Spontaneous Freeze-Out of Dark Matter from an Early Thermal Phase Transition

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In collaboration with

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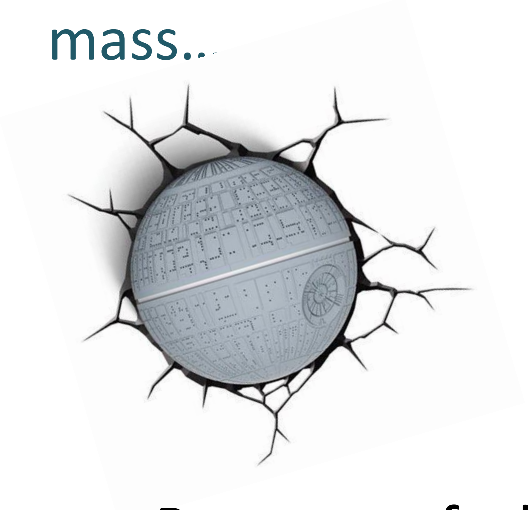
Pheno 2020

Dark Matter is often thought of as being a cold object with constant mass…



… however we know that this is not the case for most of the particles in cosmology !

- Presence of a high temperature
- Thermal effects $\longrightarrow \langle h \rangle = 0$
- All particles massless at large T

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Dark Matter is often thought of as being a cold object with constant mass…

… however we know that this is not the case for most of the particles in cosmology !

- Presence of a high temperature
- Thermal effects $\longrightarrow \langle h \rangle = 0$
- All particles massless at large T

- Interactions between the dark sector and the thermal bath
- Mass terms in the dark sector might vary with T

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What's new there ?

The idea of Temperature-dependent mass has already been used for different purposes :

- VAMPs (interaction DM-DE) [Aderson, Carroll '97] [Rosenfeld '05] [Rosenfeld, Franca '04]
- Flip-Flop Vev mechanisms [Baker, Breitbach, Kopp, Mittnacht ,18] [Baker, Mittnacht '18]
- Forbidden Freeze-In [Darmé, Hryczuk, Karamitros, Roszkowski '19]
- Super-Cool DM [Hambye, Strumia, Teresi '18]
- Superheavy WIMPS [Hui, Stewart '95]
- Filtered Dark Matter [Baker, Kopp, Long '19]
- …

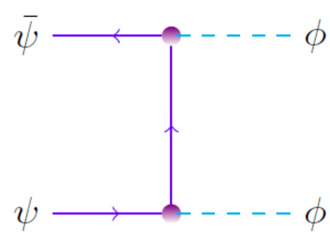
Our paper : What about the usual Vanilla WIMPs?

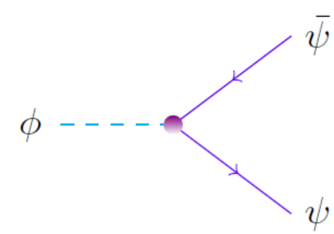
A simple toy model :

Fermionic Dark Matter + Dark Higgs

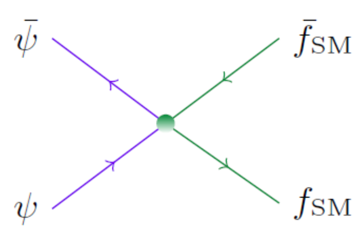
$$\mathcal{L}_{\rm tree} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm dark} + \mathcal{L}_{\rm int} \ ,$$

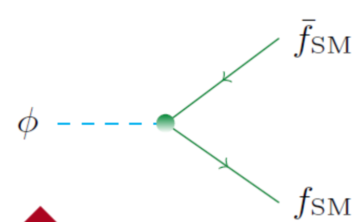
$$\mathcal{L}_{\rm dark} = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - y\phi\bar{\psi}\psi - \mathcal{V}_{\rm tree}(\phi) + \mathcal{L}^{\rm c.t.}_{\rm dark} \ ,$$





$\mathcal{L}_{\rm int}$: Interaction between the dark sector and SM particles





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Naive approach :

1) Find out the potential minimum $V'(\phi_{min}) = 0 \ \rightarrow m_{DM} = y\ \phi_{min}$

2) Solve Boltzmann Equation with a constant DM mass

3) Scan over parameters to obtain the correct relic abundance.

How do thermal effects modify this story?

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1 Zero-temperature

$$\mathcal{L}_{\rm tree} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm dark} + \mathcal{L}_{\rm int} \,,$$

$\mathcal{L}_{\rm int}$: Interaction between DM particles ψ and SM particles

$$\mathcal{L}_{\rm dark} = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - y\phi\bar{\psi}\psi - \mathcal{V}_{\rm tree}(\phi) + \mathcal{L}_{\rm dark}^{\rm c.t.} \,,$$

$$m_\psi(\langle\phi\rangle) = y\langle\phi\rangle$$

$$\mathcal{V}_{\rm tree}(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \,,$$

