Simulating Multi-Jet Events at Hadron Colliders using Forward Branching Phase Space Generators

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Parallel Talk



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Introduction

A Projective Phase Space Generator for Hadronic Vector Boson Plus One Jet Production Tinghua Chen, Terrance M. Figy (Wichita State U.), Walter T. Giele (Fermilab) Jul 8, 2019 - 18 pages •FERMILAB-PUB-19-300-T •e-Print: arXiv:1907.03893 [hep-ph] | PDF A Forward Branching Phase Space Generator for Hadron colliders Terrance M. Figy (Wichita State U.), Walter T. Giele (Fermilab) Jun 25, 2018 - 16 pages •JHEP 1810 (2018) 203 •(2018-10-31) •DOI: 10.1007/JHEP10(2018)203 •FERMILAB-PUB-18-187-T •e-Print: arXiv:1806.09678 [hep-ph] PDF



History

1) A Forward Branching Phase-Space Generator By Walter T. Giele, Gerben C. Stavenga, Jan-Christopher Winter. arXiv:1106.5045 [hep-ph].

2) The Matrix Element Method at Next-to-Leading Order By John M. Campbell, Walter T. Giele, Ciaran Williams. arXiv:1204.4424 [hep-ph]. <u>10.1007/JHEP11(2012)043</u>. JHEP 1211 (2012) 043.

3) Extending the Matrix Element Method to Next-to-Leading Order By John M. Campbell, Walter T. Giele, Ciaran Williams. arXiv:1205.3434 [hep-ph].

4) Finding the Higgs boson in decays to \$Z \gamma\$ using the matrix element method at Next-to-Leading Order By John M. Campbell, R.Keith Ellis, Walter T. Giele, Ciaran Williams. arXiv:1301.7086 [hep-ph]. <u>10.1103/PhysRevD.87.073005</u>. Phys.Rev. D87 (2013) no.7, 073005.

5) Event-by-event weighting at next-to-leading order By Ciaran Williams, John M. Campbell, Walter T. Giele. arXiv:1311.5811 [hep-ph]. <u>10.22323/1.197.0037</u>.PoS RADCOR2013 (2013) 037.

6) Improved Partonic Event Generators at Lepton Colliders By Walter T. Giele. arXiv:1504.02137 [hep-ph]



History

 Matrix Element Method at NLO for (anti-)\$\mathbf{k_t}\$-jet algorithms By Manfred Kraus, Till Martini, Peter Uwer. arXiv:1901.08008 [hep-ph].<u>10.1103/PhysRevD.100.076010</u>. Phys.Rev. D100 (2019) no.7, 076010.

2) The Matrix Element Method at next-to-leading order QCD using the example of single top-quark production at the LHC By Till Martini. arXiv:1807.06859 [hep-ph]. <u>10.18452/19288</u>.

3) The Matrix Element Method at next-to-leading order QCD for hadronic collisions: Single top-quark production at the LHC as an example application By Till Martini, Peter Uwer. arXiv:1712.04527 [hep-ph]. <u>10.1007/JHEP05(2018)141</u>. JHEP 1805 (2018) 141.

4) Single top-quark production with the Matrix Element Method in next-to-leading order accuracy By Till Martini, Peter Uwer. arXiv:1709.04656 [hep-ph]. <u>10.22323/1.297.0136</u>. PoS DIS2017 (2018) 136.

5) The Matrix Element Method at Next-to-Leading Order Accuracy By Till Martini, Peter Uwer. arXiv:1511.07150 [hep-ph]. <u>10.5506/APhysPolB.46.2143</u>. Acta Phys.Polon. B46 (2015) no.11, 2143.

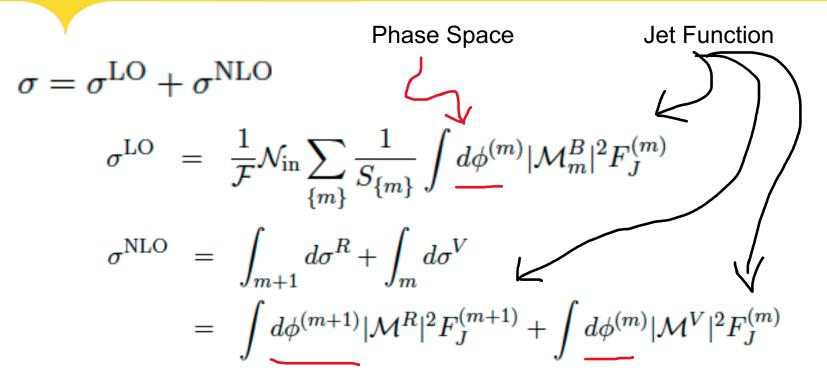
6) Extending the Matrix Element Method beyond the Born approximation: Calculating event weights at next-to-leading order accuracy

By Till Martini, Peter Uwer. arXiv:1506.08798 [hep-ph]. <u>10.1007/JHEP09(2015)083</u>. JHEP 1509 (2015) 083.



