Gravitational Waves from Cosmological Phase Transitions in an Expanding Universe

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Based on ongoing work by:
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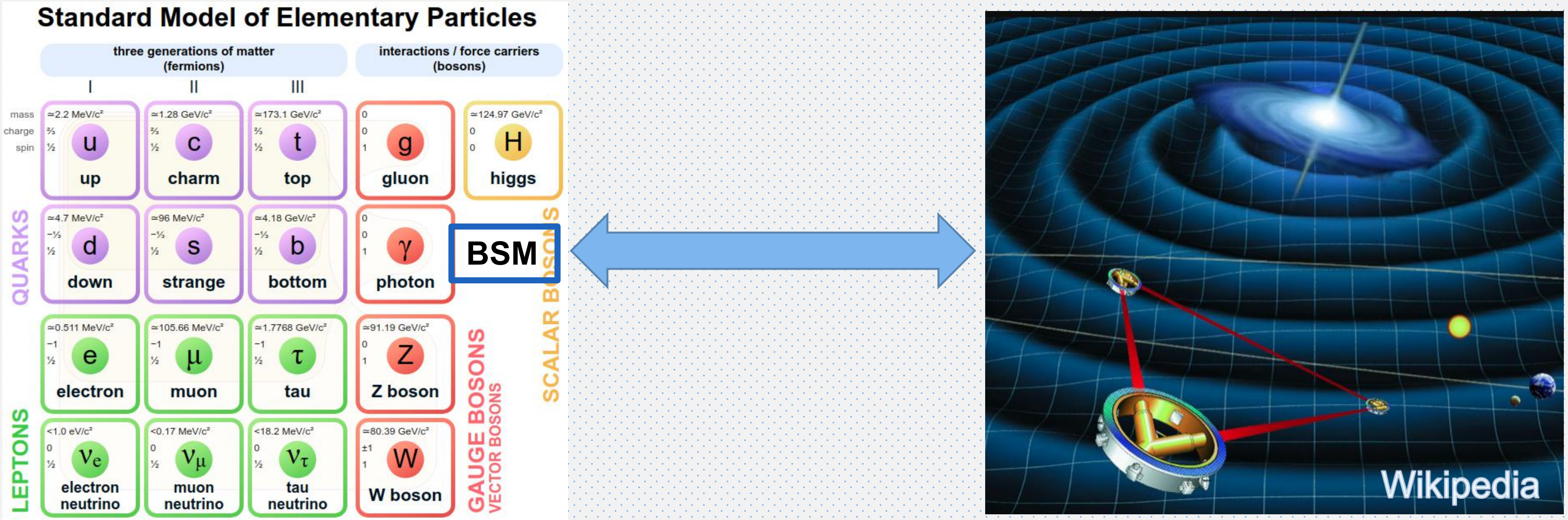
Motivations

- Gravitational waves as a new method of probing particle physics
- Gravitational waves as cosmic witnesses (PT, cosmic strings, etc)

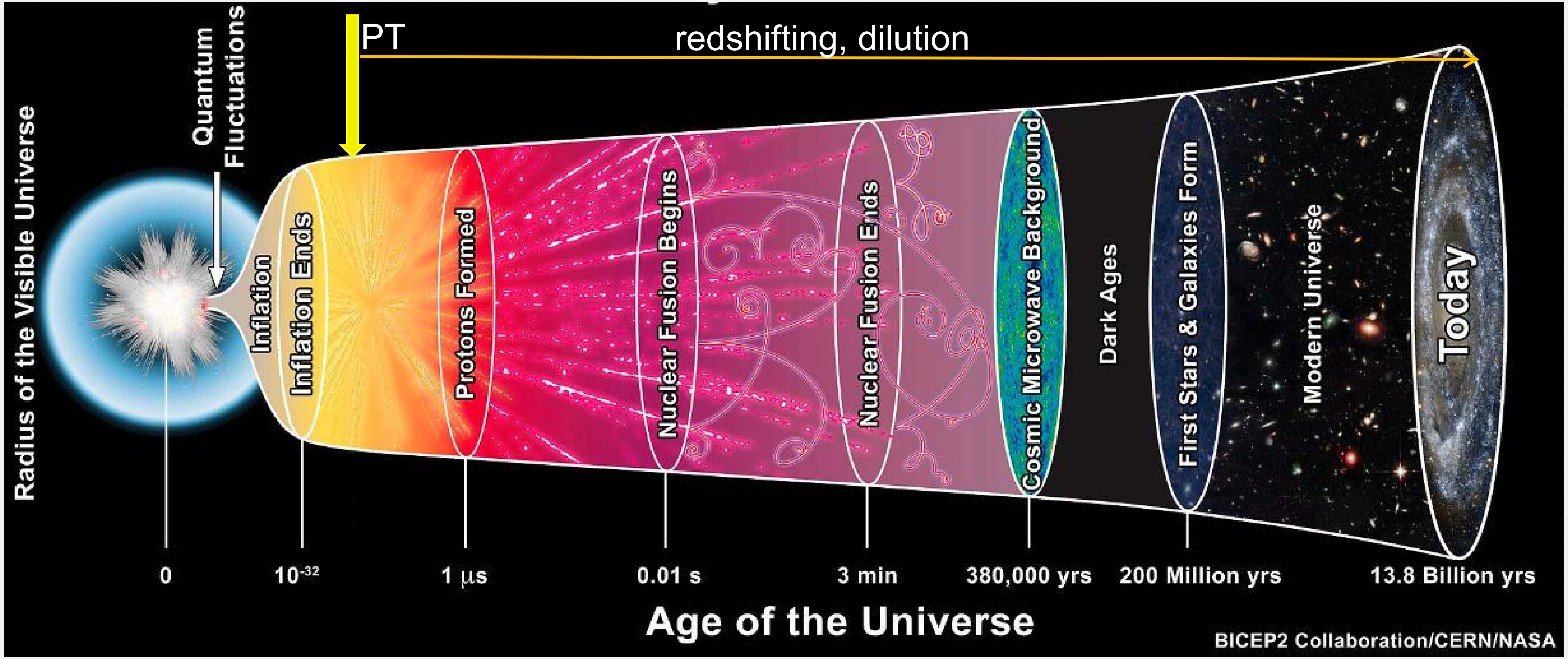
  Early matter domination(string moduli), Kination, Intermediate Inflationary stage(supercooling), etc