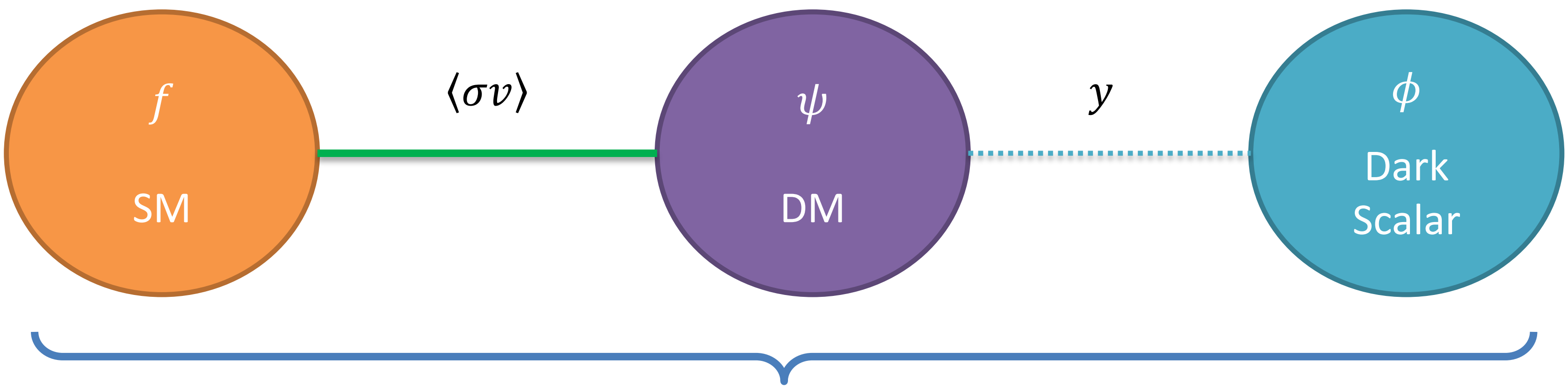
$$m_0(\phi)^2 = -\mu^2 + \frac{\lambda}{2}\phi^2$$

But this is in the vacuum, at $T = 0$ …

Zero Temperature Coleman-Weinberg Potential

$$\mathcal{V}_{\rm CW}(\phi) + \mathcal{V}^{\rm c.t.}_{\rm dark}(\phi) = \frac{m_0(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_0(\phi)^2}{Q^2}\right) - \frac{3}{2}\right] - n_F\,\frac{m_\psi(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_\psi(\phi)^2}{Q^2}\right) - \frac{3}{2}\right]$$

2 Finite-temperature



Thermal equilibrium : $T \longrightarrow \mathcal{V}_{\text{eff}}^{\text{th}}(T, \phi)$

$$\mathcal{V}_{\text{eff}}^{\text{th}}(T,\phi) = \mathcal{V}_{\text{tree}}(\phi) + \mathcal{V}_{\text{CW}}(\phi) + \mathcal{V}_{\text{dark}}^{\text{c.t.}}(\phi) + \mathcal{F}(T,\phi)$$

Zero Temperature CW

1-loop correction to the free energy

ψ and ϕ assumed to be thermalized

$$\mathcal{V}_{\text{eff}}^{\text{th}}(T,\phi) = \mathcal{V}_{\text{tree}}(\phi) + \mathcal{V}_{\text{CW}}(\phi) + \mathcal{V}_{\text{dark}}^{\text{c.t.}}(\phi) + \mathcal{F}(T,\phi)$$

$$\begin{aligned}\mathcal{V}_{\text{CW}}(\phi) + \mathcal{V}_{\text{dark}}^{\text{c.t.}}(\phi) = & \frac{m_0(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_0(\phi)^2}{Q^2}\right) - \frac{3}{2}\right] \\ & - n_F \frac{m_\psi(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_\psi(\phi)^2}{Q^2}\right) - \frac{3}{2}\right]\end{aligned}$$

$$\mathcal{F}(T,\phi) = \frac{T^4}{2\pi^2}\left[J_B\left(\frac{m_0(\phi)^2}{T^2}\right) - n_F\, J_F\left(\frac{m_\psi(\phi)^2}{T^2}\right)\right]$$

$$J_{\substack{B\\F}}\left(\frac{m^2}{T^2}\right) = \int_0^{+\infty} du\, u^2 \log\left(1 \mp e^{-\sqrt{u^2+m^2/T^2}}\right)$$

See e.g. [Mariano Quiros '99]

