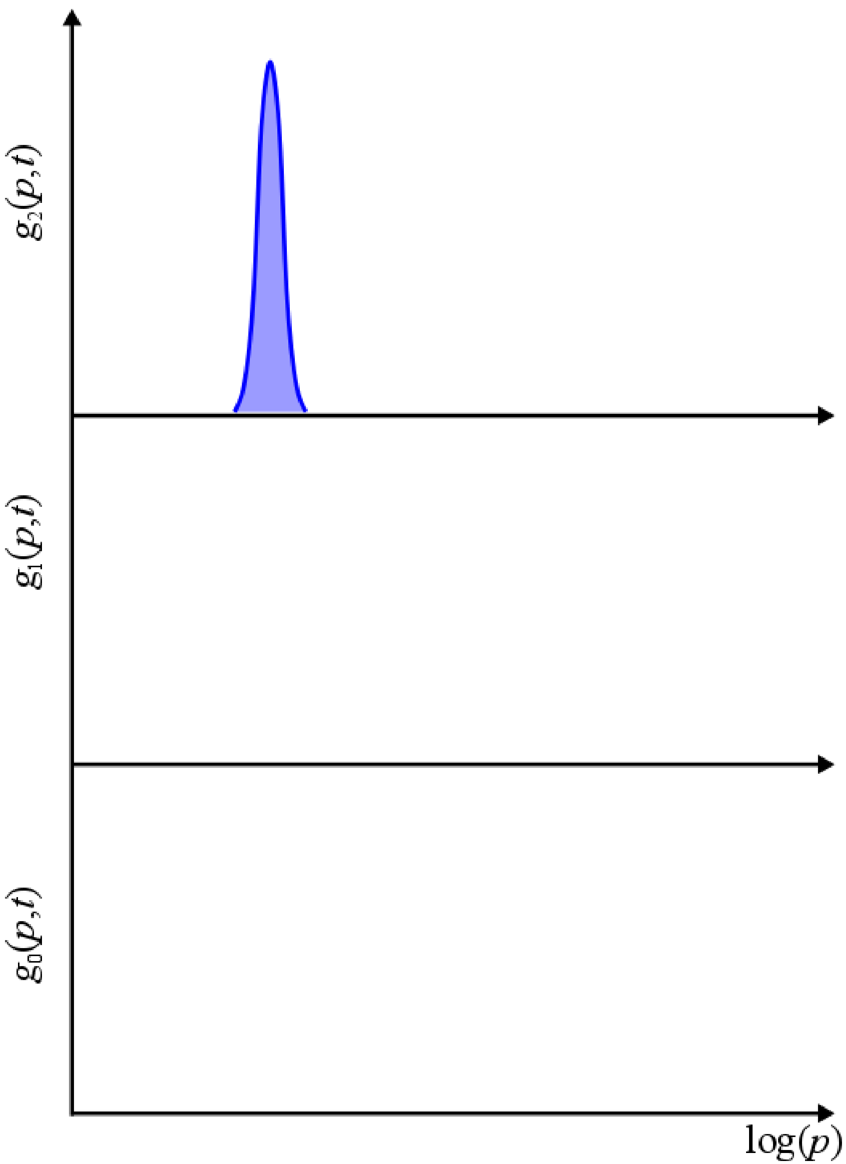
- Thus, as t increases, the $g(p,t)$ distribution retains the same overall profile, which simply redshifts undistorted to lower values of $\log(p)$.

- In other words, $g(p,t)$ is carried along like a cucumber on a ***conveyor belt***, moving to lower and lower $\log(p)$ at speed $|d\log p/dt| = H(t)$, but retaining a fixed shape.



Dark-Sector Dynamics and Non-Trivial $g(p)$

- Things become more interesting, of course, when sources and sinks are included, such as those associated with ***particle decays***.



- Consider a dark sector involving three scalars ϕ_0, ϕ_1, and ϕ_2 with similar quantum numbers and masses $m_2 > 2m_1 > 4m_0$.

- For purposes of illustration, let's assume

$$\boxed{\mathrm{BR}(\phi_2 \to \phi_1\phi_0) \approx \mathrm{BR}(\phi_1 \to \phi_0\phi_0) \approx 1}$$

- We'll also work in the ***instantaneous-decay approximation***, wherein each ϕ_i decays completely at its lifetime τ_i.
- We'll also assume that ϕ_2 is initially the only state populated.

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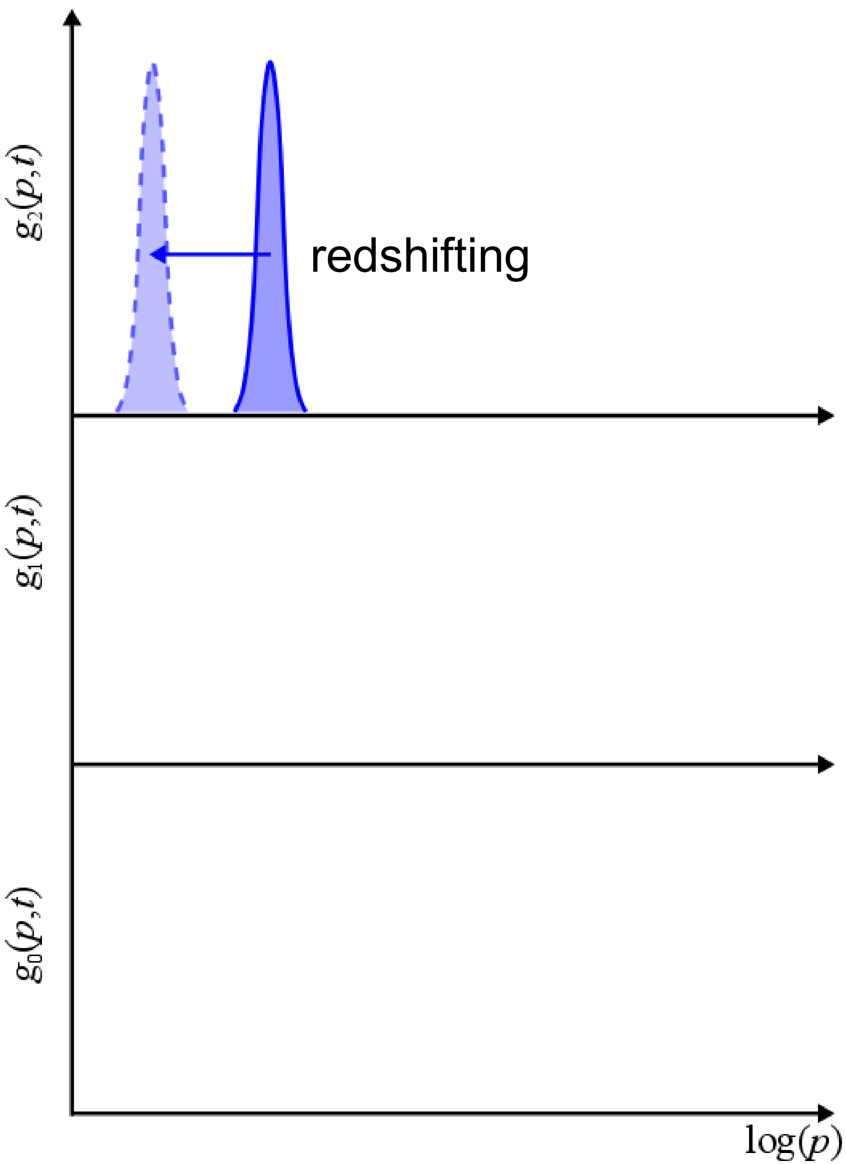

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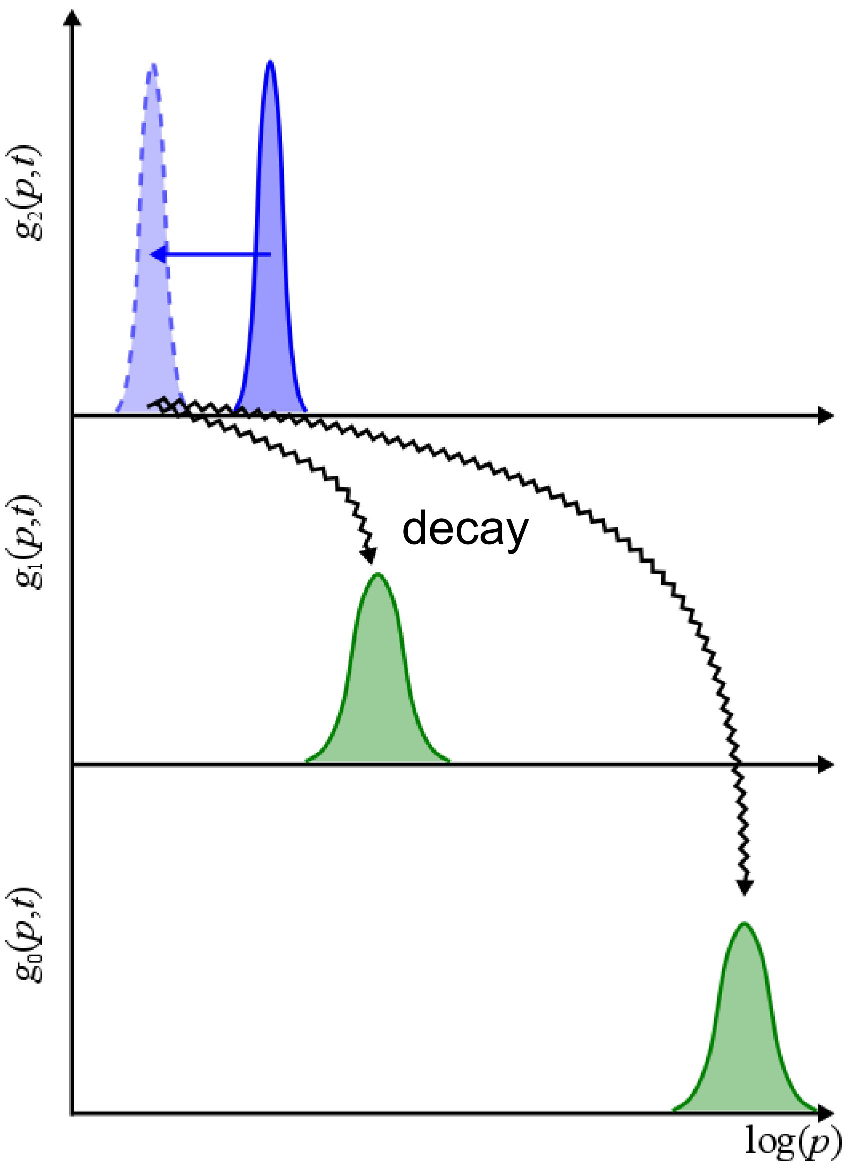

