Gravitational Waves from Cosmological Phase Transitions in an Expanding Universe

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Based on ongoing work by:
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Motivations

- Gravitational waves as a new method of probing particle physics
- Gravitational waves as cosmic witnesses (PT, cosmic strings, etc)
  - Early matter domination (string moduli), Kination, Intermediate Inflationary stage (supercooling), etc
- Calculations (simulations) previously done in Minkowski spacetime
Do we need a new simulation?

How will the properties of the PT and GW be modified?
Gravitational Waves from Cosmic Phase Transition

- Bubble Collisions
- Sound Waves in Plasma
- MagnetoHydrodynamic Turbulence (see Kakhniashvili’s talk)

Dynamics of Phase Transition: An Example

\[ T \propto a^{-\gamma} \]

\( \gamma = 1 \) (radiation domination)

\( \gamma = 3/8 \) (matter domination with entropy injection)

\( k_M \) (matter content at \( T_c \), normalized by radiation)

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**False Vacuum Fraction**

\[ g(T, T) \]

\[ T(\text{GeV}) \]

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**Number of Bubbles per Hubble Volume**

\[ \frac{\chi_b}{H^3} \]

\[ T(\text{GeV}) \]
Formalism

\[ ds^2 = -dt^2 + a^2 (\delta_{ij} + h_{ij}(x)) dx^2 \]

Tensor Mode

\[ \langle h_{ij}(t, \mathbf{q}) \dot{h}_{ij}(t, \mathbf{k}) \rangle = (2\pi)^{-3} \delta^3 (\mathbf{k} + \mathbf{q}) P_h (k, t) \]

GW Spectrum

\[ \frac{d \rho_{GW}(t)}{d \ln k} = \frac{1}{64 \pi^3 G} k^3 P_h (t, k) \]

Energy-momentum conservation (hydrodynamic limit)

Einstein equation

\[ h''_q + 2 \frac{a'}{a} h'_q + q^2 h_q = 16\pi G a^2 \pi^T_q \]

Source evolutions

Plasma (relativistic species), Matter (non-relativistic), Scalar field, EM
Formalism

\[ ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(x))dx^2 \]

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Source evolutions

Plasma (relativistic species), Matter (non-relativistic), Scalar field, EM

Energy-momentum conservation (hydrodynamic limit)

neglect backreaction

solve with Green’s function
Behavior of the Source

- Equations of motion can be obtained by simply rescaling of Minkowski counterpart scalar field is a problem.
- For sufficiently small vacuum energy, velocity profile for one bubble unchanged.
- Sound waves (fluctuations of energy, pressure, velocity)

Scalar field and EM neglected

\[
(a^4 S^i)' + \nabla \cdot (a^4 S^i \mathbf{v}) + \partial_i (a^4 p) = 0, \quad S^i = \gamma^2 (\epsilon + p) v^i
\]

\[
(a^4 \epsilon \gamma)' + [\gamma' + \nabla \cdot (\gamma \mathbf{v})](a^4 p) + \nabla \cdot (a^4 \epsilon \gamma \mathbf{v}) = 0,
\]

\[
\gamma^2 (v' + \frac{1}{2} \mathbf{v} \cdot \nabla v^2) [a^4 (\epsilon + p)] + v(a^4 p)' + \mathbf{v} \cdot \nabla (a^4 p) = 0
\]

Conformal time

Special relativistic Hydrodynamics
Equations of motion can be obtained by simply rescaling of Minkowski counterpart scalar field is a problem.

For sufficiently small vacuum energy, velocity profile for one bubble unchanged.

Sound waves (fluctuations of energy, pressure, velocity)

\[
T_{ij} = a^2 \left[ p \delta_{ij} + (p + \epsilon) \gamma^2 v^i v^j \right]
\]
\[
T_{i0} = a \left[ -(p + \epsilon) \gamma^2 v^i \right],
\]
\[
T_{00} = \gamma^2 (\epsilon + pv^2).
\]

\[
\pi^{f}_{ij}(k, \eta) = \frac{a^4_*}{a^4(\eta)} \tilde{\pi}^{f}_{ij}(k\eta)
\]

Calculate total velocity field.
The velocity field is the linear superposition of those surrounding all bubbles.

Can be carried out in expanding universe context, but need change of variables.

Need statistical properties of bubbles for power spectrum.

\[ v^i(n, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} \left[ v^i_q e^{-i\omega \eta + i\mathbf{q} \cdot \mathbf{x}} + v^i_q e^{i\omega \eta - i\mathbf{q} \cdot \mathbf{x}} \right] \]

conformal time

\[ v^i_q = \sum_{n=1}^{N_b} v^i_{q(n)} \]

Hindmarsh, 120, 071301 (2018)
Hindmarsh, Hijazi, JCAP 12 (2019) 062
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**Velocity Field obtained with the Sound Shell Model**
Velocity Field obtained with the Sound Shell Model

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![Autocorrelation of the source](image)
More details on numerics: see Daniel Vagie’s talk.
Summary

- Analyzed PT in an expanding universe (non-standard comic histories)
- Source evolution takes similar form for small vacuum energy
- Generalized GW formalism in an expanding universe
Thanks!