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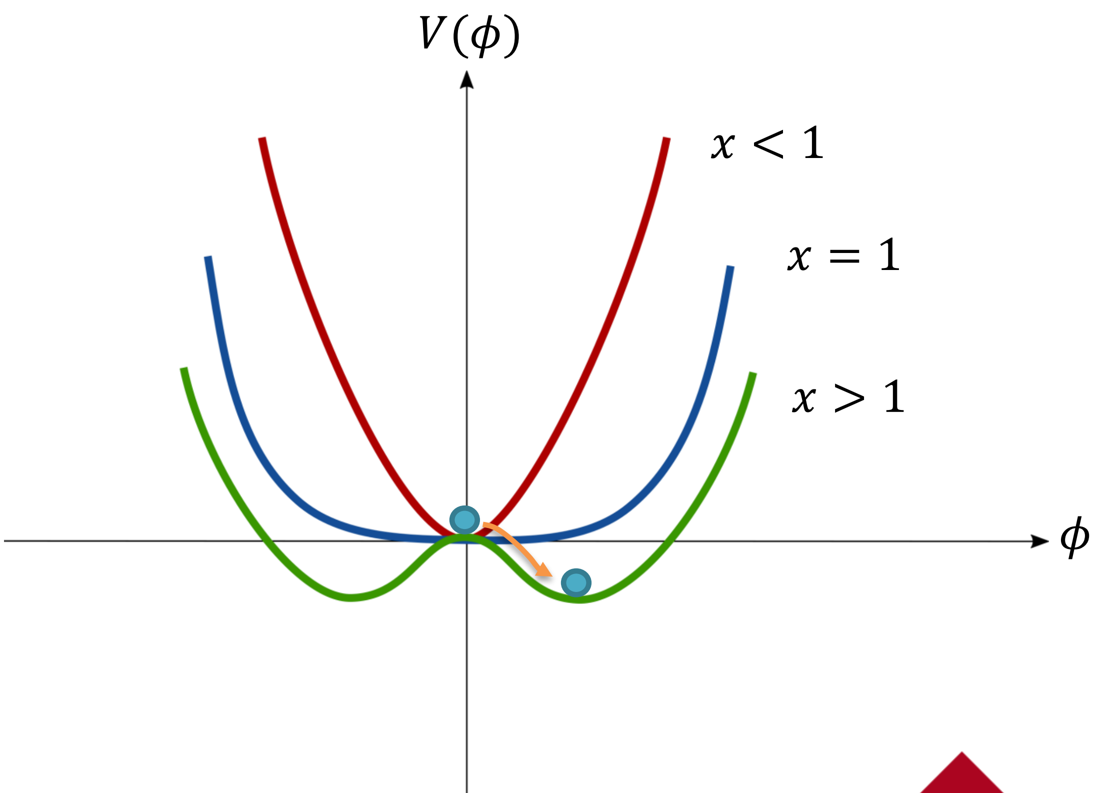
$$x = \frac{T_c}{T}$$

$$\mathcal{V}_{\rm eff}^{\rm th}(x,\phi) = \mathcal{V}_0(x) - \frac{\mu_{\rm eff}(x)^2}{2}\phi^2 + \frac{\lambda_{\rm eff}(x)}{4!}\phi^4$$



2nd order phase transition

$x < 1$ DM massless

$x > 1$ DM acquires mass

→ Spontaneous Freeze Out ?

3 Temperature-dependent mass spectrum

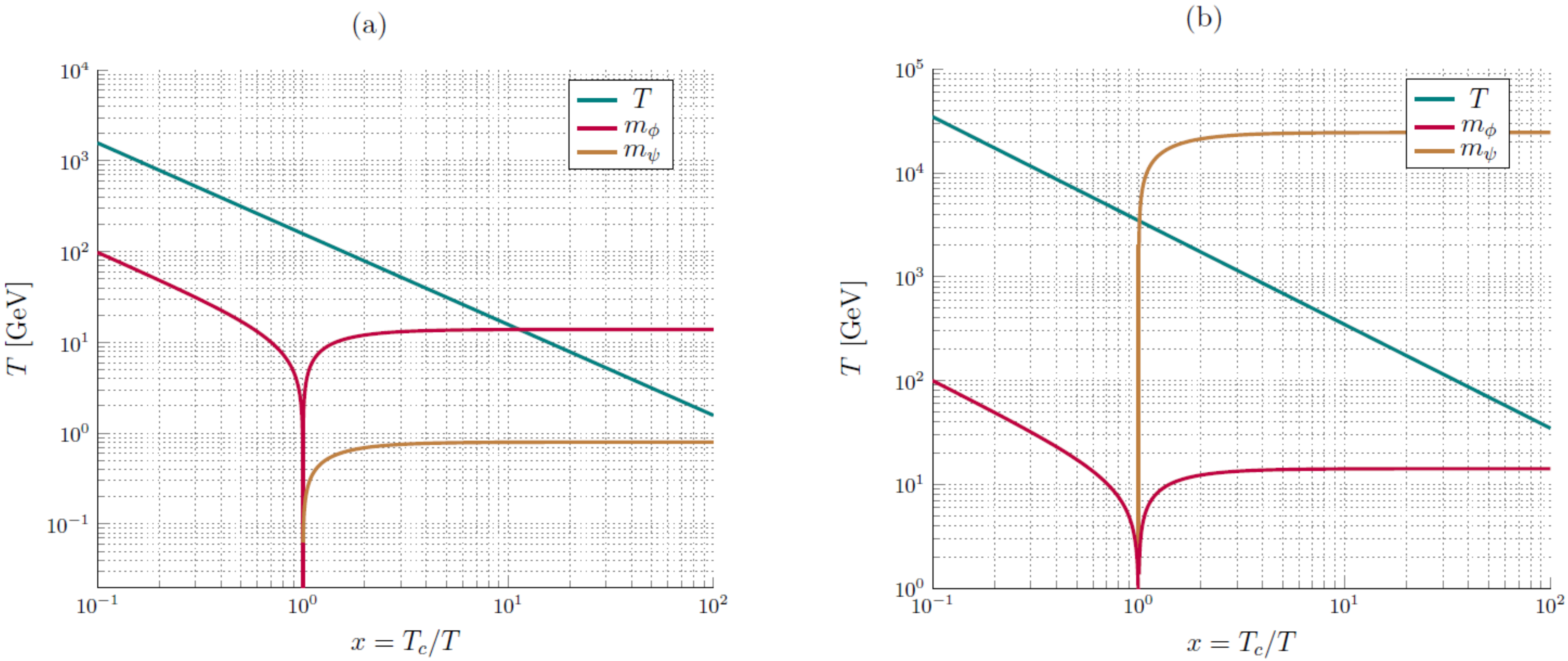


FIG. 3: Examples reproducing (a) the usual freeze out and (b) spontaneous freeze-out scenarios in the case where $y = 10^{-2}$, $\mu = 10$ GeV and, respectively, $\lambda = 10^3 y^2 = 0.1$ and $\lambda = 10^{-2} y^4 = 10^{-10}$.

