Thermal Loop Effects on Large-Scale Curvature Perturbation in the Higgs Inflation

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JHEP **04** (2020) 163 **arXiv:1907.04857**

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May 04, 2020 PHENO 2020

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Outline

Cosmic Inflation

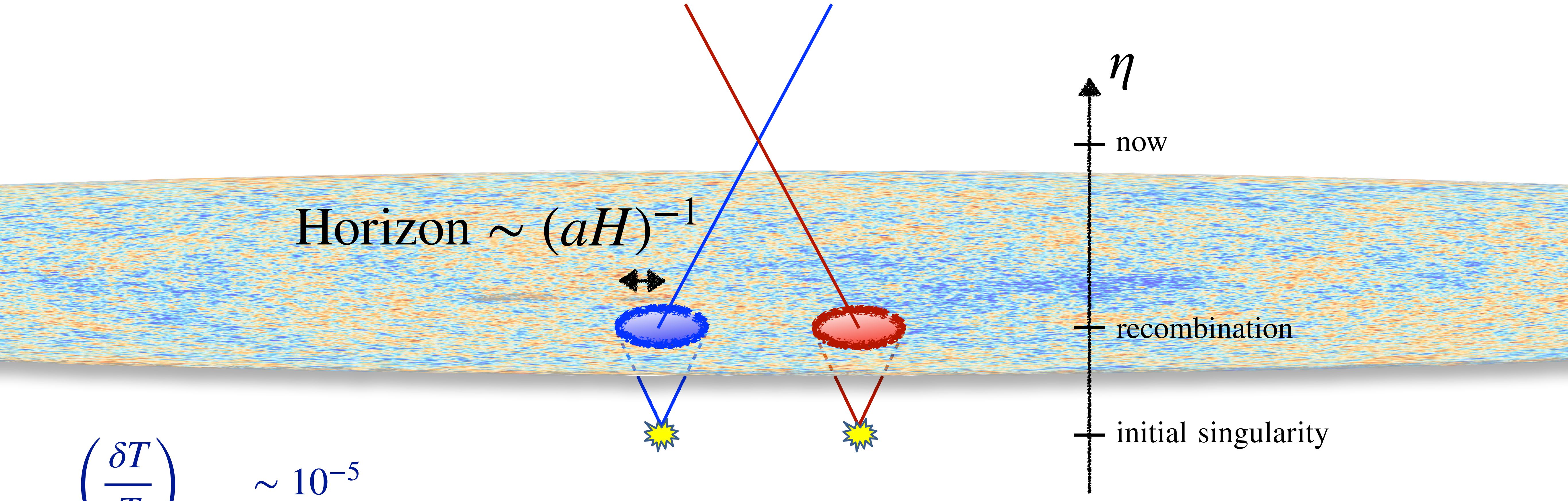
Higgs Inflation

Quantum Loop Corrections

Curvature Perturbation

Summary

Cosmic Inflation

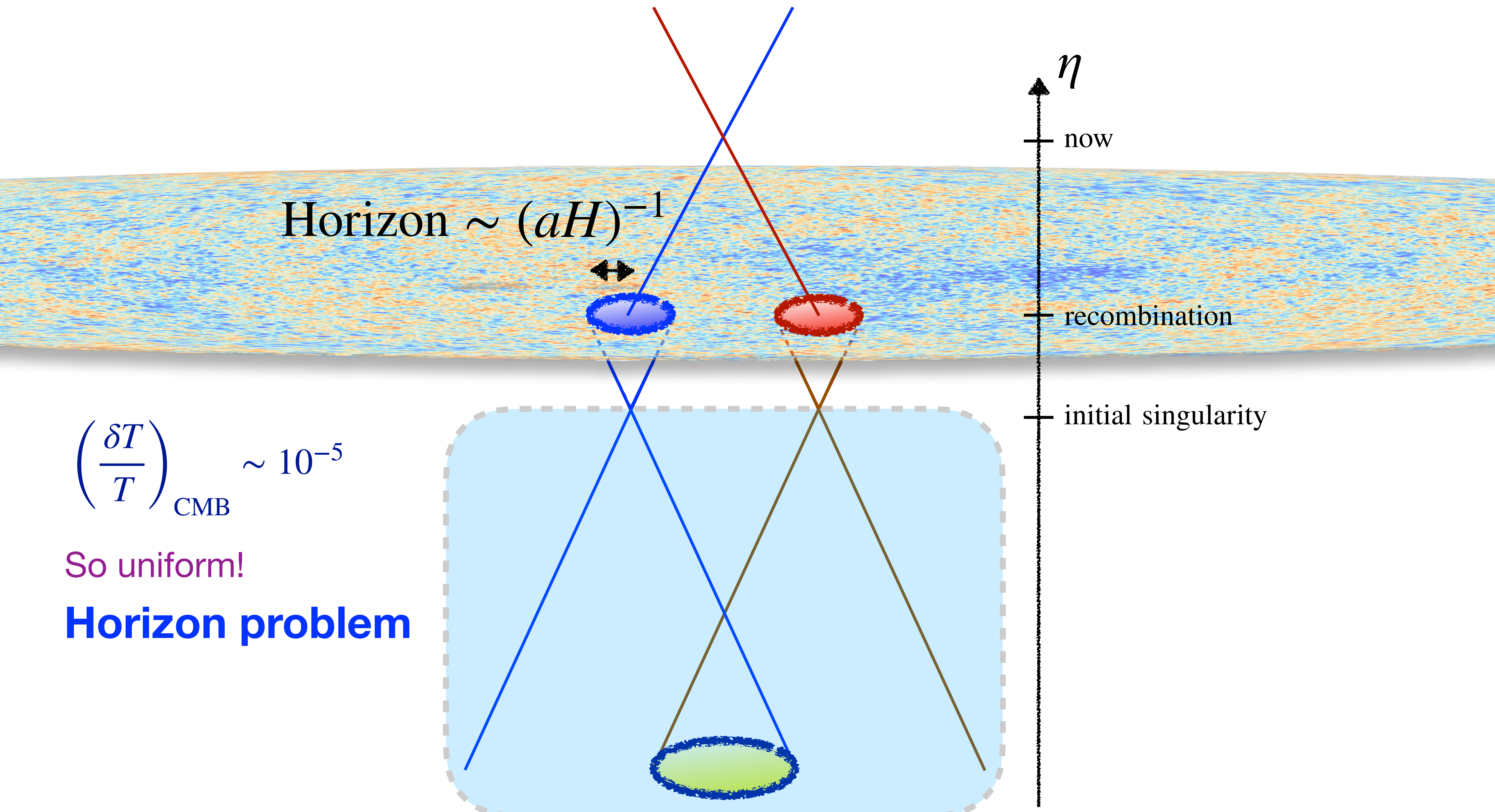


$$\left(\frac{\delta T}{T}\right)_{\rm CMB} \sim 10^{-5}$$

So uniform!

Horizon problem

Cosmic Inflation



$$\left(\frac{\delta T}{T}\right)_{\text{CMB}} \sim 10^{-5}$$

So uniform!

Horizon problem

Cosmic inflation

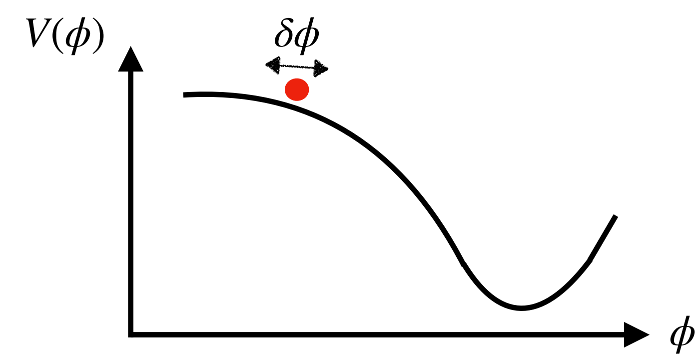
Cosmic Inflation

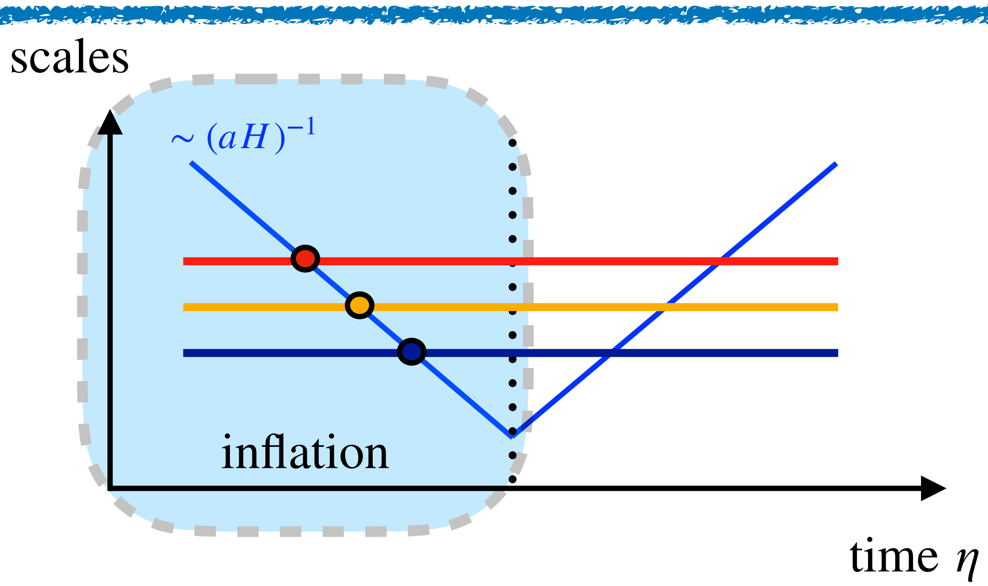
Theory

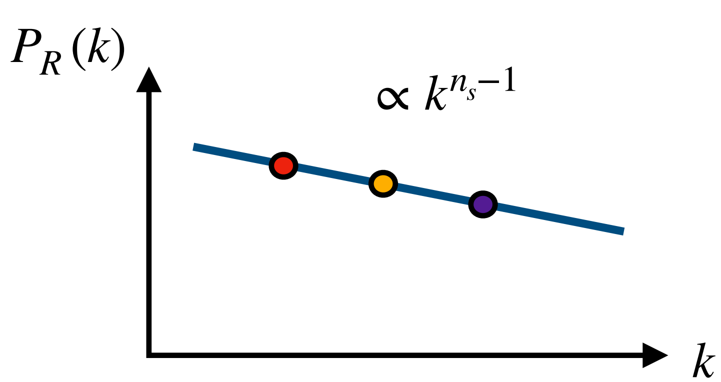
flat potential + large friction = slow-roll inflaton

