Towards Minimal SU(5)

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Outline

- Introduction to $SU(5)$ Model Building
- Towards Realistic $SU(5)$
- A Novel $SU(5)$ Proposal
- Summary

$SU(5)$ Model Building

SU_5 irreps → smallest group that contains SM
Minimal choice (rank 4)

Dynkin label	Dimension (name)	l (index)	Quintality	SU_4 singlets	$SU_2 \times SU_3$ singlets
(1000)	5	1	1	1	0
(0100)	10	3	2	0	1
(2000)	15				
(1001)	24				
(0003)	35				
(0011)	40				
(0101)	45				
(0020)	50				
(2001)	70				
(0004)	70′				
(0110)	75	50	0	0	1*
(0012)	105	91	1	0	0
(2010)	126	105	0	0	0
(5000)	126′	210	0	1	0

$SU_5 \supset SU_2 \times SU_3 \times U_1$

$5 = (2,1)(3) + (1,3)(-2)$

$10 = (1,1)(6) + (1,\bar{3})(-4) + (2,3)(1)$

$15 = (3,1)(6) + (2,3)(1) + (1,6)(-4)$

$24 = (1,1)(0) + (3,1)(0) + (2,3)(-5) + (2,\bar{3})(5) + (1,8)(0)$

$35 = (4,1)(-9) + (3,\bar{3})(-4) + (2,\bar{6})(1) + (1,\overline{10})(6)$

$40 = (2,1)(-9) + (2,3)(1) + (1,\bar{3})(-4) + (3,\bar{3})(-4) + (1,8)(6) + (2,\bar{6})(1)$

$45 = (2,1)(3) + (1,3)(-2) + (3,3)(-2) + (1,\bar{3})(8) + (2,\bar{3})(-7) + (1,\bar{6})(-2) + (2,8)(3)$

$50 = (1,1)(-12) + (1,3)(-2) + (2,\bar{3})(-7) + (3,\bar{6})(-2) + (1,6)(8) + (2,8)(3)$

$70 = (2,1)(3) + (4,1)(3) + (1,3)(-2) + (3,3)(-2) + (3,\bar{3})(8) + (2,6)(-7) + (2,8)(3) + (1,15)(-2)$

$70' = (5,1)(-12) + (4,\bar{3})(-7) + (3,\bar{6})(-2) + (2,\overline{10})(3) + (1,\overline{15'})(8)$

$75 = (1,1)(0) + (1,3)(10) + (2,3)(-5) + (1,\bar{3})(-10) + (2,\bar{3})(5) + (2,\bar{6})(-5) + (2,6)(5) + (1,8)(0) + (3,8)(0)$

Slansky 1981

Georgi-Glashow Model

- Fermions

$$\overline{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \quad \mathbf{10}_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_2^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}.$$

- Scalars

$$\mathbf{24}_H : \quad SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$\mathbf{5}_H : \quad SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

Georgi, Glashow 1974

Georgi-Glashow Model

- GG model: $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H$

✗ $M_d = M_e^T$ Georgi, Jarlskog 1979

✗ $M_\nu = 0$

✗ Gauge coupling unification

In this talk

- Towards Realistic Models

✓ Renormalizable models

✗ Non-renormalizable models

Towards Realistic Models

- $\overline{\mathbf{5}}_{\boldsymbol{F}\,i} + \mathbf{10}_{\boldsymbol{F}\,i} + \mathbf{5}_{\boldsymbol{H}} + \mathbf{24}_{\boldsymbol{H}} + \mathbf{45}_{\boldsymbol{H}}$ Georgi, Jarlskog 1979

