The electroweak parton distribution functions

Part I: General considerations

Keping Xie

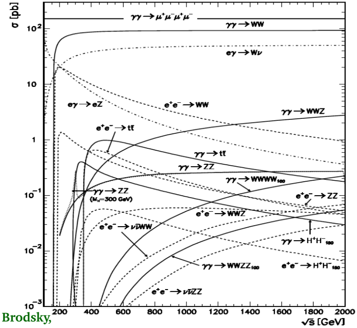
Pittsburgh Partcile-physics, Astrophysics, and Cosmology Center,
Department of Physics and Astronomy,
University of Pittsburgh, PA 15260, USA

Pheno 2020 @ Pittsburgh

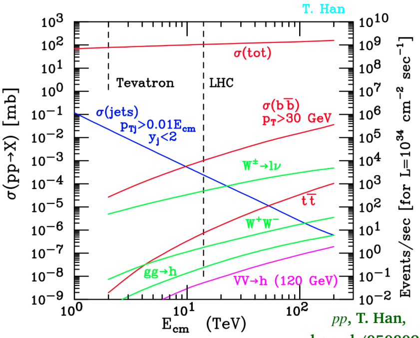
Ongoing work with Tao Han and Yang Ma.
Please also see Yang Ma's talk for Part II.

Why EW PDFs?

- Theoretical significance: the EW PDFs are very different from the quark/gluon PDFs in QCD.
  - The EW theory is chiral
  - Spontaneous Symmetry breaking: $(B, W^i) \to (\gamma, W^\pm/Z)$
    gauge boson mass effect (low-E) and symmetry restoration (high-E)
  - Different from the QCD confinement (color-charged particles cannot be directly observed), the weak-charged particles (non-singlets, such as electron or neutrino) can be directly observed.
- Practical application for future colliders (e^-e^+ and pp)



e^+e^-, S. Brodsky, SLAC-PUB-11581



pp, T. Han, hep-ph/0508097

Factorization formalism

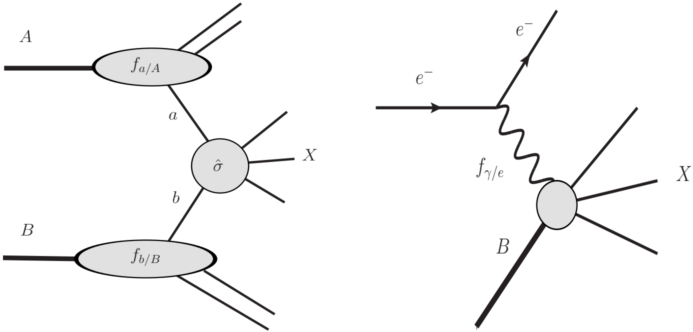
- For a process $AB \to X$, the beam cross section can be written in a factorized form:

$$\sigma(AB \to X) = \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X) + \cdots .$$

- Example: Equivalent Photon Approximation (Weizäicker-Williams)

$$\sigma(e^- B \to e^- X) \approx \int \mathrm{d}x f_{\gamma/e}(x, Q) \hat{\sigma}(\gamma B \to X),$$

where the photon is treated as a parton, like the quark or gluon in QCD.



Parton distribution functions (PDFs)

The PDF $f(x,Q^2)$ is the probability of finding a collinear parton with a momentum fraction x inside the initial beam.

- The perturbative part is determined by the DGLAP evolution:

$$\frac{\mathrm{d}}{\mathrm{d}\log Q} f_i(x,Q) = \frac{\alpha}{\pi} \sum_j \int_x^1 \frac{\mathrm{d}z}{z} P_{i/j}(x/z) f_j(z,Q).$$

- The non-perturbative part: such as PDFs at a initial scale $f_i(x,Q_0)$
  - Global fits to data: CT, MMHT, NNPDF etc.
  - Lattice QCD approach: see K. Liu's talk and etc.

In the EW (or QED) theory: the non-perturbative effect is negligible.
PDFs can be obtained perturbatively through the DGLAP resummation.

- In QED, the initial conditions at $Q_0 = m_e$:

$$f_{e/e}(x,Q_0) = \delta(1-x),\ f_{\bar{e}/e}(x,Q_0) = f_{\gamma/e}(x,Q_0) = 0.$$

  The first order resummation gives EPA [Weizsäcker '34, Williams '34]:
  $f_{\gamma/e}(x,Q) \sim \alpha \frac{1+(1-x)^2}{x} \log(Q/m_e)$
- Similarly in EW theory, the first order resummation gives the Effective W Approximation (EWA) [Kane et. al. '84, Dawson '85].

