Non-linearly Realized Discrete Symmetries

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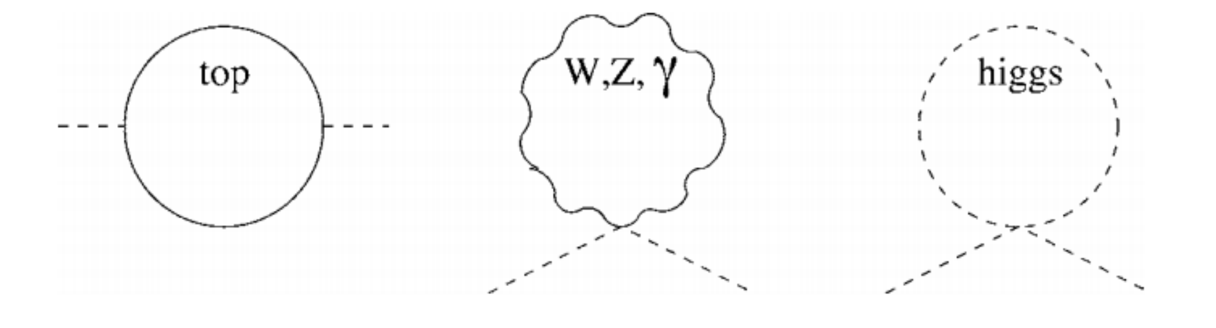
hep-ph/2005.XXXXX: SD, A. Hook

Overview

- Introduction
- Example with A_4 Symmetry
- Invariant Analysis

Introduction



- The Higg’s mass has a quadratic sensitivity to the high mass scales.
- Declare Higgs to be a Goldstone Boson of a larger symmetry.
- Break the symmetry by a ‘tiny’ amount.
- Quadratic divergence often reappears.

Georgi, Kaplan, Phys. Lett. B 136 (1984) 183

Arkani-Hamed et al, JHEP07(2002)

Chacko et al, Phys. Rev. Lett. 96, 231802

Introduction

- Eliminate the quadratic sensitivity from the effective potential
- Still preserve some potential. More suppressed, the better
- Bonus point: Even with O(1) coupling
- Solution: Goldstone of Discrete Symmetry

Introduction

- Not a theory of Higgs (yet)

- We will study, given a Yukawa coupling, how large is the associated Potential?

- Normal scenario : $V(\pi) \sim y^2 \Lambda^2 \pi^2$

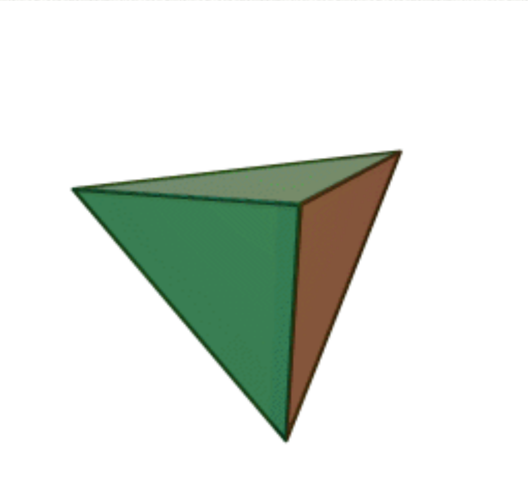
- Discrete Goldstone Boson ($n =$ Group dependent number)