- Calculations(simulations) previously done in Minkowski spacetime



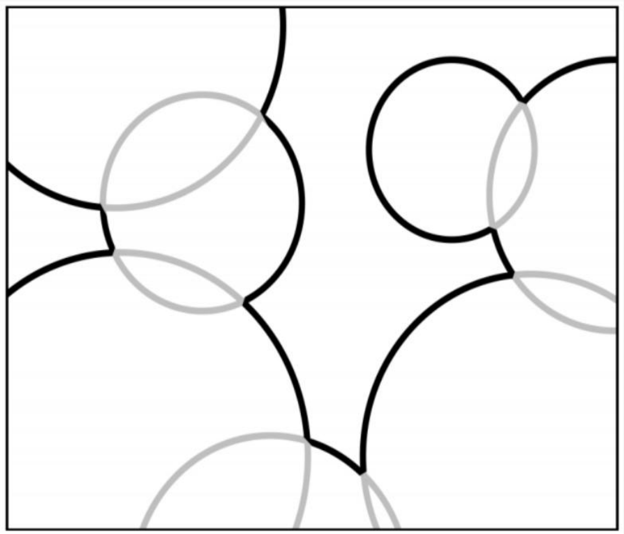
Modifications in an Expanding Universe

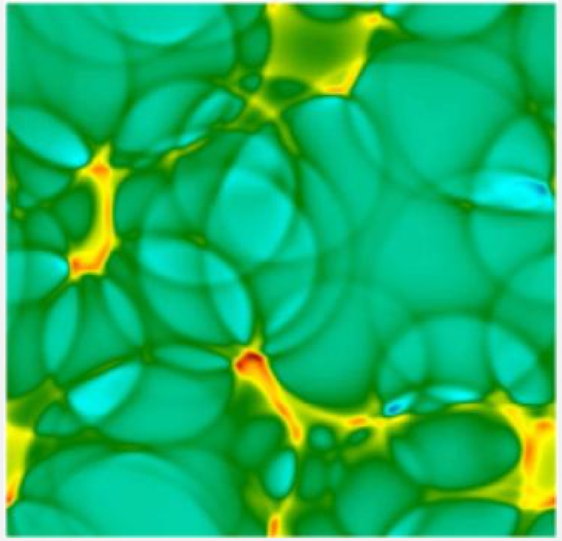
- Do we need a new simulation?
- How will the properties of the PT and GW be modified?



