Gravitational Production of Darkest Matter:
(Non-adiabatic Production of Ultralight Dark Matter)

Based on PRD 101, 083516 (2020)

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Ultralight Dark Matter: Axion-like Particles

- In modern usage often refers to a light scalar or pseudoscalar BSM degree of freedom
  - Example: Fuzzy Dark Matter
    - L. Hui *et al.* (2017)
    - $m \sim 10^{-22}$ eV $\quad \lambda \sim$ kpc
    - Forms Halo-sized BE condensate whose quantum properties may resolve "small-scale structure problems"
- Production Mechanisms:
  - Decay of heavy field
  - Thermal production
  - Misalignment Mechanism
- Question: Can we exploit expanding spacetime phenomenology to produce ALPs?

The Friedmann-Robertson-Walker Metric

- A Spatially Flat Expanding Universe is described by the FRW Metric:

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$$

- The scale factor must obey Friedmann's Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = H_0^2 \left[\frac{\Omega_M}{a^3(t)} + \frac{\Omega_R}{a^4(t)} + \Omega_\Lambda\right]$$

$$H_0 = 1.5 \times 10^{-42}\,\mathrm{GeV} \quad ; \quad \Omega_M = 0.308 \quad ; \quad \Omega_R = 5 \times 10^{-5} \quad ; \quad \Omega_\Lambda = 0.692$$

Values as determined by C.L. Bennett *et al.* (2013)

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4 Important Consequences:

1. **Homogeneous (Momentum Conservation)**
2. **Isotropic (Angular Momentum Conservation)**
3. **Time-Dependence (Energy is Not Conserved)**
4. **Conformal to Minkowski (dt = a dη)**

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The Adiabatic Approximation

- WKB Ansatz: $$g_k(\eta) = \frac{e^{-i\int_{\eta_i}^{\eta} W_k(\eta')\,d\eta'}}{\sqrt{2\,W_k(\eta)}} \qquad W_k^2(\eta) = \omega_k^2(\eta) - \frac{1}{2}\left[\frac{W_k''(\eta)}{W_k(\eta)} - \frac{3}{2}\left(\frac{W_k'(\eta)}{W_k(\eta)}\right)^2\right]$$

- Adiabatic Expansion: $$W_k^2(\eta) = \omega_k^2(\eta)\left[1 - \frac{1}{2}\frac{\omega_k''(\eta)}{\omega_k^3(\eta)} + \frac{3}{4}\left(\frac{\omega_k'(\eta)}{\omega_k^2(\eta)}\right)^2 + \cdots\right]$$

- The Physical Character of this Expansion: $$\frac{\omega_k'(\eta)}{\omega_k^2(\eta)} = \frac{H(t)}{\gamma_k^2(t)\,E_k(t)}$$

Non-adiabatic Window for Ultralight DM

- Adiabatic Criterion: $\frac{\omega_k^{'}(\eta)}{\omega_k^2(\eta)} \ll 1$

- During Radiation Domination Epoch: $\frac{\omega_k^{'}(\eta)}{\omega_k^2(\eta)} = \frac{a'(\eta)}{m\, a^2(\eta)} = \frac{\sqrt{\Omega_R} H_0}{m\, a^2(\eta)} \ll 1 \;\rightarrow a(\eta) \gg \frac{10^{-17}}{\sqrt{m_{\text{eV}}}}$

- Non-adiabatic window closes: $a(\eta) \gg 10^{-6}\sqrt{\frac{10^{-22}\text{eV}}{m}}$
  - Window closes late into RD!