A Review NLO Calculations

[https://arxiv.org/abs/0803.2231v1]





A Review of NLO Calculations: The Jet Function

Observables are required to be infrared safe, i.e. the observable's value does not depend on the number of soft/collinear final-state hadrons.

 $F_J^{(m+1)} \to F_J^{(m)}$



A Review of NLO Calculations: Subtraction Methods

$$\sigma^{\text{NLO}} = \sigma^{\text{NLO}\{m+1\}} + \sigma^{\text{NLO}\{m\}}$$

=
$$\int_{m+1} [d\sigma^R - d\sigma^A] + \int_m [d\sigma^V + d\sigma^B \otimes \mathbf{I}] \qquad d\sigma^A = d\phi^{(m+1)}(p;Q) \sum_{\text{dipoles}} (\mathcal{D} \cdot F^{(m)})(p)$$

$$\sum_{\text{dipoles}} (\mathcal{D} \cdot F^{(m)})(p) = \left\{ \sum_{i,j} \left[\sum_{k \neq i,j} \mathcal{D}_{ij,k}(p) F_J^{(m)}(\widehat{p}) + \mathcal{D}_{ij}^a(p) F_J^{(m)}(\widehat{p}) + \mathcal{D}_{ij}^b(p) F_J^{(m)}(\widehat{p}) \right] + \sum_i \left[\sum_{k \neq i} \mathcal{D}_k^{ai}(p) F_J^{(m)}(\widehat{p}) + \mathcal{D}^{ai,b}(p) F_J^{(m)}(\widehat{p}) + (a \leftrightarrow b) \right] \right\}. \quad (65)$$

There is a mismatch between the dipole kinematics and the born kinematics.



A Review of NLO Calculations: Subtraction Methods

$$\sigma^{\text{NLO}} = \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V}$$

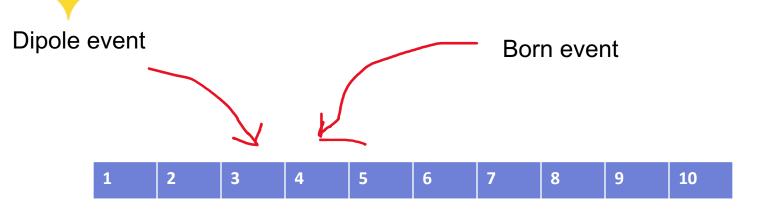
= $\int d\phi^{(m+1)} |\mathcal{M}^{R}|^{2} F_{J}^{(m+1)} + \int d\phi^{(m)} |\mathcal{M}^{V}|^{2} F_{J}^{(m)}$
$$\sum_{\text{dipoles}} (\mathcal{D} \cdot F^{(m)})(p) = \left\{ \sum_{i,j} \left[\sum_{k \neq i,j} \mathcal{D}_{ij,k}(p F_{J}^{(m)}(\tilde{p}) + \mathcal{D}_{ij}^{a}(p) F_{J}^{(m)}(\tilde{p}) + \mathcal{D}_{ij}^{b}(p) F_{J}^{(m)}(\tilde{p}) \right] + \sum_{i} \left[\sum_{k \neq i} \mathcal{D}_{k}^{ai}(p) F_{J}^{(m)}(\tilde{p}) + \mathcal{D}^{ai,b}(p) F_{J}^{(m)}(\tilde{p}) + (a \leftrightarrow b) \right] \right\}. \quad (65)$$

There is a mismatch between the dipole kinematics and the born kinematics.



A Review of NLO Calculations:

A typical run of a calculation using subtraction methods.

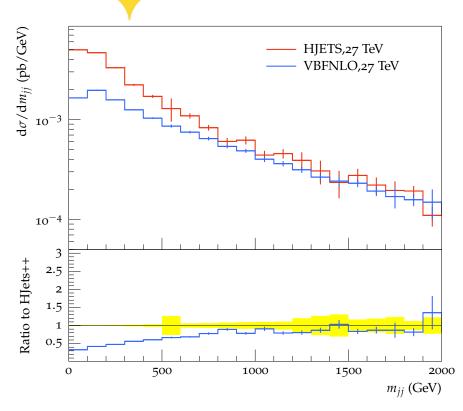


Histogram bins



A Review of NLO Calculations:

A typical run of a calculation using subtraction methods.



Missed binning is an issue. Possible remedies:

- Bin Smearing (?)
- Fuzzy Cuts (?)
- Cuts on the dipole phase space (?)
- Higher Statistics resulting in higher costs.
- Does anybody know what will happen at NNLO?