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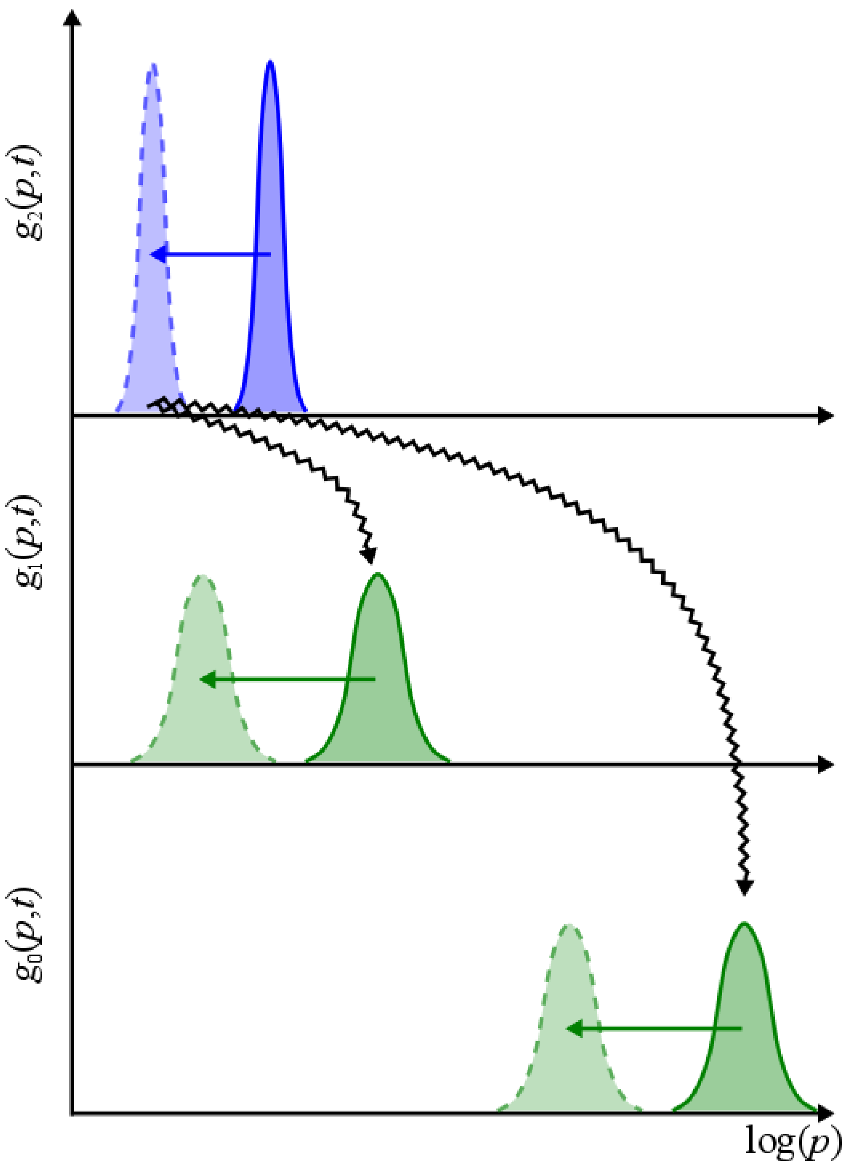

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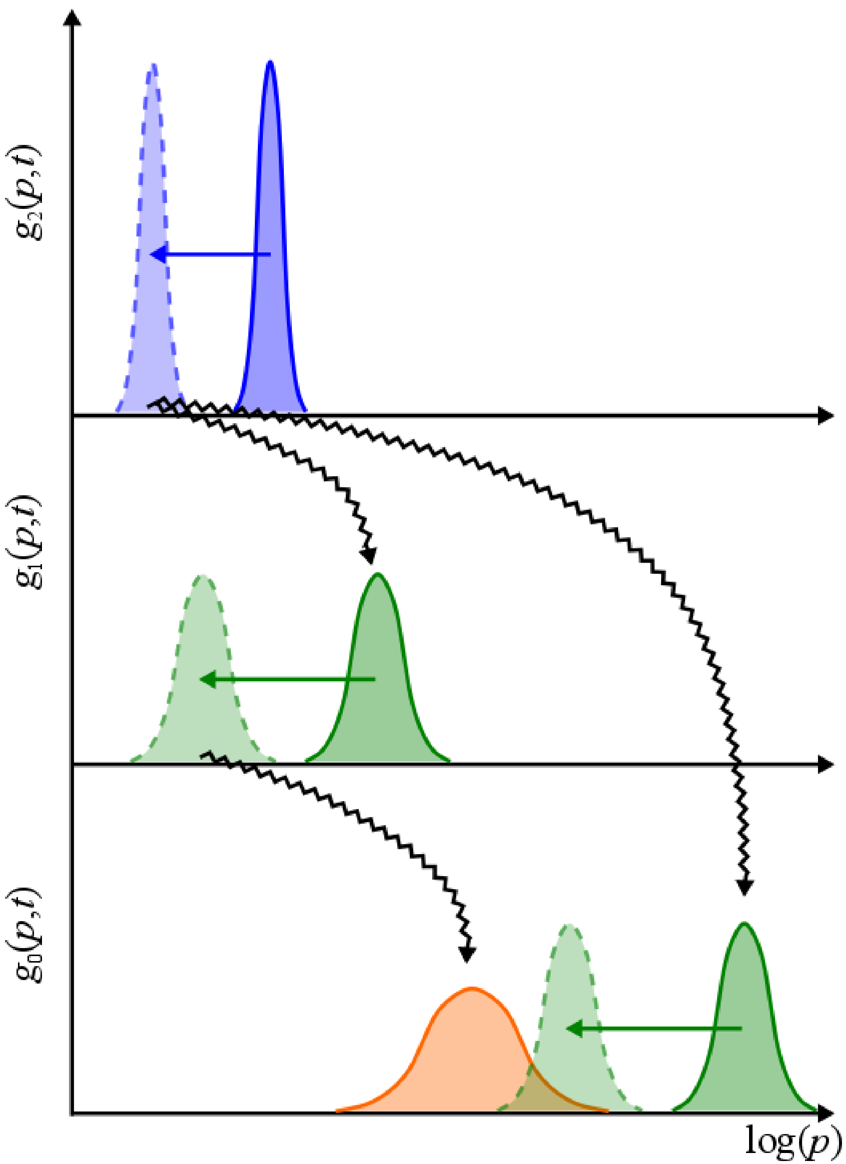