- Yukawa Lagrangian: Dorsner, Perez 2006

$$\mathcal{L}_Y = Y_{1,ij}\overline{\mathbf{5}}_{\boldsymbol{F}\,i}^{\ \alpha}\ \mathbf{10}_{\boldsymbol{F}\,\alpha\beta,j}\ \ \mathbf{5}^{*\ \beta}_{H} \ +\ Y_{2,ij}\ \overline{\mathbf{5}}_{\boldsymbol{F}\,i}^{\ \delta}\ \mathbf{10}_{\boldsymbol{F}\,\alpha\beta,j}\ \mathbf{45}^{*\ \alpha\beta}_{H\delta} + \epsilon^{\alpha\beta\gamma\delta r}\left(Y_{3,ij}\ \mathbf{10}_{\boldsymbol{F}\,\alpha\beta,i}\ \mathbf{10}_{\boldsymbol{F}\,\gamma\delta,j}\ \mathbf{5}_{\boldsymbol{H}\,r}\ +\ Y_{4,ij}\ \mathbf{10}_{\boldsymbol{F}\,\alpha\beta,i}\mathbf{10}_{\boldsymbol{F}\,m\gamma,j}\mathbf{45}_{\boldsymbol{H}\,\delta r}^{\ m}\right)$$

Towards Realistic Models

- $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H$

✓ $M_d \neq M_e^T$

✗ $M_\nu = 0$

✓ Gauge coupling unification

✓ Proton decay (safe)

Neutrino Mass: Tree level

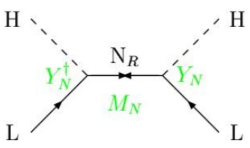
- Type-I seesaw: $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H + \mathbf{1}_{F\,i}$

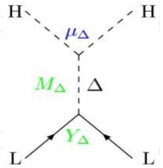
- Type-II seesaw: $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H + \mathbf{15}_H$

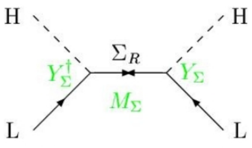
  Dorsner, Perez 2005; Dorsner, Mocioiu 2008

- Type-I+III seesaw: $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H + \mathbf{24}_{F\,j}$

  Bajc, Senjanovic 2007; Perez 2007



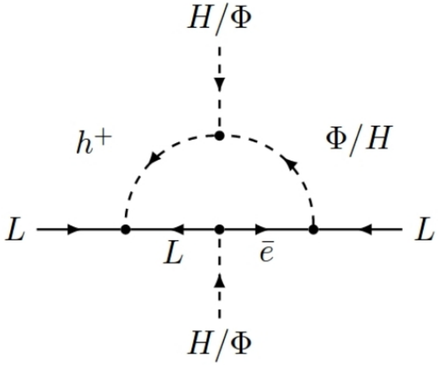




Neutrino Mass: 1-loop

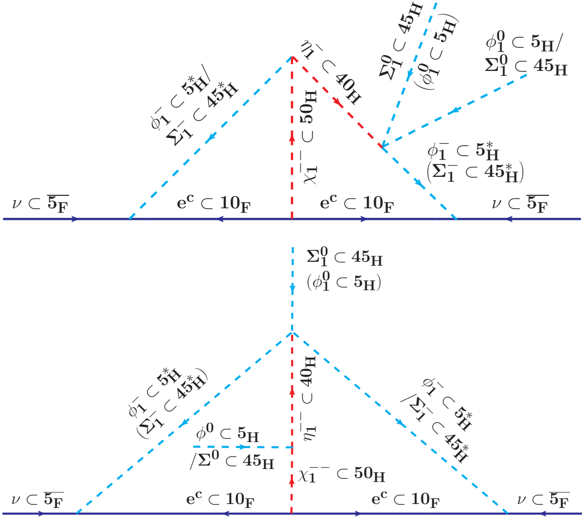
- Zee-mechanism: $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H + \mathbf{10}_H$

  Wolfenstein 1980; Barbieri, Nanopoulos, Wyler 1981; Perez, Murgui 2016



Neutrino Mass: 2-loop

- $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{45}_H + \mathbf{40}_H + \mathbf{50}_H$ Saad 2019



Minimal Renormalizable Model?