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The « Spontaneous » Freeze Out Regime

$$\kappa = m_\psi(x_{\rm FO})/T_{\rm FO}.$$

$$x_{\rm FO} \gg \kappa : \quad \text{Constant-Mass Freeze Out}\,,$$
$$x_{\rm FO} \simeq \kappa : \quad \text{Spontaneous Freeze Out}\,.$$

$$\lambda \gg n_F y^2 \quad \Longrightarrow \quad x_{\rm FO} \simeq \mathcal{O}\Big(\frac{2\kappa}{y}\Big) \gg \kappa$$

$$\lambda \ll n_F y^2 \Longrightarrow x_{\rm FO} \simeq \left[1 + \kappa^2\Big(\frac{4\lambda}{n_F y^4} + \frac{3}{\pi^2}\log x_{\rm FO}\Big)\right]^{1/2}$$

4 Solve **Boltzmann Equation for DM** production

In order to compute the relic abundance: need to specify $\mathcal{L}_{\rm int}$

Consider SI interactions : $\mathcal{O}_V = \bar{\psi}\gamma_\mu\psi\bar{f}\gamma^\mu f \quad \text{and} \quad \mathcal{O}_S = \bar{\psi}\psi\bar{f}f$

$$\langle\sigma v\rangle_V \simeq \frac{G_V^2}{2\pi}\left(1 + \frac{x^{-1}T_c}{m_\psi(x)}\right) m_\psi^2(x)\,,$$

$$\langle\sigma v\rangle_S \simeq \frac{3G_S^2}{8\pi} x^{-1}T_c m_\psi(x)\,.$$

$$\Omega h^2 = n_F \frac{m_\psi(x_0)s_0}{6H_0^2M_p^2} Y_\psi^0$$

$$Y_\psi = n_\psi/s$$

$$\frac{\mathrm{d}Y_\psi}{dx} = \frac{\langle\sigma v\rangle s}{xH}(Y_{\psi,\mathrm{eq}}^2 - Y_\psi^2)$$

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Results : Educated guess?

- For a given DM mass TODAY m_ψ^0 : Requesting Ωh^2=0.12 → Fixed Yield Y_ψ^0

- DM velocity at FO $\langle v^2 \rangle \sim T_{\rm FO}/m_\psi = \kappa^{-1}$ model independent : $m_\psi(x_{\rm FO}) < m_\psi^0$ → $T_{\rm FO}$ smaller in our case !

- At Freeze Out : $n_\psi^{\rm FO}\langle\sigma v\rangle_{\rm FO} = H_{\rm FO}$

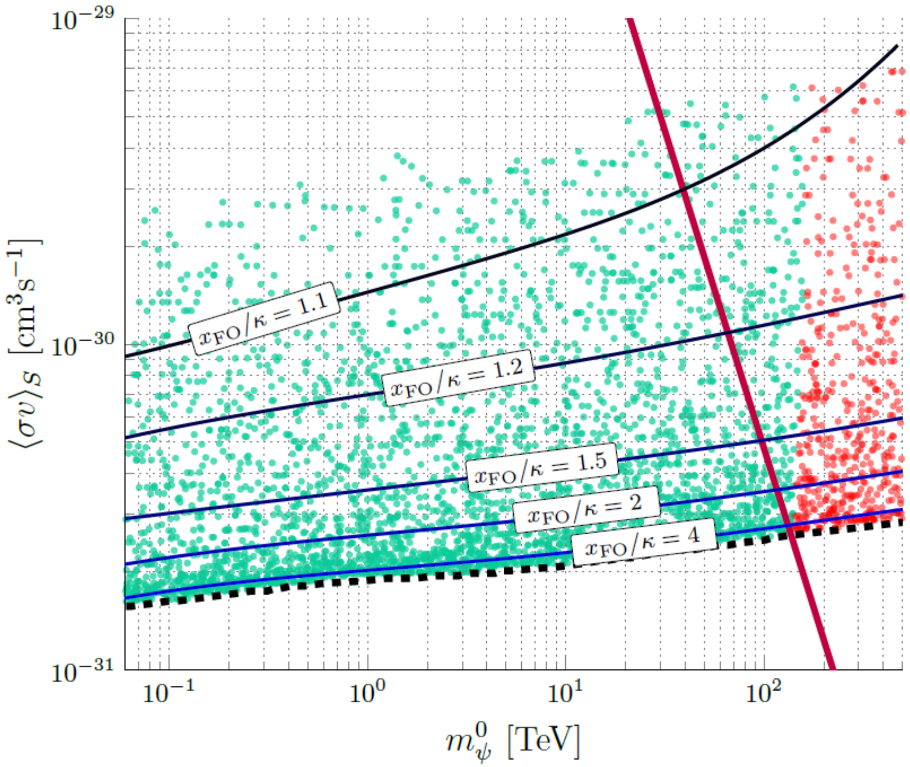
→ $Y_\psi^{\rm FO}\langle\sigma v\rangle_{\rm FO} \propto T_{\rm FO}^{-1}$ therefore

Even bigger today if $\langle\sigma v\rangle$ is an increasing function of the DM mass