$\Rightarrow$ Accelerated expansion

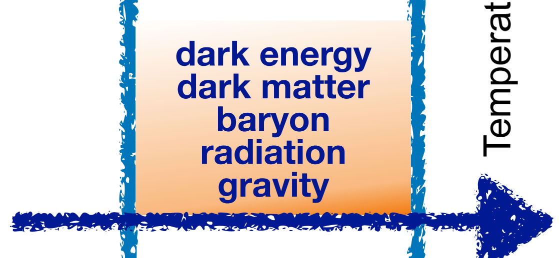
quantum fluctuations ⟶ **curvature perturbations**

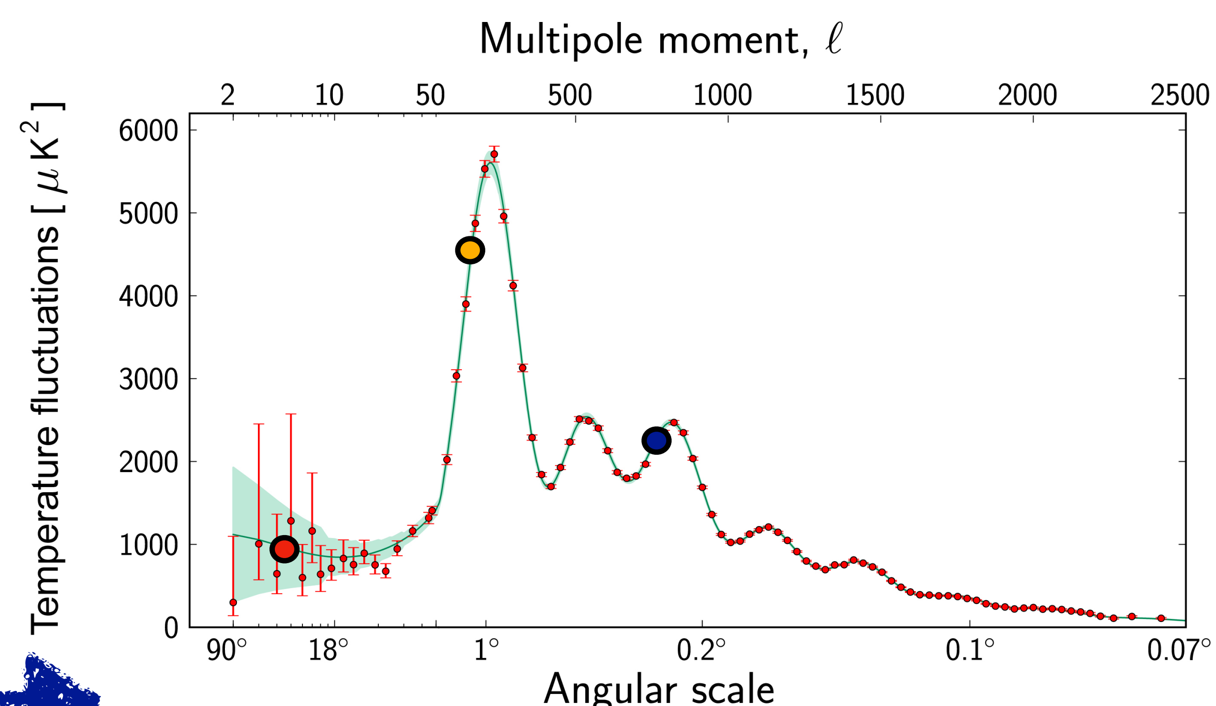






primordial power spectrum



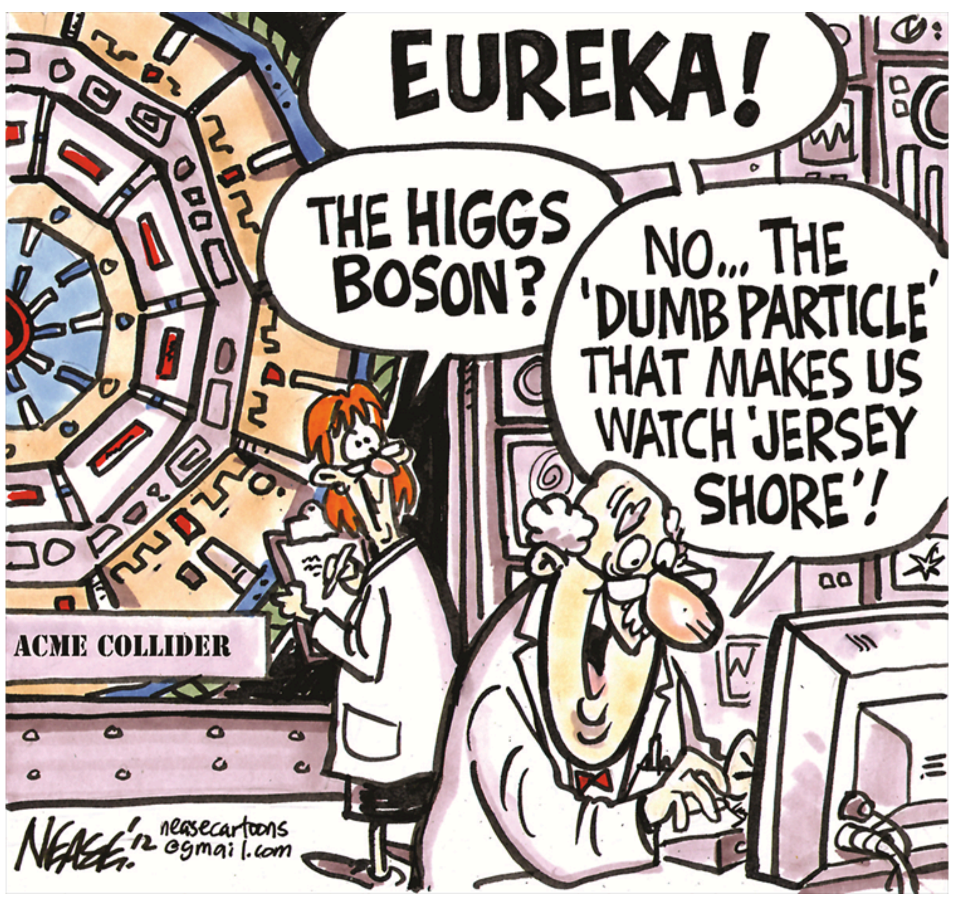


CMB angular power spectrum

Cosmic Inflation

Interesting theory. But…

- What exactly is the inflaton?
- How to understand the nature of a flat potential?
- Can Standard Model particles help us answer these questions?



Picture by Steve Nease

Picture by Pablo Stanley

Higgs Inflation

A naïve attempt to take the Higgs boson as an inflaton would generally fail

- **Higgs potential (tree level):**

$$V_0(h) = \frac{\lambda}{4}(h^2 - v^2)^2$$

- **Cosmology:**

$$\left(\frac{\delta T}{T}\right)_{\text{CMB}} \sim \sqrt{\lambda} \quad \Rightarrow \quad \lambda \sim 10^{-13}$$

- **Standard Model:**

big mismatch

$$\lambda = m_h^2/2v^2 \simeq 10^{-1}$$

Higgs Inflation

Bezrukov & Shaposhnikov, Phys. Lett. B 659 (2008)

Jordan frame

(The frame with non-minimal coupling)

Conformal Transformation

$$g_{\mu\nu} \quad \to \quad \widetilde{g}_{\mu\nu} = f(h)\, g_{\mu\nu}$$

Einstein frame

(The frame without non-minimal coupling)

Jordan frame

Action

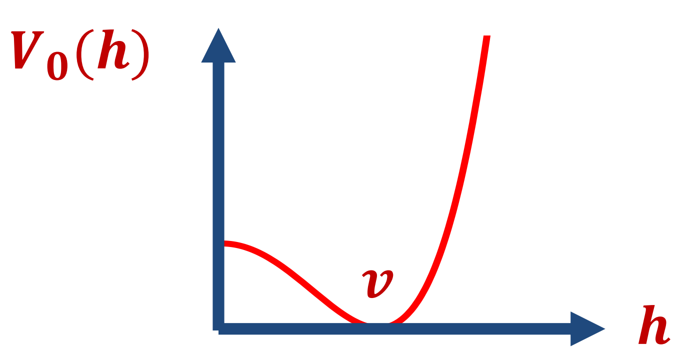
$$S_J = \int d^4x\sqrt{-g}\,\left[\frac{M^2}{2} f(h)R - \frac{1}{2}g^{\mu\nu}\partial_\mu h\,\partial_\nu h - V_0(h)\right]$$

$$f(h) = 1 + \frac{\xi}{M^2}h^2$$

quadratic non-minimal coupling

Potential

$$V_0(h) = \frac{\lambda}{4}(h^2 - v^2)^2$$



Einstein frame

Action S_E

$$S_E = \int d^4x\sqrt{-\widetilde{g}}\,\left[\frac{M_P^2}{2}\widetilde{R} - \frac{1}{2}\widetilde{g}^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - U_0(\chi)\right]$$