Traditional unpolarized PDFs

- In general, the complete factorized form of a process $(A \to V)B \to X$

$$\sigma = \sum_{\lambda,s_1,s_2} f_{V_\lambda/A_{s_1}} \hat{\sigma}(V_\lambda B_{s_2} \to X).$$

- In a vector-like theory (QED or QCD), we have the parity conservation:

$$f_{V_+/A_+} = f_{V_-/A_-}, f_{V_+/A_-} = f_{V_-/A_+}, \hat{\sigma}(V_+B_+) = \hat{\sigma}(V_-B_-), \hat{\sigma}(V_+B_-) = \hat{\sigma}(V_-B_+).$$

- For unpolarized beam A and B, the factorized from can be written in an unpolarized form:

$$\sigma = f_{V/A}\hat{\sigma}(VB), \; f_{V/A} = (1/2)\sum_{\lambda,s_1} f_{V_\lambda/A_{s_1}}, \; \hat{\sigma}(VB) = (1/4)\sum_{\lambda,s_2}\hat{\sigma}(V_\lambda B_{s_2}).$$

Example: EPA $f_{\gamma_\lambda/e_s^\pm}(x,Q) = \frac{1}{4\pi^2} e^2 P_{\gamma_\lambda/e_s^\pm}(x)\log(Q/m_e)$ (universal coupling e).

	e_L^-	e_R^-	$\langle e^- \rangle$	e_L^+	e_R^+	$\langle e^+ \rangle$
γ_-	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$
γ_+	$\frac{(1-x)^2}{x}$	$\frac{1}{x}$	$\frac{1+(1-x)^2}{2x}$	$\frac{1}{x}$	$\frac{(1-x)^2}{x}$	$\frac{1+(1-x)^2}{2x}$
$\sum \gamma_\lambda$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$	$\frac{1+(1-x)^2}{x}$

EW PDFs must be polarized: $f_{V_\lambda}(f_{e_s})$

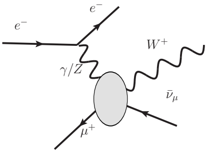
- The chiral EW theory violates parity. In general,

  $$f_{V_+/A_+} \neq f_{V_-/A_-},\ f_{V_+/A_-} \neq f_{V_-/A_+},\ \hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-),\ \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)$$

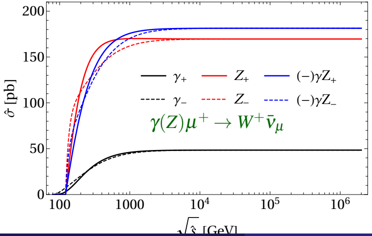
  Example: EZA $f_{Z_\lambda/e_s^\pm}(x,Q) = \frac{1}{4\pi^2} g_s^2 P_{Z_\lambda/e_s^\pm}(x) \log(Q/m_Z)$ (chiral coupling $g_{L,R}$)

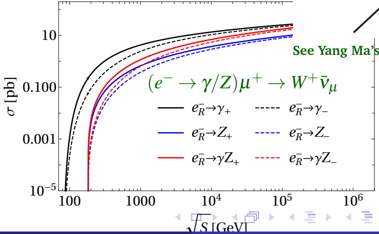
	e_L^-	e_R^-	e_L^+	e_R^+
Z_-	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$
Z_+	$g_L^2 \frac{(1-x)^2}{x}$	$g_R^2 \frac{1}{x}$	$g_L^2 \frac{1}{x}$	$g_R^2 \frac{(1-x)^2}{x}$

$$g_L = \frac{g_2}{c_W}\left(-\frac{1}{2} + s_W^2\right) < 0,\ g_R = \frac{g_2}{c_W} s_W^2 > 0.$$



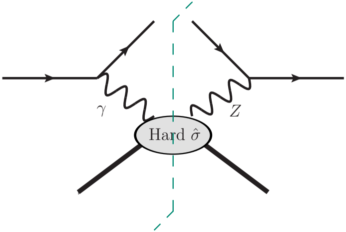
- We need polarized PDFs, generally in two scenarios: (1) polarized beams; (2) chiral interactions (EW);





See Yang Ma's talk for more details

The coherence: $\gamma Z_T(BW^3)$, hZ_L



The interference gives the mixed PDFs [Bauer '17, '18, Manohar '18 , Tao '16.]

$$f_{\gamma Z} \sim \langle \Omega | A^{\mu\nu} Z_{\mu\nu} | \Omega \rangle + \text{h.c.},$$

Similarly for the f_{hZ_L}.

