# Revisiting $X(3872) \to D^0\bar{D}^0\pi^0$ in XEFT

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with

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**(Phys.Rev.D 101 (2020) 5, 054024)**

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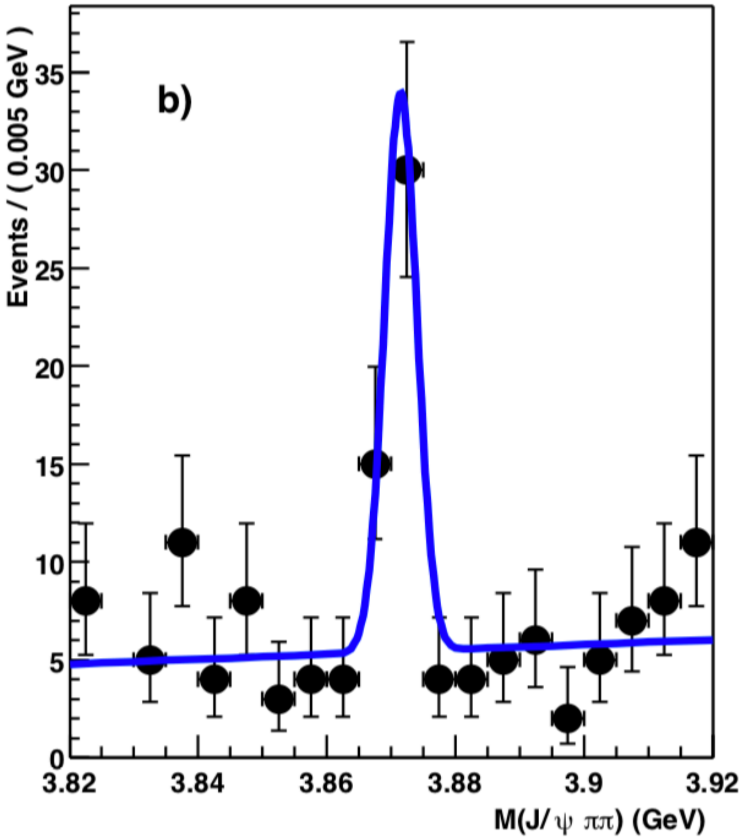
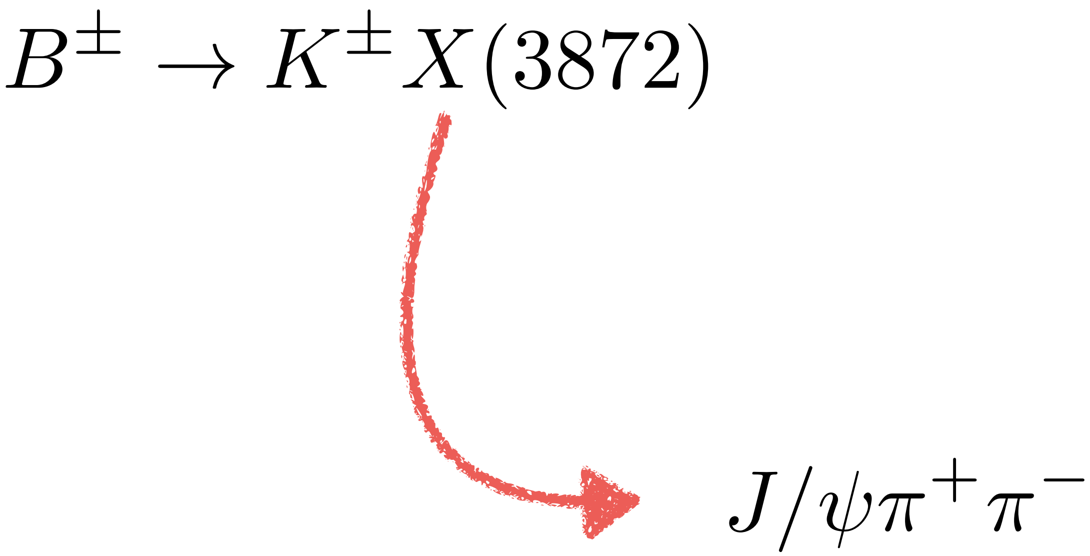
# Outline

