Valence and Connected Sea Partons from Hadronic Tensor and LaMET

- Deficiency of the conventional definition of the valence parton.
- Hadronic Tensor from Euclidean Path Integral
- Quasi-PDF from LaMET
- Valence and Sea Partons in NNLO Evolution

Pheno Symposium , May 4, 2020

Questions

- Is the definition $\mathrm{q}_i^v \equiv q_i - \overline{q}_i$ for the valence parton valid or adequate in NNLO evolution?

- Why should strange has valence distribution $q_s^- \equiv q_s^v = s - \overline{s}$ (?)

Evolution Equations

S. Moch et al., hep/0403192,0404111
A. Cafarella et al., 0803.0462

NNLO

$$dq_i / dt = \sum_k (P_{ik} \otimes q_k + P_{i\bar{k}} \otimes q_{\bar{k}}) + P_{ig} \otimes g;$$

$$d\bar{q}_i / dt = \sum_k (P_{\bar{i}k} \otimes q_k + P_{\bar{i}\bar{k}} \otimes q_{\bar{k}}) + P_{\bar{i}g} \otimes g;$$

$$dg / dt = \sum_k (P_{gk} \otimes q_k + P_{g\bar{k}} \otimes q_{\bar{k}}) + P_{gg} \otimes g.$$

→

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + \frac{P_{ns}^s}{N_f} \otimes \Sigma_v;$$

where $q_i^- \equiv q_i - \bar{q}_i$, $\Sigma_v \equiv \sum_k (q_k - \bar{q}_k)$,

and $P_{ns}^s \sim O(\alpha_s^3)$

Valence u can affect the evolution of valence d ?

Also: $q_s^- \equiv q_s^v = s - \bar{s}$ (?)

Hadronic Tensor in Euclidean Path-Integral Formalism

- DIS in Minkowski space

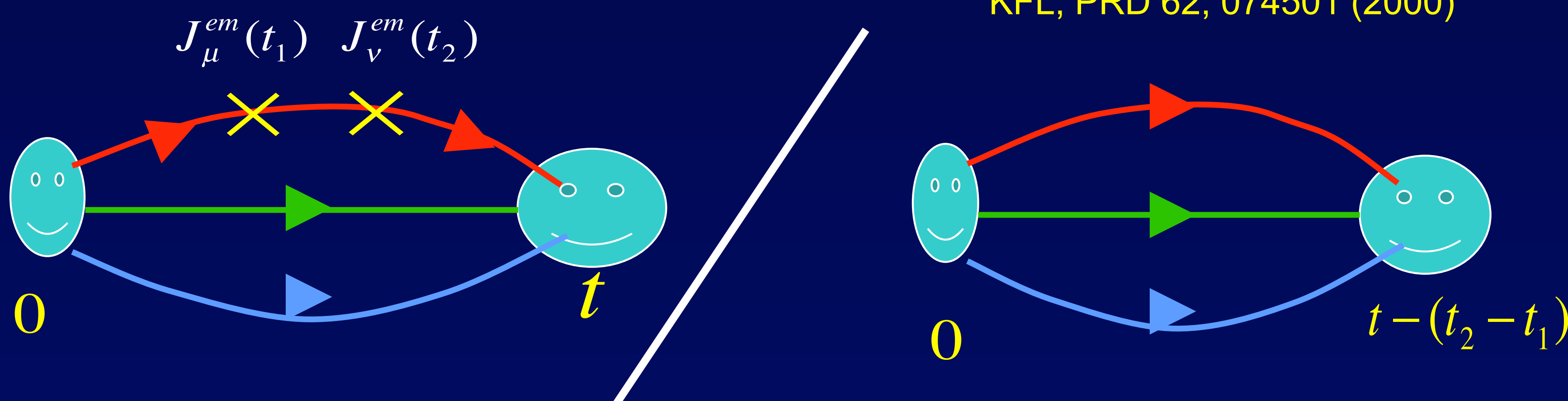
$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu} = \left\langle N(\vec{p})\right|\int \frac{d^4x}{4\pi}\, e^{iq\cdot x} J_\mu(x) J_\nu(0) \left| N(\vec{p})\right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n \int \prod_{i=1}^{n}\left[\frac{d^3 p_i}{(2\pi)^3 2E_{pi}}\right](2\pi)^3 \delta^4(p_n - p - q) < N(\vec{p}) \mid J_\mu \mid n >< n \mid J_\nu \mid N(\vec{p}) >_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)



Inverse Laplace transform

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau\, e^{\nu\tau}\, \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$