DM Model: Assumptions and Production

- light scalar field:
  - Spectator during Inflation
  - Uncoupled to SM
  - Only gravitational interactions

- Low momentum modes:
  - $\lambda \gg \dfrac{1}{H(\eta_R)}$
  - All astrophysically relevant scales

- Instantaneous reheating

- Non-adiabaticity during RD

- No Backreaction

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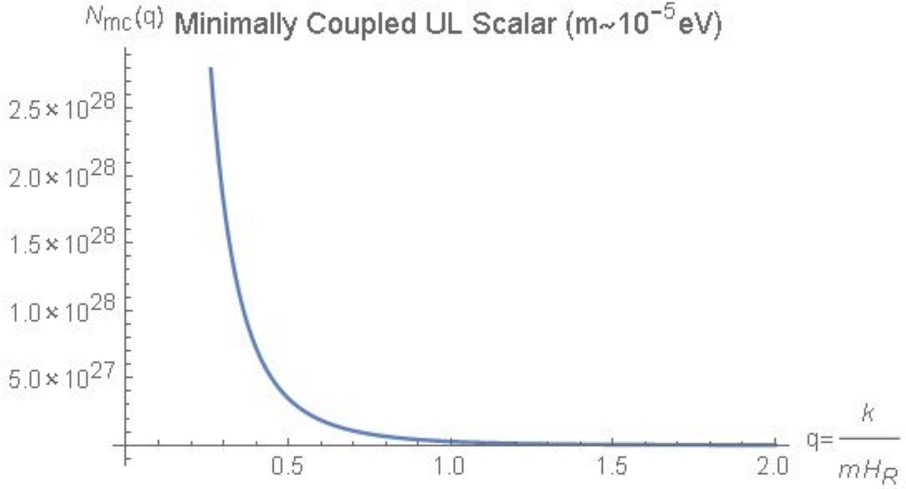
1. Solve Equations of Motion exactly during inflationary epoch.
   a. Initial conditions: Field is in ground state of Bunch-Davies vacuum.
2. Solve Equations of Motion exactly during RD.
   a. Modes asymptotically approach adiabatic evolution.
3. Enforce continuity.
   a. Matching coefficients yield particle production.
   b. Bogoliubov Coefficients
4. Compute the energy momentum tensor during MD (with adiabatic expansion)

Particle Production



- Whenever the Bogoliubov Coefficients are non-zero, there will be particle production!
- For minimally coupled case, the distribution is peaked at low momentum
  - Not quite Bose-Einstein Condensate
  - Consistent with low momentum mode assumption

Produced Dark Matter Abundance

- Compute the Energy Momentum Tensor and extract energy density and pressure at onset of Matter Domination
  - EMT is continuous throughout expansion history
  - Adiabatic approximation obtains
  - Compute to 4th order, and renormalize/regularize to handle divergences (Anderson and Parker 1987)

- To leading adiabatic order:

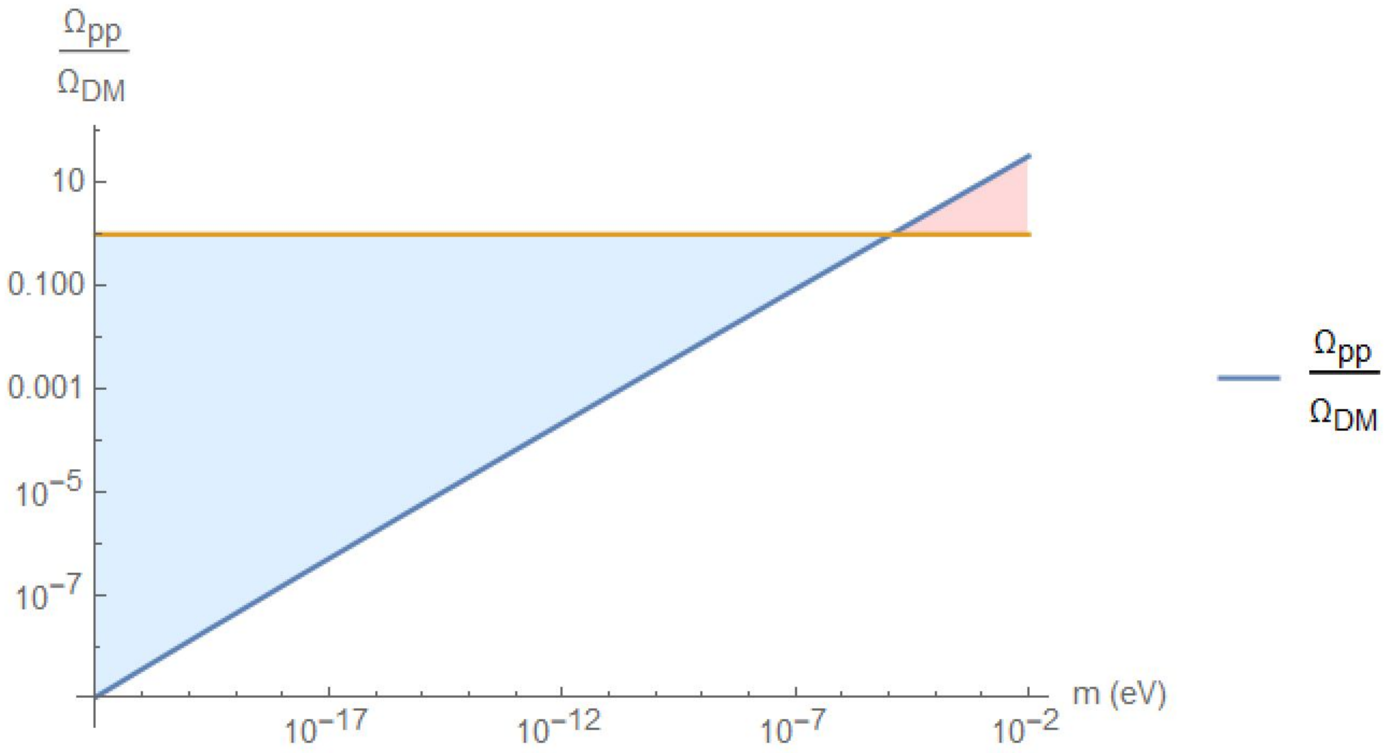
$$\rho^{(pp)}(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_0^\infty k^2 \mathcal{N}_k \, \omega_k(\eta) dk$$

$$P^{(pp)}(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_0^\infty \frac{1}{3} k \, v_k(\eta) \, \mathcal{N}_k \, k^2 dk \quad v_k = \frac{k}{\omega}$$

- Introduce upper and lower cutoff:

$$k_{\min} \simeq H_0 \quad k_{\max} \leq m \, a_{eq}$$

Produced Dark Matter Abundance



Conclusions

- For m ~ 10^{-5} eV correct relic abundance! (**Axions!**)
    - w ~ 10^{-14} (Cold, light dark matter)
    - Free streaming length ~ 100 pc (comparable with WIMP halos)
    - Truly the “darkest” of matter: only gravitational interactions!

- New production mechanism for ***ultralight*** (m < 1 eV) scalar dark matter!
    - Decay of heavy field
    - Thermal production
    - Misalignment Mechanism
    - **Non-adiabatic Gravitational Production**
        - Modest assumptions for this effect to be present in a scalar field theory!
        - Two parameter model: (particle mass, scale of inflation)
- Open questions:
    - What is the phenomenology of models with multiple production mechanisms?
    - What happens in the fermionic case? (see: arXiv:2005.00391)

Extra Slides

DM Model and Assumptions

- Inflation spectator, light scalar field:

$$S = \int d^3x \, d\eta \, \frac{1}{2} \left[\chi'^2 - (\nabla\chi)^2 - \mathcal{M}^2(\eta)\, \chi^2 \right]$$

$$\mathcal{M}^2(\eta) = m^2 \, a^2(\eta) - \frac{a''(\eta)}{a(\eta)} (1 - 6\xi)$$

- Low momentum modes:

$$k\eta_R \ll 1 \qquad \lambda \gg \frac{1}{H(\eta_R)}$$

- Instantaneous reheating:

$$a_{ds}(\eta_R) = \frac{1}{H_{ds}\eta_R} = H_R \eta_R \qquad H_R = \sqrt{\Omega_R} H_0 \;\; ; \frac{H_{ds}}{M_{pl}} < 2.5 \times 10^{-5}$$

- No Backreaction

Solving Equations of Motion

- Solve Equations of Motion during inflationary epoch.
  a. Ground state of Bunch-Davies vacuum as initial conditions
     i. **IN states**
  b. Hankel function solutions: $g_k^{<}(\eta)$
- Solve Equations of Motion during RD.
  a. Modes asymptotically approach adiabatic evolution.
     i. **OUT states**
  b. Weber function solutions
  c. Construct General Solution: $g_k^{>}(\eta) = A_k\, f_k(\eta) + B_k\, f_k^*(\eta)$
  d. Enforce continuity. Matching coefficients yield particle production.