- Review XEFT
- XEFT and Power Counting
- NLO Calculations of X decay rate and Numerical Results
- Conclusion

# X(3872)



Belle Collaboration,
Phys.Rev.Lett. 91, 262001



$$B^{\pm} \to K^{\pm} X(3872)$$

$$J/\psi\pi^{+}\pi^{-}$$

$$B_X = M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)\text{MeV}$$

- Assume X(3872) a DD* bound state in our analysis

$$X \to D^0 \overline{D}^0 \pi^0$$

$$\frac{B\left(X(3872) \to D^0 \bar{D}^0 \pi^0\right)}{B\left(X(3872) \to \pi^+\pi^- J/\psi\right)} = 9.4^{+3.6}_{-4.3}$$

# XEFT Lagrangian

- Matching to HHChPT:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{XEFT}} = &\sum_{\boldsymbol{\phi}=\boldsymbol{D},\bar{\boldsymbol{D}}} \boldsymbol{\phi}^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{D^{*0}}}\right)\boldsymbol{\phi} + \sum_{\phi=D,\bar{D}} D^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{D^0}}\right)D + \pi^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M_{\pi^0}} + \delta\right)\pi \\
&+ \left[\frac{\bar{g}}{F_\pi}\frac{1}{\sqrt{2M_{\pi^0}}}\left(D\boldsymbol{D}^\dagger\cdot\boldsymbol{\nabla}\pi + \bar{D}^\dagger\bar{\boldsymbol{D}}\cdot\boldsymbol{\nabla}\pi^\dagger\right) + \mathrm{H.c.}\right] \\
&- \frac{C_0}{2}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger\cdot\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right) \\
&+ \left[\frac{C_2}{16}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger\cdot\left(\bar{\boldsymbol{D}}(\overleftrightarrow{\nabla})^2 D + \boldsymbol{D}(\overleftrightarrow{\nabla})^2\bar{D}\right) + \mathrm{H.c.}\right] \\
&+ \left[\frac{B_1}{\sqrt{2}}\frac{1}{\sqrt{2M_{\pi^0}}}\left(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D}\right)^\dagger\cdot D\bar{D}\boldsymbol{\nabla}\pi + \mathrm{H.c.}\right]
\end{aligned}
$$

$$
\boxed{+\frac{C_\pi}{2M_{\pi^0}}\left(D^\dagger\pi^\dagger D\pi + \bar{D}^\dagger\pi^\dagger\bar{D}\pi\right) + C_{0D}D^\dagger\bar{D}^\dagger D\bar{D}\,, \qquad \textbf{Missing Before}}
$$

S. Fleming, M. Kusunoki,
T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

# XEFT Power Counting

- ✦ P.C. parameter Q (dynamical mom of X(3872)):

$$\{p_D, p_{D^*}, p_\pi, \mu, \gamma_0\} = \mathcal{O}(Q)$$

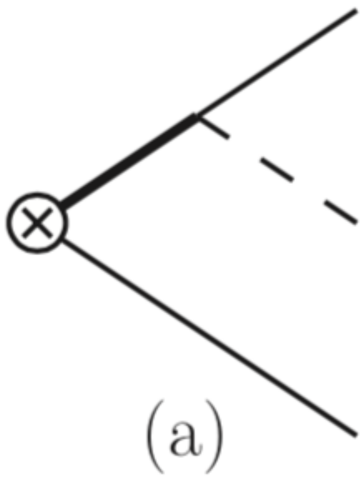
$$\mu = \sqrt{\Delta^2 - M_{\pi^0}^2} \simeq 44\text{MeV} \quad \text{with} \quad \Delta \equiv M_{D^*} - M_D$$

$$\gamma_0 = \sqrt{2\mu_0 B_X}$$

LOOP: $\int d^4p \sim Q^5$ Propagator: $\frac{1}{p^2 - \gamma^2} \sim Q^{-2}$

- ✦ Dynamic pion: NR & Perturbative

$$D^* \to D + \pi \quad \text{with} \quad M_{D*} - M_D - M_\pi \sim 7\text{MeV}$$