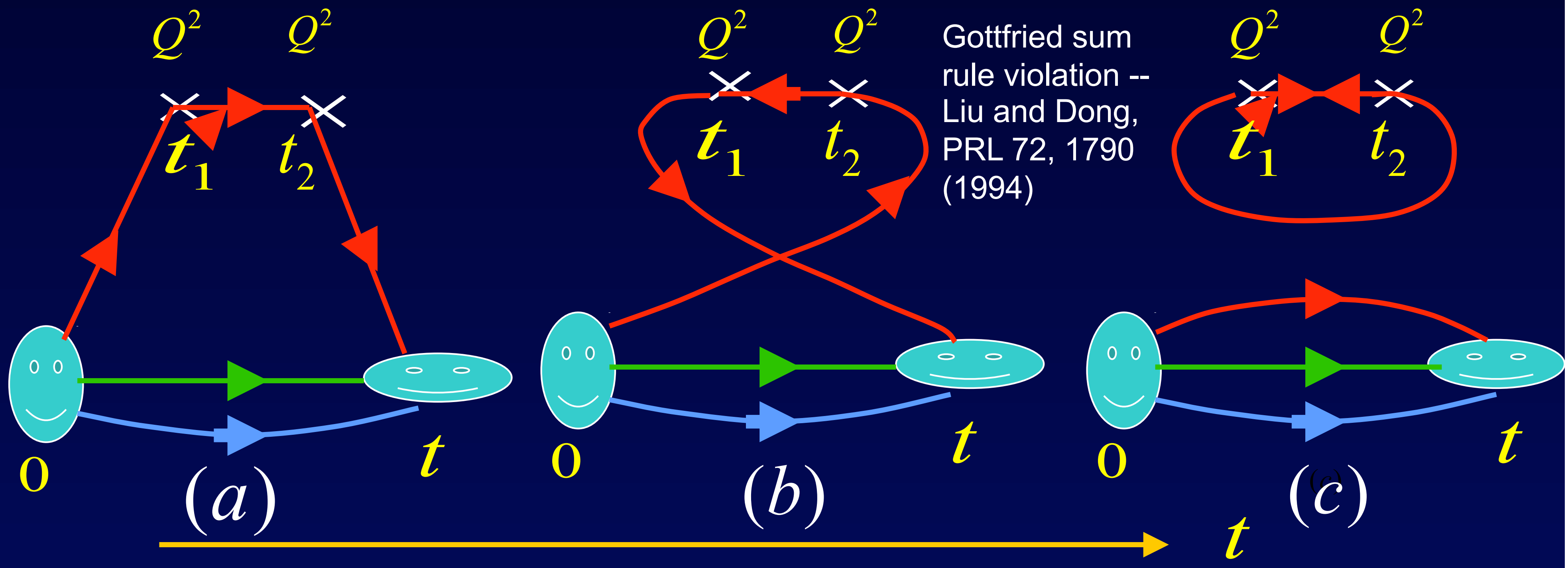
Hadronic Tensor

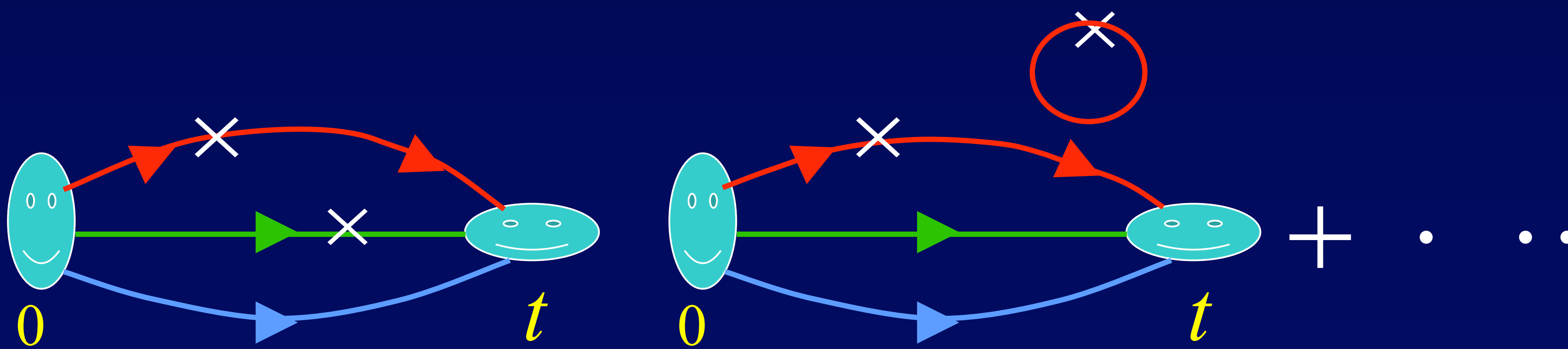
$q = q_V + q_{CS}$

Connected sea $\overline{q}_{CS}$