Gravitational Waves from Cosmic Phase Transition

- Bubble Collisions
- Sound Waves in Plasma → dominant in plasma
- MagnetoHydrodynamic Turbulence (see Kakhniashvili's talk)





Hindmarsh, et al,PRL112,041301(2013), PRD92,123009(2015)

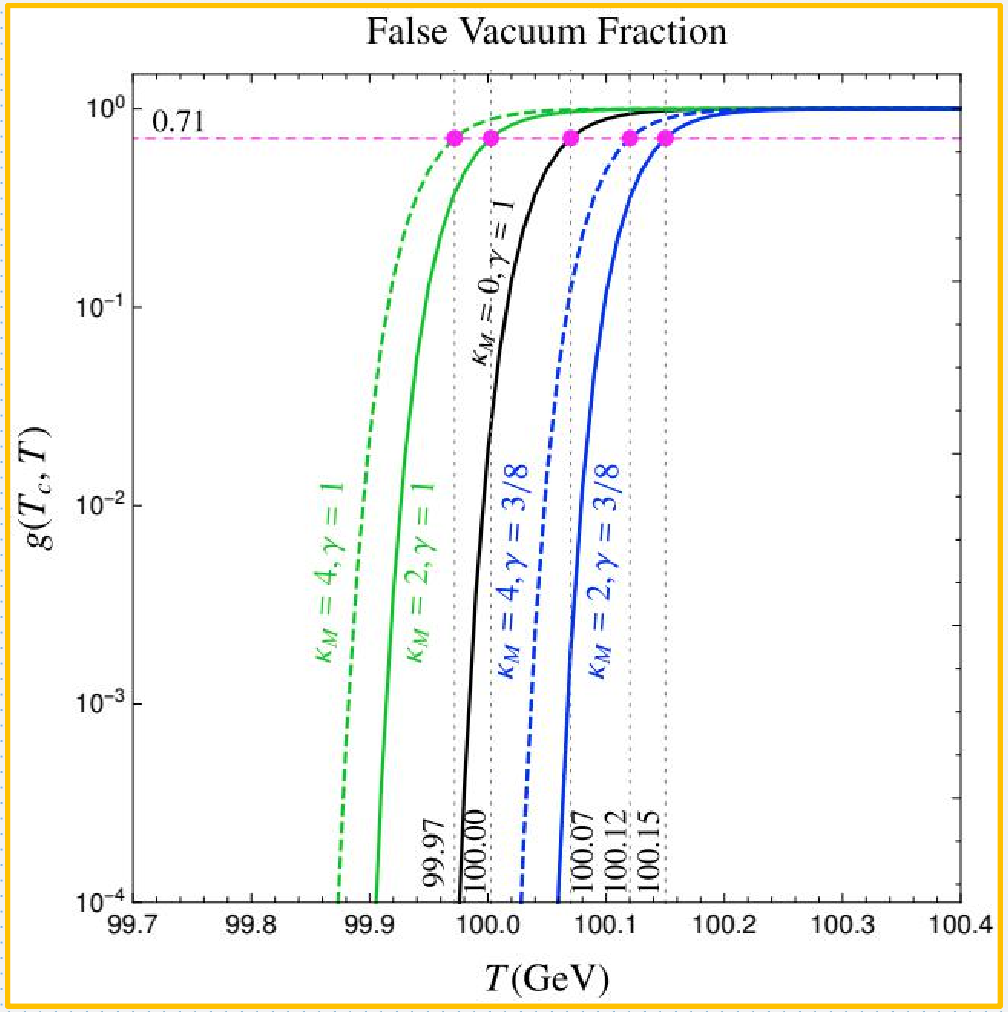
Dynamics of Phase Transition: An Example