$$V(\pi) \sim \frac{(yf)^n}{\Lambda^{n-4}} \cos\left(\frac{n\pi}{f}\right)$$

Example with A_4 Symmetry

$A_4 =$ Group of even permutations of four objects 12 elements

Equivalent to proper rotations of a regular Tetrahedron



Irreducible representations

1
1'
1''
3

Ishimori et al, PTPS 2010

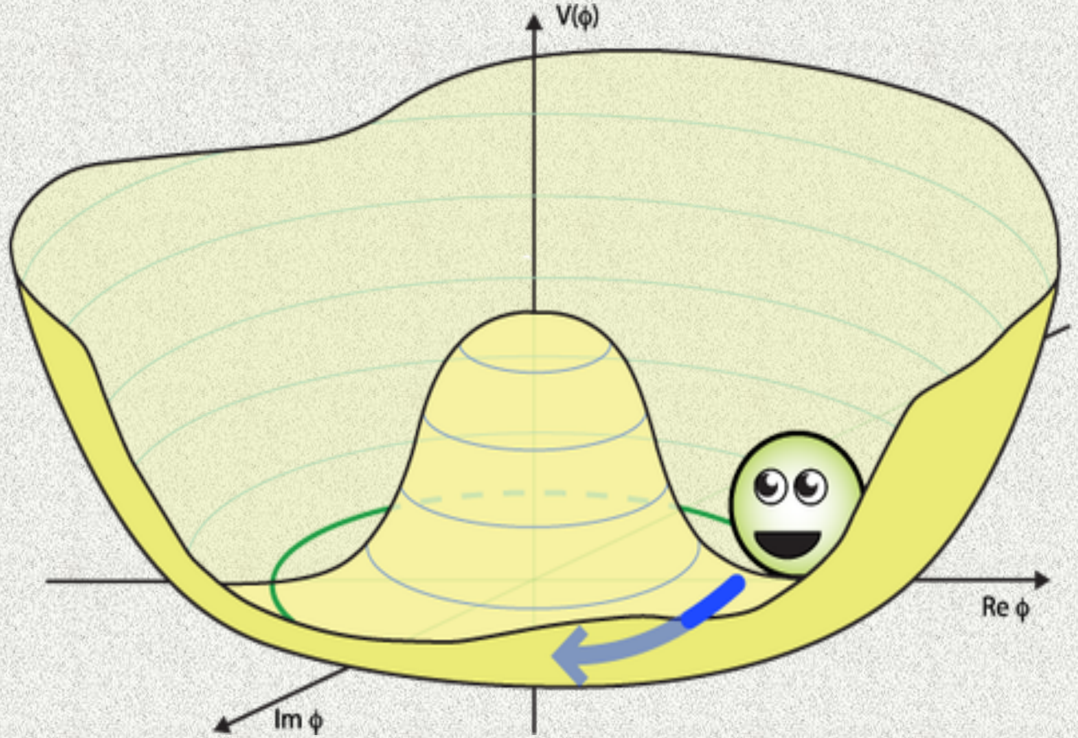
Example with A_4 Symmetry

$$\mathcal{L}_{\text{tree}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{V}(\phi)} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{kin}} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \overline{\Psi}(i\gamma^\mu\partial_\mu)\Psi$$

$$\mathcal{L}_{\text{V}(\phi)} = \frac{m^2}{2}\phi^T\phi - \frac{\lambda}{4}(\phi^T\phi)^2$$

$V(\phi)$ has accidental global $SO(3)$ symmetry



$$\phi = \exp\left[\frac{1}{f}\begin{pmatrix} 0 & 0 & \pi_1 \\ 0 & 0 & \pi_2 \\ -\pi_1 & -\pi_2 & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad f = \frac{m^2}{\lambda}$$