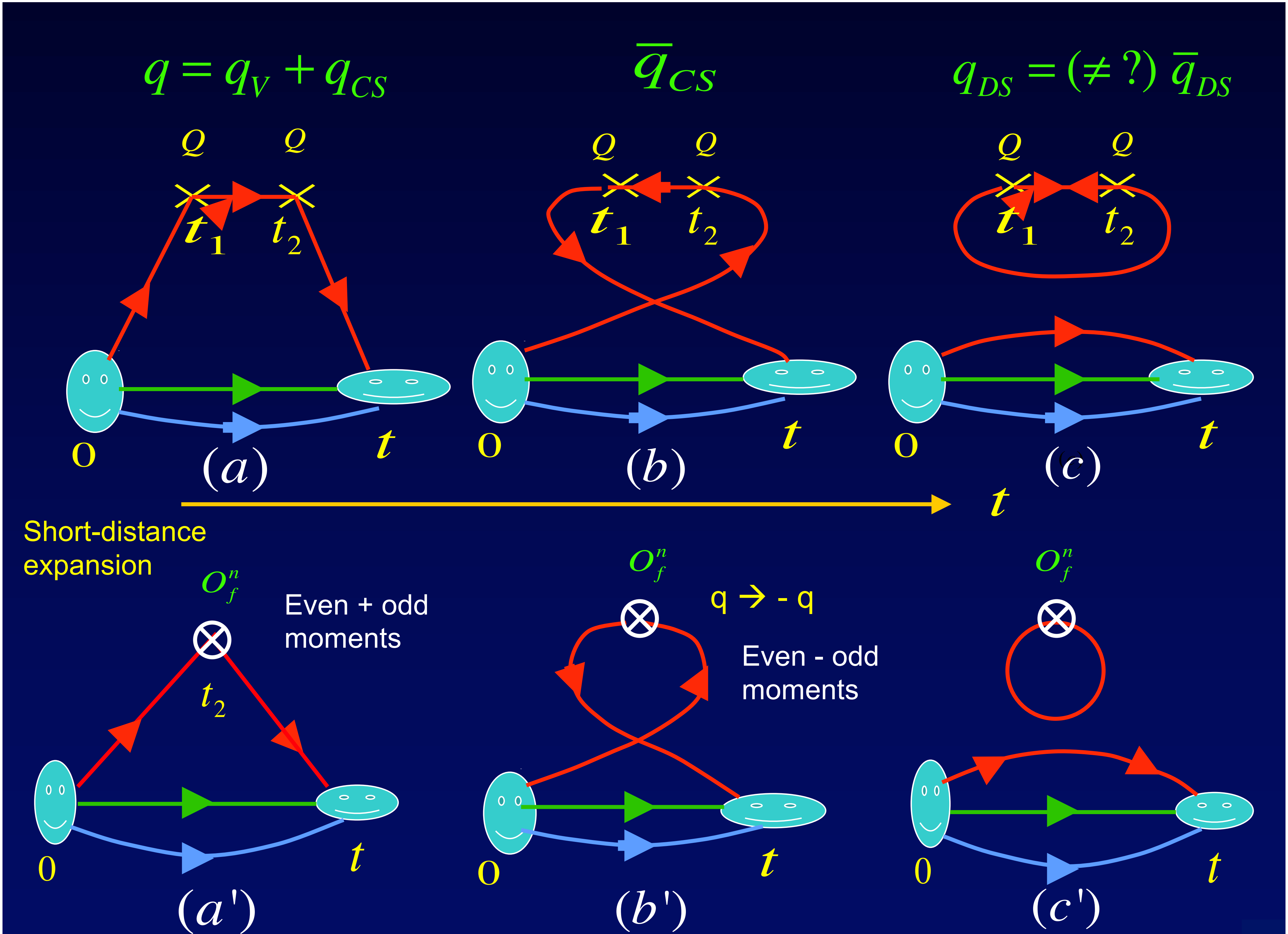
Disconnecrted sea $q_{DS} = (\neq ?)\ \overline{q}_{DS}$

Gottfried sum rule violation -- Liu and Dong, PRL 72, 1790 (1994)