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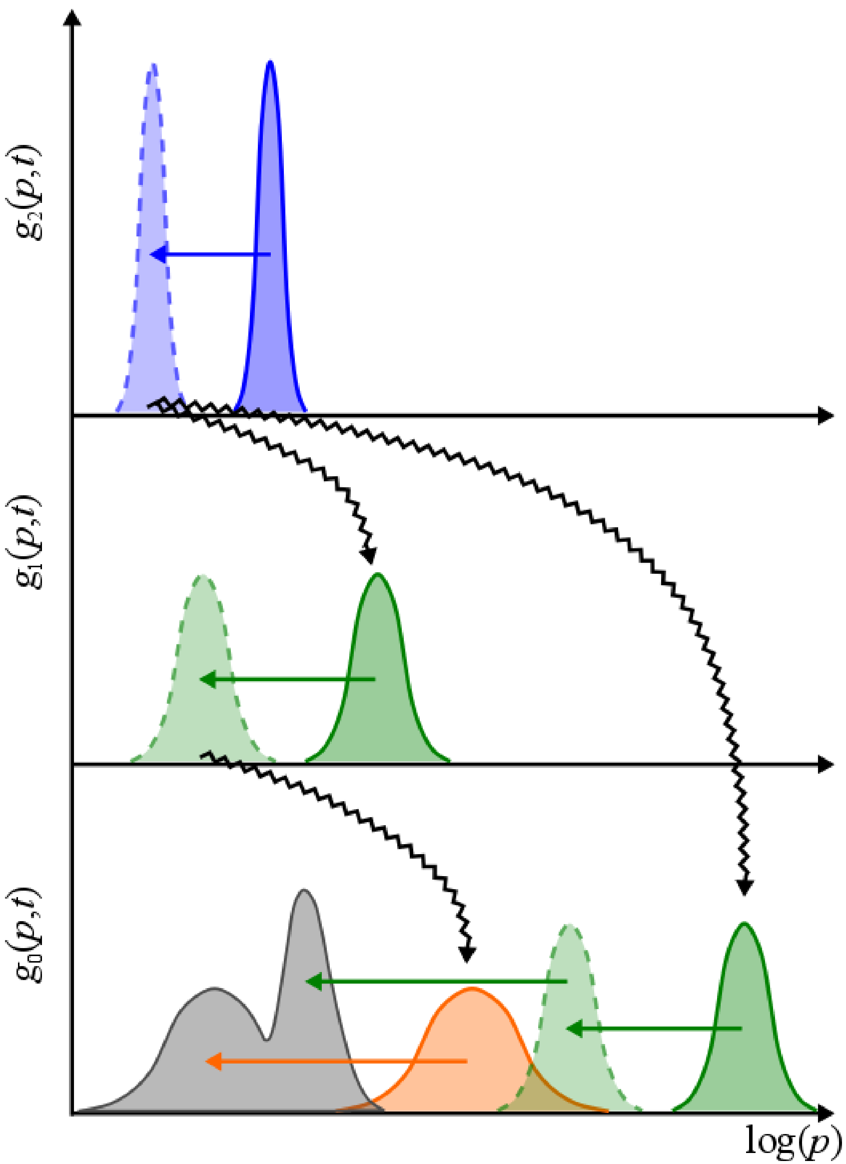

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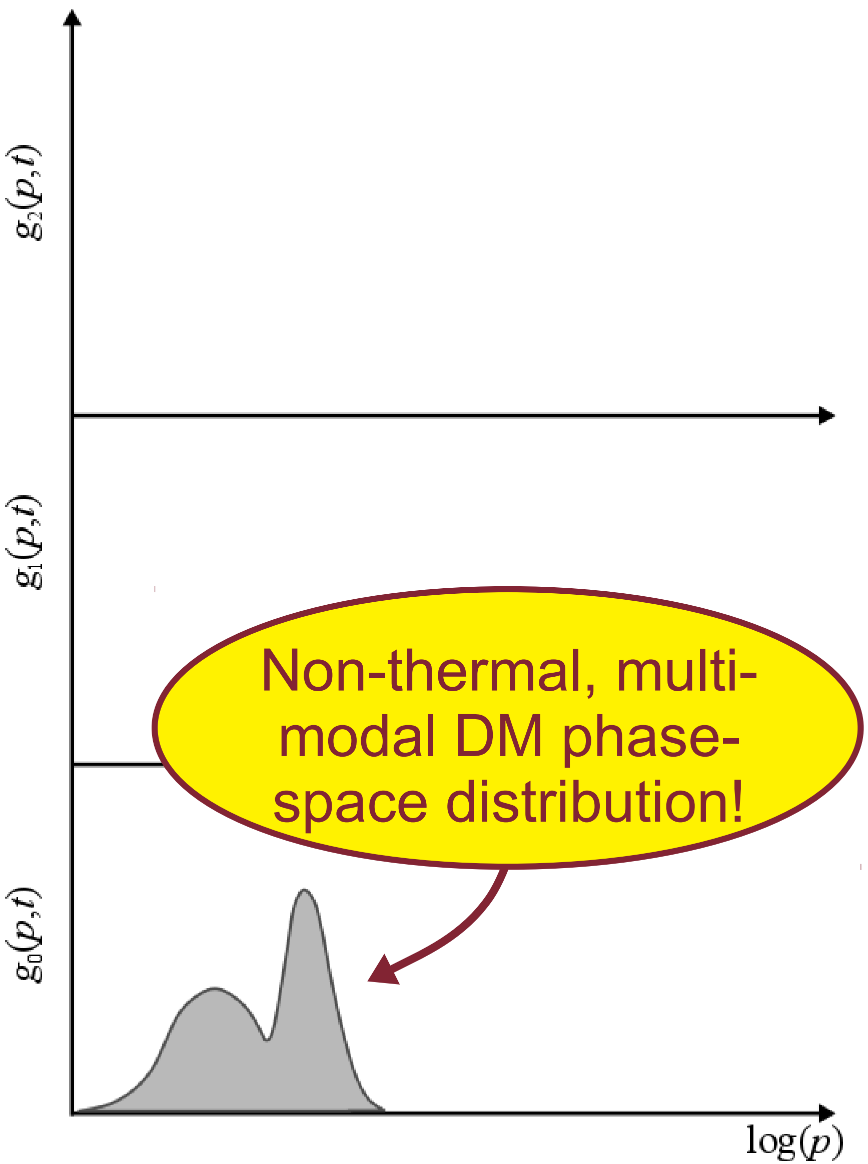

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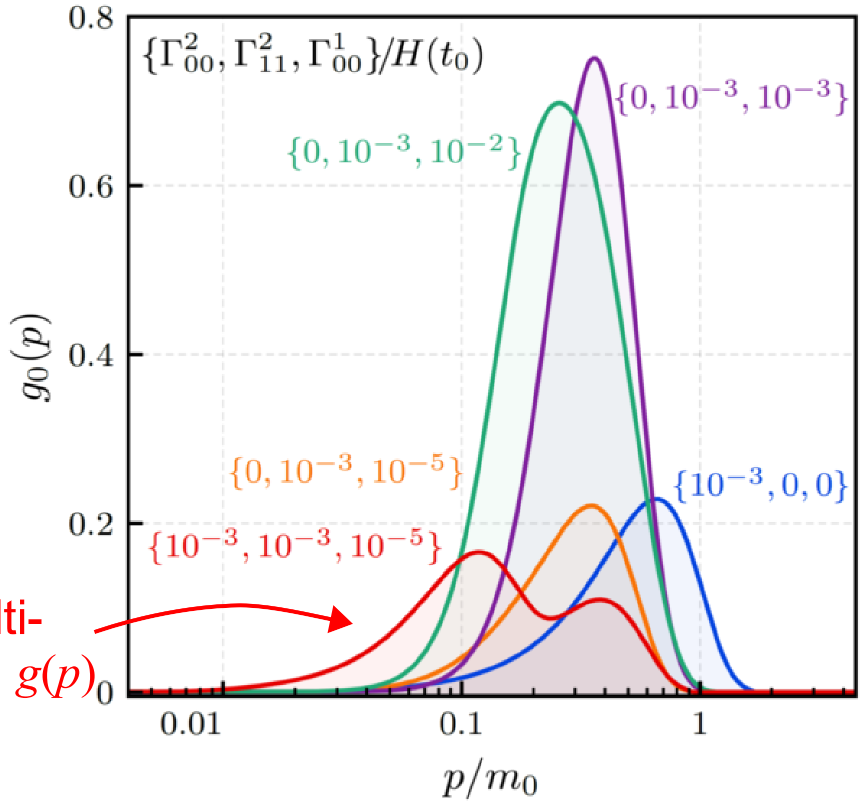
Dark-Sector Dynamics and Non-Trivial *g*(*p*)

- Generally speaking, we expect multi-modal phase-space distributions to arise in dark-sector decay scenarios wherein...
  - ***Multiple decay pathways*** to the lightest (stable) state exist.
  - Decay pathways with substantial overall branching fractions have ***significantly different timescales***.
  - Scattering rates in the dark sector are sufficiently low that states ***don't have time to thermalize***.



- These conclusions remain robust even when we include all relevant physical effects (exponential decay, time-dilation, etc.).
- Let's therefore investigate how we might obtain evidence of such multi-modality in the matter power specturm.

Full numerical results for three-field scenarios with:

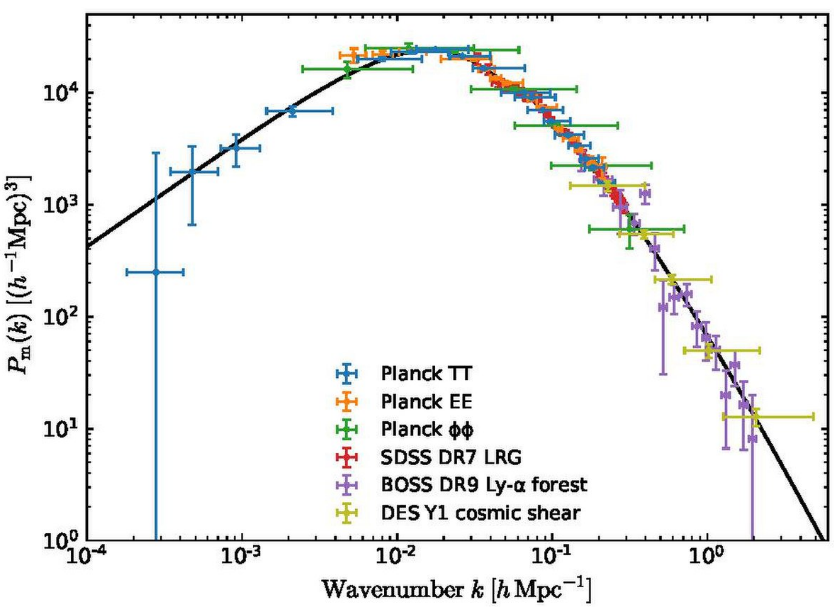
$$m_2/7 = m_1/3 = m_0$$

$$\Gamma^{\ell}_{m\ell} \equiv \Gamma(\phi_\ell \to \phi_m \phi_n)$$

Deciphering the Matter Power Spectrum

- The ***matter power spectrum*** provides an observational handle on the velocity distribution of DM particles.
- Dark matter particles with sufficient speed can escape from gravitational potential wells as they form, leading to a ***suppression of structure*** on small scales.