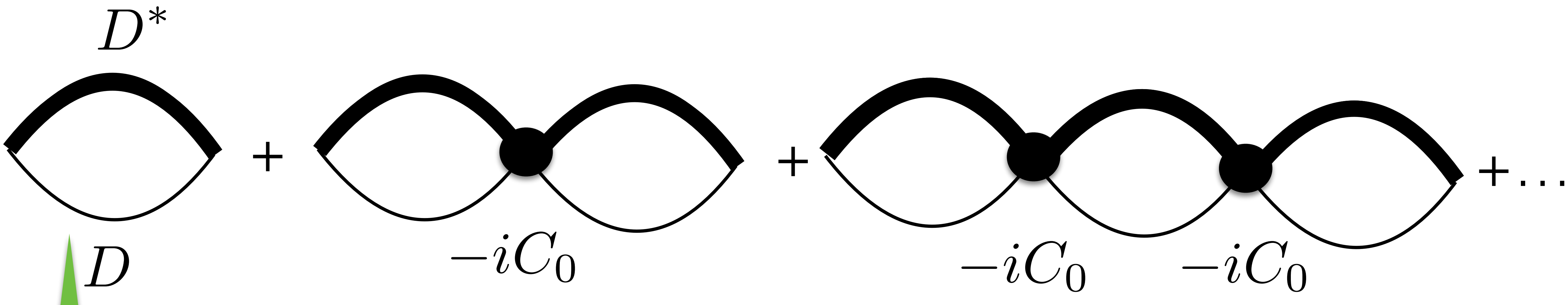
$$\frac{g^2\mu_0\mu}{8\pi F_\pi^2} \simeq \frac{1}{20} \cdots \frac{1}{10}$$

S. Fleming, M. Kusunoki,
T. Mehen, Phys. Rev. D **76**, 034006 (2007)

# XEFT Power Counting

- ✦ $C_0$ treated non-perterbatively, generate X(3872) bound state

$$-\frac{C_0}{2}\left(\overline{\boldsymbol{D}}D+\boldsymbol{D}\overline{D}\right)^{\dagger}\cdot\left(\overline{\boldsymbol{D}}D+\boldsymbol{D}\overline{D}\right)$$



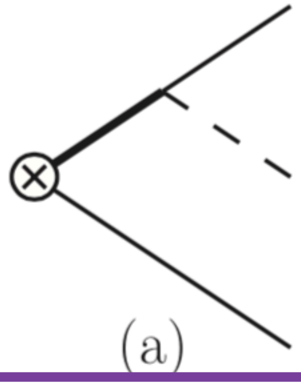
$$\Sigma_0\left(-B_X\right)\sim Q^5Q^{-4}=Q^1$$

