A $\mathcal{T}_{13}$ Family Symmetry Model for Quarks and Leptons

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Outline

- An Asymmetric Yukawa Texture
  - bottom-up approach using $SU(5)$ GUT
    `arXiv:1805.10684, M.H.R., P. Ramond, B. Xu`
- Yukawa Texture from a Discrete Family Symmetry
  - based on $\mathcal{T}_{13} = \mathcal{Z}_{13} \rtimes \mathcal{Z}_3$
    `arXiv:1907.10698, M.J. Pérez, M.H.R., P. Ramond, A.J. Stuart, B. Xu`
- Tribimaximal Seesaw Mixing from $\mathcal{T}_{13}$
  - predicts "normal ordering" of light neutrino masses
    `arXiv:2001.04019, M.J. Pérez, M.H.R., P. Ramond, A.J. Stuart, B. Xu`

A Marriage between the EW and Seesaw Sectors?

Lepton mixing angles in the PMNS matrix are

$$\theta_{12} = 33.65^\circ, \quad \theta_{23} = 45.57^\circ, \quad \theta_{13} = 8.34^\circ$$

[PDG 2018]

$$\mathcal{U}_{PMNS} = \underbrace{\mathcal{U}^{(-1)^\dagger}}_{\text{EW sector}} \underbrace{\mathcal{U}_{TBM}}_{\text{Seesaw sector}}$$

$$\mathcal{U}_{TBM} \equiv \mathcal{R}(\theta_{12} = 35.3^\circ),\ \mathcal{R}(\theta_{23} = 45^\circ),\ \mathcal{R}(\theta_{13} = 0^\circ),\ \delta = 0^\circ$$

Can the Electroweak sector generate enough "Cabibbo Haze"?

Hunting the 'Minimal' Texture

What is the 'minimal' Yukawa texture that

- reproduces the GUT-scale mass relations, the CKM mixing angles, and
- generates *enough* 'Cabibbo haze' for the reactor angle?

The Asymmetric Texture `arXiv:1805.10684`

$$Y^{(\frac{2}{3})} \sim \text{ diag } (\lambda^8, \lambda^4, 1),$$

$$Y^{(-\frac{1}{3})} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \quad Y^{(-1)} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

$$\mathcal{U}_{Seesaw} = \text{diag } (1, 1, e^{i\delta})\, \mathcal{U}_{TBM}$$

$$a = c = \frac{1}{3}, \quad g = A, \quad b = A\sqrt{\rho^2 + \eta^2}, \quad d = \frac{2a}{g} = \frac{2}{3A}, \quad \cos\delta = 0.2$$

The Asymmetric Texture arXiv:1805.10684

$$Y^{(-\frac{1}{3})} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \; Y^{(-1)} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

- One phase to rule them all:
  $|\delta| = 78^\circ$: all angles within 1σ of PDG 2019 fit

Predicts leptonic CP violation: $|\delta_{CP}| = 1.32\pi$

consistent with PDG 2019 fit $\delta_{CP} = 1.37^{+0.18}_{-0.16}\pi$, to be measured at DUNE and Hyper-K in ~ 10 years [arXiv:1807.10334, 1805.04163].

- Asymmetric terms: $(13 - 31)$, by $\mathcal{O}(\lambda)$, not negligible!
- Zero subdeterminant: The determinant of the [13] submatrix vanishes, so that

$$\det Y^{(-\frac{1}{3})} = \det Y^{(-1)}$$

Family Symmetry arXiv:1907.10698

Suppose $SU(5)$ matter fields $F \sim \bar{\mathbf{5}}$ and $T \sim \mathbf{10}$ are triplets of some discrete family symmetry group, G_f.

F and T must be different triplets of G_f

- $Y^{(-1/3)}$ and $Y^{(-1)}$ comes from $F \otimes T = (\bar{\mathbf{5}}, \mathbf{r}) \otimes (\mathbf{10}, \mathbf{s})$
- $3 \times 3 \Rightarrow$ either symmetric or antisymmetric
  We need a group with at least two different triplets!
- Candidates: $\mathcal{S}_4$ (order 24), $\Delta(27)$ (order 27), $\mathcal{T}_{13}$ (order 39)

$\mathbf{s} \otimes \mathbf{s}$ must distinguish diagonal from off-diagonal

- $Y^{(2/3)}$ comes from $T \otimes T = (\mathbf{10}, \mathbf{s}) \otimes (\mathbf{10}, \mathbf{s})$

Only $\mathcal{T}_{13}$ survives!