Forward Branching Phase Space Generator

The philosophy is to optimize both the jet function and phase space at the same time. The clever idea is to construct the real emission phase space from the born phase space. This approach has been advocated by the N-jettiness slicing.

$$\int \underline{d\phi^{(m+1)}} |\mathcal{M}^R|^2 \underline{F_J^{(m+1)}}$$

N-jettiness Subtractions for NNLO QCD Calculations By Jonathan Gaunt, Maximilian Stahlhofen, Frank J. Tackmann, Jonathan R. Walsh. arXiv:1505.04794 [hep-ph]. <u>10.1007/JHEP09(2015)058</u>.JHEP 1509 (2015) 058.



Forward Branching Phase Space Generator

The first step is to partition the phase of the real radiation according to a jet algorithm. For this we need to define the jet algorithm.

- 1. Resolution function: $d_{ij} = d(p_i, p_j)$
- 2. Partons with the smallest resolution will be clustered until

 $\min_{ij}(d_{ij}) > d_{cut}$ (exclusive jet cross section) or a certain jet multiplicity is reached.

The second step is to introduce a partition of one:

$$1 = \sum_{j=1}^{n-1} \theta(\hat{d}_{jn} = \hat{d}_{\min})$$

 $\hat{d}_{\min} = \min_{ij}(\hat{d}_{ij}) = \min_{ij}(d(\hat{p}_i, \hat{p}_j))$



Forward Branching Phase Space Generator

The dcut can also be included.

$$1 = \sum_{j=1}^{n-1} \theta(\hat{d}_{jn} = \hat{d}_{\min}) \left(\theta(\hat{d}_{\min} < \hat{d}_{\operatorname{cut}}) + \theta(\hat{d}_{\min} > \hat{d}_{\operatorname{cut}}) \right)$$

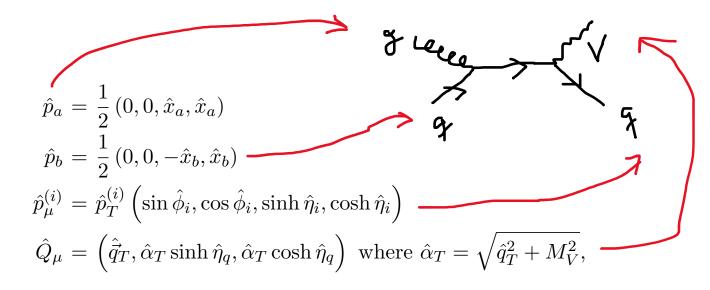
$$d\Phi_n(Q; \{\hat{p}\}) = d\Phi_{n-1}(Q; \{p\}) \times d\Phi_{\text{fbps}}^{\text{excl}}(\{\hat{p}\}|\{p\})$$

$$d\Phi_{\rm fbps}^{\rm excl}(\{\hat{p}\}|\{p\}) = \sum_{j=1}^{n-1} \theta(\hat{d}_{jn} = \hat{d}_{\min})\theta(\hat{d}_{\min} < \hat{d}_{\rm cut})d\Phi_{\rm fbps}^{[j]}(\{\hat{p}\}|\{p\})$$

Now, to work out the phase space maps.



Born Phase Space (V+1 parton kinematics)





Born Phase Space (V+1 parton kinematics)

$$d\hat{x}_a d\hat{x}_b d\Phi(\hat{p}_a \hat{p}_b; \hat{Q}, \{\hat{p}\}_n) = \frac{1}{(16\pi^3)^{n+1}} \frac{2}{S} \left(\prod_{i=1}^n d\hat{p}_T^{(i)} d\hat{\eta}_i d\hat{\phi}_i \times \hat{p}_T^{(i)} \right) \times d\hat{\eta}_q \times \Theta(1 - \hat{x}_1) \Theta(1 - \hat{x}_2) ,$$
(2.5)

with

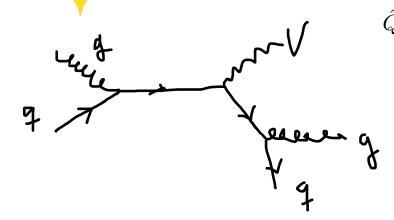
$$\hat{\vec{q}}_{T} = -\sum_{i=1}^{n} \hat{\vec{p}}_{T}^{(i)}$$

$$\hat{x}_{a} = \frac{1}{\sqrt{S}} \left(\hat{\alpha}_{T} e^{\eta_{q}} + \sum_{i=1}^{n} \hat{p}_{T}^{(i)} e^{\hat{\eta}_{i}} \right)$$

$$\hat{x}_{b} = \frac{1}{\sqrt{S}} \left(\hat{\alpha}_{T} e^{-\eta_{q}} + \sum_{i=1}^{n} \hat{p}_{T}^{(i)} e^{-\hat{\eta}_{i}} \right) ,$$
(2.6)