- ✦ Pole at $B_X$

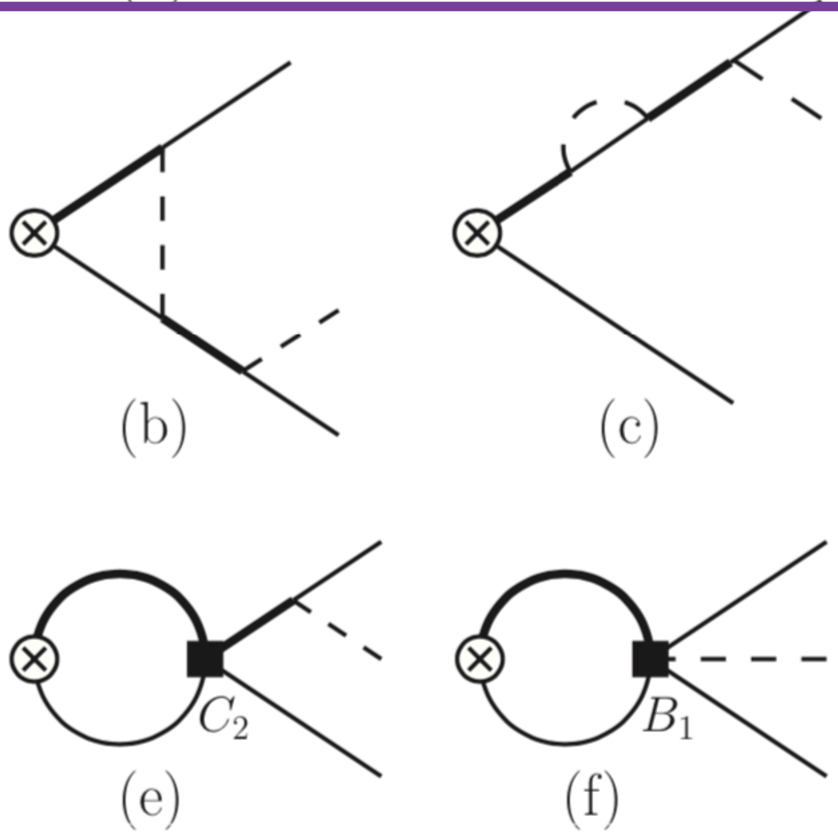
$$1+C_0\Sigma_0\left(-B_X\right)=0\rightarrow C_0\sim Q^{-1}$$

# XEFT Power Counting

$\mathbf{D}D^\dagger \cdot \nabla\pi$



$$\mathcal{O}\left(Q/Q^2\right) = \mathcal{O}\left(Q^{-1}\right)$$



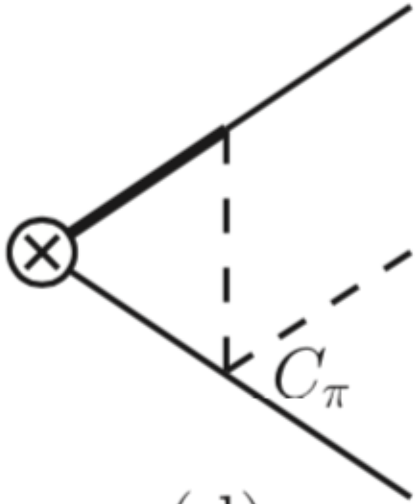
(b): $\mathcal{O}(Q^3Q^{-8}Q^5) = \mathcal{O}(Q^0)$

Similar for others

S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

# XEFT Power Counting

The diagram: $\mathcal{O}(QQ^{-6}Q^{5}) = \mathcal{O}(Q^{0})$

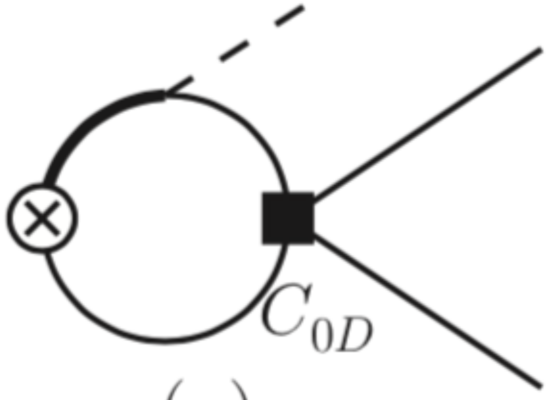


(d)

Num. HH**χ**PT: $i\mathcal{A}_{h_0,h_1} = i\frac{2}{3}(6h_0 + h_1)\frac{M_\pi^2}{F_\pi^2} \sim 0.65$

Match to $\frac{C_\pi}{2M_{\pi^0}}\left(D^\dagger\pi^\dagger D\pi + \overline{D}^\dagger\pi^\dagger\overline{D}\pi\right) \rightarrow C_\pi = \mathcal{O}(Q^0)$

FK. Guo, C. Hanhart, S. Kreward, Phys. Lett. B666, 251–255 (2008)
FK. Guo, C. Hanhart, Ulf-G. Meissner Eur. Phys. J. A40, 171–179 (2009)