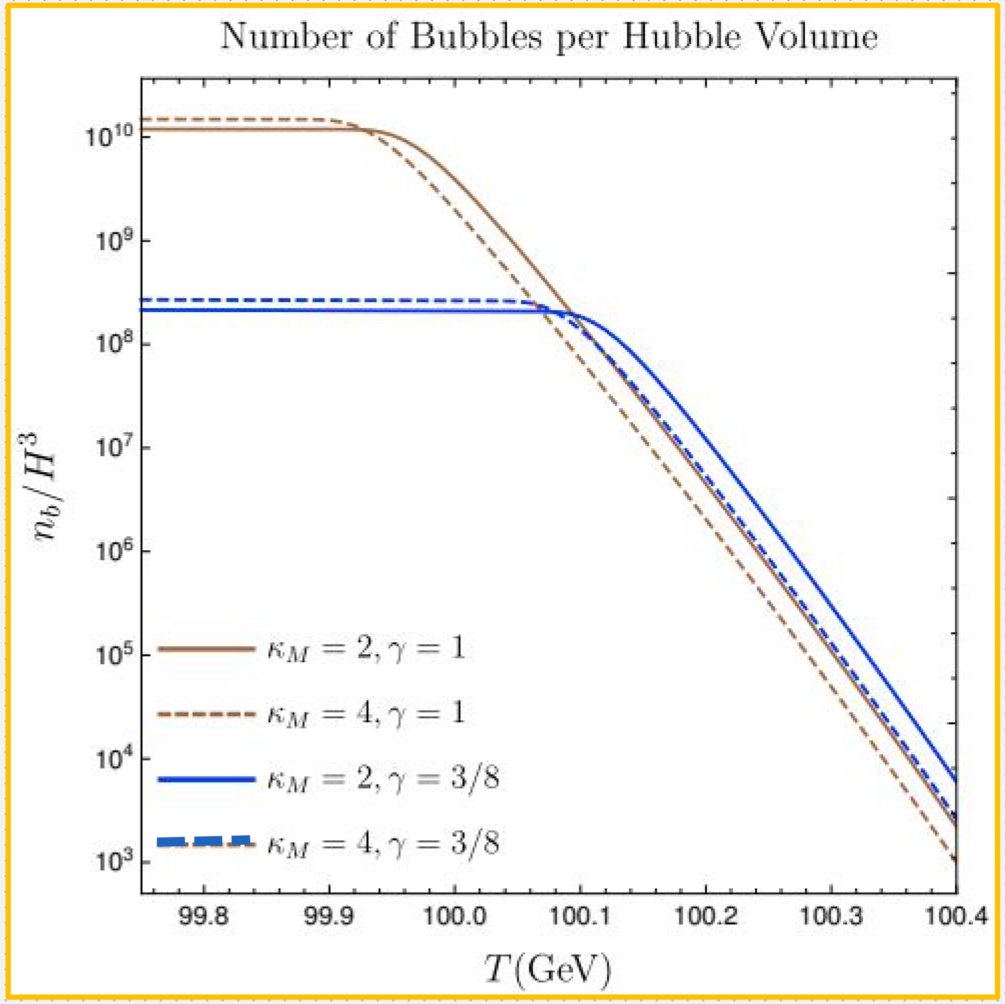
$T \propto a^{-\gamma}$

$\gamma = 1$(radiation domination)

$\gamma = 3/8$(matter domination with entropy injection)

κ_M(matter content at T_c normalized by radiation)





Formalism

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$$

Tensor Mode

$$\langle \dot{h}_{ij}(t, \mathbf{q})\dot{h}_{ij}(t, \mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k} + \mathbf{q})P_{\dot{h}}(k, t)$$

$$\frac{d\rho_{\mathrm{GW}}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t, k)$$

GW Spectrum

Einstein equation

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

Source evolutions

Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM

Energy-momentum conservation (hydrodynamic limit)

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$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

neglect backreaction
solve with Green's function

Source evolutions

Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM
Energy-momentum conservation (hydrodynamic limit)

Behavior of the Source

- Equations of motion can be obtained by simply rescaling of Minkowski conterpart
  scalar field is a problem
- For sufficiently small vacuum energy, velocity profile for one bubble unchanged.
- Sound waves(fluctuations of energy, pressure, velocity)

scalar field and EM neglected

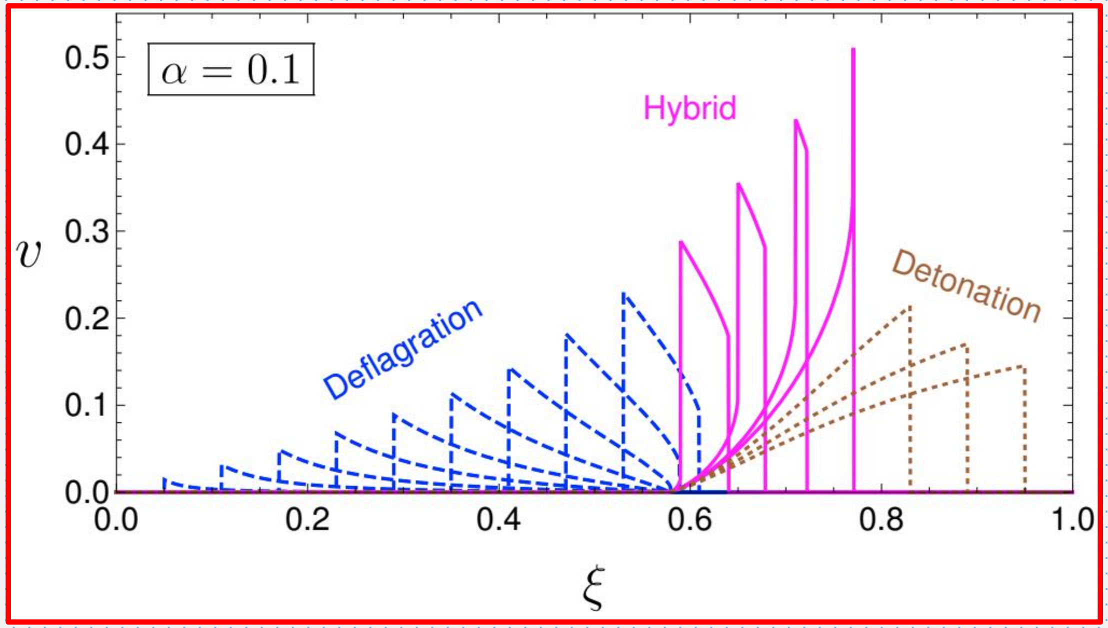
$$(a^4 S^i)' + \nabla \cdot (a^4 S^i \mathbf{v}) + \partial_i (a^4 p) = 0, \quad S^i = \gamma^2 (\epsilon + p) v^i$$