- The (pure) PDFs f_γ, f_Z are positive definite
- The Mixed PDF $f_{\gamma Z}$ can be either positive or negative.

$$f_{\gamma Z/e_L} \sim (-eg_L) > 0,\ f_{\gamma Z/e_R} \sim (-eg_R) < 0.$$

- The interference partonic cross section

$$\hat{\sigma}_{\gamma Z} \sim \int \mathrm{d}\Phi (\mathcal{M}_\gamma^\dagger \mathcal{M}_Z + \mathcal{M}_Z^\dagger \mathcal{M}_\gamma)$$

  In our example $\gamma(Z)\mu^+ \to W^+ \bar{\nu}_\mu$, $\hat{\sigma}_{\gamma Z} < 0$.
- The mixed PDF contribution can be either constructive or destructive.

$$\sigma_{\gamma Z/e_L} > 0, \quad \sigma_{\gamma Z/e_R} < 0, \quad \text{if } \hat{\sigma}_{\gamma Z} > 0,$$
$$\sigma_{\gamma Z/e_L} < 0, \quad \sigma_{\gamma Z/e_R} > 0, \quad \text{if } \hat{\sigma}_{\gamma Z} < 0.$$

The (B, W) basis

- Rotation: $B = c_W A - s_W Z,\ W^3 = s_W A + c_W Z,$

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}.$$

  In the high energy limit, (B, W) basis approaches to EW unbroken phase: symmetry restoration and massless gauge boson (QCD-like)
- Taking EPA and EZA:

$$f_{\gamma/e} \sim \log(Q/m_e) = L_\gamma,\ f_{Z/e} \sim f_{\gamma Z/e} \sim \log(Q/m_Z) = L_Z,$$

  the EW PDFs in (B, W) basis become

$$f_{B/e_R} \sim L_\gamma(L_Z),\ f_{W^3/e_R} \sim f_{BW^3/e_R} \sim \log(m_Z/m_e) = L_\gamma - L_Z.$$

- In the gauge basis, e_R does not couple to W^3.
  If we choose $L_Z = \log(Q/m_Z)$ for both EPA and EZA (scale variation), we get exact cancellation in γ/Z for W^3: $f_{W^3/e_R} = f_{BW^3/e_R} = 0$.
- The non-zero $f_{W^3/e_R}, f_{BW^3/e_R}$ are constants ($\sim L_\gamma - L_Z$) due to the uncanceled residues in symmetry breaking.

Impact of Mixed PDFs: $(e^- \to \gamma/Z)\mu^+ \to W^+\bar{\nu}_\mu$

The partonic amplitudes $\gamma(Z)\mu^+ \to W^+\bar{\nu}_\mu$ in the (γ, Z) and (B, W^3) bases

$$\mathcal{M}_B = c_W \mathcal{M}_\gamma - s_W \mathcal{M}_Z,\ \mathcal{M}_{W^3} = s_W \mathcal{M}_\gamma + c_W \mathcal{M}_Z.$$

In the high energy limit, partonic cross sections approach to constants.

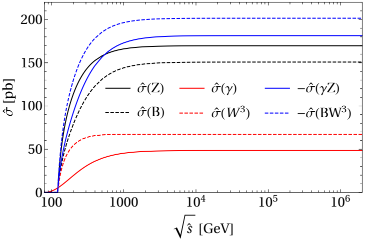
$$\hat{\sigma}_\gamma = \frac{g_2^4 s_W^2}{16\pi m_W^2}, \quad \hat{\sigma}_Z = \frac{g_2 c_W^2}{16\pi m_W^2}, \quad \hat{\sigma}_{\gamma Z} = -\frac{g_2^2 s_{2W}}{16\pi m_W^2},$$

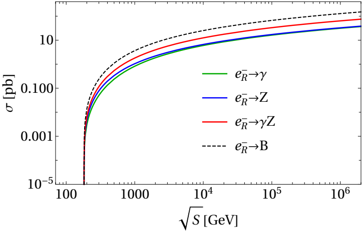
$$\hat{\sigma}_B = \frac{g_2^4 s_{2W}^2}{16\pi m_W^2}, \quad \hat{\sigma}_{W^3} = \frac{g_2^4 c_{2W}^2}{16\pi m_W^2}, \quad \hat{\sigma}_{BW^3} = -\frac{g_2^4 s_{4W}}{16\pi m_W^2}.$$

With a universal log $L_Z = \log(Q/m_Z)$ for all EW PDFs, the high-E beam cross section ratio

$$\begin{aligned}&\sigma_{B/e_R^-} : \sigma_{\gamma/e_R^-} : \sigma_{Z/e_R^-} : \sigma_{\gamma Z/e_R^-}\\ =&\hat{\sigma}_B f_{B/e_R^-} : \hat{\sigma}_\gamma f_{\gamma/e_R^-} : \hat{\sigma}_Z f_{Z/e_R^-} : \hat{\sigma}_{\gamma Z} f_{\gamma Z/e_R^-}\\ =&4:1:1:2.\end{aligned}$$

Constructive $\sigma(\gamma Z/e_R) > 0 \Longleftarrow f_{\gamma Z/e_R} < 0, \hat{\sigma}(\gamma Z) < 0.$





- $\sigma(e_R \to W^3/BW^3) = 0$ due to $f_{(B)W^3/e_R} = 0$.