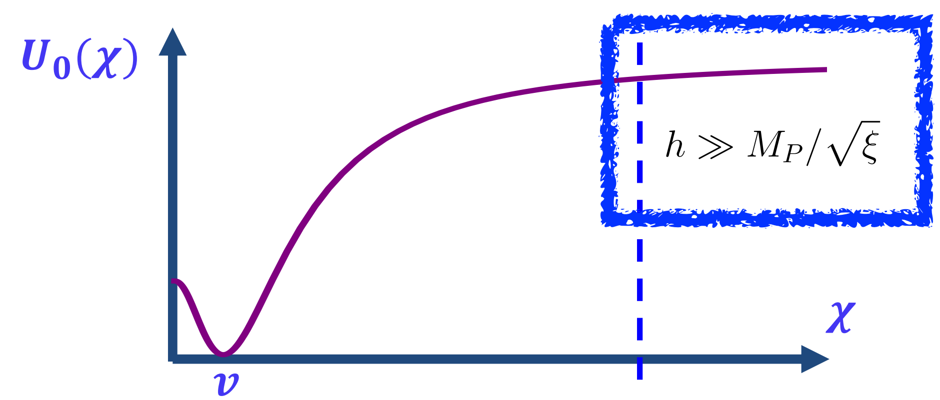
Effective field χ

$$\frac{d\chi}{dh} = \left(\frac{f + 3M_P^2 \cdot f'^2/2}{f^2}\right)^{1/2}$$

Effective potential $U_0(\chi)$

$$U_0\,(h(\chi)) = f^{-2}V_0(h)$$

$$\approx \frac{\lambda M_P^4}{4\,\xi^2}\left[1 - \exp\left(\frac{-2\,\chi}{\sqrt{6}M_P}\right)\right]^2 \quad (h \gg M_P/\xi)$$



The potential is flatten in large-field regime

Higgs Inflation

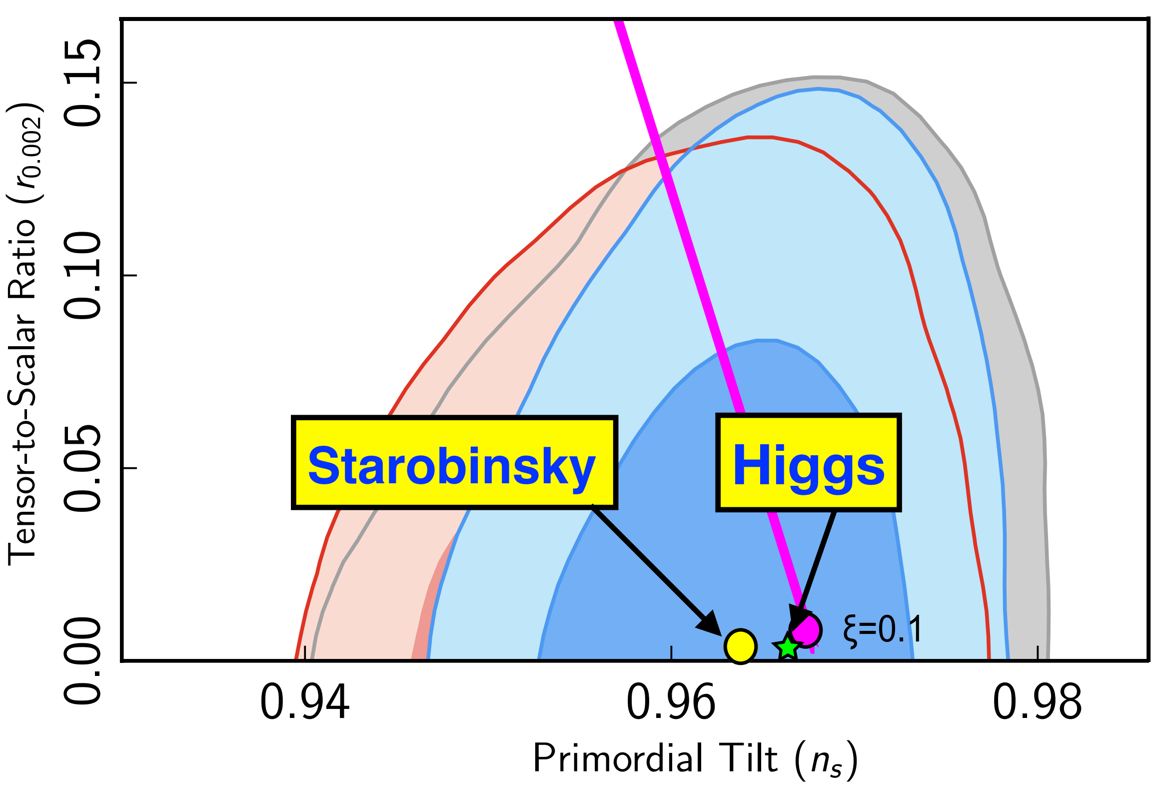
Bezrukov & Shaposhnikov, Phys. Lett. B 659 (2008)

Einstein frame

(The frame without non-minimal coupling)

$$\left(\frac{\delta T}{T}\right)_{\rm CMB} \sim \frac{\sqrt{\lambda}}{\xi} \quad \Rightarrow \quad \xi \approx 4.7\times 10^4 \sqrt{\lambda}$$

- **Higgs boson = inflaton**
- **Flat potential: a consequence of the conformal transformation**



Bezrukov, Class. Quant. Grav. 30 (2013) 214001

Action S_E

$$S_E = \int d^4x\sqrt{-\tilde{g}}\,\left[\frac{M_P^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - U_0(\chi)\right]$$

Effective field χ

$$\frac{d\chi}{dh} = \left(\frac{f + 3M_P^2\cdot f'^2/2}{f^2}\right)^{1/2}$$

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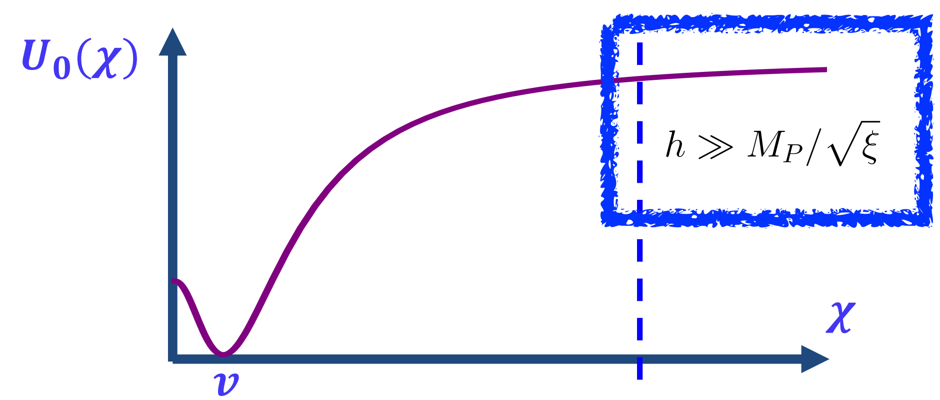


The potential is flatten in large-field regime

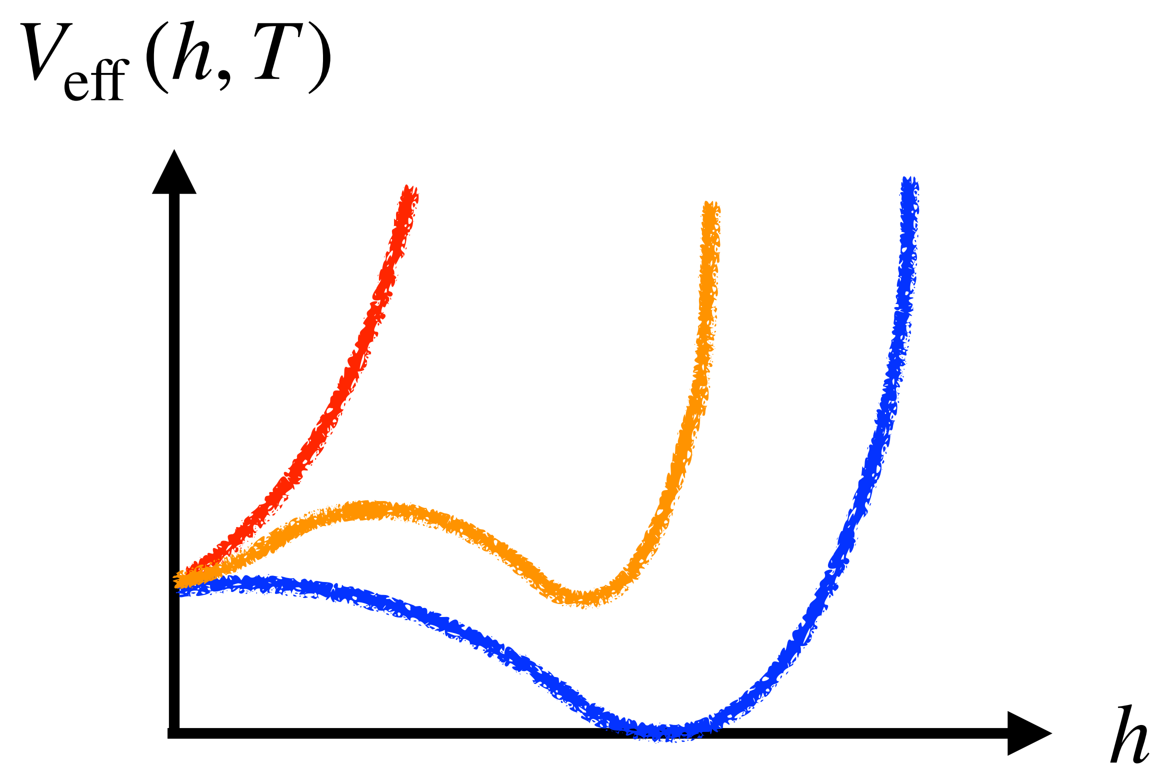
Can Higgs inflation and SM naturally predict other observable features in CMB?

Finite-temperature field theory:

The Higgs field would acquire thermal corrections to its free energy density due to the loop interaction with SM particles in a heat bath, leading to a temperature-dependent effective potential.