$$(a^4 \epsilon \gamma)' + [\gamma' + \nabla \cdot (\gamma \mathbf{v})](a^4 p) + \nabla \cdot (a^4 \epsilon \gamma \mathbf{v}) = 0,$$

$$\gamma^2 (v' + \frac{1}{2} \hat{\mathbf{v}} \cdot \nabla v^2)[a^4 (\epsilon + p)] + v(a^4 p)' + \hat{\mathbf{v}} \cdot \nabla (a^4 p) = 0$$

conformal time

special relativistic Hydrodynamics



Behavior of the Source

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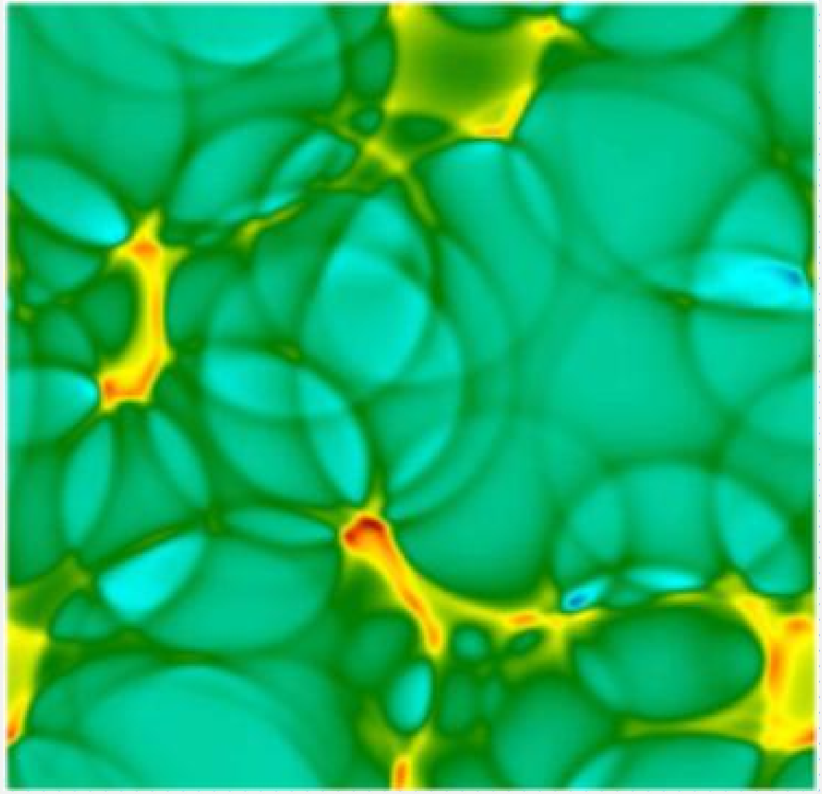
$$T_{ij} = a^2 \left[p\delta_{ij} + (p+\epsilon)\gamma^2 v^i v^j \right]$$
$$T_{i0} = a \left[-(p+\epsilon)\gamma^2 v^i \right],$$
$$T_{00} = \gamma^2(\epsilon + p v^2).$$

$$\pi^f_{ij}(k,\eta) = \frac{a_*^4}{a^4(\eta)} \tilde{\pi}^f_{ij}(k\eta)$$

calculate total velocity field

Velocity Field obtained with the Sound Shell Model

- The velocity filed is the linear superposition of those surrounding all bubbles
- Can be carried out in expanding universe context, but need change of variables
- Need statistical properties of bubbles for power spectrum