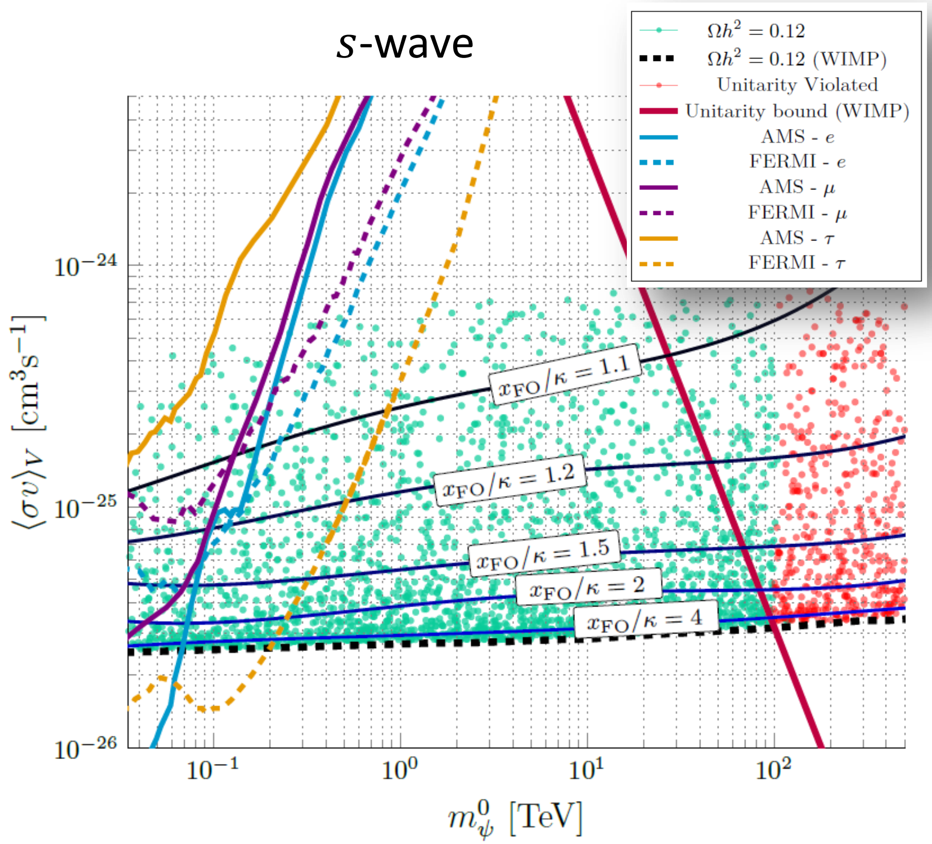
for smaller FO temperature, the value of **the annhilation cross section has to be larger AT FREEZE OUT**.

Interactions with the SM

p-wave



s-wave



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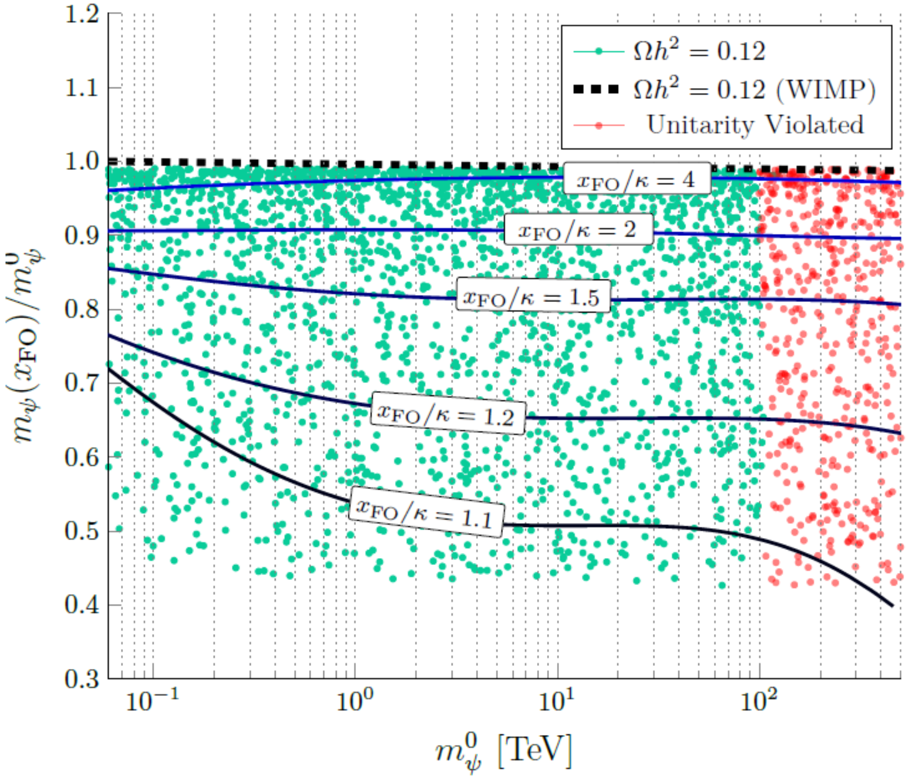