Classic example:

Electroweak phase transition (EWPT) at $T \sim 100\ \mathrm{GeV}$



Do we need to bother non-zero temperature during inflation?

Common folklore: No, because…

- The embedding physics of inflation is unknown.
- Any non-zero temperature T prior to inflation drops exponentially once inflation begins.

However, we find that…

- The thermal loop correction to the Higgs inflation can leave significant imprint on CMB!
- Temperature effect could be particularly important to the Higgs inflation.

Quantum Loop Corrections

- It is unclear whether perturbative calculations of Standard Model are valid up to the inflationary region of the Higgs inflation.

- (Barbón & Espinosa 2009; Burgess, Lee & Trott 2010) point out that the theory of inflation with non-minimal coupling suffers from the problem that a UV cutoff exists at $\Lambda \sim M_p/\xi \sim \Lambda_{\rm inf}$.

- (Bezrukov, Magnin, Shaposhnikov & Sibiryakov 2010) shows that the UV cutoff could be background-dependent, making the EFT valid up to M_P during inflation.

- Radiative correction of Higgs inflation to 1-loop has been studied by (Geroge, Mooij & Postma 2014; Hamada, Kawai, Nakanishi & Oda 2017), and (Bezrukov & Shaposhnikov 2009; Allison 2014) has extended the analysis to the 2-loop level.

Quantum Loop Corrections

Thermal effective potential at 1-loop:

Dolan & Jackiw, PRD 9 (1974) 3320;
Carrington, PRD 45 (1992) 2933;
Kolb & Turner, Front. Phys. 69 (1990) 1

- Describes the **Helmholtz free energy** of the system

$$\Delta V_{T,i}(h,T) = g_i \frac{T^4}{2\pi^2} \cdot F_i(m_i,\, T)$$

$$F_{b/f}(m,\, T) = \pm \int_0^\infty dq\, q^2 \ln\left[1 \mp \exp\left(-\sqrt{q^2 + \frac{m^2}{T^2}}\right)\right]$$

boson/fermion loop contribution

- Can be used to derive other thermodynamical variables

Entropy

$$s = -\frac{\partial}{\partial T}\Delta V_T(h,\, T)$$

Internal energy

$$\rho_T = \Delta V_T(h,T) + Ts$$

Pressure

$$\mathcal{P}_T = -\Delta V_T(h,\, T)$$

Quantum Loop Corrections

Computation of the 1-loop effective potential in Einstein frame

$$U_{1,\,\mathrm{eff}}\left(\chi,\widetilde{T}\right)=U_0(\chi)+\Delta U_{\mathrm{CW}}\left(\chi\right)+\Delta U_T\left(\chi,\widetilde{T}\right)$$

1-loop correction

- **Tree-level inflationary potential:** (the flat inflationary potential)

$$U_0(\chi)\approx\frac{\lambda M_P^4}{4\,\xi^2}\left[1-\exp\left(\frac{-2\,\chi}{\sqrt{6}M_P}\right)\right]^2$$

(The "tilde" denotes the quantities defined in Einstein frame)

- **Zero-temperature (1-loop): Coleman-Weinberg effective potential**

$$\Delta U_{\mathrm{CW}}\sim\frac{1}{16\pi^2}\widetilde{m}^4\sim\frac{1}{16\pi^2}\frac{y^4M_P^4}{\xi^2}\lesssim\mathcal{O}\left(10^{-2}\right)\cdot\frac{\lambda M_P^4}{4\,\xi^2}\sim\mathcal{O}\left(10^{-2}\right)\cdot U_0(\chi)$$

(during inflation: not important compared to the tree-level potential)

- **Finite-temperature (1-loop): thermal effective potential**

$$\Delta U_T\left(\chi,\,\widetilde{T}\right)=\frac{\widetilde{T}^4}{2\pi^2}\sum_i g_iF_i\left(\widetilde{m}_i,\,\widetilde{T}\right)$$

Assumption:
The Higgs field was immersed in a heat bath with all SM DOFs before inflation

Curvature Perturbation

As long as the slow-roll condition of Higgs holds:

$$\dot{\chi}_c{}^2 \ll U_0(\chi) \text{ and } \Delta U_T + \widetilde{T}\tilde{s} \ ,$$

we can find the primordial power spectrum by the conventional approach:

$$P_{\mathcal{R}}(k) = \frac{1}{8\pi^2 M_P^2} \frac{\widetilde{H}^2}{\varepsilon}\bigg|_{k=\tilde{a}\widetilde{H}} \overset{\text{(roughly)}}{\propto} \frac{\left\{U_0 + \left(\Delta U_T + \widetilde{T}\tilde{s}\right)\right\}^2}{\rho + \mathcal{P}}\bigg|_{k=\tilde{a}\widetilde{H}} \sim \frac{U_0(\chi)^2}{\dot{\chi}_c^2 + \widetilde{T}\tilde{s}}\bigg|_{k=\tilde{a}\widetilde{H}}$$

The EOS of the Higgs is modified by an additional Ts term, which does not exist in the conventional model.

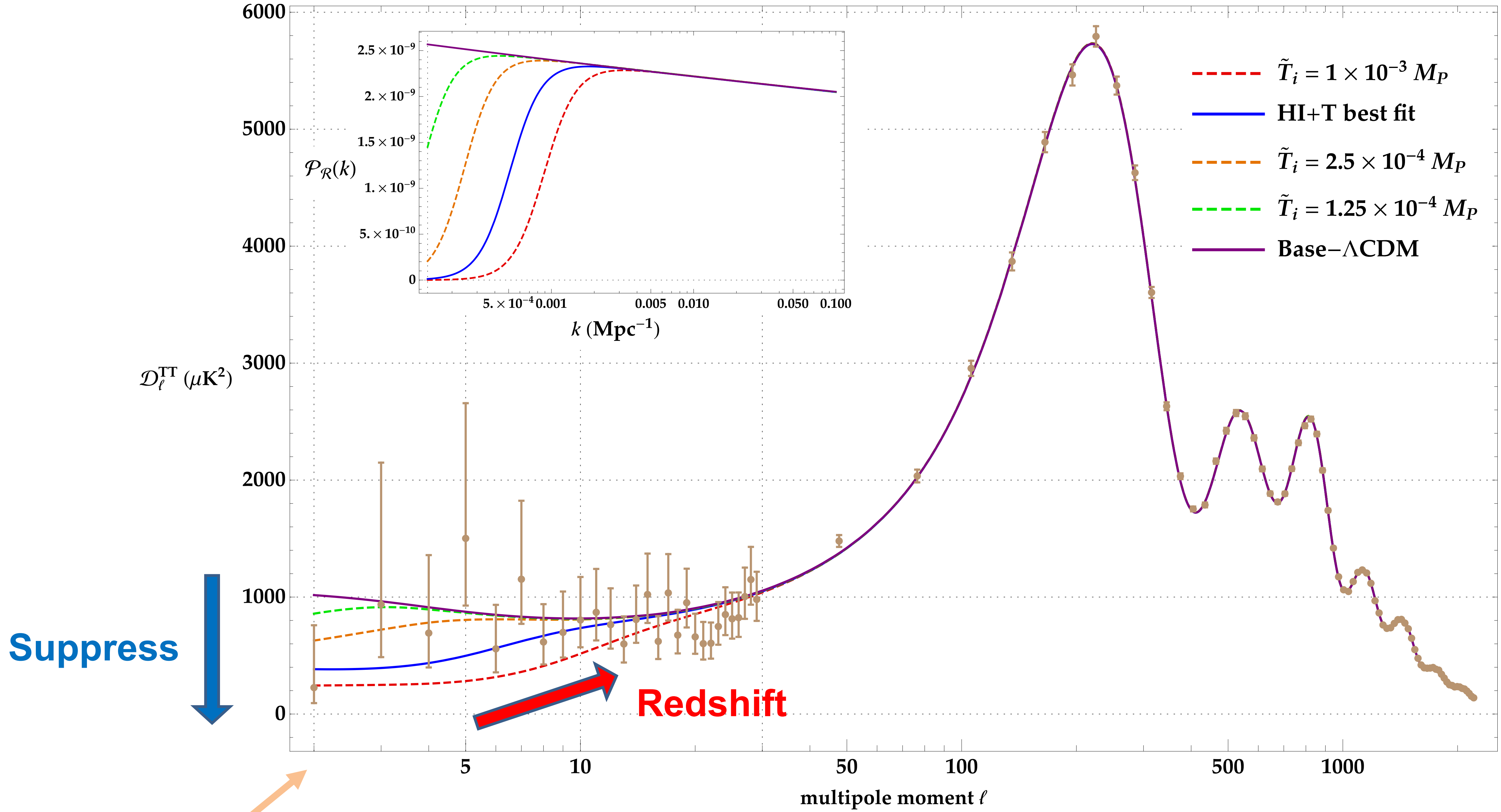
$$\rho = \dot{\chi}_c{}^2/2 + U_{1,\mathrm{eff}} + \widetilde{T}\tilde{s}$$

$$\mathcal{P} = \dot{\chi}_c{}^2/2 - U_{1,\mathrm{eff}}$$

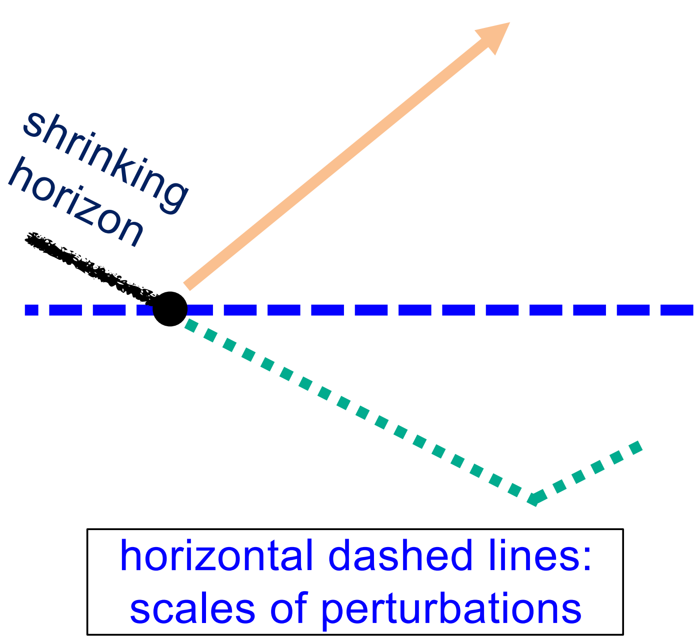
- The entropy of the heat bath will significantly modify the primordial power spectrum of curvature perturbation!
- In high-temperature limit:

$$P_{\mathcal{R}} \approx \frac{5}{4\pi^4 M_P^4}\left[U_0 - \widetilde{T}^2 \frac{\partial}{\partial \widetilde{T}}\left(\frac{\Delta U_T}{\widetilde{T}}\right)\right]^2 \left(2\sum_b g_b + \frac{7}{4}\sum_f g_f\right)^{-1} \widetilde{T}^{-4}$$