Final State Forward Branching Phase Space



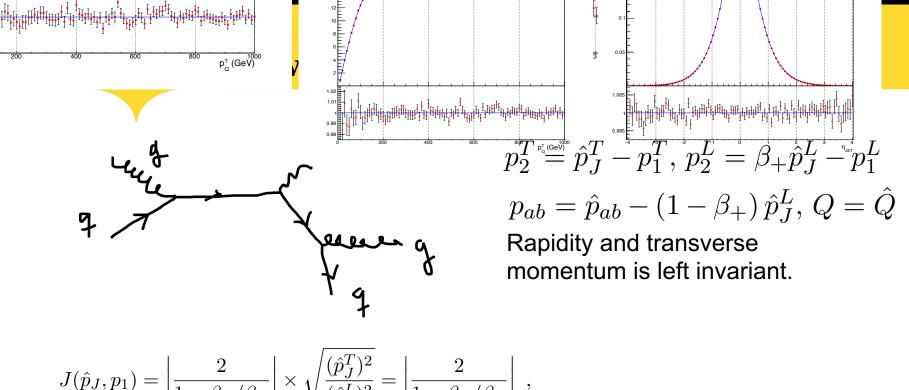
$$\hat{Q} = Q, \ \hat{p}_{ab} = \hat{p}_a + \hat{p}_b = p_a + p_b + \alpha p_{12}^L,$$

 $\hat{p}_J = p_{12} + \alpha p_{12}^L, \ p_{12}^L = (p_1 + p_2)_L$

Rapidity and transverse momentum is left invariant.

$$d\Phi_3^{\text{FINAL}}(p_a, p_b; Q, p_1, p_2) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\,\,\delta(p_1^2)\right] \times J(\hat{p}_J, p_1) \,\,.$$

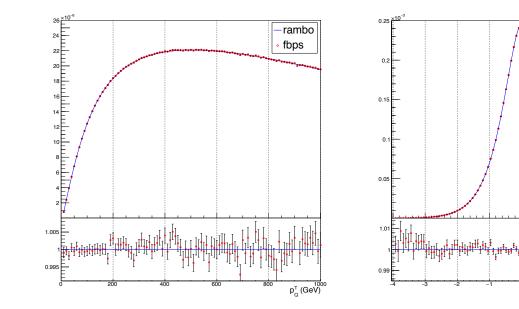




$$\beta_{\pm} = \frac{(\hat{p}_{J}^{L} \cdot p_{1}^{L}) \pm \sqrt{(\hat{p}_{J}^{L})^{2}} + 2(\hat{p}_{J}^{L})^{2}(\hat{p}_{J}^{T} \cdot p_{1}^{T}) + (\hat{p}_{J}^{L} \cdot p_{1}^{L})^{2}}{(\hat{p}_{J}^{L})^{2}}$$



Final State Forward Branching Phase Space



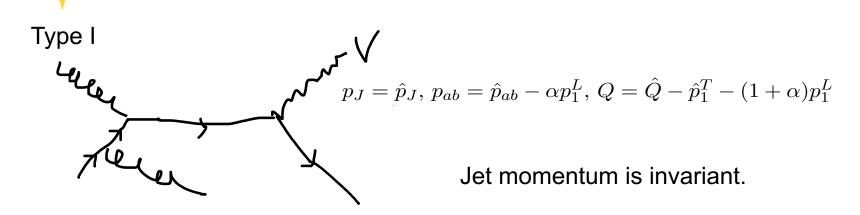


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 $\eta_{_{\text{JET}}}$

fbps

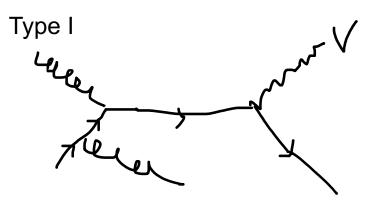
Initial State Forward Branching Phase Space – Type I



$$d\Phi_3^{\text{INIT,I}}(p_a, p_b; Q, p_J, p_1) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\,\,\delta(p_1^2)\right] \times J(\hat{Q}, p_1) \,\,.$$



Initial State Forward Branching Phase Space – Type I



$$Q_T = \hat{Q}_T - p_1^T, \, Q_L = \hat{Q}_L - (1+\alpha)p_1^L$$

$$p_{ab} = \hat{p}_{ab} - \alpha p_1^L, \, p_J = \hat{p}_J$$

Jet momentum is invariant.