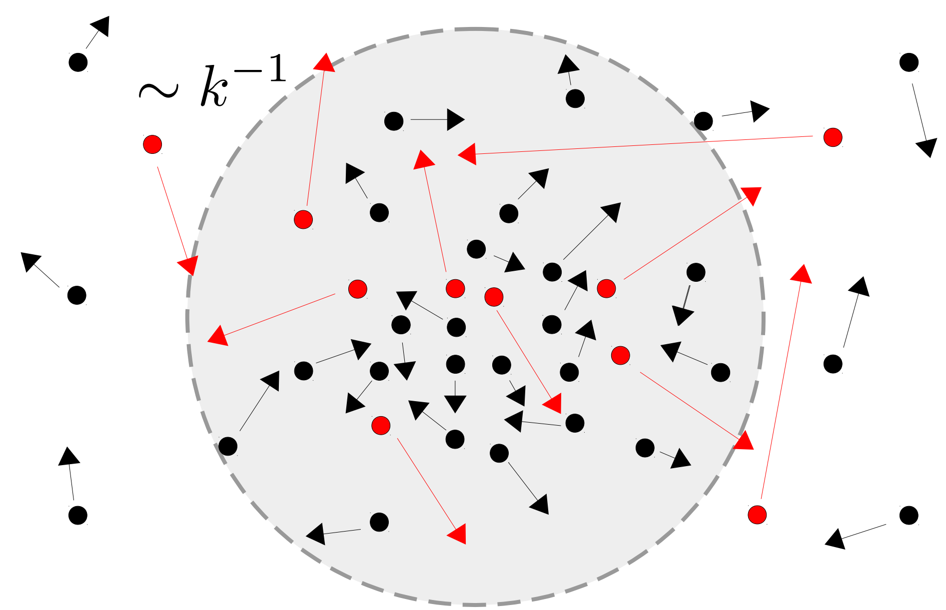
Linear Matter Power Spectrum



- Departures from a purely CDM cosmology can be expressed in terms of the ***transfer function*** $T(k)$, where

$$\boxed{T^2(k) = P(k)/P_{\mathrm{CDM}}(k)}$$

Particles moving at sufficient speeds can escape potential wells of size $\sim k^{-1}$



Our Approach

- The free-streaming horizon for a particle of mass m and present-day momentum p in an expanding universe is

$$k_{\rm hor}(p) \equiv \xi \left[\int_{t_{\rm prod}}^{t} v(p,t)\frac{dt}{a(t)}\right]^{-1} = \xi\left[\int_{a_{\rm prod}}^{1} \frac{da}{Ha^2}\frac{p}{\sqrt{p^2+m^2a^2}}\right]^{-1}$$

$\mathcal{O}(1)$ constant

- The usual approach (e.g., for warm DM) is to define a single "free-streaming-horizon" scale $k_{\rm FSH}$ using the ***average*** DM velocity $\langle v(t)\rangle$:

$$k_{\rm FSH} \sim \left[\int_{t_{\rm prod}}^{t} \langle v(t)\rangle \frac{dt}{a(t)}\right]^{-1}$$

- By contrast, we shall consider a somewhat unorthodox procedure in which we regard $k_{\rm hor}(p)$ as a ***functional map*** between p and k.

- We can use this map to define a ***phase-space distribution in k-space*** which correspond to $g(p)$ in momentum space.

Inverse of $k_{\rm hor}(p)$

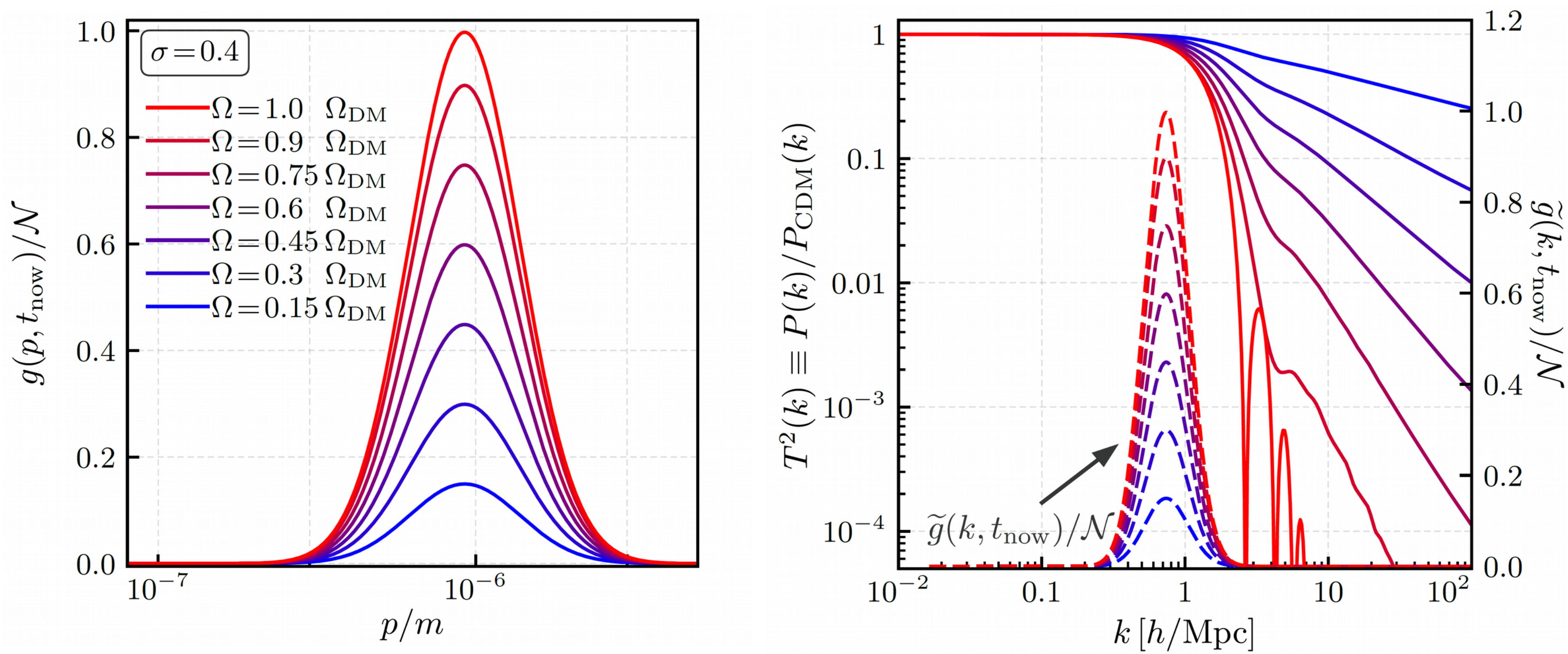
$$\widetilde{g}(k) \equiv g\big(k_{\rm hor}^{-1}(k)\big)\underbrace{\left|\frac{d\log p}{d\log k}\right|}_{\text{Jacobian}}$$

Relating $g(p)$ to $T^2(k)$

- Let's first consider the case of a simple $g(p,t_{\rm now})$ which consists of a single log-normal peak with average momentum $\langle p \rangle$ and width σ.
- We'll begin simply by fixing $\langle p \rangle$ and σ and ***varying the normalization*** of the peak, assuming that the rest of $\Omega_{\rm DM}$ is made up by cold DM.
- Increasing the abundance Ω associated with the peak, we find...

DM ***acoustic oscillations*** (the "wiggles") become more pronounced.