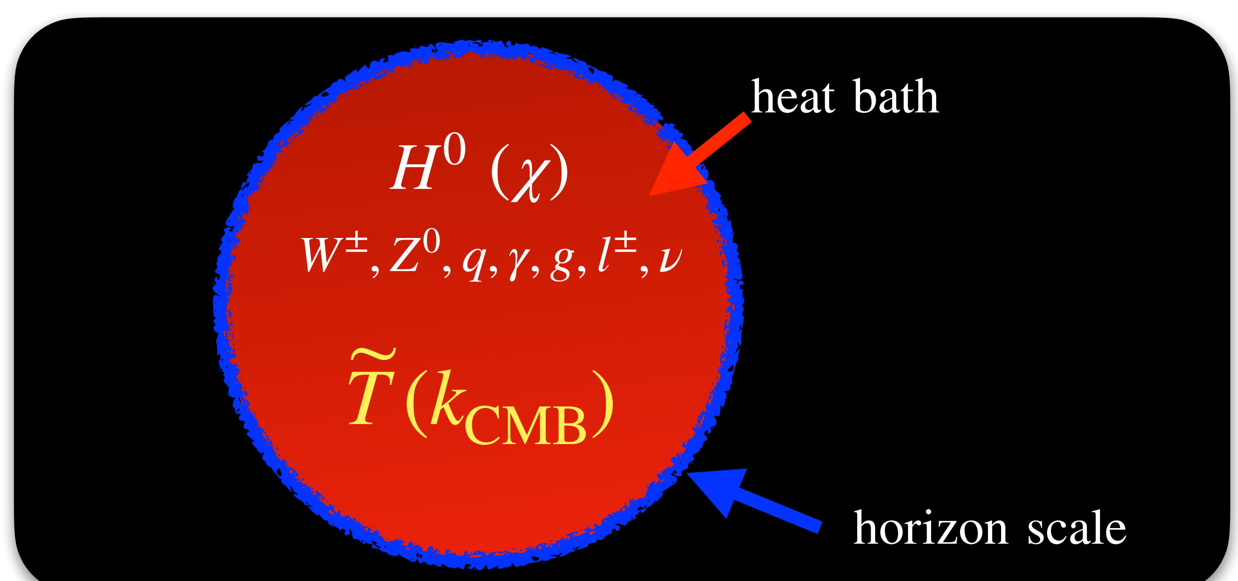
$$\begin{cases} \widetilde{T}_i^4 \ll U_0(\chi) \ \Rightarrow \ P_{\mathcal{R}}(k_{\mathrm{CMB}}) \propto \widetilde{T}_i^{-4} & \textbf{Suppress} \\ \widetilde{T}_i \sim 10^{-3}\ M_P & \\ \widetilde{T}_i^4 \gg U_0(\chi) \ \Rightarrow \ P_{\mathcal{R}}(k_{\mathrm{CMB}}) \propto \widetilde{T}_i^4 & \textbf{Enhance} \end{cases}$$



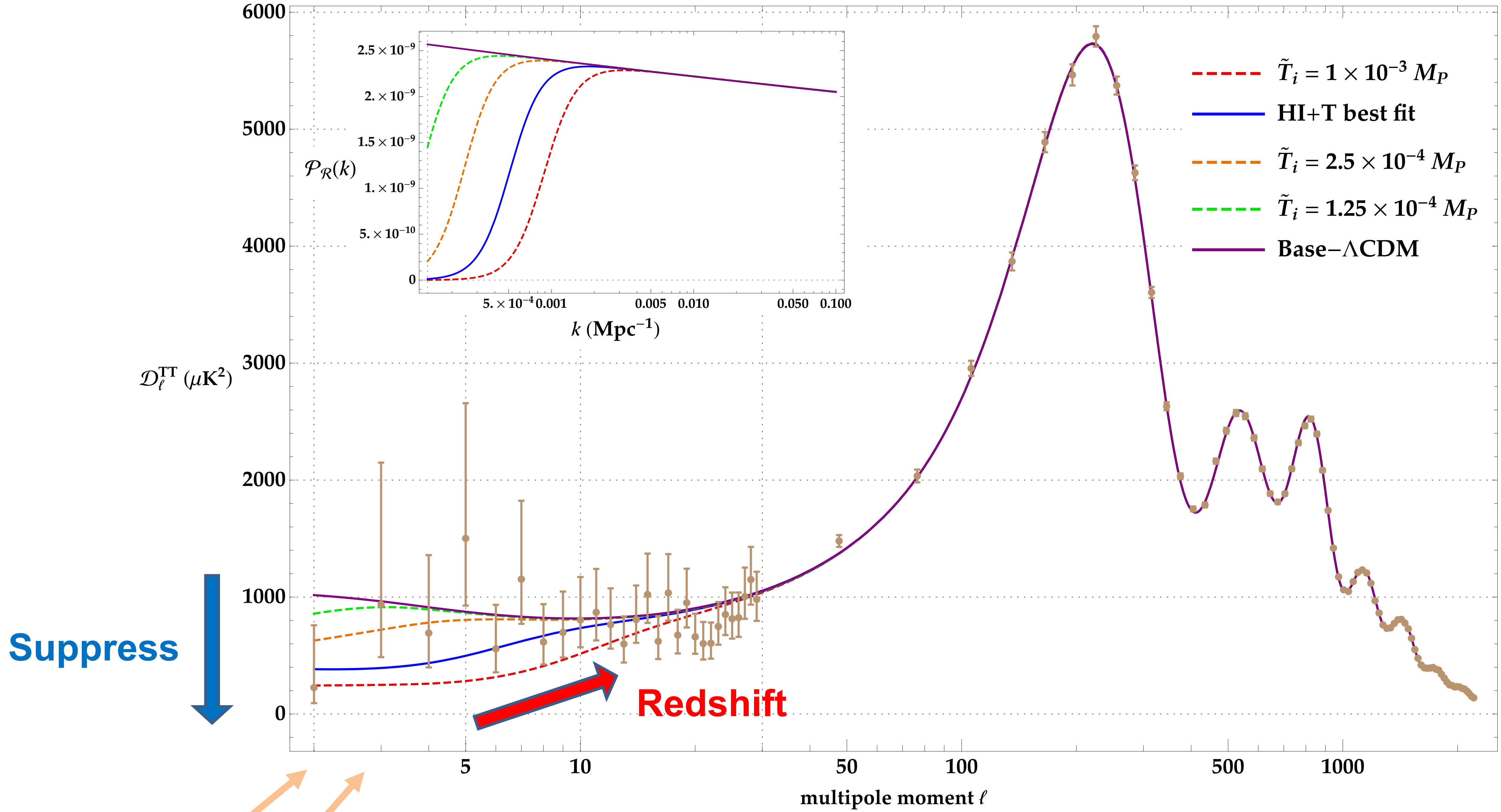
$\tilde{T}_i = 1 \times 10^{-3}\ M_P$
HI+T best fit
$\tilde{T}_i = 2.5 \times 10^{-4}\ M_P$
$\tilde{T}_i = 1.25 \times 10^{-4}\ M_P$
Base$-\Lambda$CDM
$\mathcal{D}_\ell^{TT}$ (μK^2)
multipole moment ℓ
$\mathcal{P}_{\mathcal{R}}(k)$
k (Mpc^{-1})
Suppress
Redshift



shrinking horizon
horizontal dashed lines: scales of perturbations



heat bath
$H^0\ (\chi)$
$W^\pm, Z^0, q, \gamma, g, l^\pm, \nu$
$\tilde{T}\,(k_{CMB})$
horizon scale



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Suppress
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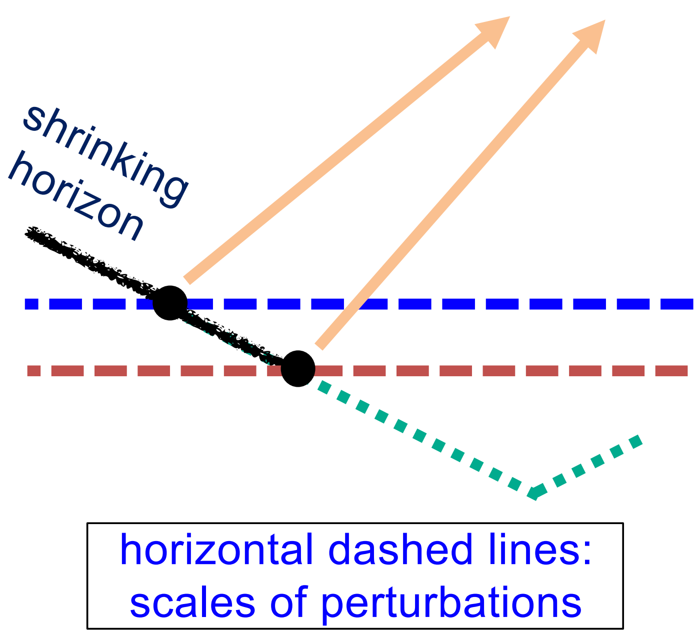