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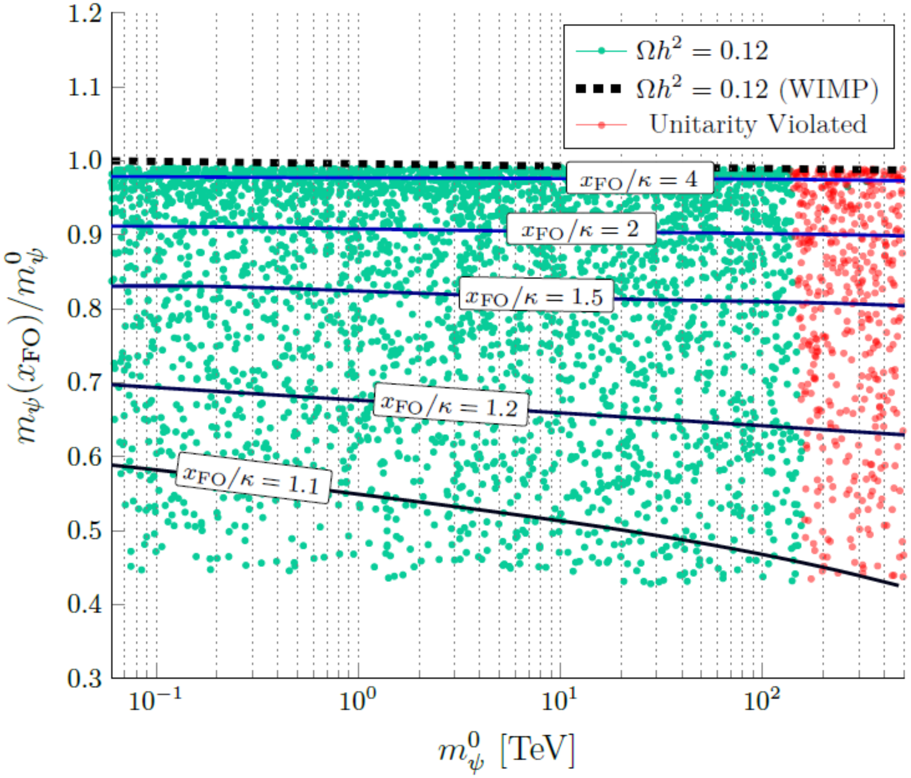

Conclusion

- Masses in the dark sector might be generated by the spontaneous breaking of some global symmetry

- While DM particles are in thermal equilibrium, thermal corrections to the scalar potential associated with such SSB might be significant and restore the symmetry above some critical temperature

- The 2nd order phase transition taking place at $T = T_c$ might provoke the Spontaneous Freeze Out (SFO) of dark matter particles before their mass reached their asymptotic value

- The cross section necessary to generate the correct DM relic abundance is typically larger than in the usual WIMP scenario

- Unitarity bounds on the WIMP mass might be overshot thanks to the dynamical evolution of the dark matter mass

Thank you very much!

ψ and ϕ assumed to be thermalized

$$\mathcal{V}_{\rm eff}^{\rm th}(T,\phi) = \mathcal{V}_{\rm tree}(\phi) + \mathcal{V}_{\rm CW}(\phi) + \mathcal{V}_{\rm dark}^{\rm c.t.}(\phi) + \mathcal{F}(T,\phi)$$

$$\begin{aligned}\mathcal{V}_{\rm CW}(\phi) + \mathcal{V}_{\rm dark}^{\rm c.t.}(\phi) = &\frac{m_0(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_0(\phi)^2}{Q^2}\right) - \frac{3}{2}\right] \\ &- n_F \frac{m_\psi(\phi)^4}{64\pi^2}\left[\log\left(\frac{m_\psi(\phi)^2}{Q^2}\right) - \frac{3}{2}\right]\end{aligned}$$

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$$J_B\left(\frac{m^2}{T^2}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{\frac{3}{2}} - \frac{1}{32}\frac{m^4}{T^4}\log\frac{m^2}{16\alpha T^2} + \mathcal{O}\left(\frac{m^6}{T^6}\right)$$

$$J_F\left(\frac{m^2}{T^2}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m^2}{T^2} - \frac{1}{32}\frac{m^4}{T^4}\log\frac{m^2}{\alpha T^2} + \mathcal{O}\left(\frac{m^6}{T^6}\right)$$

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Non-analiticity

$$J_B\left(\frac{m^2}{T^2}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{\frac{3}{2}} - \frac{1}{32}\frac{m^4}{T^4}\log\frac{m^2}{16\alpha T^2} + \mathcal{O}\left(\frac{m^6}{T^6}\right)$$

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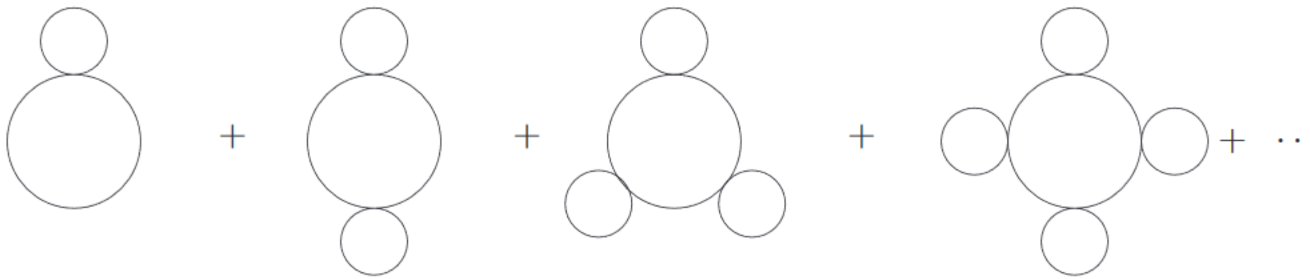
Need to take into account « Daisy » diagrams