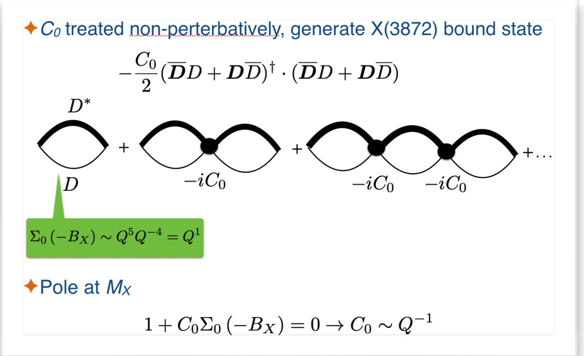


(g)

The diagram: $\mathcal{O}(QQ^{-6}Q^{5}) = \mathcal{O}(Q^{0})$

To estimate $C_{0D}$, assume there is a bound state near $D\bar{D}$ threshold:

$$C_{0D} \sim 1\ \text{fm}^2$$



✦ $C_0$ treated non-perterbatively, generate X(3872) bound state

$$-\frac{C_0}{2}(\overline{\boldsymbol{D}}D + \boldsymbol{D}\overline{D})^\dagger \cdot (\overline{\boldsymbol{D}}D + \boldsymbol{D}\overline{D})$$

$\Sigma_0(-B_X) \sim Q^5Q^{-4} = Q^1$

✦ Pole at $M_X$

$$1 + C_0\Sigma_0(-B_X) = 0 \rightarrow C_0 \sim Q^{-1}$$

FK. Guo, C.Hidalgo-Duque, J. Nieves, A. Ozpineci, Manuel Pavón Valderrama, Eur. Phys. J. C (2014) 74:2885

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

# Decay Rate of X ➟ $D\bar{D}\pi$

$$\left(\frac{\bar{g}\mu_0}{F_\pi}C_2\left(\Lambda_{\mathrm{PDS}}\right)+B_1\left(\Lambda_{\mathrm{PDS}}\right)\right)\left(\Lambda_{\mathrm{PDS}}-\gamma\right)=\pm r_0^3$$

$$C_2\left(\Lambda_{\mathrm{PDS}}\right)=\frac{2\pi}{\mu_0}\frac{r_0}{2}\frac{1}{\left(\Lambda_{\mathrm{PDS}}-\gamma\right)^2}$$

$$\begin{aligned}
\frac{d\Gamma_{\mathrm{NLO}}}{dp_D^2\,dp_{\bar{D}}^2}=&\frac{d\Gamma_{\mathrm{LO}}}{dp_D^2\,dp_{\bar{D}}^2}\left(1+\frac{\bar{g}^2\mu_0\gamma}{3\pi F_\pi^2}\left(\frac{4\gamma^2-\mu^2}{4\gamma^2+\mu^2}\right)+\boxed{C_2\left(\Lambda_{\mathrm{PDS}}\right)\frac{\mu_0\gamma\left(\gamma-\Lambda_{\mathrm{PDS}}\right)^2}{\pi}}\right)\\
&-\frac{\bar{g}\gamma}{8\sqrt{2}\pi^3F_\pi}\boxed{\left(\sqrt{2}\frac{\bar{g}\mu_0}{F_\pi}C_2\left(\Lambda_{\mathrm{PDS}}\right)-B_1\left(\Lambda_{\mathrm{PDS}}\right)\right)\left(\Lambda_{\mathrm{PDS}}-\gamma\right)}\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\\
&+\frac{1}{8\pi^2}\frac{\bar{g}^4}{F_\pi^4}\frac{\gamma}{M_{\pi^0}}\left[\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\frac{p_D^2\tilde{I}_1^{(2)}\left(p_D\right)}{P_D^2+\gamma^2}+\frac{p_{\bar{D}}^2\tilde{I}_1^{(2)}\left(p_{\bar{D}}\right)}{P_{\bar{D}}^2+\gamma^2}\right)\right]\\
&+\frac{1}{8\pi^2}\frac{\bar{g}^4}{F_\pi^4}\frac{\gamma}{M_{\pi^0}}\left[\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\frac{\vec{p}_D\cdot\vec{\epsilon}_X\vec{p}_D\cdot\vec{p}_\pi}{p_D^2+\gamma^2}\left(I(p_D)-2I^{(1)}(p_D)+I_0^{(2)}(p_D)\right)\right.\right.\\
&\left.\left.+\frac{\vec{p}_{\bar{D}}\cdot\vec{\epsilon}_X\vec{p}_{\bar{D}}\cdot\vec{p}_\pi}{p_{\bar{D}}^2+\gamma^2}\left(I(p_{\bar{D}})-2I^{(1)}(p_{\bar{D}})+I_0^{(2)}(p_{\bar{D}})\right)\right)\right]
\end{aligned}$$