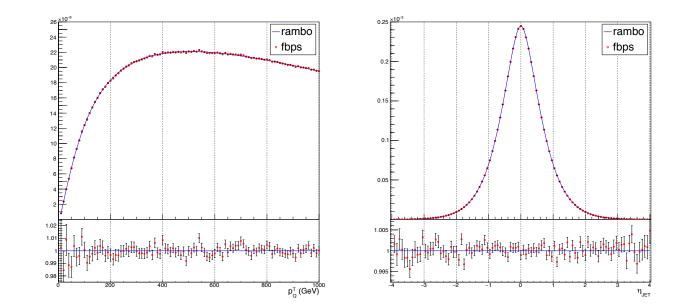
$$J(\hat{Q}, p_1) = \sqrt{\frac{(p_1^L)^4 - 2(p_1^L)^2(p_1^T \cdot Q_T) + (p_1^L \cdot Q_L)^2}{(p_1^L)^4 + 2(p_1^L)^2(p_1^T \cdot \hat{Q}_T) + (\hat{p}_1^L \cdot \hat{Q}_L)^2}}$$

where

$$\alpha = \frac{(p_1^L \cdot \hat{Q}_L) - (p_1^L)^2 - \sqrt{(p_1^L)^4 + 2(p_1^L)^2(p_1^T \cdot \hat{Q}_T) + (p_1^L \cdot \hat{Q}_L)^2}}{(p_1^L)^2}$$

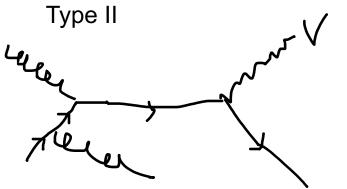


Initial State Forward Branching Phase Space – Type I





Initial State Forward Branching Phase Space - Type II



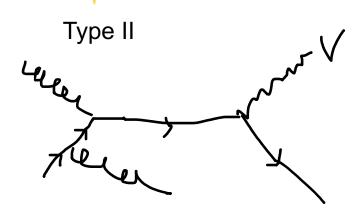
 $Q = \hat{Q}, \ p_{ab} = \hat{p}_{ab} - \alpha p_1^L, \ p_J = \hat{p}_J - p_1^T - (1+\alpha)p_1^L$

Vector boson momentum is invariant.

$$d\Phi_3^{\text{INIT,I}}(p_a, p_b; Q, p_J, p_1) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\,\,\delta(p_1^2)\right] \times J(\hat{Q}, p_1) \,\,.$$



Initial State Forward Branching Phase Space - Type II



$$p_J^T = \hat{p}_J^T - p_1^T, \ p_J^L = \hat{p}_J^L - (1+\alpha)p_1^L,$$
$$p_{ab} = \hat{p}_{ab} - \alpha p_1^L, \ Q = \hat{Q}$$

Vector boson momentum is invariant.

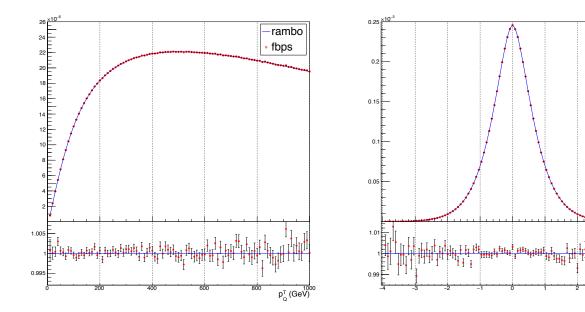
$$J(\hat{p}_J, p_1) = \sqrt{\frac{(p_1^L)^4 - 2(p_1^L)^2(p_1^T \cdot p_J^T) + (p_1^L \cdot p_J^L)^2}{(p_1^L)^4 + 2(p_1^L)^2(p_1^T \cdot \hat{p}_J^T) + (p_1^L \cdot \hat{p}_J^L)^2}},$$

where

$$\alpha = \frac{(p_1^L \cdot \hat{p}_J^L) - (p_1^L)^2 - \sqrt{(p_1^L)^4 + 2(p_1^L)^2(p_1^T \cdot \hat{p}_J^T) + (p_1^L \cdot \hat{p}_J^L)^2}}{(p_1^L)^2}$$



Initial State Forward Branching Phase Space - Type II





rambo

 η_{JET}

fbps

$$\Delta_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

$$1 = \Theta(R - \Delta_{12}) + \Theta(\Delta_{12} - R)$$

0-jet, 1-jet, 2-jet

$$1 = \Theta(R - \Delta_{12}) + \Theta(\Delta_{12} - R) \times \left(\Theta(p_{\min}^T - p_1^T) + \Theta(p_1^T - p_{\min}^T)\right) \times \left(\Theta(p_{\min}^T - p_2^T) + \Theta(p_2^T - p_{\min}^T)\right) \ .$$