SU_5 irreps

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(2000)	15	7	2	1	0
(1001)	24	10	0	1*	1*
(0003)	35	28	2	1	0
(0011)	40	22	2	0	0
(0101)	45 ???	24	1	0	0
(0020)	50	35	1	0	1
(2001)	70	49	1	1	0
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Slansky 1981

Minimality Criteria

✓ least number of parameters

✓ lowest dimensional representations

✗ non-renormalizable operators

✗ singlets

A Novel $SU(5)$ Proposal

SU_5 irreps

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Dorsner, Saad 2019

A Novel $SU(5)$ Proposal

- $\overline{\mathbf{5}}_{F\,i} + \mathbf{10}_{F\,i} + \mathbf{5}_H + \mathbf{24}_H + \mathbf{35}_H + \mathbf{15}_F + \overline{\mathbf{15}}_F$

- Decompositions

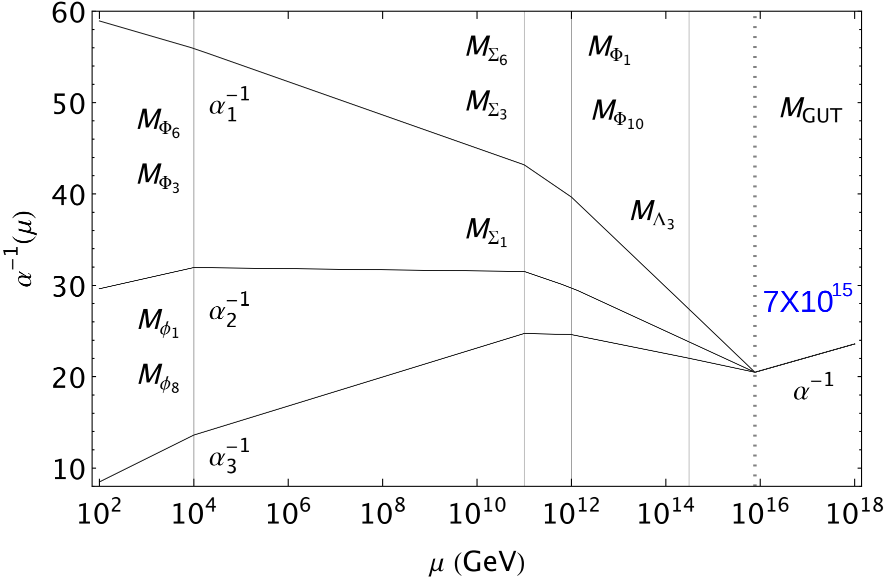
$$
\begin{aligned}
\mathbf{5}_H &\equiv \Lambda = \Lambda_1(1,2,1/2) + \Lambda_3(3,1,-1/3) \\
\mathbf{24}_H &\equiv \phi = \phi_0(1,1,0) + \phi_1(1,3,0) + \phi_3(3,2,-5/6) \\
&\qquad + \phi_{\overline{3}}(\overline{3},2,5/6) + \phi_8(8,1,0) \\
\mathbf{35}_H &\equiv \Phi = \Phi_1(1,4,-3/2) + \Phi_3(\overline{3},3,-2/3) + \Phi_6(\overline{6},2,1/6) \\
&\qquad + \Phi_{10}(\overline{10},1,1) \\
\mathbf{15}_F &\equiv \Sigma = \Sigma_1(1,3,1) + \Sigma_3(3,2,1/6) + \Sigma_6(6,1,-2/3)
\end{aligned}
$$

Gauge Coupling Unification

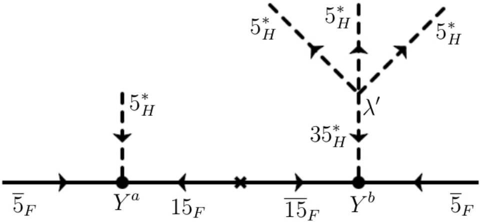
Highly non-trivial:

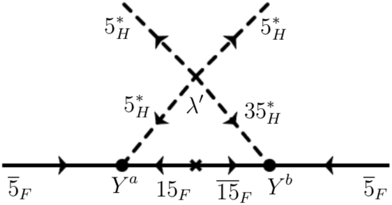
- $M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1}$
- $M^2_{\Phi_{10}} = M^2_{\Phi_1} - 3M^2_{\Phi_3} + 3M^2_{\Phi_6}$
- $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV (proton decay)
- $M_{\mathrm{GUT}} \geq 5 \times 10^{15}$ GeV (proton decay)
- $M_k \geq 10$ TeV ($k =$ any BSM state)
- ν-mass requires specific M_{Σ_1} and M_{Φ_1}
  (rules out most of the parameter space consistent with unification)