$$\mathcal{V}_{\text{eff}}^{\text{th}}(T,\phi) \equiv \mathcal{V}_{\text{1-loop}}^{\text{th}}(T,\phi) + \mathcal{V}_{\text{ring}}^{\text{th}}(T,\phi)$$

$$\mathcal{V}_{\text{ring}}^{\text{th}}(T,\phi) = \frac{T}{12\pi}\left[\left(m_0(\phi)^2\right)^{\frac{3}{2}} - \left(m_0(\phi)^2 + \Pi_\phi(T)\right)^{\frac{3}{2}}\right]$$

$$\Pi_\phi(T) = \frac{T^2}{24}(\lambda + n_F y^2)$$ Leading term of $\partial^2\mathcal{F}/(\partial\phi)^2$

L. Dolan and R. Jackiw, Phys. Rev. **D9**, 3320 (1974).
M. E. Carrington, Phys. Rev. **D45**, 2933 (1992).
C. Delaunay, C. Grojean, and J. D. Wells, JHEP **04**, 029 (2008), arXiv:0711.2511 [hep-ph].

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$$Q = \pi e^{-\gamma_{\rm E}} T_c\,,$$

$$T_c = \frac{2\sqrt{6}\,\mu}{\sqrt{\lambda + n_F y^2}} \sqrt{\frac{1 - \frac{\sqrt{6}}{8\pi}\,\xi + \frac{\log 2}{8\pi^2}\,\lambda}{1 - \frac{\sqrt{6}}{4\pi}\,\xi}}\,, \qquad \xi \equiv \frac{\lambda}{\sqrt{\lambda + n_F y^2}}$$

$$x = \frac{T_c}{T}$$

$$\mathcal{V}_{\rm eff}^{\rm th}(x,\phi) = \mathcal{V}_0(x) - \frac{\mu_{\rm eff}(x)^2}{2}\phi^2 + \frac{\lambda_{\rm eff}(x)}{4!}\phi^4$$

$$\mu_{\rm eff}(x)^2 = \mu^2\left[\left(1 - \frac{\sqrt{6}}{8\pi}\,\xi + \frac{\log 2}{8\pi^2}\,\lambda\right)\left(1 - \frac{1}{x^2}\right) - \frac{\lambda}{16\pi^2}\log x\right],$$

$$\lambda_{\rm eff}(x) = \lambda\left(1 - \frac{3\sqrt{6}}{8\pi}\,\xi + \frac{3\log 2}{8\pi^2}\,\lambda\right) + \frac{3}{16\pi^2}\left(4 n_F y^4 - \lambda^2\right)\log x\,.$$

The « Spontaneous » Freeze Out Regime

- « Time scale » needed for the Freeze Out to happen after DM becomes non-relativistic

$$\kappa = m_\psi(x_{\rm FO})/T_{\rm FO}.$$

- « Time scale » between the phase transition ($x = 1$) and the Freeze Out

$$x_{\rm FO}^2 = 1 + \left(\frac{4\kappa^2 \left[\lambda + \frac{3}{16\pi^2}(4n_F y^4 - \lambda^2)\log x_{\rm FO}\right]}{y^2(\lambda + n_F y^2)} + \frac{\lambda}{16\pi^2}\log x_{\rm FO} \right)(1 + \mathcal{O}(\xi)),$$

$$\lambda \gg n_F y^2 \quad \Longrightarrow \quad x_{\rm FO} \simeq \mathcal{O}\left(\frac{2\kappa}{y}\right) \gg \kappa$$

$$\lambda \ll n_F y^2 \Longrightarrow x_{\rm FO} \simeq \left[1 + \kappa^2\left(\frac{4\lambda}{n_F y^4} + \frac{3}{\pi^2}\log x_{\rm FO}\right)\right]^{1/2}$$

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The « Spontaneous » Freeze Out Regime

$$\text{When } x_{\rm FO} \gg \kappa, \quad 1 \ll x \leqslant x_{\rm FO} : \quad m_\phi(x) \simeq m_\phi^{\rm tree}\,, \qquad m_\psi(x) \simeq m_\psi^{\rm tree}\,.$$

$$\text{When } x_{\rm FO} \lesssim \kappa, \quad 1 \leqslant x \leqslant x_{\rm FO} : \quad m_\phi(x) \simeq m_\phi^{\rm tree}\sqrt{1-\frac{1}{x^2}}\,,$$

$$m_\psi(x) \simeq y\mu\sqrt{\frac{6}{\lambda + \frac{3n_F}{4\pi^2}y^4\log x}}\sqrt{1-\frac{1}{x^2}}\,.$$

$x_{\rm FO} \gg \kappa$: Constant-Mass Freeze Out ,

$x_{\rm FO} \simeq \kappa$: Spontaneous Freeze Out .

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Consequences

- Models of WIMP already excluded by detection EVEN MORE ruled out in the SFO regime.

→ DM interacting with SM quarks essentially excluded

→ DM interacting with SM lepton could be detected soon

- The unitarity bound gets modified by the masses dynamics

$$\sigma_J v_{rel} < \frac{4\pi(2J+1)}{{m_\psi}^2 v_{rel}}$$

$$(\sigma_J v_{rel})^{FO}{}_{max} < (\sigma_J v_{rel})^{SFO}{}_{max}$$

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Important Hypothesis/Details

- While DM freezes out, the DM particles ψ, the dark Higgs ϕ, and SM fermions **are assumed to be in thermal equilibrium**.