$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} \left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}} e^{i\omega\eta - i\mathbf{q}\cdot\mathbf{x}} \right]$$

conformal time

$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

Hindmarsh, 120, 071301 (2018)
Hindmarsh, Hijazi, JCAP 12 (2019) 062

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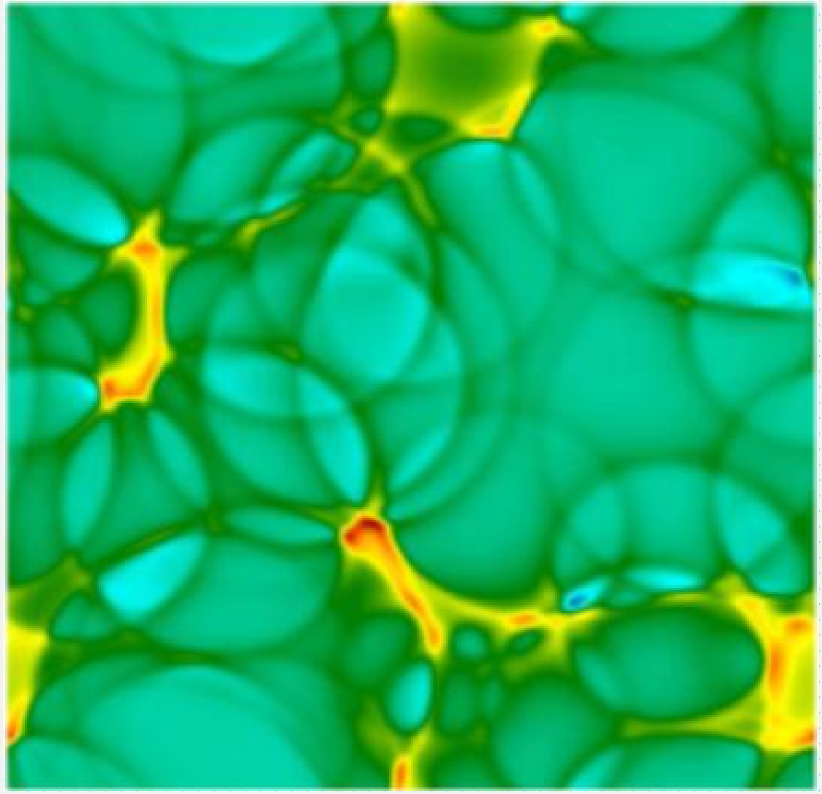