Generating the Asymmetric Term

$Y^{(-\frac{1}{3})} \leftarrow FTH_{\bar{\mathbf{5}}}\varphi^{(-\frac{1}{3})}$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}_{\mathbf{3}_1} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} F_3T_2 \\ F_1T_1 \\ F_2T_3 \end{pmatrix}_{\mathbf{3}_1} \oplus \begin{pmatrix} F_3T_1 \\ F_1T_3 \\ F_2T_2 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} F_3T_3 \\ F_1T_2 \\ F_2T_1 \end{pmatrix}_{\mathbf{3}_2}$$

$\mathcal{T}_{13}$ can dial *individual* matrix elements!

$Y^{(\frac{2}{3})} \leftarrow TT\bar{H}_{\mathbf{5}}\varphi^{(\frac{2}{3})}$

$$\begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} T_3T_3 \\ T_2T_2 \\ T_1T_1 \end{pmatrix}_{\bar{\mathbf{3}}_1} \oplus \begin{pmatrix} T_3T_2 \\ T_2T_1 \\ T_1T_3 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} T_2T_3 \\ T_1T_2 \\ T_3T_1 \end{pmatrix}_{\bar{\mathbf{3}}_2}$$

Diagonals are distinguished from off-diagonals!

Generating the Zero Subdeterminant

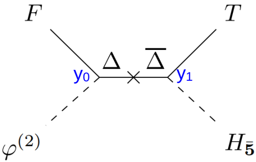
$$\begin{pmatrix} bd\lambda^4 & \times & b\lambda^3 \\ \times & \times & \times \\ d\lambda & \times & 1 \end{pmatrix} = \begin{pmatrix} b\lambda^3 \cdot d\lambda & \times & b\lambda^3 \cdot 1 \\ \times & \times & \times \\ d\lambda & \times & 1 \end{pmatrix}$$

$$(33)\ \text{term} \rightarrow FTH_{\bar{\mathbf{5}}}\varphi^{(1)}$$

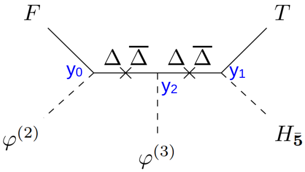
$$(31)\ \text{term} \rightarrow FTH_{\bar{\mathbf{5}}}\varphi^{(2)}$$

$$(13)\ \text{term} \rightarrow FTH_{\bar{\mathbf{5}}}\varphi^{(1)}\varphi^{(3)}$$

$$(11)\ \text{term} \rightarrow FTH_{\bar{\mathbf{5}}}\varphi^{(2)}\varphi^{(3)}$$



(a) (31) term



(b) (11) term

TBM Mixing from $\mathcal{T}_{13}$ arXiv:2001.04019

- Introduce three right handed neutrinos $\bar{N} \sim (\mathbf{1}, \mathbf{3}_2)$
- Dirac Yukawa matrix, $Y^{(0)}$: $F\bar{N}\bar{H}_{\mathbf{5}}\varphi_{\mathcal{A}}$, $\varphi_{\mathcal{A}} \sim (\mathbf{1}, \bar{\mathbf{3}}_2)$
- Majorana matrix, $\mathcal{M}$: $\bar{N}\bar{N}\varphi_{\mathcal{B}}$, $\varphi_{\mathcal{B}} \sim (\mathbf{1}, \mathbf{3}_2)$
- Seesaw matrix, $\mathcal{S} = Y^{(0)}\ \mathcal{M}^{-1}\ {Y^{(0)}}^T$
- For $\langle\varphi_{\mathcal{B}}\rangle \propto (1,1,1)$ and $\langle\varphi_{\mathcal{A}}\rangle \propto (-e^{i\delta},1,1)$,

$$\mathcal{S} \propto \mathcal{U}_{TBM}(\delta)\ \mathrm{diag}\left(-1, \frac{1}{2}, -1\right)\ \mathcal{U}_{TBM}^{T}(\delta).$$