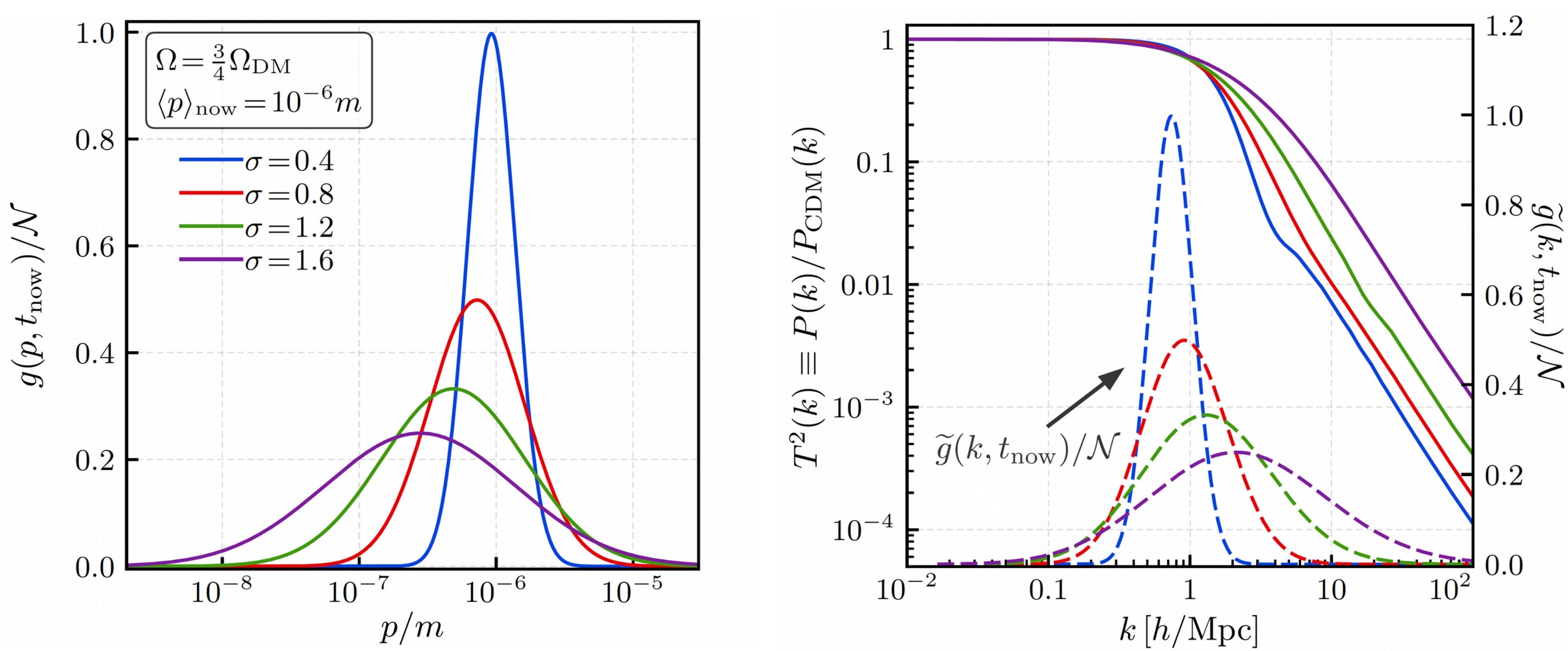
The ***slope*** of $T^2(k)$ at k above the peak in $\tilde{g}(k,t_{\rm now})$ increases.



Relating $g(p)$ to $T^2(k)$

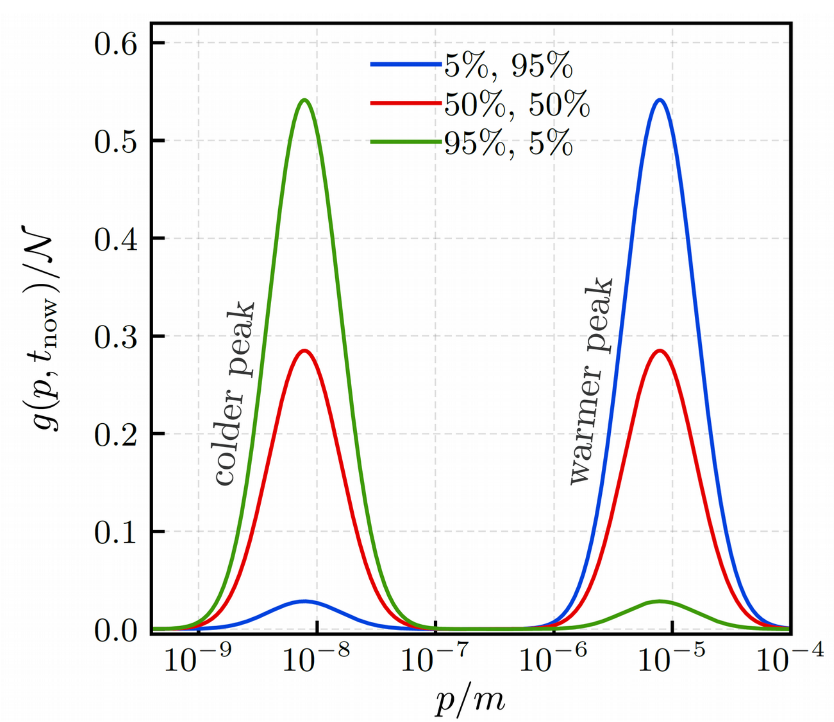
- Now let's hold Ω and $\langle p \rangle$ fixed and ***vary the width*** σ of the peak.
- Different values of σ lead to different amounts of suppression in $T^2(k)$ for k above the peak in $\tilde{g}(k,t_{\text{now}})$, but essentially ***identical slopes***!

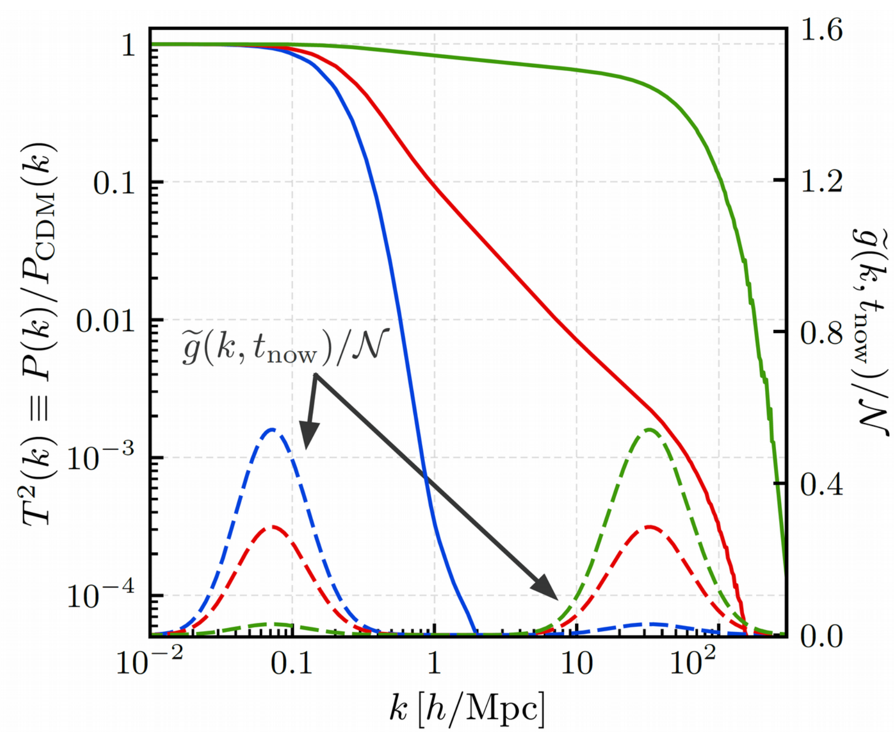
The abundance associated with a peak in $\tilde{g}(k,t_{\text{now}})$ correlates not with supression in $T^2(k)$, but rather the slope.



Relating $g(p)$ to $P(k)$

- In order to test this conjecture, let's consider a distribution consisting of ***two log-normal peaks*** with the same σ, but different values of $\langle p \rangle$.
- Here, $\Omega_{\rm DM}$ is partitioned entirely between the two peaks (no extra cold DM component). We vary their relative normalizations.
- Indeed, once again, we observe that the abundance associated with a peak in $\widetilde{g}(k,t_{\rm now})$ is correlated with the ***change in the slope*** of $T^2(k)$.