  Can that be true for the scalar?

- Before the phase transition $m_\psi = 0$ so $\phi \to \psi\psi$ allowed and maintains equilibrium as long as $\Gamma_\phi > H$

- After the phase transition, in the SFO case ($\lambda \ll n_F y^2$) :

$$1 \leqslant x \leqslant x_{\rm FO}: \quad \frac{m_\psi(x)^2}{m_\phi(x)^2} \simeq \frac{3y^2}{\lambda + \frac{3n_F}{4\pi^2} y^4 \log x} \gg 1$$

so $\phi \to \psi\psi$ forbidden…

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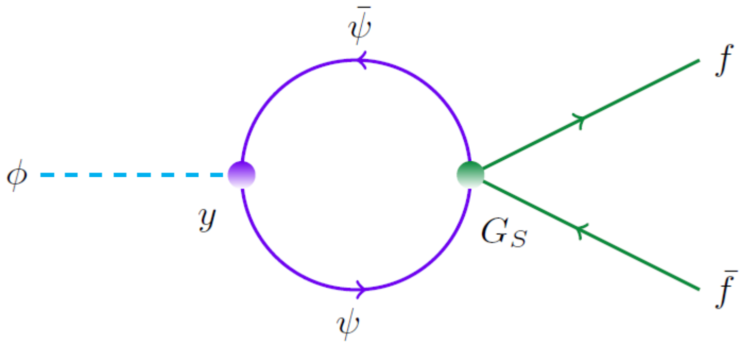
Important Hypothesis/Details

→ Need the scalar to remain in thermal equilibrium while DM is freezing out…

In the scalar operator case :



Otherwise a coupling to the Higgs can always be present.

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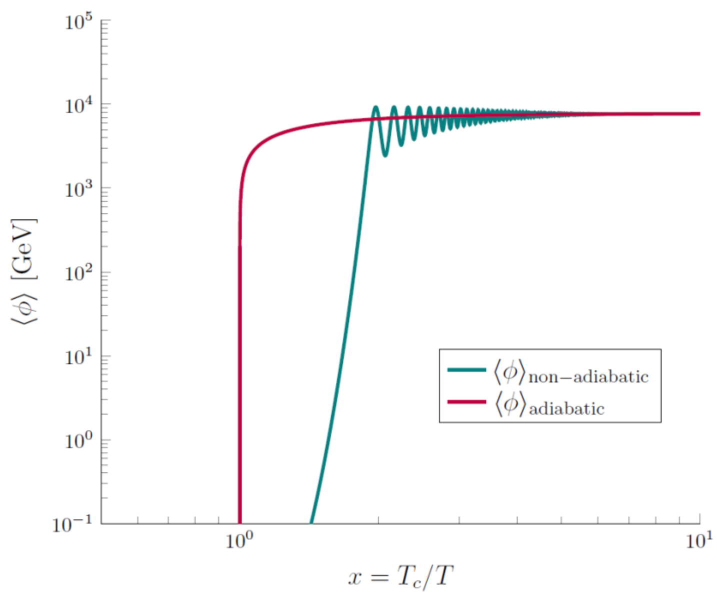
Important Hypothesis/Details

→ We have assumed the scalar to constantly track its minimum. In practice, its destabilization will take some time and delay the phase transition time.

This would favor even smaller FO temperatures, therefore the effect we have described is UNDER-estimated.



→ Presence of a large decay width Γ_ϕ
Also necessary to damp the scalar oscillations → avoid moduli cosmological problem…

Decoupling of Dark Matter And Scalar Potential

After Dark Matter freezes out

Conservation of Stress-Energy tensor:

$$\dot{\rho}_\phi + (\Gamma_\phi + 3H)(\rho_\phi + P_\phi) = -\dot{\phi}\,\frac{d\mathcal{V}_{\rm dust}}{d\phi}$$

$$\dot{\rho}_{\rm dust} + 3H(\rho_{\rm dust} + P_{\rm dust}) = \dot{\phi}\,\frac{d\mathcal{V}_{\rm dust}}{d\phi}$$

$\rho_{\rm dust} = n_\psi y|\phi|\,, \quad P_{\rm dust} = 0$ and $\boxed{\dfrac{d\mathcal{V}_{\rm dust}}{d\phi} = \mathrm{sign}(\phi) y n_\psi}$

$$\rightarrow \quad \boxed{\ddot{\phi} + 3H\dot{\phi} = -\frac{d\mathcal{V}_{\rm eff}}{d\phi} - \frac{d\mathcal{V}_{\rm dust}}{d\phi}}$$

Backreaction disappears with expansion…

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Parameter Constraint

Existence of the minimum at zero temperature :

$$\langle\phi\rangle \simeq \frac{4\pi\mu}{y^2\sqrt{-n_F W\left(-\frac{16\pi^2\mu^2\exp(-1-8\pi^2\lambda/3n_F y^4)}{n_F Q^2 y^2}\right)}}$$

W Lambert function

Demanding the minimum to be real :

$$\frac{\lambda}{y^4} > \frac{3n_F}{8\pi^2}\left(\log\frac{2}{3} + 2\gamma_{\rm E}\right) \simeq 0.03\, n_F \longrightarrow x_{\rm FO}/\kappa \gtrsim 1$$

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