- Introduce a fourth right handed neutrino $\bar{N}_4 \sim (\mathbf{1}, \mathbf{1})$ with no Yukawa interaction
- New Majoarna operator $\bar{N}\bar{N}_4\varphi_z$, with $\langle\varphi_z\rangle \propto (1,-2,1)$

Light Neutrino Masses

Prediction: normal ordering

$$m_{\nu_1} = 27.6, \quad m_{\nu_2} = 28.9, \quad m_{\nu_3} = 57.8 \text{ meV}$$

- $\sum_i |m_{\nu_i}| = 114.3$ meV compared to Planck: $\sum_i |m_{\nu_i}| < 120$ meV [arXiv:1807.06209].
- Combining data from Euclid and LSST to DESI and WFIRST, the error bound on $\sum_i |m_{\nu_i}|$ will be constrained to $8 - 11$ meV.
- Normal ordering is preferred above 3σ by Super-K, T2K and NOvA [arXiv:1710.09126].
- DUNE and Hyper-K will resolve the correct mass ordering beyond 5σ in $5 - 7$ yrs [arXiv:1807.10334, 1805.04163].

Neutrinoless Double Beta Decay

- Dirac $\not{CP}$ Jarlskog-Greenberg Invariant, $|\mathcal{J}| = 0.028$
- Majorana Invariants, $|\mathcal{I}_1| = 0.106, \quad |\mathcal{I}_2| = 0.011$

Prediction for $0\nu\beta\beta$

$$|m_{\beta\beta}| = 13.02 \text{ or } 25.21 \text{ meV}$$

compared to $|m_{\beta\beta}| \leqslant 61 - 165$ meV by KamLAND-Zen [arXiv:1605.02889]

- Our predictions are expected to be tested in several next generation experiments [J.Phys.Conf.Ser. 1390 (2019) 1, 012048]:

Experiment	Sensitivity (meV)	Experiment	Sensitivity (meV)
LEGEND	11 - 28	SNO+-II	20 - 70
nEXO	8 - 22	AMoRE-II	15 - 30
CUPID	6 - 17	PandaX-III	20 - 55

Summary

- An $SU(5) \times \mathcal{T}_{13}$ model that covers both quarks and leptons
- Asymmetry and zero-subdeterminant conditions are implemented without *fine-tuning*
- Prediction for CP violation: $|\delta_{CP}| = 1.32\pi$
- Prediction for light neutrino masses: Normal ordering
  $m_{\nu_1} = 27.6,\ m_{\nu_2} = 28.9,\ m_{\nu_1} = 57.8$ meV
- Prediction for $0\nu\beta\beta$: $|m_{\beta\beta}| = 13.02$ or 25.21 meV

Thank You!

Backup slides

$\mathcal{T}_{13}$ Group Theory

- $\mathcal{T}_{13} = \mathcal{Z}_{13} \rtimes \mathcal{Z}_3$ has two generators a and b
- Presentation: $\langle a, b \mid a^{13} = b^3 = I, bab^{-1} = a^3 \rangle$
- Order $= 13 \times 3 = 39$
- Three one dimensional irreps $\mathbf{1}$, $\mathbf{1'}$, $\bar{\mathbf{1}}'$ and four three dimensional irreps $\mathbf{3_1}$, $\bar{\mathbf{3}}_\mathbf{1}$, $\mathbf{3_2}$, $\bar{\mathbf{3}}_\mathbf{2}$

  $1^2 + 1^2 + 1^2 + 3^2 + 3^2 + 3^2 + 3^2 = 39$
- $\mathbf{1} : a = 1, b = 1$; $\mathbf{1'} : a = 1, b = \omega$; $\bar{\mathbf{1}}' : a = 1, b = \omega^2$, where $\omega = e^{i2\pi/3}$

$\mathcal{T}_{13}$ Group Theory (contd.)

- b permutes the components of the triplets
- a assigns $\mathcal{Z}_{13}$ charges:

$$\mathbf{3_1}: \quad (\rho,\, \rho^3,\, \rho^9), \quad \mathbf{3_2}: \quad (\rho^2,\, \rho^6,\, \rho^5)$$

  where $\rho^{13} = 1$.
- Diagonal elements are naturally singled out in Kronecker products of triplets:

$$\begin{aligned} \mathbf{3_1} \otimes \mathbf{3_1} &= \bar{\mathbf{3}}_\mathbf{1} \oplus \mathbf{3_2} \oplus \bar{\mathbf{3}}_\mathbf{1}, \\ \mathbf{3_2} \otimes \mathbf{3_2} &= \bar{\mathbf{3}}_\mathbf{2} \oplus \bar{\mathbf{3}}_\mathbf{1} \oplus \bar{\mathbf{3}}_\mathbf{2}, \\ \mathbf{3_1} \otimes \mathbf{3_2} &= \mathbf{3_2} \oplus \mathbf{3_1} \oplus \bar{\mathbf{3}}_\mathbf{2}. \end{aligned}$$

$\mathcal{T}_{13}$ Group Theory (contd.)