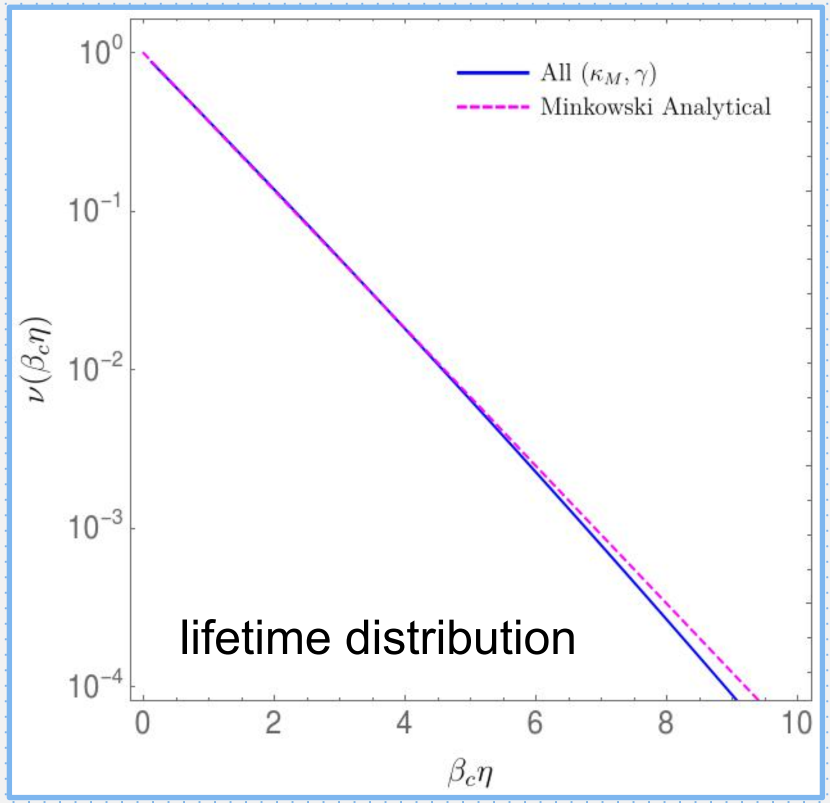

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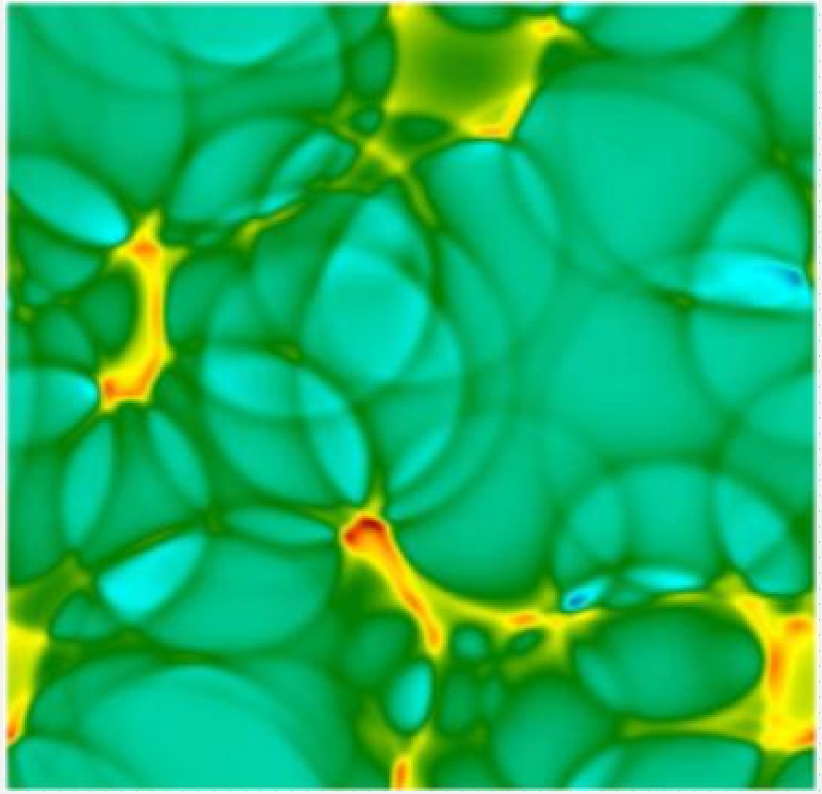