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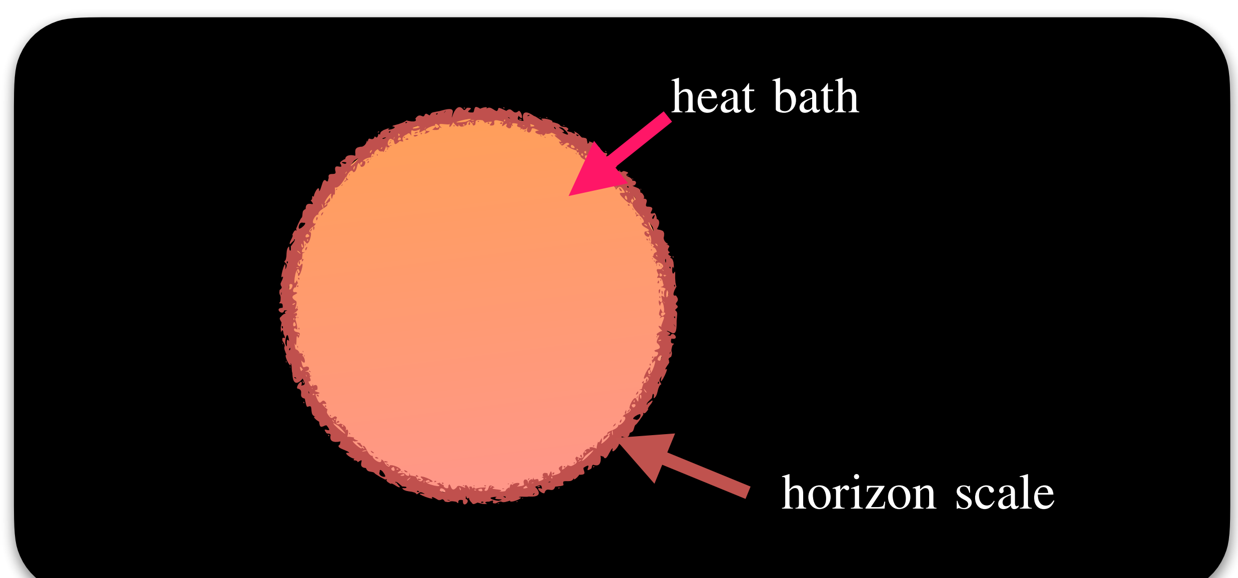
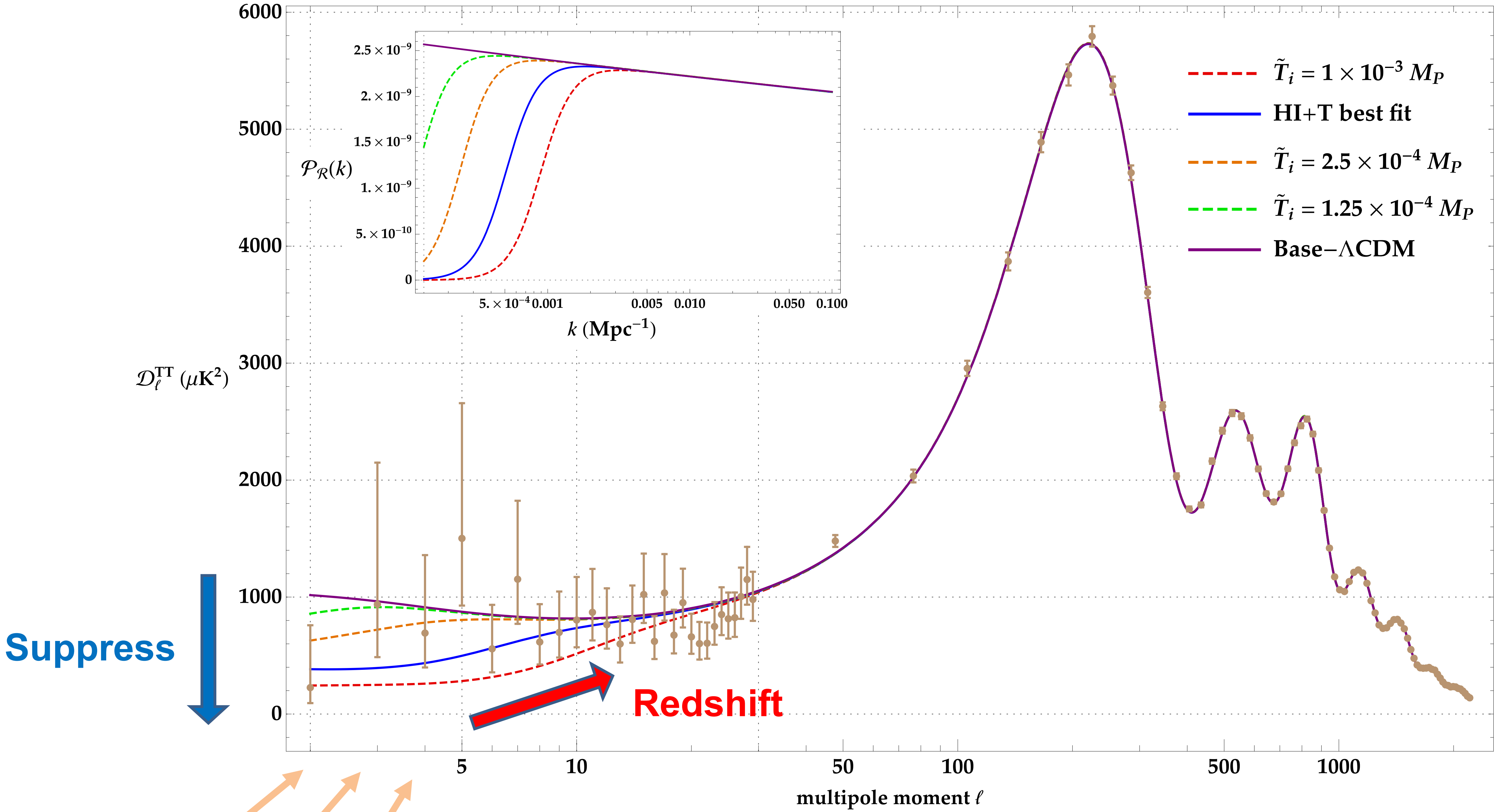


heat bath
horizon scale



$\tilde{T}_i = 1 \times 10^{-3}\ M_P$
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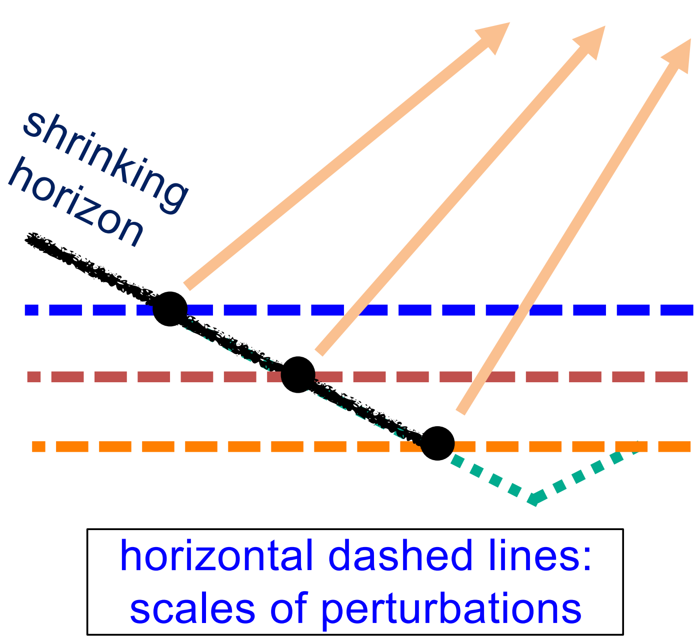
Base−ΛCDM
$\mathcal{P}_{\mathcal{R}}(k)$
k (Mpc^{-1})
$\mathcal{D}_\ell^{TT}$ (μK^2)
multipole moment ℓ
Suppress
Redshift