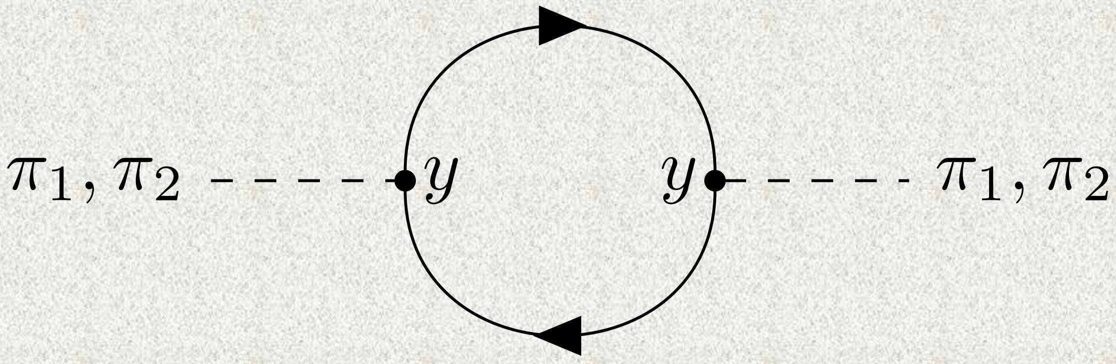
Example with A_4 Symmetry

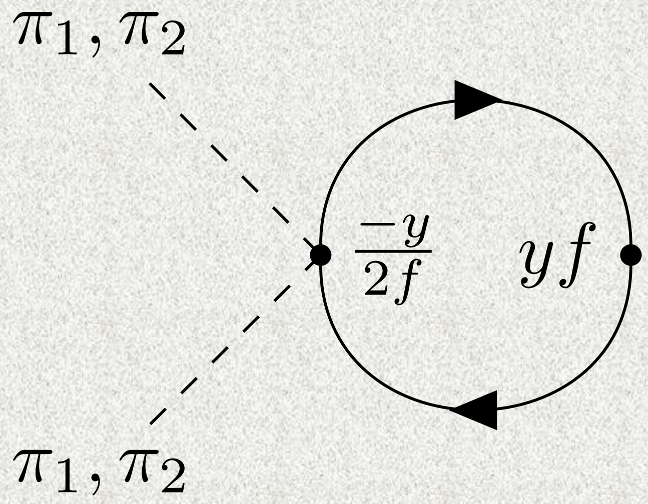
$$\mathcal{L}_{\text{int}} = \phi_{\mathbf{3}} \overline{\Psi}_{\mathbf{3}} \Psi_{\mathbf{3}} = \left[y_s \begin{pmatrix} \{\overline{\Psi}_2 \Psi_3\} \\ \{\overline{\Psi}_3 \Psi_1\} \\ \{\overline{\Psi}_1 \Psi_2\} \end{pmatrix} + y_a \begin{pmatrix} [\overline{\Psi}_2 \Psi_3] \\ [\overline{\Psi}_3 \Psi_1] \\ [\overline{\Psi}_1 \Psi_2] \end{pmatrix} \right] \cdot \phi$$

$$\{\Psi_i, \Psi_j\} = \Psi_i \Psi_j + \Psi_j \Psi_i$$
$$[\Psi_i, \Psi_j] = \Psi_i \Psi_j - \Psi_j \Psi_i$$

$$\mathcal{L}_{\text{int}} = y\pi_1 \left(\overline{\Psi}_2 \Psi_3 + \overline{\Psi}_3 \Psi_2\right) + y\pi_2 \left(\overline{\Psi}_3 \Psi_1 + \overline{\Psi}_1 \Psi_3\right) + yf \left(1 - \frac{1}{2} \frac{\pi_1^2 + \pi_2^2}{f^2}\right) \left(\overline{\Psi}_1 \Psi_2 + \overline{\Psi}_2 \Psi_1\right) + \mathcal{O}(\pi^3)$$

Quadratic Divergence cancels!





Example with A_4 Symmetry

Symmetry, symmetry, symmetry.

$$L_{\text{int}} = M^{IJ}\overline{\Psi}_I \Psi_J \qquad M = y \begin{pmatrix} 0 & \phi_3 & \phi_2 \\ \phi_3 & 0 & \phi_1 \\ \phi_2 & \phi_1 & 0 \end{pmatrix}$$