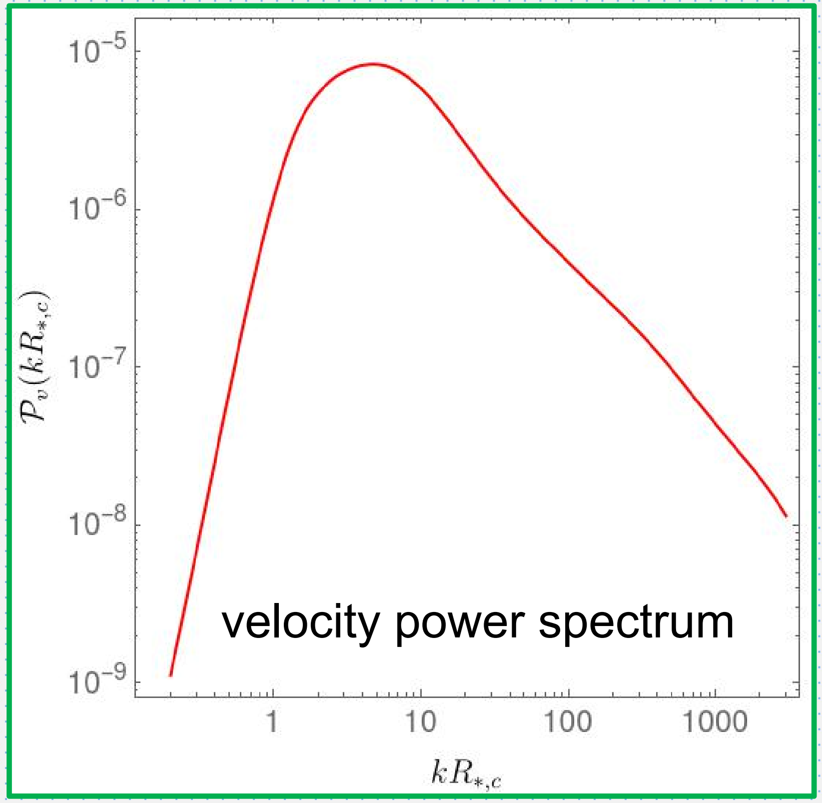

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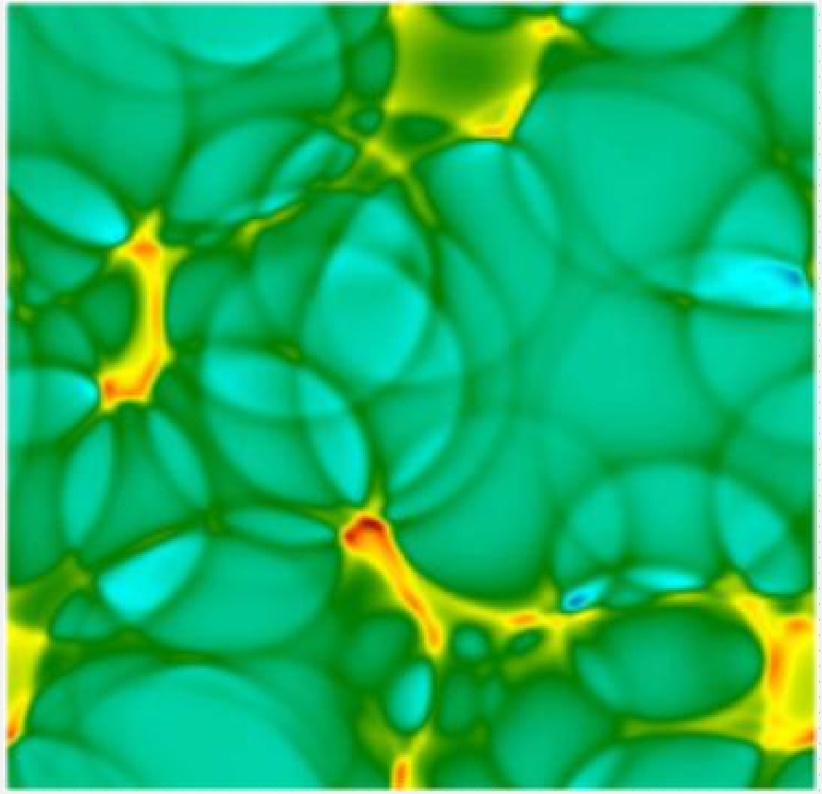

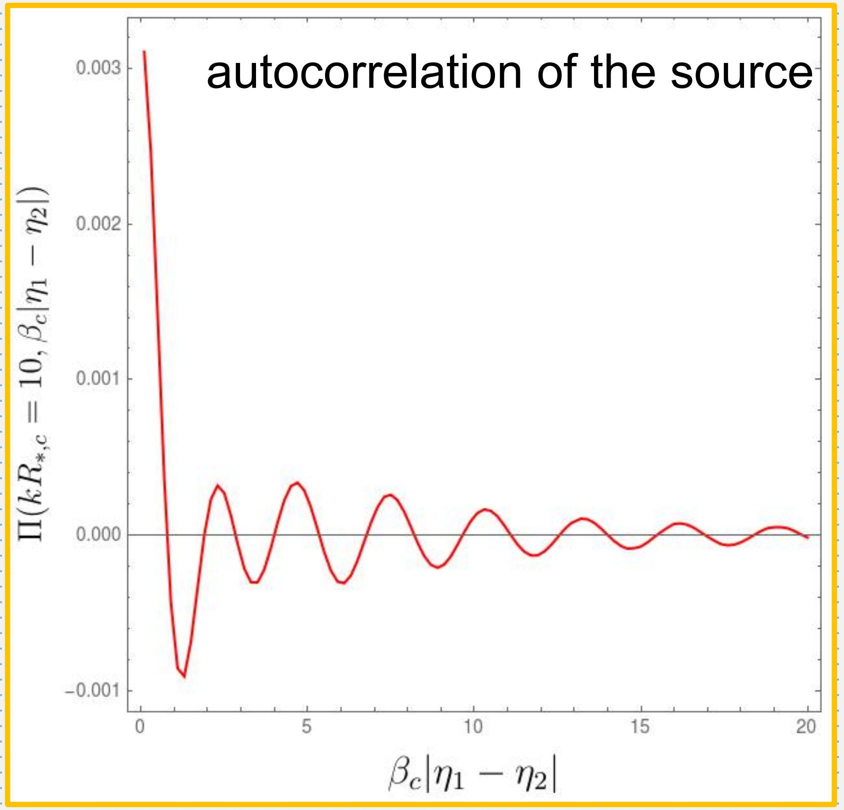

GW Power Spectrum

$$\mathcal{P}_{\rm GW}(y, kR_{*,c}) = \frac{[16\pi G\,(\bar{\epsilon}+\bar{p})\,\bar{U}_f^2]^2}{24\pi^2 H^2}\frac{1}{y^4}(kR_{*,c})(a_0 R_{*,c})^2\int_{y_0}^{y} dy_1' \int_{y_0}^{y} dy_2' \,\frac{\mathcal{G}_2(\tilde{y},\tilde{y}_1',\tilde{y}_2')}{(y_1')^2(y_2')^2}\tilde{\Pi}^2\left(kR_{*,c},\left|\frac{\eta_1-\eta_2}{R_{*,c}}\right|\right)$$

$$y = \frac{a}{a_0}$$

correct scaling for deep-inside-horizon modes

source dilution

rescaled source correlator

More details on numericals: see Daniel Vagie’s talk.

Summary

- Analyzed PT in an expanding universe(non-standard comic histories)
- Source evolution takes similar form for small vacuum energy
- Generazlied GW formalism in an expanding universe

Thanks!