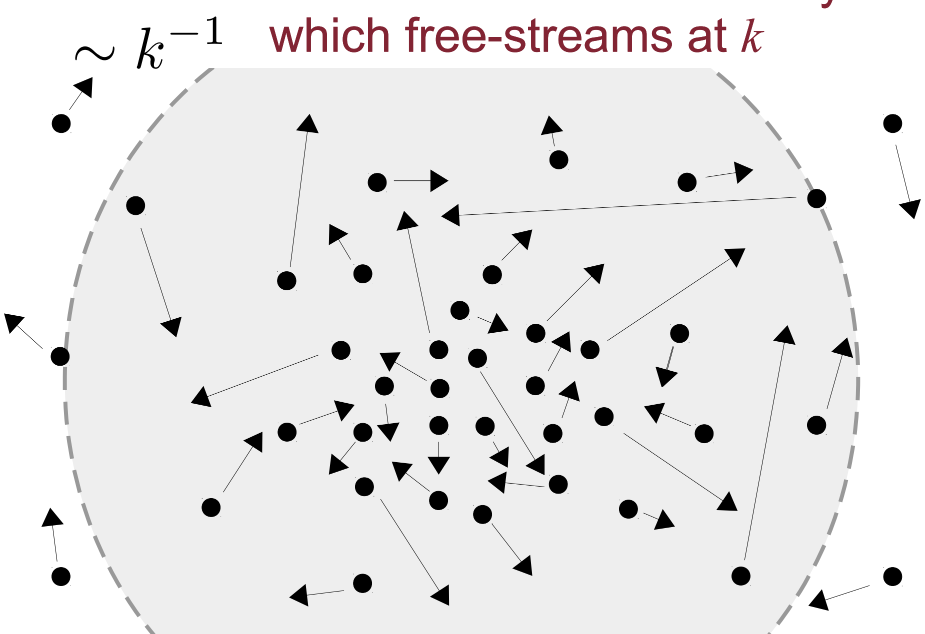
The Hot-Fraction Function

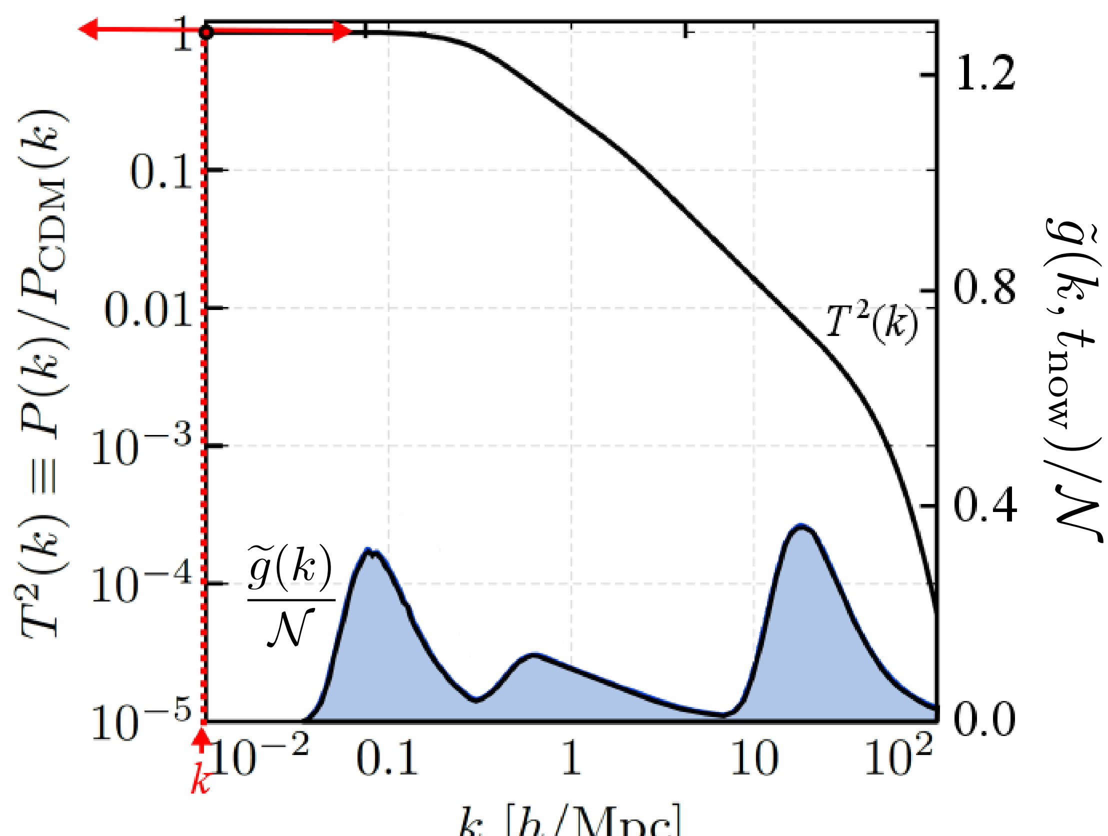
- The slope of the transfer function at a given value of k seems to correlate with the the total ***number density of particles which can free-stream*** at that value of k – particles with momenta $p > k_{\text{hor}}^{-1}(k)$.

- Motivated by these empirical findings, let us define the "***hot-fraction function***" $F(k)$ as follows:

$$\boxed{F(k) = \frac{\int_{-\infty}^{\log k} \widetilde{g}(k) d\log k'}{\int_{-\infty}^{\infty} \widetilde{g}(k) d\log k'}}$$

Fraction of DM number density which free-streams at k





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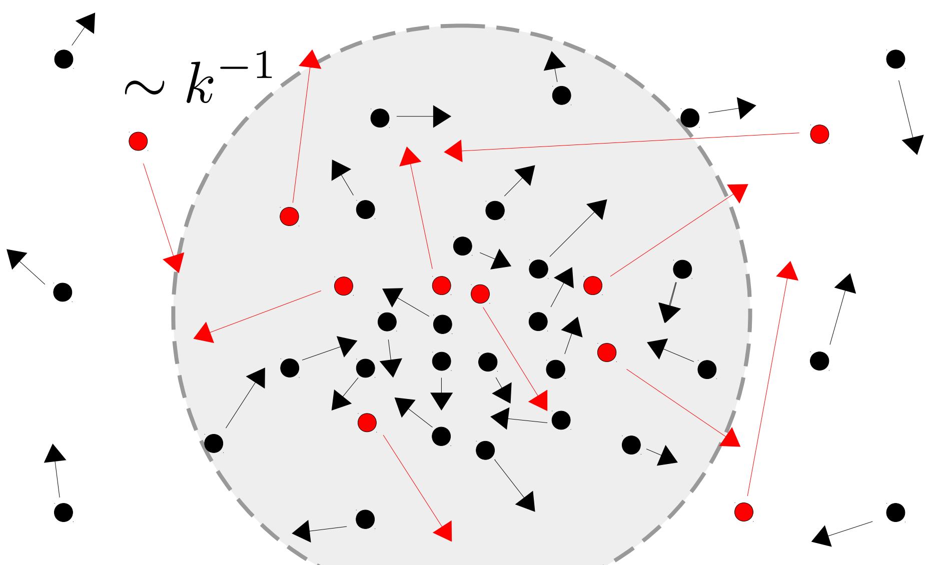

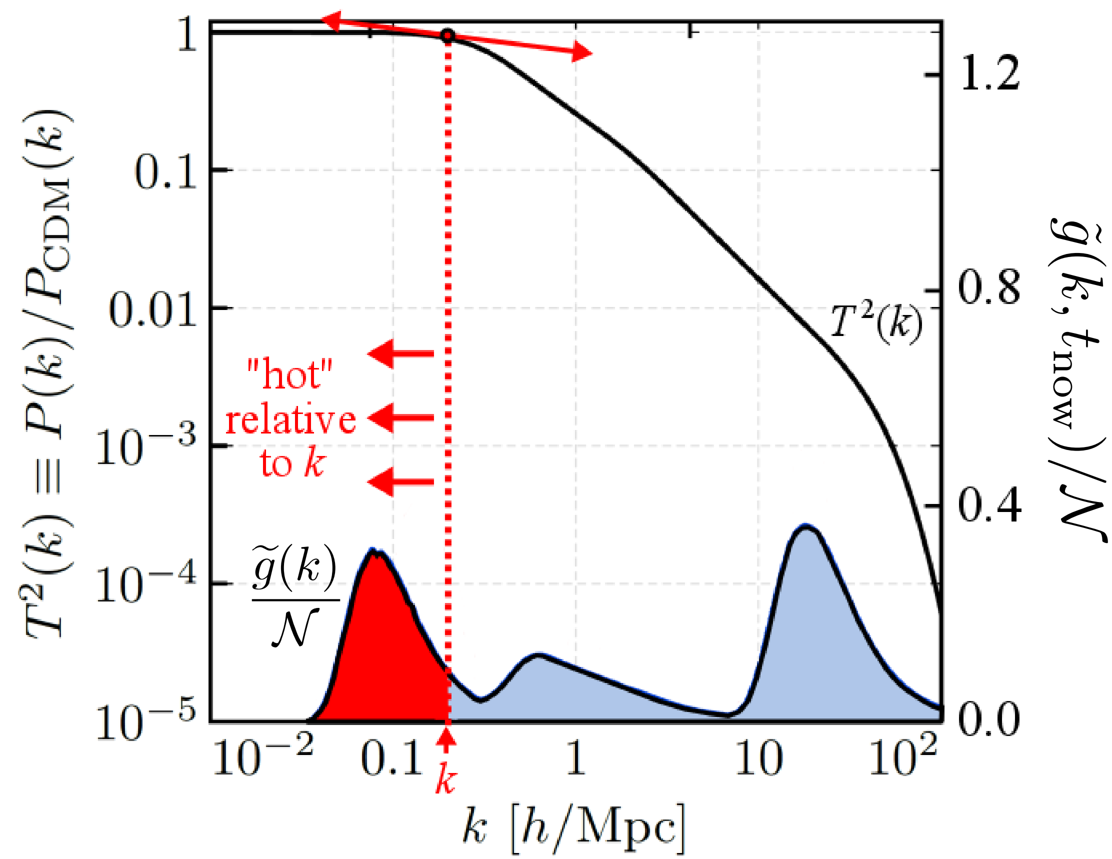

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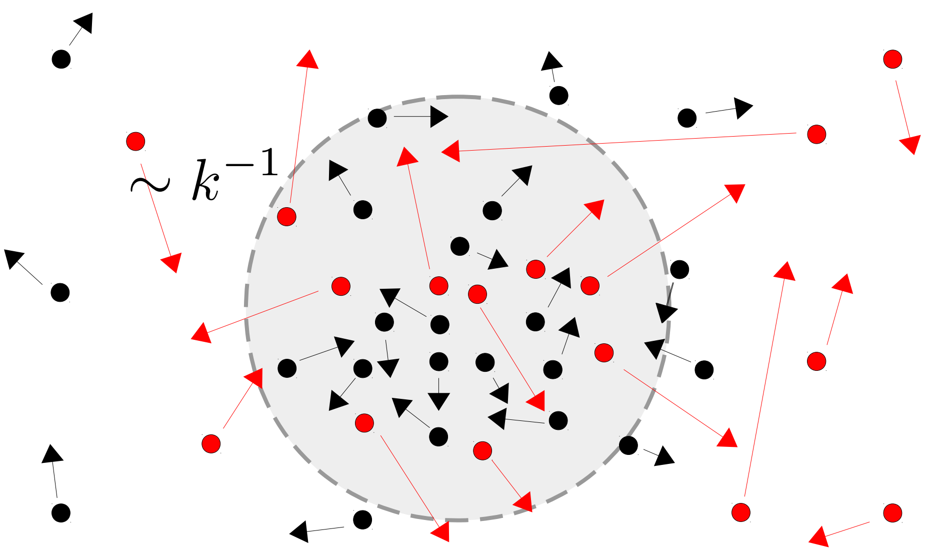