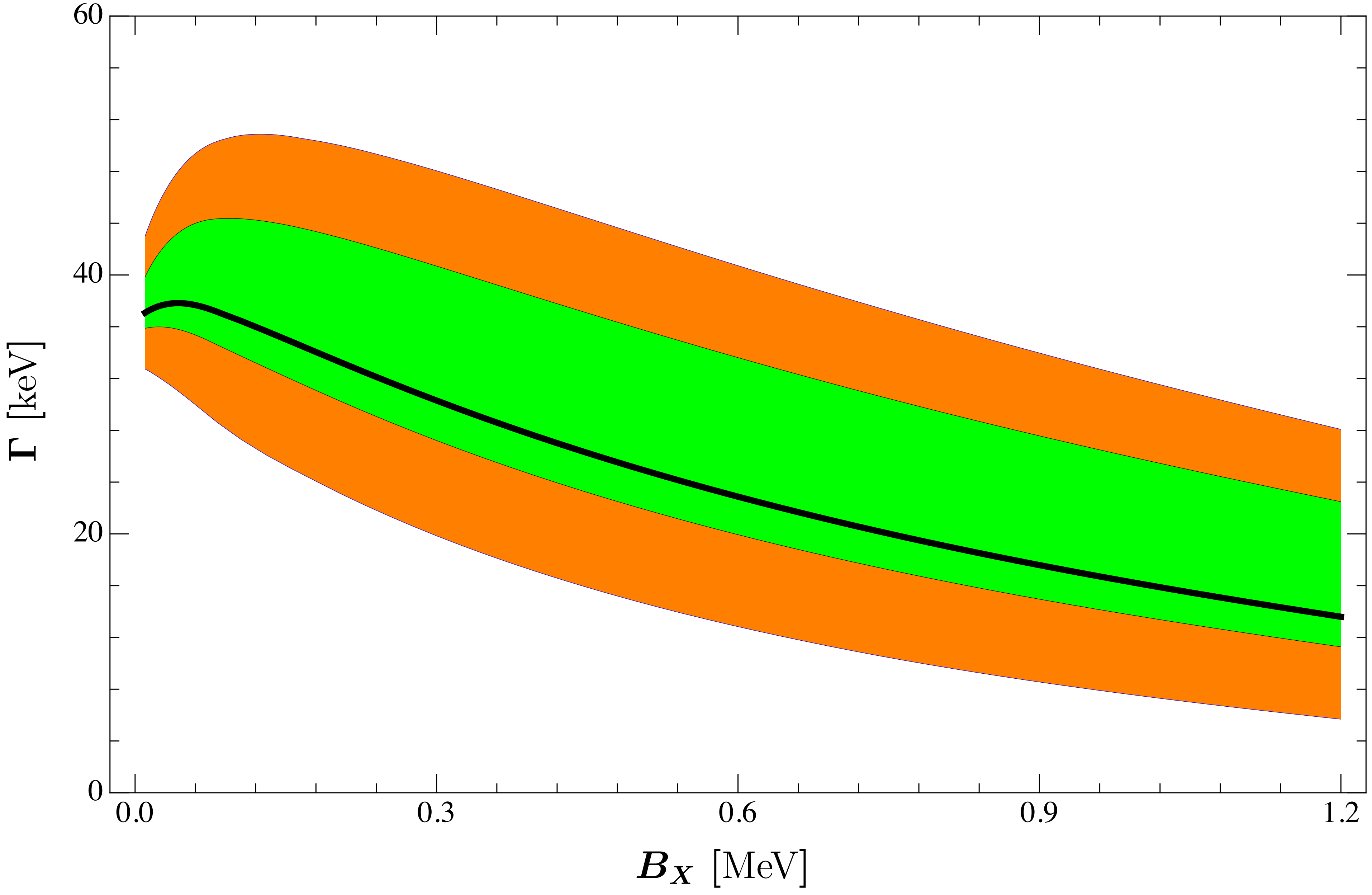
$$\boxed{\begin{aligned}
&+\frac{C_\pi\bar{g}^2\gamma}{16\pi^2F_\pi^2M_{\pi^0}\mu_0}\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right)\left(\vec{p}_D\cdot\vec{\epsilon}_X\left(I^{(1)}(p_D)-I(p_D)\right)+\vec{p}_{\bar{D}}\cdot\vec{\epsilon}_X\left(I^{(1)}(p_{\bar{D}})-I(p_{\bar{D}})\right)\right)\\
&+\frac{C_{0D}\bar{g}^2\gamma}{4\pi^2F_\pi^2\mu_0}\left(\vec{p}_\pi\cdot\vec{\epsilon}_X\right)^2I(p_\pi)\left(\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar{D}}^2+\gamma^2}\right).
\end{aligned}}$$