Consider a triplet $\psi = \{\psi^1, \psi^2, \psi^3\} \sim \mathbf{3_1}$

Under the generator b,
$\{\psi^1, \psi^2, \psi^3\} \to \{\psi^2, \psi^3, \psi^1\}$ cyclic permutation of the components

Under the generator a,
$\{\psi^1, \psi^2, \psi^3\} \to \{\rho\psi^1, \rho^3\psi^2, \rho^9\psi^3\}$ the components gain $\mathcal{Z}_{13}$ charges

We can determine the Clebsch-Gordan decompositions by simply following the action of the a generator!

$\mathcal{T}_{13}$ Clebsch-Gordan Decomposition Table

$$\mathbf{3_1} \otimes \mathbf{3_1} \to \begin{pmatrix} \psi^{11} \\ \psi^{22} \\ \psi^{33} \end{pmatrix}_{\mathbf{3_2}}, \quad \begin{pmatrix} \psi^{23} \\ \psi^{31} \\ \psi^{12} \end{pmatrix}_{\mathbf{\bar{3}_1}}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\mathbf{\bar{3}_1}}$$

$$\mathbf{3_2} \otimes \mathbf{3_2} \to \begin{pmatrix} \psi^{22} \\ \psi^{33} \\ \psi^{11} \end{pmatrix}_{\mathbf{\bar{3}_1}}, \quad \begin{pmatrix} \psi^{23} \\ \psi^{31} \\ \psi^{12} \end{pmatrix}_{\mathbf{\bar{3}_2}}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\mathbf{\bar{3}_2}}$$

$$\mathbf{3_1} \otimes \mathbf{\bar{3}_1} \to \begin{pmatrix} \psi^{1}_{2} \\ \psi^{2}_{3} \\ \psi^{3}_{1} \end{pmatrix}_{\mathbf{\bar{3}_2}}, \quad \begin{pmatrix} \psi^{2}_{1} \\ \psi^{3}_{2} \\ \psi^{1}_{3} \end{pmatrix}_{\mathbf{3_2}}, \quad (\psi^1_1 + \psi^2_2 + \psi^3_3)_{\mathbf{1}},$$

$$(\psi^1_1 + \omega\psi^2_2 + \omega^2\psi^3_3)_{\mathbf{1'}}, \quad (\psi^1_1 + \omega^2\psi^2_2 + \omega\psi^3_3)_{\mathbf{\bar{1}'}}$$

$\mathcal{T}_{13}$ Clebsch-Gordan Decomposition Table

$$\mathbf{3_2} \otimes \bar{\mathbf{3}}_\mathbf{2} \to \begin{pmatrix} \psi_2^3 \\ \psi_3^1 \\ \psi_1^2 \end{pmatrix}_{\bar{\mathbf{3}}_\mathbf{1}}, \quad \begin{pmatrix} \psi_3^2 \\ \psi_1^3 \\ \psi_2^1 \end{pmatrix}_{\mathbf{3_1}}, \quad (\psi_1^1 + \psi_2^2 + \psi_3^3)_\mathbf{1},$$

$$(\psi_1^1 + \omega\psi_2^2 + \omega^2\psi_3^3)_{\mathbf{1}'}, \quad (\psi_1^1 + \omega^2\psi_2^2 + \omega\psi_3^3)_{\bar{\mathbf{1}}'}$$

$$\mathbf{3_1} \otimes \mathbf{3_2} \to \begin{pmatrix} \psi^{33} \\ \psi^{11} \\ \psi^{22} \end{pmatrix}_{\mathbf{3_1}}, \quad \begin{pmatrix} \psi^{31} \\ \psi^{12} \\ \psi^{23} \end{pmatrix}_{\bar{\mathbf{3}}_\mathbf{2}}, \quad \begin{pmatrix} \psi^{32} \\ \psi^{13} \\ \psi^{21} \end{pmatrix}_{\mathbf{3_2}}$$