Equations of Motion in Three Regimes

Inflationary Epoch

- Initial Conditions define "in" states.
- Solution is Bessel Functions.

$$g_k(\eta) \xrightarrow[\eta \to -\infty]{} \frac{e^{-ik\eta}}{\sqrt{2k}}$$

$$g_k^{<}(\tau) = \frac{1}{2}\sqrt{-\pi\tau}\, e^{i\frac{\pi}{2}(\nu+1/2)}\, H_\nu^{(1)}(-k\tau)$$

$$\tau = \eta - 2\eta_R$$

Radiation Dominated Epoch

- Solution is Weber Functions
- Construct General Solution

$$f_k(\eta) = \frac{1}{(8mH_R)^{1/4}}\left[\frac{1}{\sqrt{\kappa}} W[\alpha; x] - i\sqrt{\kappa}\, W[\alpha; -x]\right]$$

$$g_k^{>}(\eta) = A_k\, f_k(\eta) + B_k\, f_k^{*}(\eta)$$

$$g_k^{<}(\eta_R) = g_k^{>}(\eta_R) \;\; ; \;\; \frac{d}{d\eta} g_k^{<}(\eta)|_{\eta_R} = \frac{d}{d\eta} g_k^{>}(\eta)|_{\eta_R}$$

Matter Dominated Epoch

- Adiabatic Approximation holds!
- Asymptotic "out" states are particle states of today

$$h_k(\eta) \simeq \frac{e^{-i\omega(\eta_0)(\Delta\eta)}}{\sqrt{2\omega(\eta_0)}}$$

$$g_k^{\gg}(\eta) = C_k\, h_k(\eta) + D_k\, h_k^{*}(\eta)$$

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Particle Production

- How does Particle Production emerge?

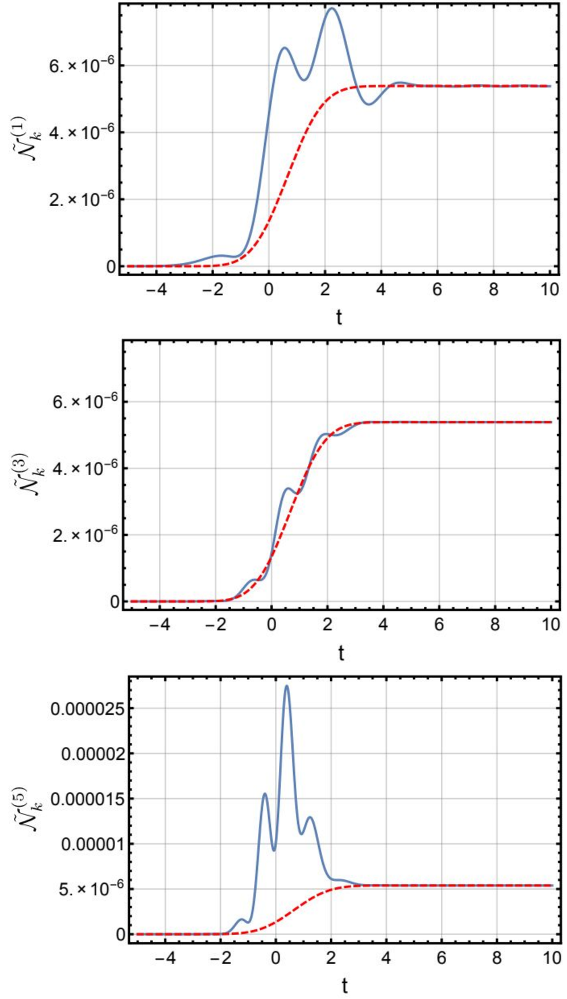
$$\chi(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[a_{\vec{k}}\, g_k^{>}(\eta)\, e^{-i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^{\dagger}\, g_k^{*>}(\eta)\, e^{i\vec{k}\cdot\vec{x}} \right] = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[b_{\vec{k}}\, f_k(\eta)\, e^{-i\vec{k}\cdot\vec{x}} + b_{\vec{k}}^{\dagger}\, f_k^{*}(\eta)\, e^{i\vec{k}\cdot\vec{x}} \right]$$