Cat's ears diagrams are suppressed by O(1/Q²). $q_i^- = q_i^{v+cs} - \overline{q}_i^{cs} + q_i^{ds} - \overline{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \overline{q}_i^{ds}$



$q = q_V + q_{CS}$
$\overline{q}_{CS}$
$q_{DS} = (\neq ?)\ \overline{q}_{DS}$
Q
Q
t_1
t_2
0
t
(a)
(b)
(c)
t
Short-distance
expansion
O_f^n
t_2
Even + odd
moments
q → - q
Even - odd
moments
(a')
(b')
(c')

Quasi-PDF (LaMET)

X. Ji, PRL, 110, 262002 (2013)

§ Take the large-P_z limit:

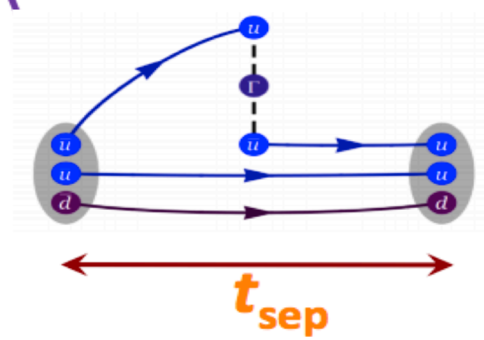
$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

$$+\mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

$x = k^z/P^z$

Lattice z coordinate

Nucleon momentum $P^\mu = \{P^0,0,0,P^z\}$



Product of lattice gauge links

- At $P^z \longrightarrow \infty$ limit, twist-2 parton distribution is recovered
- For finite P^z, corrections are needed

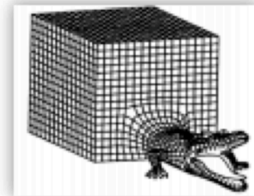
Xiangdong Ji, this Thursday; HWL et al in progress

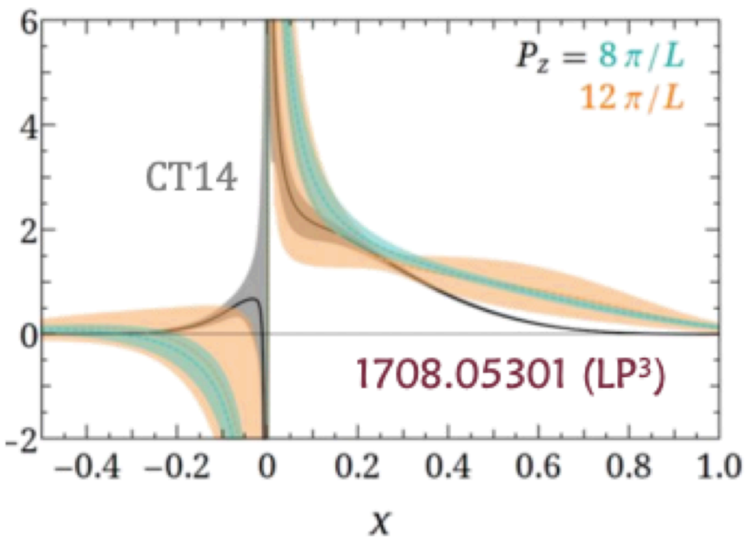
Quasi PDF results from LP3 and ETMC

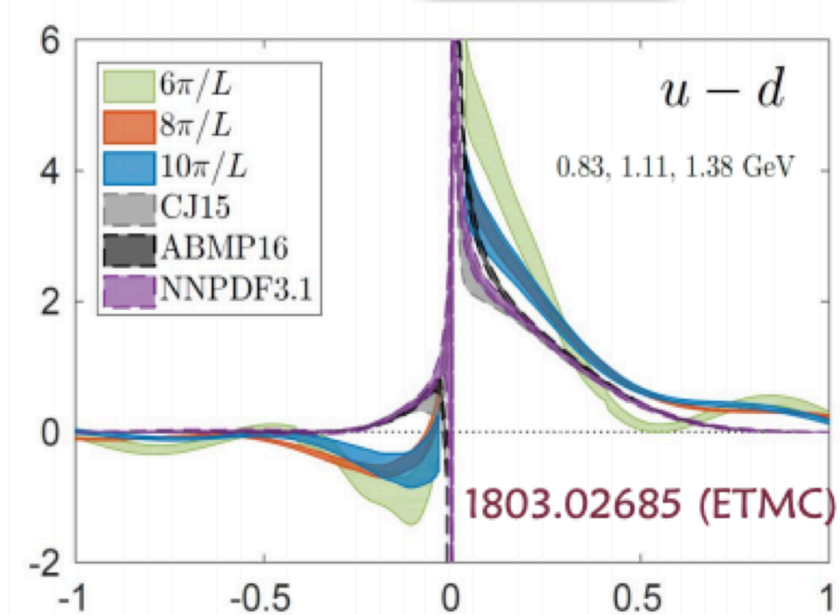
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed







Not use any parametrization form like $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$

Large Momentum Approach: Quasi-PDF, Pseuso-PDF, Lattice Cross Secton

- Factorization Theorem for Large Momentum Effective Theory (LaMET) via OPE – Izubuchi et al., PRD 98, 0-56004 (2018)

- Quasi-PDF

$$\tilde{q}(x,\mu/P^z) = \int_{-1}^{1} dy\, C\left(\frac{x}{y}, \frac{\mu}{|y|P^z}\right) q(y,\mu)$$

$$q(x>0) = q^{v+cs}(x), \quad q(x<0) = -\bar{q}^{cs}(x)$$

- Moments

$$\begin{aligned} a_{n+1}(CI) &= \int_{-1}^{1} x^n q(x) dx \\ &= \int_0^1 x^n q^{v+cs}(x) dx + (-)^{n+1} \int_0^1 \bar{q}^{cs}(x) dx \end{aligned}$$

NNLO Evolution equations separating CS from the DS partons -- 11 eqs. for the most general case

K.F. Liu, PRD 96, 033001 (2017), arXiv: 1703.04690

$$dq_i^{v+cs}/dt = P_{ii}^c \otimes q_i^{v+cs} + P_{i\bar{i}}^c \otimes \bar{q}_i^{cs};$$

$$d\bar{q}_i^{cs}/dt = P_{\bar{i}\bar{i}}^c \otimes \bar{q}_i^{cs} + P_{\bar{i}i}^c \otimes q_i^{v+cs};$$

$$dq_i^{ds}/dt = \sum_k (P_{ik}^{cd} \otimes q_k^{ds} + P_{i\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{ik}^d \otimes q_k^{v+cs} + P_{i\bar{k}}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$d\bar{q}_i^{ds}/dt = \sum_k (P_{\bar{i}\bar{k}}^{cd} \otimes \bar{q}_k^{ds} + P_{\bar{i}k}^{cd} \otimes q_k^{ds} + P_{\bar{i}k}^d \otimes q_k^{v+cs} + P_{\bar{i}\bar{k}}^d \otimes \bar{q}_k^{cs}) + P_{ig} \otimes g;$$

$$dg/dt = \sum_k [P_{gk} \otimes (q_k^{v+cs} + q_k^{ds}) + P_{g\bar{k}} \otimes (\bar{q}_k^{cs} + \bar{q}_k^{ds}) + P_{gg} \otimes g.$$

$$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$$

Comments $q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$

- CS and DS are explicitly separated, leading to more equations (11 vs 7) which can accommodate $s \neq \bar{s},\ \mathrm{u}^{ds} \neq \bar{\mathrm{u}}^{ds}$
- There is no flavor-changing evolution of the valence partons.

$$dq_i^- / dt = P_{qq}^- \otimes q_i^- + P_{ds}^- \otimes \sum_k (q_k - \bar{q}_k);$$

is the sum of two equations

$$dq_i^v / dt = P_{qq}^- \otimes q_i^v, \quad q^v \equiv q^{v+cs} - \bar{q}^{cs}$$

$$d(q_i^{ds} - \bar{q}_i^{ds}) / dt = \sum_k P_{ik}^{cd-} \otimes (q_k^{ds} - \bar{q}_k^{ds}) + \sum_k P_{ds}^{d-} \otimes q_k^v$$

- Once the CS is separated at one Q^2 , it will remain separated at other Q^2.
- Gluons can split into DS, but not to valence and CS.
- It is necessary to separate out CS from DS when quark and antiquark annihilation (higher twist) is included in the evolution eqs. (Annihilation involves only DS.)

Summary and Challenges

- Euclidean Path-integral formulation of the hadronic tensor and PDF with large momentum approach give a precise definition of the parton degrees of freedom from QCD. It avoids the deficiency of the phenomenological definition of the valence.

- It would be useful to carry out global fitting with the connected sea and disconnected sea separated so that the direct comparison with lattice calculation of the moments can be made.

- Together with experiments on LHC and EIC and global fitting of PDF, lattice QCD calculations of hadron structure (proton spin and mass decomposition, moments of PDFs, form factors, etc.) can advance our understanding of the nucleon properties in more detail.