Gauge Coupling Unification



Neutrino Mass





Neutrino Mass

- $\mathcal{L} \supset \lambda' \, 5_H 5_H 5_H 35_H + Y_i^a \, 15_F \overline{5}_{F\,i} 5_H^* + Y_i^b \, \overline{15}_F \overline{5}_{F\,i} 35_H^*$

- tree-level contribution

$$(\mathcal{M}_\nu^{d=7})_{ij} = -\lambda' \frac{v_H^4}{M_{\Sigma_1} M_{\Phi_1}^2} (Y_i^a Y_j^b + Y_i^b Y_j^a)$$
$$\sim 10^{-26} \quad GeV$$

- one-loop contribution

$$(\mathcal{M}_\nu^{d=5})_{ij} = \frac{\lambda' v_H^2}{16\pi^2} \frac{(Y_i^a Y_j^b + Y_i^b Y_j^a) M_{\Sigma_1}}{M_{\Phi_1}^2 - M_H^2} \left(\frac{M_{\Phi_1}^2}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \log\left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2}\right) - \frac{M_H^2}{M_{\Sigma_1}^2 - M_H^2} \log\left(\frac{M_{\Sigma_1}^2}{M_H^2}\right)\right)$$
$$\sim 4 \times 10^{-11} \quad GeV$$

$Y^a, Y^b, \lambda' = 1$

Charged fermion masses

$$\mathcal{L} \supset Y^d_{ij} 10_{F\,i} \overline{5}_{F\,j} 5^*_H + Y^u_{ij} 10_{F\,i} 10_{F\,j} 5_H + Y^c_i 10_{F\,i} \overline{15}_F 24_H$$
$$+ Y^a_i 15_F \overline{5}_{F\,i} 5^*_H + M_\Sigma \overline{15}_F 15_F + y\, \overline{15}_F 15_F 24_H$$

- Quarks, charged leptons, neutrino masses- all connected !
- $M_u = \left(\mathbb{I}_{3\times 3} + \delta^2\ Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H Y^u$
- $M_d = \left(\mathbb{I}_{3\times 3} + \delta^2\ Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H \left(Y^d + \delta\ Y^c Y^a\right)$
- $M_e = v_H {Y^d}^T$
- $(\mathcal{M}_\nu)_{ij} = m_0 \left(Y^a_i Y^b_j + Y^b_j Y^a_i\right)$

Fit Parameters and Result

- $Y^d = (y_e, y_\mu, y_\tau)$
- $Y^a = (-0.0899, 0.551, 1)$
- $Y^b = (0.975, 2.381, 1)$
- $Y^c = -1.865 \times 10^{-7}(0.00137, 0.0942, 1)$
- $\lambda' = 0.239$

Down-type quark masses	Fit value (GeV)	ν masses	Fit value (eV)	ν mixing angles	Fit value (°)
$m_d/10^{-3}$	1.14	m_1	0	θ_{12}	34.57
$m_s/10^{-2}$	2.15	$m_2/10^{-3}$	8.70	θ_{23}	47.41
m_b	0.99	$m_3/10^{-2}$	4.99	θ_{13}	8.56

A Novel $SU(5)$ Proposal: Summary

- ✓ Gauge coupling unification (fixed by ν-mass)
- ✓ Safe from rapid proton decay
- ✓ Neutrino mass via 1-loop diagram ($m_1 = 0$)
- ✓ Correct charged fermion masses
- ✓ All fermion masses & mixings are correlated
- ✓ Only lowest dimensional representations
- ✓ Least number of Yukawa parameters
- ✓ $(1,3,0), (8,1,0), (3,3,\frac{2}{3}), (6,2,\frac{-1}{6}) \sim 1-10$ TeV (scalars)