$$b_{\vec{k}} = a_k\, A_k + a^{\dagger}_{-\vec{k}}\, B_k^{*} \;\; ; \;\; b_{\vec{k}}^{\dagger} = a_{\vec{k}}^{\dagger}\, A_k^{*} + a_{-\vec{k}}\, B_k$$

$$|A_k^2| - |B_k^2| = 1 \qquad \mathcal{N}_k = \langle 0 | b_{\vec{k}}^{\dagger} b_{\vec{k}} | 0 \rangle = |B_k|^2$$

When are the particles produced?

- The Bogoliubov coefficients in our calculation are time-independent.
- One can study time-dependent particle number (Dabrowski 2016)
  - Use adiabatic approximation within the non-adiabatic window
  - Define time-dependent coefficients
  - Particle production depends on adiabatic order considered
  - Asymptotic agreement when non-adiabatic window closes



Produced Dark Matter Abundance

- Cosmological evolution and structure formation are determined by Energy Momentum Tensor.

- Compute the EMT in terms of “particle” mode functions.
    - Average over interference terms

$$\langle 0|T_0^0|0\rangle = \rho(\eta) = \frac{1}{4\pi^2 a^4(\eta)}\int_0^\infty k^2 dk \left\{ |g_k^{'}(\eta)|^2 + \omega_k(\eta)^2 |g_k(\eta)|^2 - \left[\frac{a^{'}}{a}\left(g_k(\eta) g_k^{'*}(\eta) + g_k^{'}(\eta) g_k^*(\eta) - \frac{a^{'}}{a}|g_k(\eta)|^2\right)\right]\right\}$$

$$\langle 0|T_0^0|0\rangle = \rho(\eta) = \frac{1}{4\pi^2 a^4(\eta)}\int_0^\infty k^2 dk\,(1+2\mathcal{N}_k)\left\{ |f_k^{'}(\eta)|^2 + \omega_k(\eta)^2 |f_k(\eta)|^2 - \left[\frac{a^{'}}{a}\left(f_k(\eta) f_k^{'*}(\eta) + f_k^{'}(\eta) f_k^*(\eta) - \frac{a^{'}}{a}|f_k(\eta)|^2\right)\right]\right\}$$

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- Introduce upper and lower cutoff:

$$k_{\min} \simeq H_0 \quad k_{\max} \leq m \, a_{eq}$$

The WKB Solution

- In radiation domination, the mode function differential equation simplifies

$$\frac{d^2}{d\eta^2}\,\chi_{\vec{k}}(\eta) + \omega_k^2(\eta)\,\chi_{\vec{k}}(\eta) = 0$$

- The mode functions must be quantized

$$\chi = a\,g_k(\eta) + a_-^\dagger\,g_k^*(\eta)$$

- The WKB ansatz can then be implemented (N.D. Birrell, P. C. W. Davies)

$$g_k(\eta) = \frac{e^{-i\int_{\eta_i}^{\eta} W_k(\eta')\,d\eta'}}{\sqrt{2\,W_k(\eta)}} \qquad W_k^2(\eta) = \omega_k^2(\eta) - \frac{1}{2}\left[\frac{W_k''(\eta)}{W_k(\eta)} - \frac{3}{2}\left(\frac{W_k'(\eta)}{W_k(\eta)}\right)^2\right]$$