The quadratic part

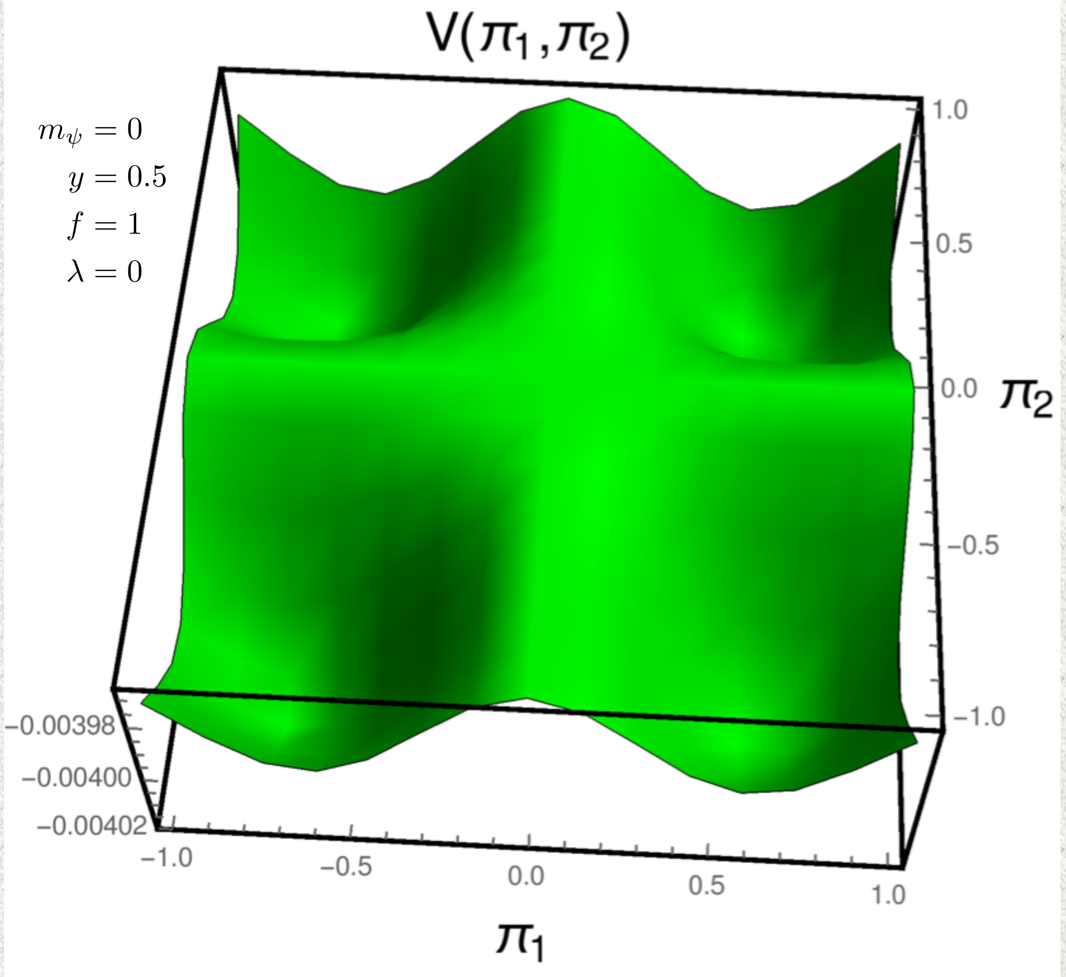
$$V_{1\text{ loop}} \supset \frac{1}{16\pi^2}\Lambda^2 \, \text{Tr}[M \cdot M^T] = \frac{1}{16\pi^2} y^2 \Lambda^2 \, 2(\phi^T \phi) \implies SO(3) \;\; \text{symmetric}$$

The total one loop Coleman-Weinberg potential*

$$V_{1\text{ loop}} = (-\frac{1}{2})\frac{1}{16\pi^2}\left[\big(\text{Tr}(M \cdot M^T)\big)^2 \left(\text{Tr}\left(\ln \frac{(M \cdot M^T)}{\mu^2}\right) - \frac{3}{2}\right)\right]$$