New Contribution

S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen,
PhysRevD.101.054024 (2020)

# Decay Rate of X ➟ $D\bar{D}\pi$



Parameters: $0 \le r_0 \le \frac{1}{100}\mathrm{MeV}^{-1}, C_\pi = (4.1 \pm 0.7) \times 10^{-3}\mathrm{MeV}^{-1}, C_{0D} = \pm 1\mathrm{fm}$

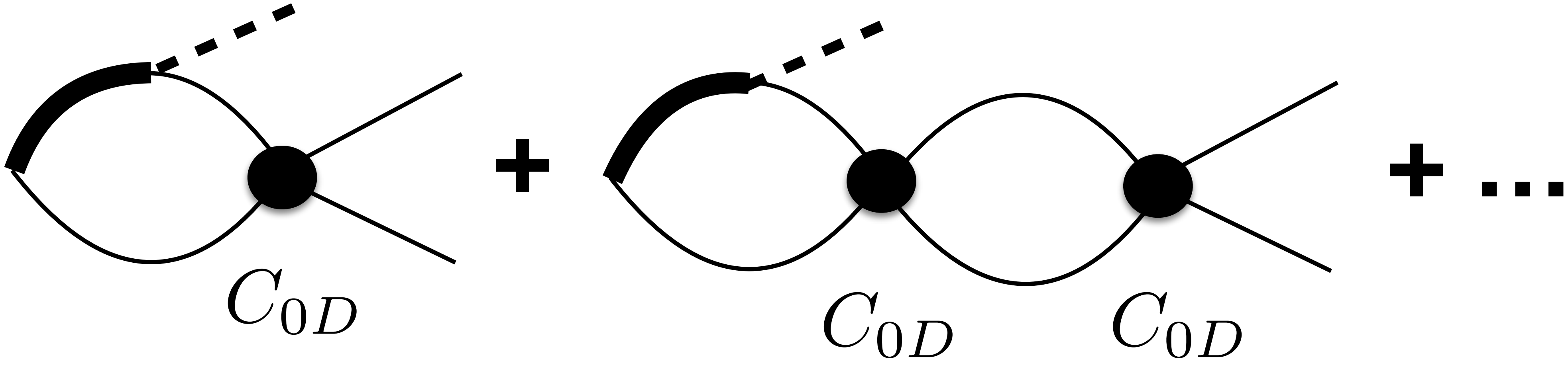
NLO Correction: $C_\pi \to \le 1\%, \quad C_{0D} \to 20\%$