See Yang Ma's talk for details of a more realistic study.

Summary

- EW PDFs extend conventional QCD/QED PDFs to the EW sector.
- The EW PDFs are polarized, as well as the hard partonic cross sections, because of the chiral nature of the EW theory.
- The interference gives mixed PDFs. The mixed PDFs can be either positive or negative, which gives either constructive or destructive contribution to the beam cross section.
- In the low energy limit, we need to match the resummed EW PDFs to the photon PDF and/or perturbative fixed-order calculation.
- In the high energy limit, we can match to the gauge basis (B, W), which asymptotically approaches EW unbroken phase, and behaves like QCD.
- EW PDFs are essential for future multiple TeV high energy colliders.

See Yang Ma's talk for a realistic case study.

EW PDFs and evolution

A brief review

- The non-cancelling IR effects: EW Sudakov double logarithms at LO, different from the QCD LO single logarithm. [M. Ciafaloni et. al. hep-ph/0001142, hep-ph/0004071]
- The EW PDF evolutions [Bauer, Webber et. al. 1712.07147, 1703.08562, Manohar et. al. 1802.08687]
- The LUX formalism for EW PDFs [Manohar et. al. 1803.06347]
- The Goldstone equivalence gauge (GEG) and its application to splitting and showering [Han et. al. 1611.00788]
- A Lorentz-covariant formalism of Goldstone equivalence [Wulzer at. al. 1911.12366]

Novel features

- Polarization effects [Bauer and Webber, 1808.08831]
- Interference: γZ_T and hZ_L mixed-state evolution
- Mass effects
  - Ultra-collinear effects
  - Matching
- Gauge non-invariant effects

Gauge boson mass effects

- The EW gauge symmetry is spontaneously broken: W, Z bosons are massive
- The ultra-collinear effects [Tao et. al. 1611.00788]:

$$\frac{\mathrm{d}P_{A\to BC}}{\mathrm{d}z\mathrm{d}k_T^2} \sim \frac{1}{16\pi^2}\frac{z\bar{z}|\mathscr{M}^{\mathrm{split}}|^2}{(k_T^2+\bar{z}m_B^2+zm_C^2-z\bar{z}m_A^2)^2}.$$

  - $|\mathscr{M}^{\mathrm{split}}|^2 \sim k_T^2$, $P \sim \mathrm{d}k_T^2/k_T^2$ gives $\log(Q^2/m^2)$ (conventional DGLAP log).
  - $|\mathscr{M}^{\mathrm{split}}|^2 \sim m^2$, $P \sim m^2\mathrm{d}k_T^2/k_T^4$
    (1) highly suppressed for $k_T \gtrsim m$; (2) enhanced for $k_T \lesssim m$: ultra-collinear
- Example: longitudinal gauge boson PDFs [Kane et. al. PLB'84, Dawson NPB'85]:

$$f_{V_L/f}(x,Q^2) \sim \alpha\frac{1-x}{x}$$

  approaches to a constant (independent of Q^2) in the high energy limit.
- $f_V(x,Q^2)=0$ when $Q<m_V$. We need a systemic matching onto the photon PDF and/or perturbative fixed-order calculation at low energy.
- In the high energy limit, we can switch to (B, W^3) basis.
- Some tricks: rescaling variables improving matching at threshold [ACOT-χ scheme, hep-ph/0110247]

Gauge non-invariant effects

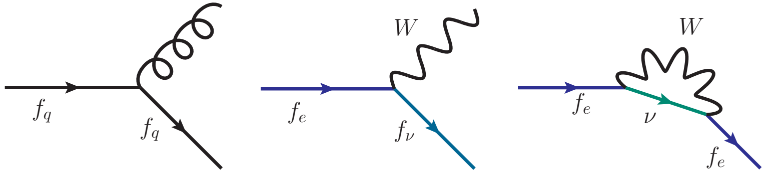
- In QCD, the color charged particles cannot be observed. The quark PDFs are defined as color singlets

$$f_q = (1/N_c)\sum_c f_{q_c}$$

The real and virtual cancellation gives leading single logarithms.

- Weak-charged particles (e.g., leptons and neutrinos) is a physical, which can be directly observed: leading double logarithm:

$$f_e(x,Q) \simeq f_e(x,Q_0)\frac{1+e^{-(\alpha/2\pi)\log^2(Q/m_W)}}{2} + f_\nu(x,Q_0)\frac{1-e^{-(\alpha/2\pi)\log^2(Q/m_W)}}{2}$$



- We need to define new PDF-like quantity: parton luminosity ensemble (PLE),

$$f_1 \sim f_e + f_\nu,\ f_3 \sim f_e - f_\nu,$$

Similar to the isospin (T) and CP basis defined in Refs. [Bauer et. al., 1703.08562, 1808.08831]