1-jet exclusive

$$1 = \Theta(R - \Delta_{12}) + \Theta(\Delta_{12} - R) \left(\Theta(p_{\min}^T - p_1^T)\Theta(p_2^T - p_{\min}^T) + \Theta(p_{\min}^T - p_2^T)\Theta(p_1^T - p_{\min}^T)\right)$$

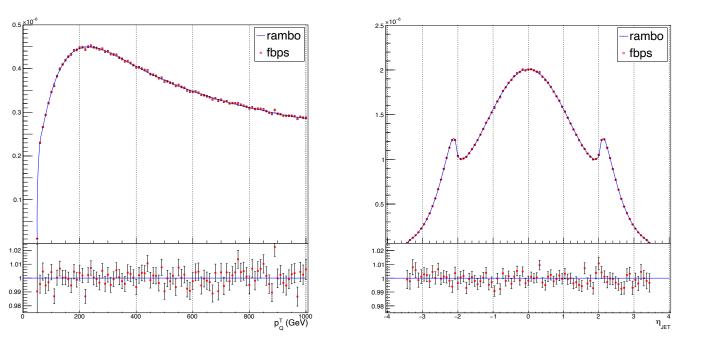


Simple Application: Generator for V+1 jet Production

$$\begin{split} d\Phi_{3}^{\text{exclusive}}(p_{a}, p_{b}; Q, p_{1}, p_{2}) &= d\Phi_{3}(p_{a}, p_{b}; Q, p_{1}, p_{2}) \\ \times \left[\Theta(R - \Delta_{12}) + \Theta(\Delta_{12} - R) \left(\Theta(p_{\min}^{T} - p_{1}^{T})\Theta(p_{2}^{T} - p_{\min}^{T}) + \Theta(p_{\min}^{T} - p_{2}^{T})\Theta(p_{1}^{T} - p_{\min}^{T})\right)\right] \\ &= d\Phi_{2}(\hat{p}_{a}, \hat{p}_{b}; \hat{Q}, \hat{p}_{J}) \times \left[\frac{dp_{1}}{(2\pi)^{3}} \,\delta(p_{1}^{2})\right] \\ \times \left[\Theta(R - \Delta_{12})J^{\text{FINAL}}(\hat{p}_{J}, p_{1})\delta(M^{\text{FINAL}}(\{\hat{p}\}_{2} \rightarrow \{p\}_{2})) \\ &+ \Theta(\Delta_{12} - R) \left(\Theta(p_{1}^{T} < p_{\min}^{T})\Theta(p_{2}^{T} > p_{\min}^{T}) + (1 \leftrightarrow 2)\right)J^{\text{INIT}}(\hat{Q}, p_{1})\delta(M^{\text{INIT}}(\{\hat{p}\}_{2} \rightarrow \{p\}_{2}))\right] \,\,. \end{split}$$



Simple Application: Generator for V+1 jet Production





V+1 jet at NLO using DYRAD

Higher order corrections to jet cross-sections in hadron colliders <u>W.T. Giele (Fermilab), E.W.Nigel Glover (Durham U.), David A.</u> <u>Kosower (CERN & Saclay)</u> •Nucl.Phys. B403 (1993) 633-670 •e-Print: <u>hep-ph/9302225</u> | <u>PDF</u>



V+1 jet at NLO using DYRAD

DYRAD MC is an NLO MC that used phase space slicing for the computation of NLO corrections in the perturbative QCD for vector boson plus 0 and 1 jet. Our starting point for the FBPS MC was DYRAD MC.



$$d\sigma^{\rm LO}(\{(p_T,\eta,\phi)_i\}) = \sum_{a,b} \frac{f_a(x_1)f_b(x_2)}{2s_{12}} \mathcal{M}_{ab}^{(0)}(\{(p_T,\eta,\phi)_i\})$$

$$d\sigma^{\text{NLO}}(\{(p_T, \eta, \phi)_i\}) = \sum_{a,b} \frac{f_a(x_1)f_b(x_2)}{2s_{12}} \mathcal{M}_{ab}^{(0)}(\{(p_T, \eta, \phi)_i\})$$
$$\times \left(1 + V(\{(p_T, \eta, \phi)_i\}) + \int dk_B \frac{s_{12}}{\hat{s}_{12}} \frac{f_a(\hat{x}_1)f_b(\hat{x}_2)}{f_a(x_1)f_b(x_2)} \frac{\mathcal{M}_{ab}^{(1)}(\{(p_T, \eta, \phi)_i\}, k_B)}{\mathcal{M}_{ab}^{(0)}(\{(p_T, \eta, \phi)_i\})}\right)$$

K-factors at the level of Born events.



Building off the DYRAD MC framework we have implemented the CUBA library (<u>http://www.feynarts.de/cuba/</u>) in order to take advantage of hyperthreading.