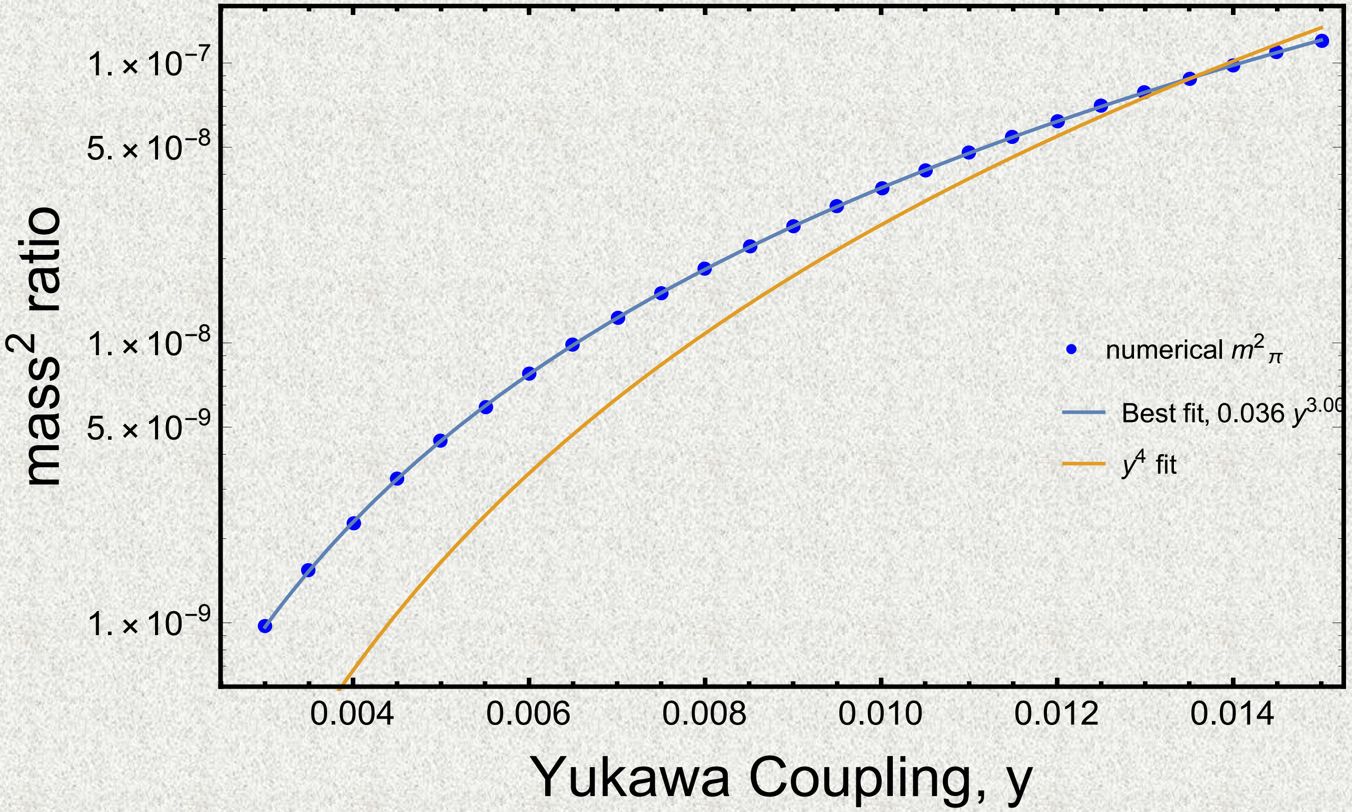
*Stephen P. Martin Phys. Rev. D **65**, 116003

Example with A_4 Symmetry



$$\langle \phi \rangle_0 = \pm \frac{f}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