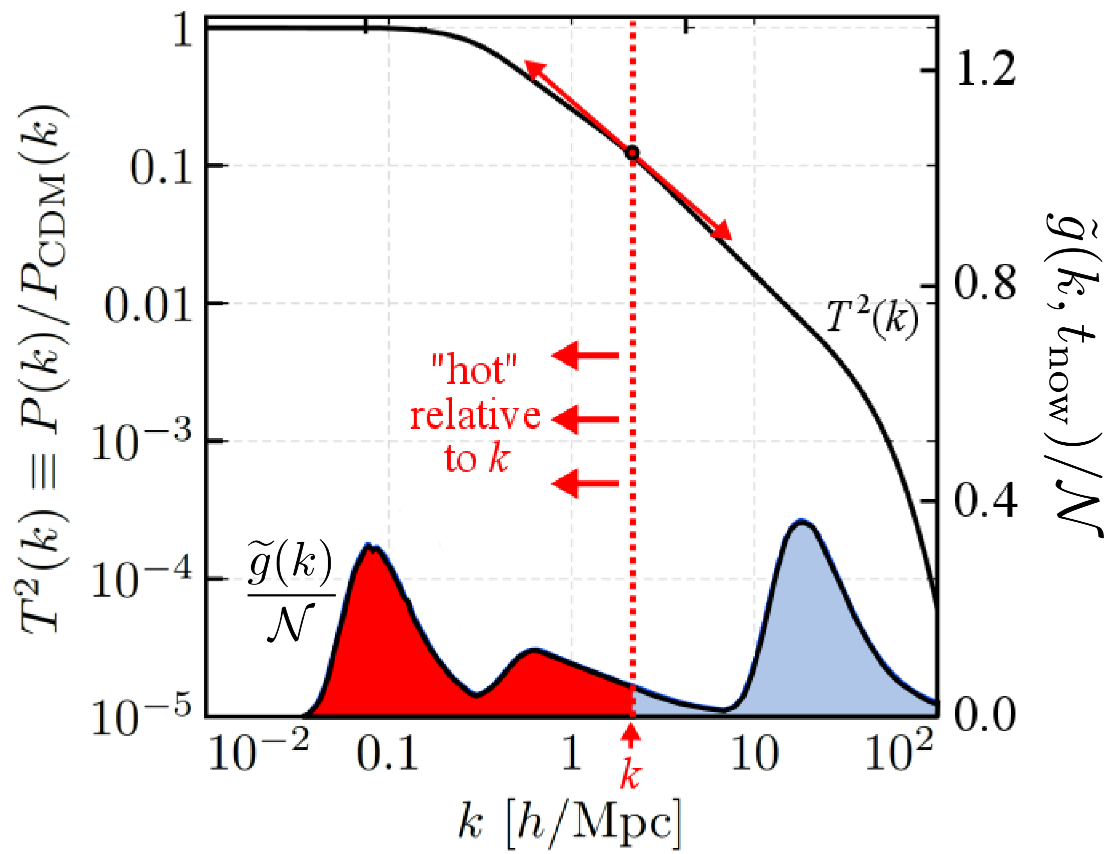

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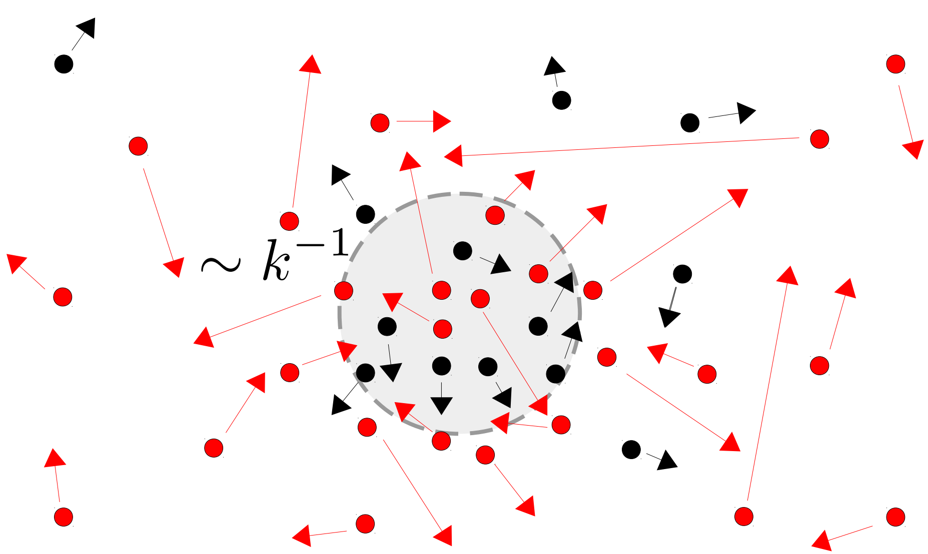