The FBPS MC allows the the computation of cross-sections and kinematics distributions at LO and NLO for the V+1 jet production at hadron colliders such as the LHC experiment. The FBPS MC can be used to compute kinematics distributions for a fixed Born jet rapidity, jet transverse momentum, and vector boson rapidity.

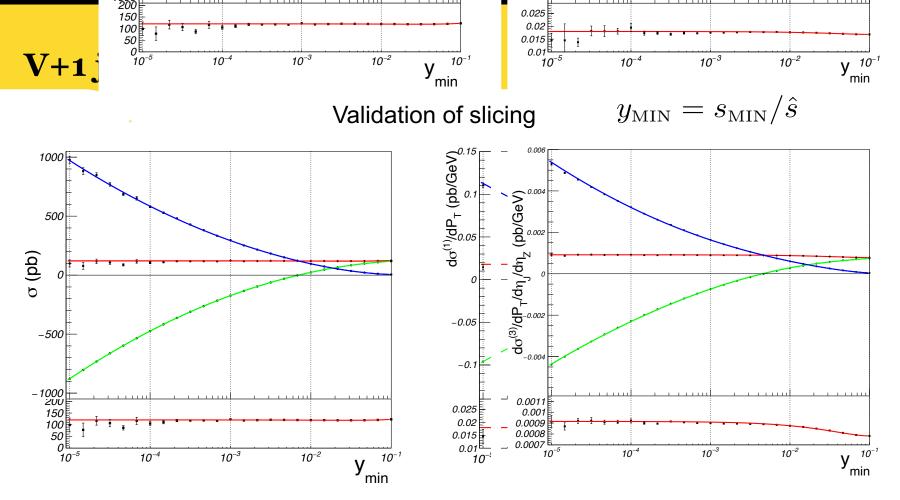
The FBPS MC uses phase space slicing for the moment.



Input parameters:

- 1. Collider Energy: 14 TeV
- 2. PDF Choice: CT14nlo
- 3. Renormalization and Factorization: Partonic CMS Energy





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 10^{-3}

 10^{-4}

y^{10⁻¹}

10⁻²

do⁽¹⁾/dP_T (pb/GeV)

n

-0.05

-0.1

0.025

0.02

0.015

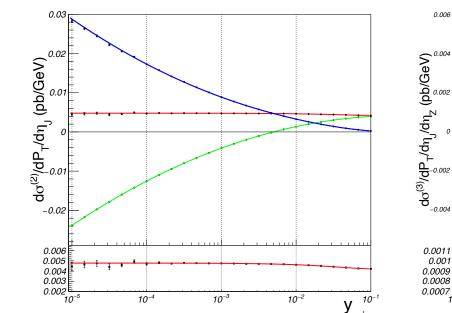
0.01^E

10⁻⁵

10-1

min

V+1 jet at NLO using FBPS



10⁻³

10⁻²

10-1

У min

-1000 200

150 100 50

0^E____ 10⁻⁵

Ŧ

10⁻⁴



0.006

-0.004

0.0011

0.001

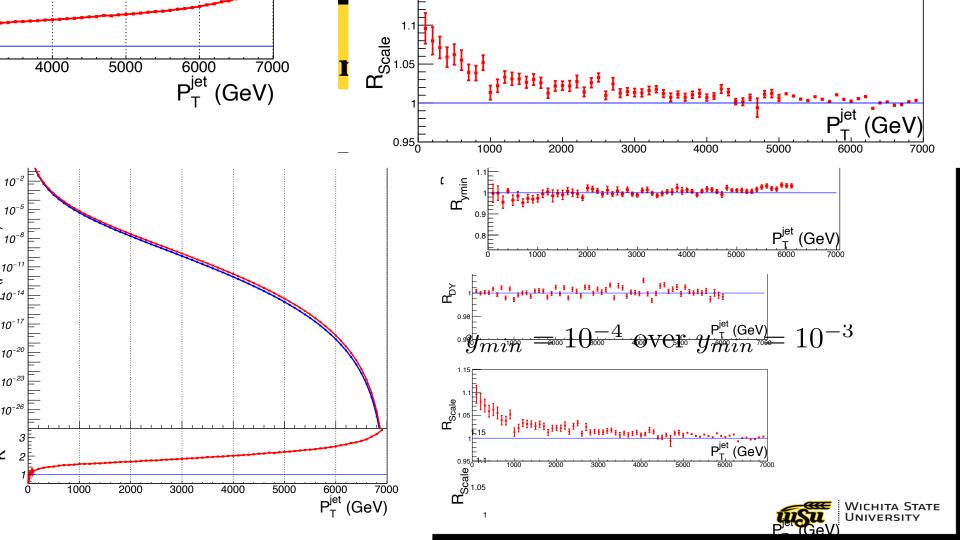
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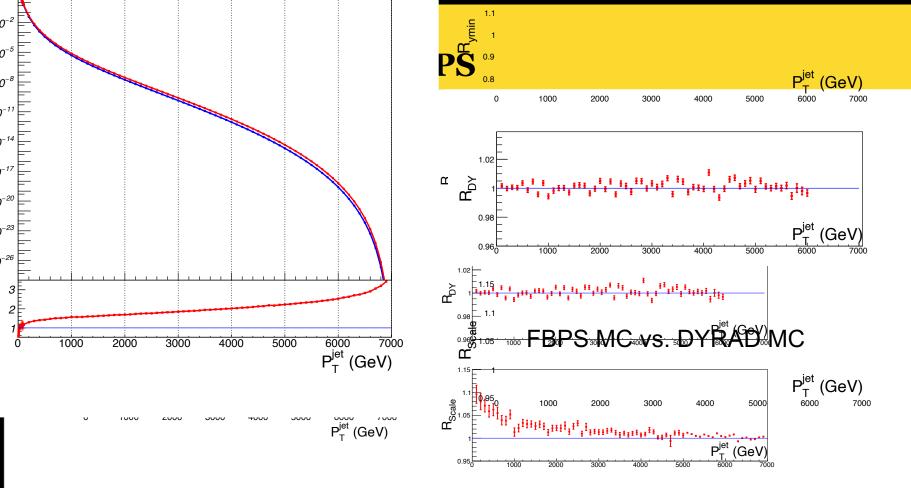
′10⁻⁵