$$\mathbf{3_1} \otimes \bar{\mathbf{3}}_\mathbf{2} \to \begin{pmatrix} \psi_1^1 \\ \psi_2^2 \\ \psi_3^3 \end{pmatrix}_{\bar{\mathbf{3}}_\mathbf{1}}, \quad \begin{pmatrix} \psi_3^2 \\ \psi_1^3 \\ \psi_2^1 \end{pmatrix}_{\bar{\mathbf{3}}_\mathbf{2}}, \quad \begin{pmatrix} \psi_1^2 \\ \psi_2^3 \\ \psi_3^1 \end{pmatrix}_{\mathbf{3_1}}$$

$\mathcal{T}_{13}$ Clebsch-Gordan Decompositions

$$\mathbf{1}' \otimes \mathbf{1}' = \bar{\mathbf{1}}'$$
$$\bar{\mathbf{1}}' \otimes \bar{\mathbf{1}}' = \mathbf{1}'$$
$$\mathbf{1}' \otimes \bar{\mathbf{1}}' = \mathbf{1}$$
$$\mathbf{3_1} \otimes \mathbf{3_1} = \bar{\mathbf{3}}_\mathbf{1} \oplus \bar{\mathbf{3}}_\mathbf{1} \oplus \mathbf{3_2}$$
$$\mathbf{3_2} \otimes \mathbf{3_2} = \bar{\mathbf{3}}_\mathbf{2} \oplus \bar{\mathbf{3}}_\mathbf{1} \oplus \bar{\mathbf{3}}_\mathbf{2}$$
$$\mathbf{3_1} \otimes \bar{\mathbf{3}}_\mathbf{1} = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3_2} \oplus \bar{\mathbf{3}}_\mathbf{2}$$
$$\mathbf{3_2} \otimes \bar{\mathbf{3}}_\mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \bar{\mathbf{1}}' \oplus \mathbf{3_1} \oplus \bar{\mathbf{3}}_\mathbf{1}$$
$$\mathbf{3_1} \otimes \mathbf{3_2} = \bar{\mathbf{3}}_\mathbf{2} \oplus \mathbf{3_1} \oplus \mathbf{3_2}$$
$$\mathbf{3_1} \otimes \bar{\mathbf{3}}_\mathbf{2} = \bar{\mathbf{3}}_\mathbf{2} \oplus \mathbf{3_1} \oplus \bar{\mathbf{3}}_\mathbf{1}$$
$$\mathbf{3_2} \otimes \bar{\mathbf{3}}_\mathbf{1} = \mathbf{3_2} \oplus \mathbf{3_1} \oplus \bar{\mathbf{3}}_\mathbf{1}$$

Table 1: $Z_{13} \rtimes Z_3$ Kronecker products

Theoretical Outlook

Why do F and T transform differently under $\mathcal{T}_{13}$?

$$SU(5)\times\mathcal{T}_{13}:\quad \overbrace{(\mathbf{10},\mathbf{3_2})}^{T}\oplus\overbrace{(\bar{\mathbf{5}},\mathbf{3_1})}^{F}\oplus\overbrace{(\mathbf{1},\mathbf{3_2})}^{\bar{N}}\oplus\overbrace{(\mathbf{1},\mathbf{1})}^{\bar{N}_4}$$

$$SO(10)\times\mathcal{T}_{13}:\qquad (\mathbf{16},\mathbf{3_2})\oplus(\mathbf{10},\mathbf{3_1})\oplus(\mathbf{1},\mathbf{1}).$$

$$\mathbf{16}=\mathbf{10_{-1}}\oplus\bar{\mathbf{5}}_\mathbf{3}\oplus\mathbf{1_{-5}},\qquad \mathbf{10}=\mathbf{5_3}\oplus\bar{\mathbf{5}}_\mathbf{-3},$$

$$E_6\supset SO(10)\times U(1):\quad \mathbf{27}=\mathbf{16_1}\oplus\mathbf{10_{-2}}\oplus\mathbf{1_4},$$

Does the $U(1)$ charge dictate which is $\mathbf{3}_1$ and which is $\mathbf{3}_2$?