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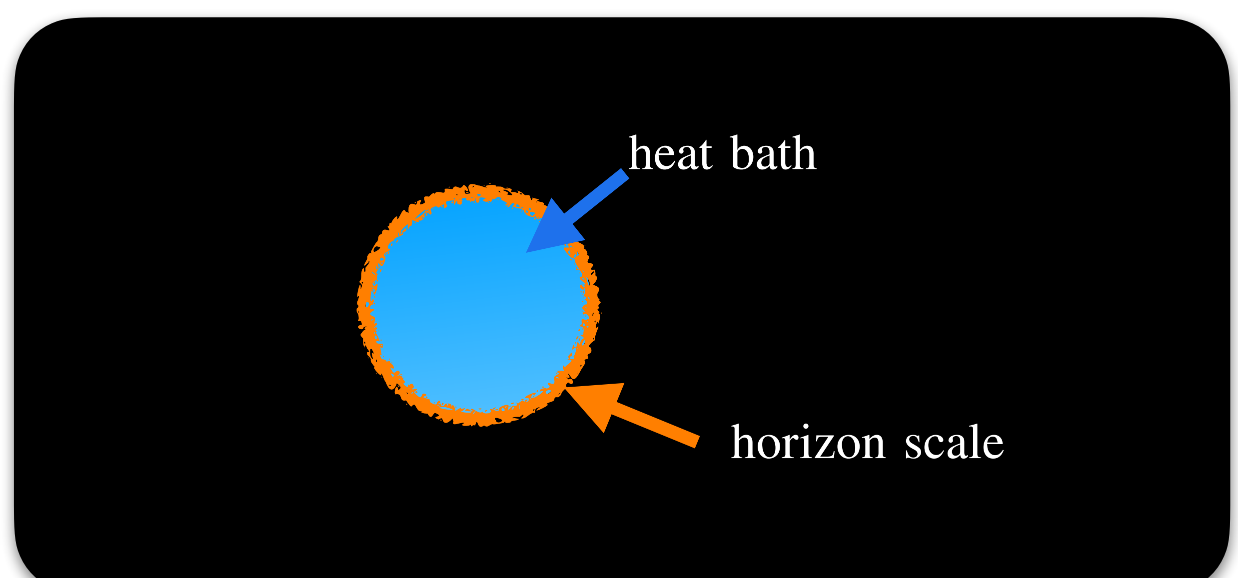
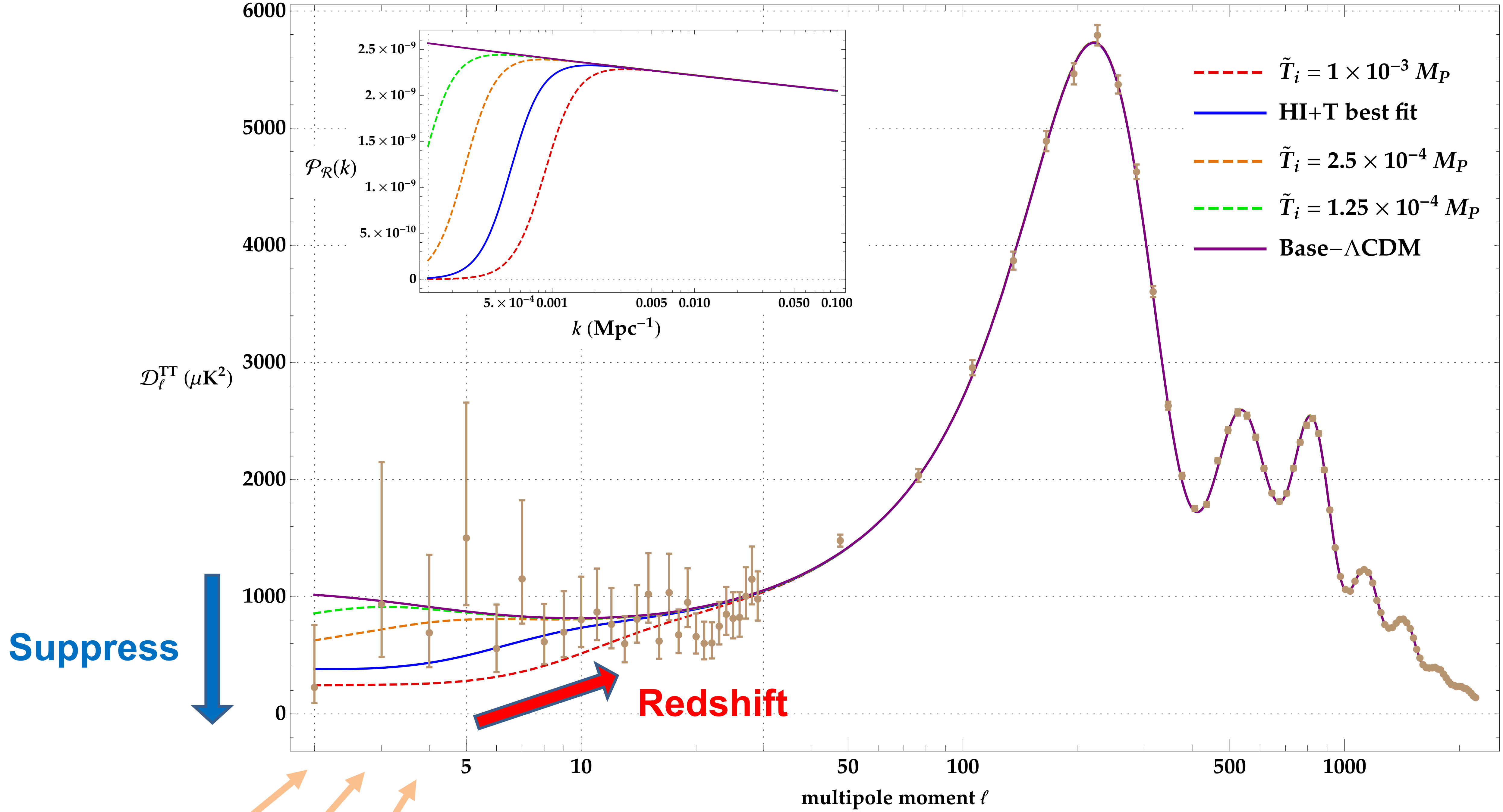
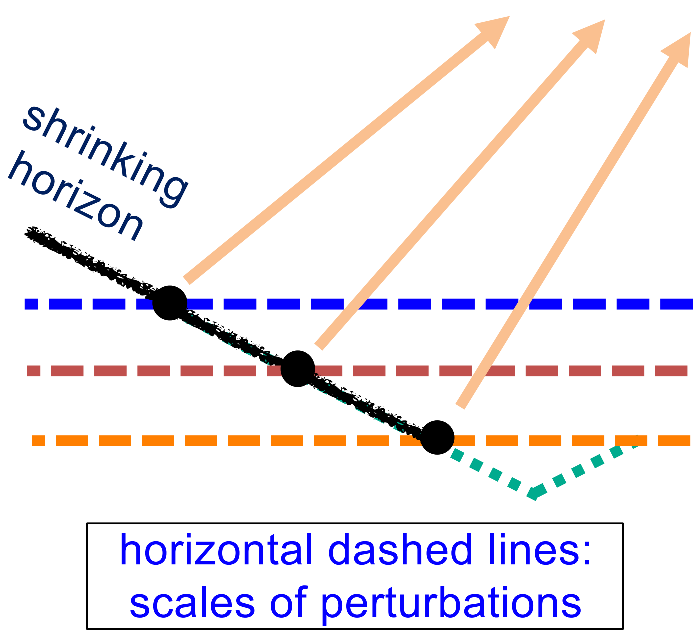
horizontal dashed lines: scales of perturbations



heat bath
horizon scale





Model	Temperature at $k_{\rm CMB}$ (M_P)	$\chi^2_{\rm theory}$ $(\ell = 2 \sim 29)$
Base-ΛCDM best fit	–	1.013
HI	0	0.971
HI + T best fit ($\widetilde{T}_i \lesssim 10^{-3}\ M_P$)	5.15×10^{-4}	0.765
HI + T best fit ($\widetilde{T}_i > 10^{-3}\ M_P$)	3.62×10^{-3}	0.762
HI + T	1.25×10^{-4}	0.918
HI + T	2.50×10^{-4}	0.829
HI + T	5.00×10^{-4}	0.766
HI + T	1.00×10^{-3}	0.871

Can thermal equilibrium be established?

Interaction rate of the Higgs and other particles:

$$\Gamma_{\text{int}} = n \langle \sigma v \rangle \approx \frac{\zeta(3)}{\pi^2} g_* \widetilde{T}^3 \cdot \frac{\alpha^2}{\widetilde{T}^2} \sim 0.1 g_* \alpha^2 \widetilde{T} \qquad g_* \sim 20$$

Cosmic expansion rate:

$$\widetilde{H} \approx \frac{1}{\sqrt{3} M_P} \left(U_0 + \frac{\pi^2}{30} \mathcal{G}_* \widetilde{T}^4 \right)^{1/2} \qquad \mathcal{G}_* \sim 100$$

Thermalization:

$$\Gamma_{\text{int}} > \widetilde{H} \qquad \Rightarrow \qquad 2 \times 10^{-4}\, M_P < \widetilde{T} < 7 \times 10^{-3}\, M_P$$

The two best-fit temperatures we have found favor the condition

Summary

- We present the calculation of the finite-temperature effective potential of Higgs inflation.
- If the Higgs field is immersed in a heat bath at the outset of inflation, the power of the curvature perturbation will be suppressed by the large entropy originating from thermal corrections.
- The precipitous drop of the temperature throughout inflation naturally explains the scale-dependent angular power spectrum at large scales $(\ell = 2 \sim 29)$.
- Planck 2018 data: the best-fit temperatures at the CMB horizon exit are $\widetilde{T}_i(k_{\rm CMB}) = 5.15 \times 10^{-4}\, M_P\,,\;\; 3.62 \times 10^{-3}\, M_P\,.$
- The Higgs inflation and the Standard Model physics can naturally predict observable features in the current CMB data.

Backup Slides

Thermal effective potential at 1-loop:

Dolan & Jackiw, PRD 9 (1974) 3320;
Carrington, PRD 45 (1992) 2933;
Kolb & Turner, Front. Phys. 69 (1990) 1