0.0007

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min







V+1 jet at NLO using FBPS in the CLOUD

Since, we can compute a local K-factor for a given Born kinematic , it is straightforward to perform single core computations via the CLOUD such as the OPEN SCIENCE GRID.

For this, we have developed a Docker container for the FBPS MC.

Hence, kinematic distributions are in fact now represented by numerical functions. There are pros and cons to this approach.



V+1 jet at NLO using FBPS in the CLOUD

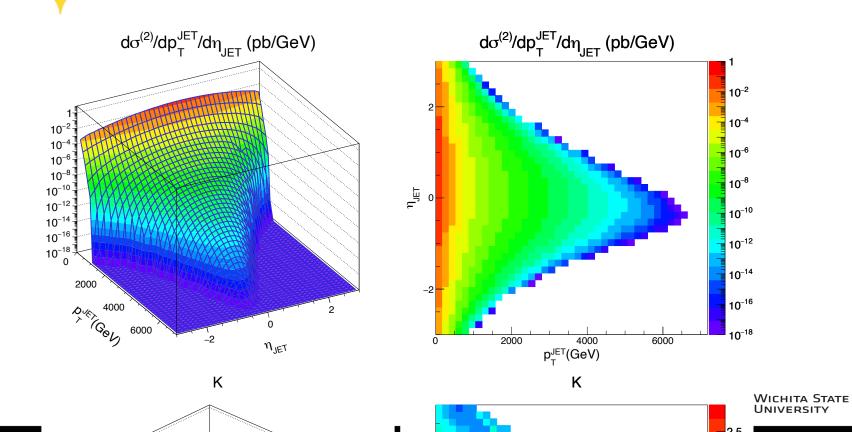
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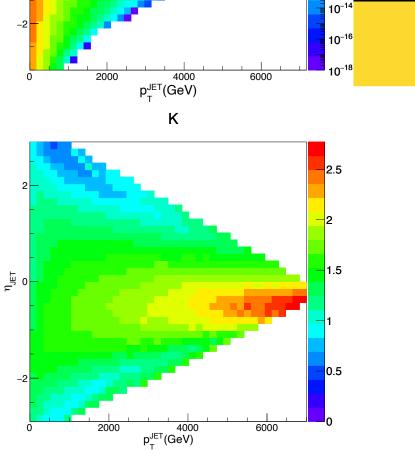


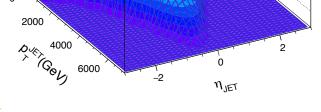
V+1 jet at NLO using FBPS in the CLOUD



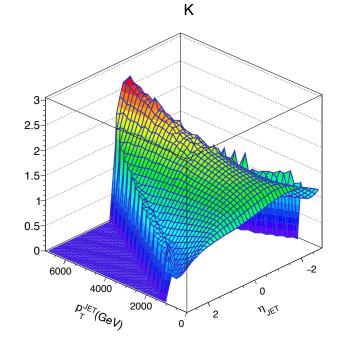








V+1 je





Outlook

- The FBPS generator allows for the re-weighting of Born events and generation of n exclusive jets.
- Issues of missed binning in histograms is absent.
- In principle one can determine local K-factors that only require a 3d integration over the phase space of the real emission contributions for a given Born kinematic.
- Future outlook: NNLO examples