Example with A_4 Symmetry



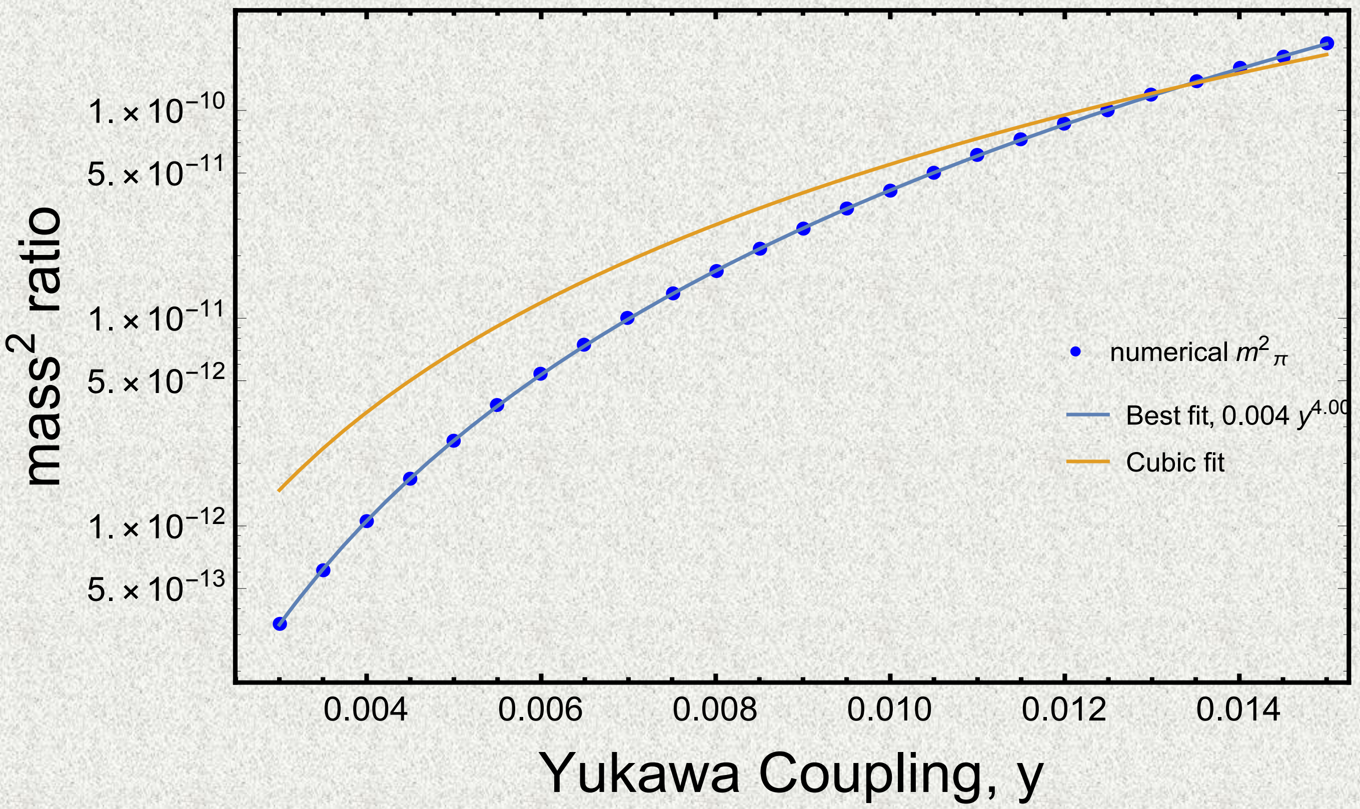
$$m_\psi \neq 0$$

$$V(\phi) \sim \phi_1\phi_2\phi_3$$

$$V(\phi) \sim V(y\phi)$$

$$m_\pi^2 \propto y^3$$

Example with A_4 Symmetry



$$m_\psi = 0$$

$$\phi \to -\phi$$

$$\phi_1\phi_2\phi_3 \not\to \phi_1\phi_2\phi_3$$

$$V(\phi) \sim \phi_1^4 + \phi_2^4 + \phi_3^4$$

$$m_\pi^2 \propto y^4$$

Invariant Analysis

- For global SO(3) symmetry, $V(\phi) = V(\phi^T \phi)$
- For A_4 , the invariants are $\phi^T \phi, \quad \phi_1 \phi_2 \phi_3, \quad \sum_i \phi_i^4$

$$\phi^6 = \alpha_1 (\phi^T \phi)^3 + \alpha_2 (\phi_1 \phi_2 \phi_3)^2 + \alpha_3 (\phi^T \phi)(\sum_i \phi_i^4) \qquad \alpha_i \in \mathbb{R}$$

- The $\phi^T \phi$ will not generate any potential for pions, so $m_\pi^2 \propto y^3$
- For a general discrete group, we need to find the lowest order invariant that breaks the continuous symmetry.

Noether, Mathematiche Annalen volume 77 (1915)

Shepherd and Todd, CJM 1954-028-3

Summary

- Nonlinear realizations of discrete symmetry forbids quadratic sensitivity in effective potential.
- The effective potential is highly suppressed.
- The degree of suppression is group dependent.

Thank you