- Describes the **Helmholtz free energy** of the system

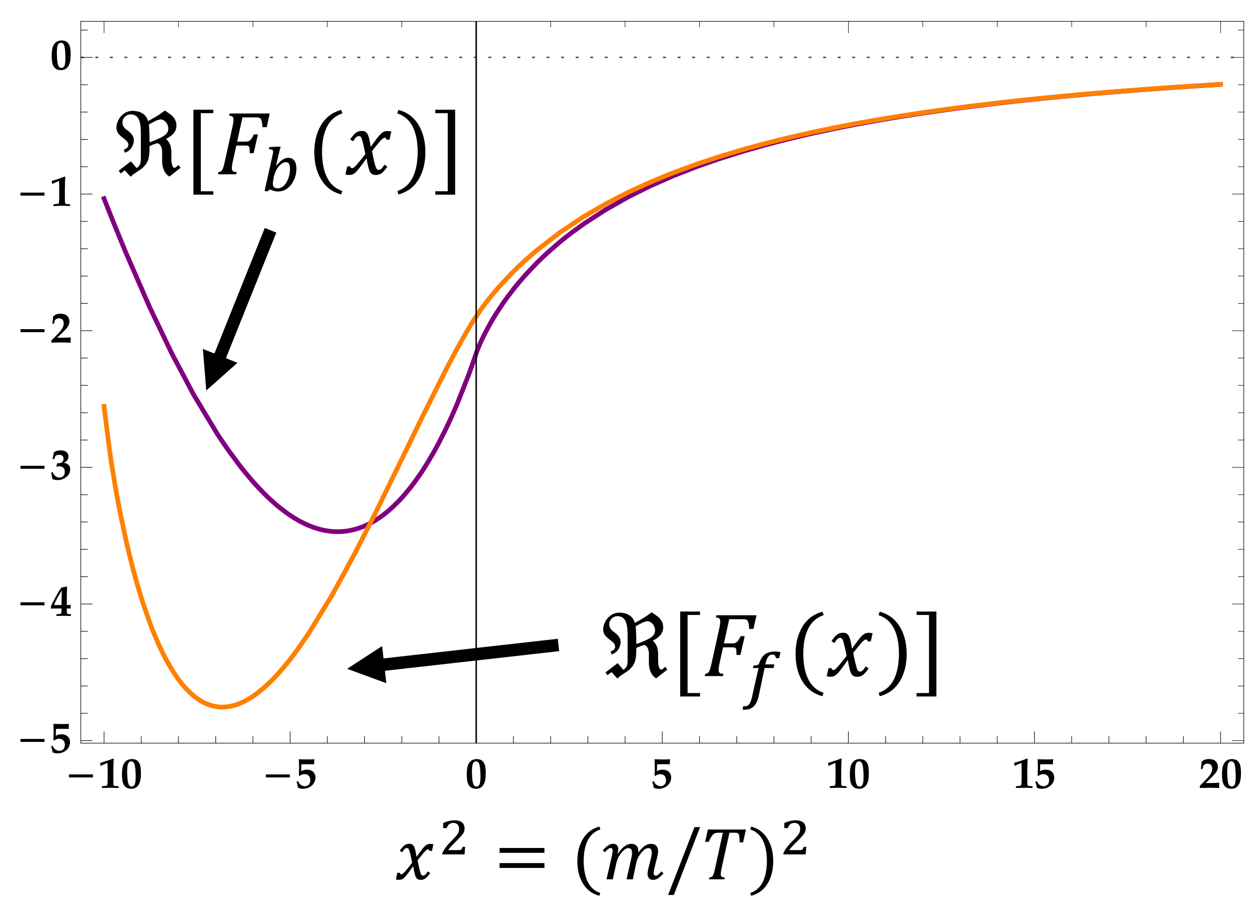
$$\Delta V_{T,i}(h,T) = g_i \frac{T^4}{2\pi^2} \cdot F_i(m_i, T)$$

$$F_{b/f}(m, T) = \pm \int_0^\infty dq\, q^2 \ln \left[1 \mp \exp\left(-\sqrt{q^2 + \frac{m^2}{T^2}} \right) \right]$$

boson/fermion loop contribution

The Higgs mass-squared is defined by the tree-level potential

$$m_H(h)^2 = \frac{d^2 V_0(h)}{dh^2}$$



Backup Slides

Prescription I (The "tilde" denotes the quantities defined in Einstein frame)

JF

$V_0(h)$

$m_i\,,\ T \propto a(t)^{-1}$

→ **CT** →

EF

$U_0\left(h(\chi)\right) = f^{-2} V_0(h)$

→

EF + Correction to EF

$$\Delta U_T\left(\chi, \tilde{T}\right) = \frac{\tilde{T}^4}{2\pi^2}\sum_i g_i F_i\left(\tilde{m}_i, \tilde{T}\right)$$

$$\tilde{m}_H(\chi)^2 = \frac{d^2 U_0(\chi)}{d\chi^2}\,, \qquad \tilde{m}_G(\chi)^2 = \frac{\lambda\, h(\chi)^2}{f^2\left[1 + 6\,\xi^2 h(\chi)^2/M_P^2\right]}$$

$$\tilde{m}_i = f(h)^{-1/2}\, m_i\,, \qquad \tilde{T} \propto \tilde{a}(\tau)^{-1} \propto f^{-\frac{1}{2}}\, a(t)^{-1}$$

Prescription II

JF

$V_0(h)$

$m_i, \ T \propto a(t)^{-1}$

→

JF + Correction to JF

$$\Delta V_T(h,T) = \frac{T^4}{2\pi^2}\sum_i g_i F_i\left(m_i, T\right) \ ?$$

$$m_H(h)^2 = \frac{d^2 V_0(h)}{dh^2}, \ m_G(h)^2 = \lambda h^2$$

$$m_i\,, \ T \propto a(t)^{-1}$$

→ **CT** →

EF + Correction to EF

$$\Delta U_T\left(h(\chi), T\right) = f^{-2}\Delta V_T(h,T)$$

Backup Slides

Prescription I (The "tilde" denotes the quantities defined in Einstein frame)

JF

$V_0(h)$

$m_i\ ,\ T \propto a(t)^{-1}$

CT →

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$U_0\left(h(\chi)\right) = f^{-2} V_0(h)$

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+ Correction to EF

$$\Delta U_T\left(\chi, \tilde{T}\right) = \frac{\tilde{T}^4}{2\pi^2}\sum_i g_i F_i\left(\tilde{m}_i, \tilde{T}\right)$$

$$\tilde{m}_H(\chi)^2 = \frac{d^2 U_0(\chi)}{d\chi^2}\ , \qquad \tilde{m}_G(\chi)^2 = \frac{\lambda\, h(\chi)^2}{f^2\left[1 + 6\,\xi^2 h(\chi)^2/M_P^2\right]}$$

$$\tilde{m}_i = f(h)^{-1/2}\, m_i\ , \qquad \tilde{T} \propto \tilde{a}(\tau)^{-1} \propto f^{-\frac{1}{2}}\, a(t)^{-1}$$

- The results of the two prescriptions for the radiative corrections at zero temperature up to two-loop level have been discussed by (Bezrukov & Shaposhnikov 2009; Allison 2014; Geroge, Mooij & Postma 2014).
- Given the fact that relatively little is known about quantum gravity, there is no obvious preference for the prescription that one should take (Bezrukov, Magnin & Shaposhnikov 2009; Allison 2014).
- Here we adopt the **prescription I** for our computation, since

(a) it can remove the uncertainty from graviton loop in JF.

(b) the inflaton field χ and its vacuum state, the Bunch-Davies vacuum, are all defined in the EF.

Backup Slides

Renormalization group running of the SM coupling constants

- **Running of the gauge and Yukawa coupling**

  Not sensitive. Most of the SM particles are relativistic in the range of the temperature we are interested in.

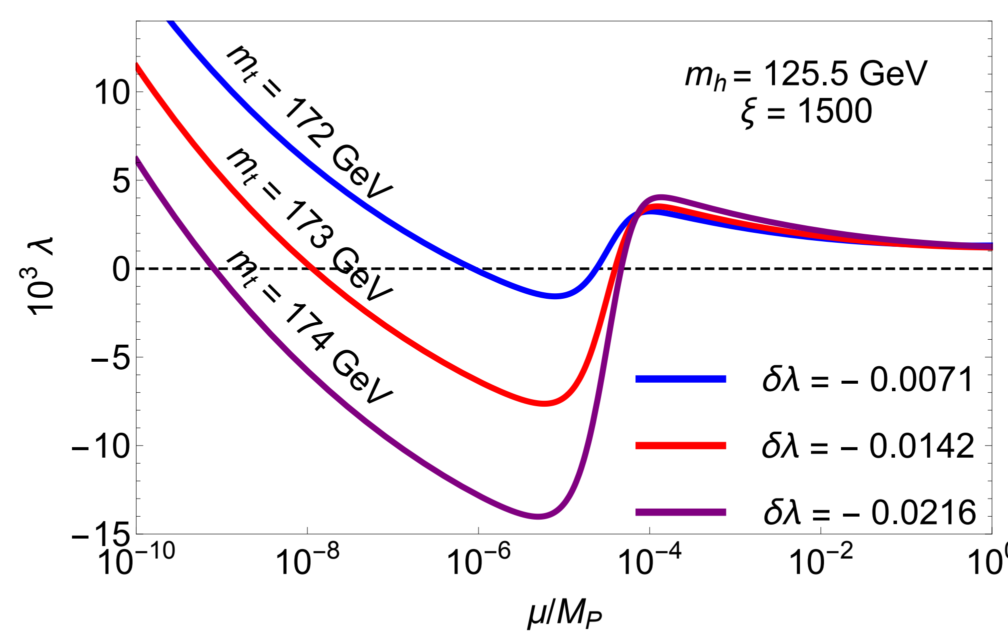
- **Running of the Higgs quartic coupling**

  The tree-level potential and 1-loop effective potential all depend on λ/ξ^2 which is fixed by the CMB normalization (recall $\xi = 4.7 \times 10^4 \sqrt{\lambda}$).

- **Higgs criticality**

  Additional renormalization effect may happen at $\chi \sim M_P/\xi$.

  Rubio, Front. Astron. Space Sci 5 (2019) 50;
  Bezrukov, Rubio & Shaposhnikov, PRD 92 (2015)



Backup Slides

Quantum instability

- **Negative Higgs mass-squared**

$$\widetilde{m}_H(\chi)^2 = \frac{d^2 U_0(\chi)}{d\chi^2} < 0$$

Complex loop effective potential

- **Decay rate of the unstable particle state**

Weinberg & Wu, PRD 36 (1987) 2474

$$\Gamma_{\rm inf} \sim {\rm Im}\,(\Delta U_{\rm CW}) \cdot \widetilde{H}_{\rm inf}^{-3} \;<\; {\rm Im}\,(\Delta U_{\rm CW}) \cdot \widetilde{H}_{\rm end}^{-3}$$

$$\Longrightarrow \quad \Gamma_{\rm inf} \ll \widetilde{H}_{\rm end} < \widetilde{H}_{\rm inf} \sim {\rm Im}\,(\Delta U_{\rm CW}) \cdot \left\{ \frac{M_P \sqrt{\lambda}}{2\sqrt{3}\,\xi} \left[1 - \exp\left(\frac{-2\,\chi_{\rm end}}{\sqrt{6} M_P} \right) \right] \right\}^{-3}$$

Not important