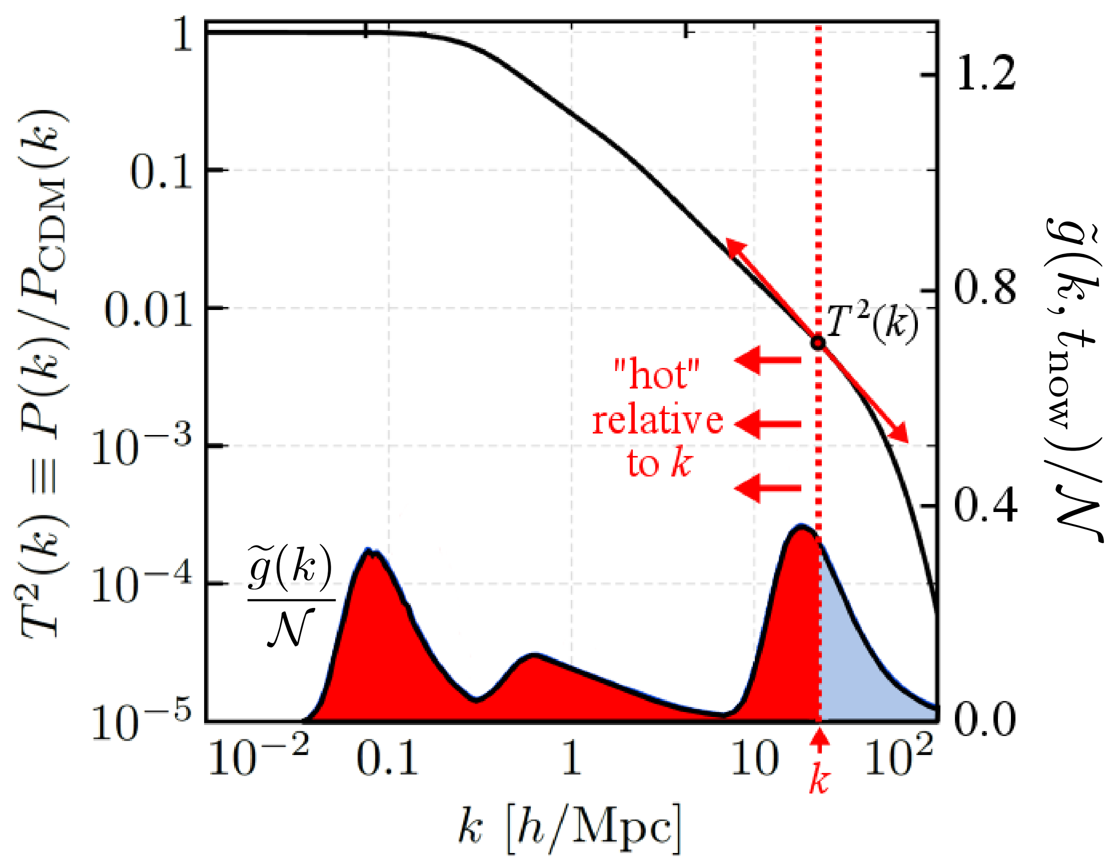

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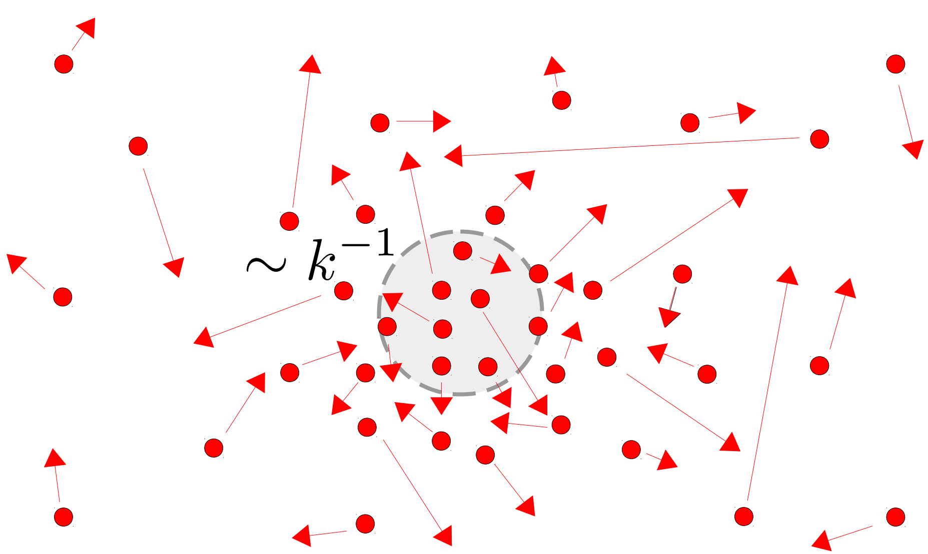
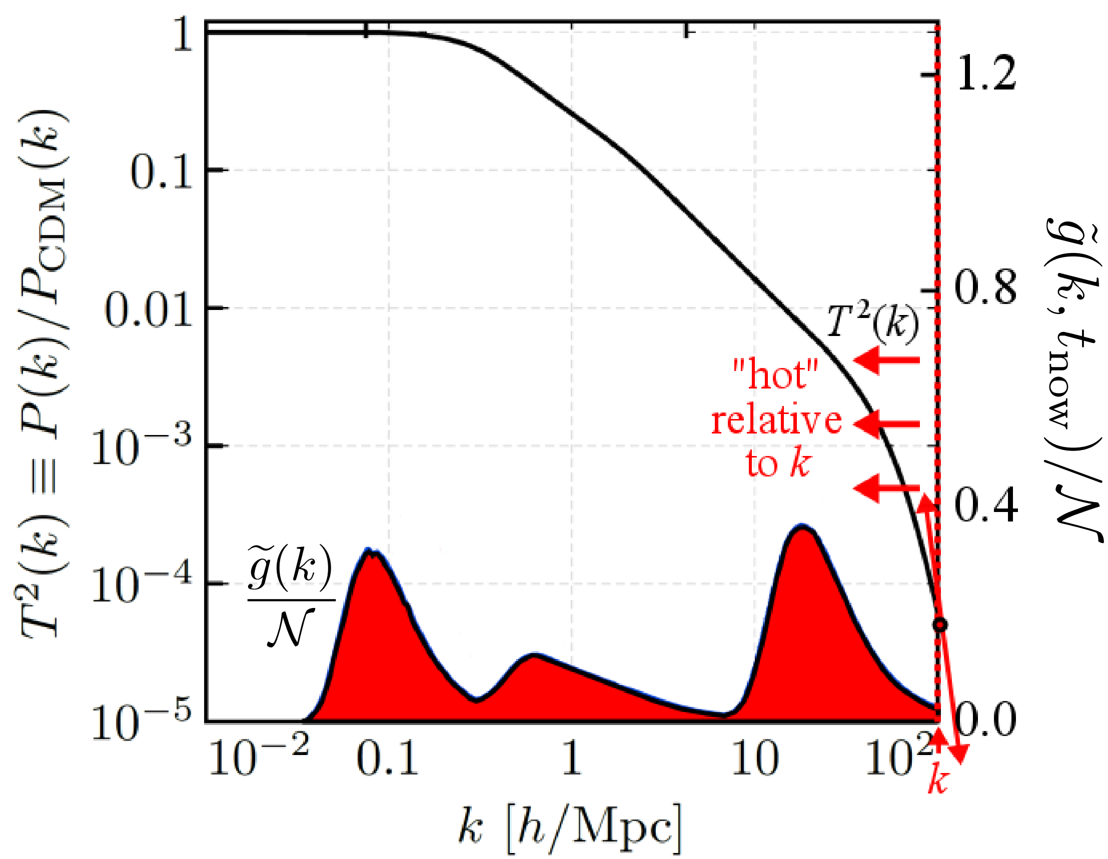




A Reconstruction Conjecture

- Our conjecture, then, is that there exists some ***invertible functional relationship*** between $F(k)$ and $T^2(k)$.
- Empirically, from numerical investigations (using CLASS) of the relationship between these two quantities, we find that

$$\boxed{\left|\frac{d\log T^2(k)}{d\log k}\right| \approx F^2(k) + \frac{3}{2}F(k)}$$

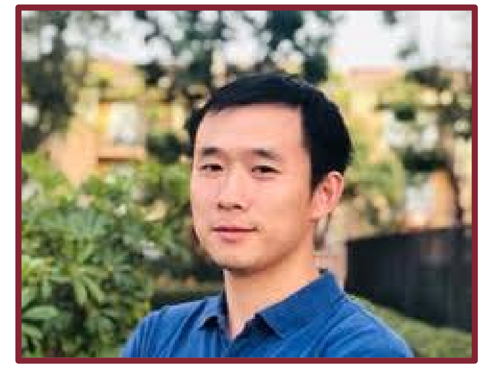
- Taking the derivative of both sides, we obtain an approximate analytic expression for reconstructing $\widetilde{g}(k)$ from $T^2(k)$.

$$\boxed{\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16} + \left|\frac{\log T^2(k)}{\log k}\right|\right)^{-1/2} \left|\frac{d^2\log T^2(k)}{(d\log k)^2}\right|}$$

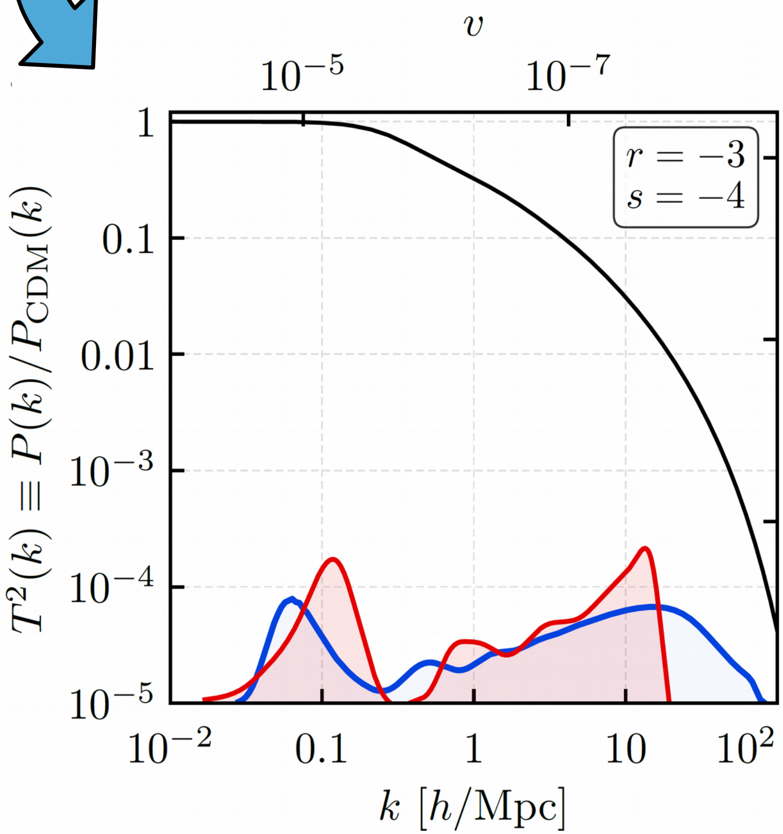
How Well Does This Work in Practice?

- In order to determine how accurately our reconstruction procedure can reproduce the DM velocity distribution from the transfer function, we need examine how well it works in practice.

- Indeed, we find that our procedure is capable of reproducing the broad-brush features of the DM velocity distribution quite robustly in the context of a concrete example model.*



* See talk by Fei Huang immediately following this talk for details.



The Upshot:

Our reconstruction procedure provides a novel and effective method for extracting detailed information about the DM velocity distribution from the matter power spectrum.

Blue: actual $\tilde{g}(k)$ distribution for the model in question

Red: reconstructed $\tilde{g}(k)$ distribution using our procedure

Summary

- Non-trivial dynamics in the early universe can lead to a complicated – and even multi-modal – DM velocity distribution, which in turn affects the shape of the matter power spectrum.
- In an effort to work backwards, we have studied the relationship between $P(k)$ and the DM velocity distribution and found that the ***slope of the transfer function*** at a given k is related to the ***fraction of the DM number density which can free-stream*** on the scale k.
- Motivated by these results, we have formulated a conjecture for ***reconstructing the dark-matter velocity distribution*** from the shape of the matter power spectrum.
- We have shown that this reconstruction conjecture can reliably capture the salient features of the velocity distribution.
- This reconstruction procedure provides a way of probing the internal dynamics within the dark sector which produced the DM through ***purely gravitational means***.