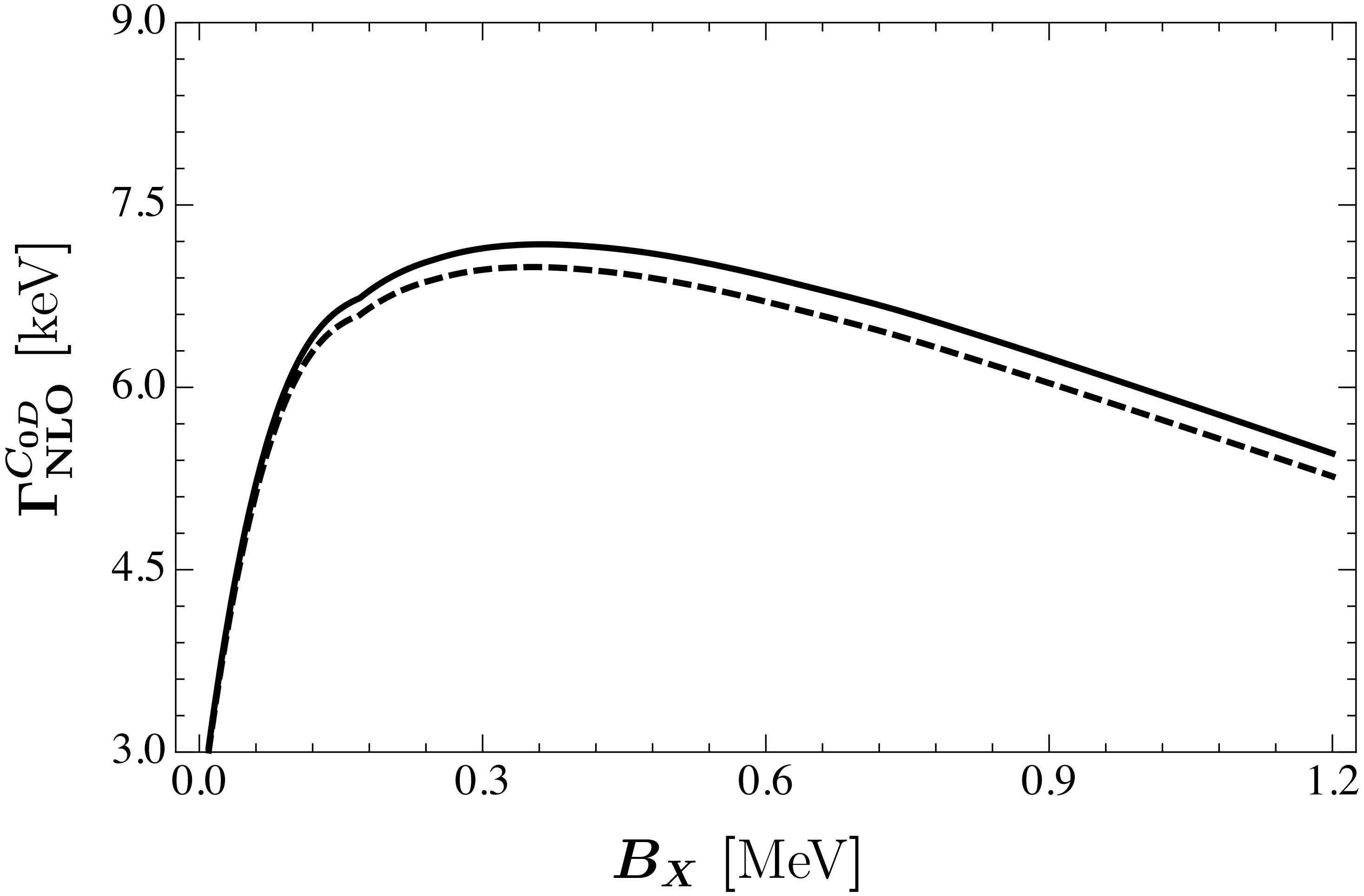
S. Fleming, M. Kusunoki, T. Mehen, Phys. Rev. D **76**, 034006 (2007)

L. Dai, FK. Guo, T. Mehen,
PhysRevD.101.054024 (2020)

# Final States Rescattering

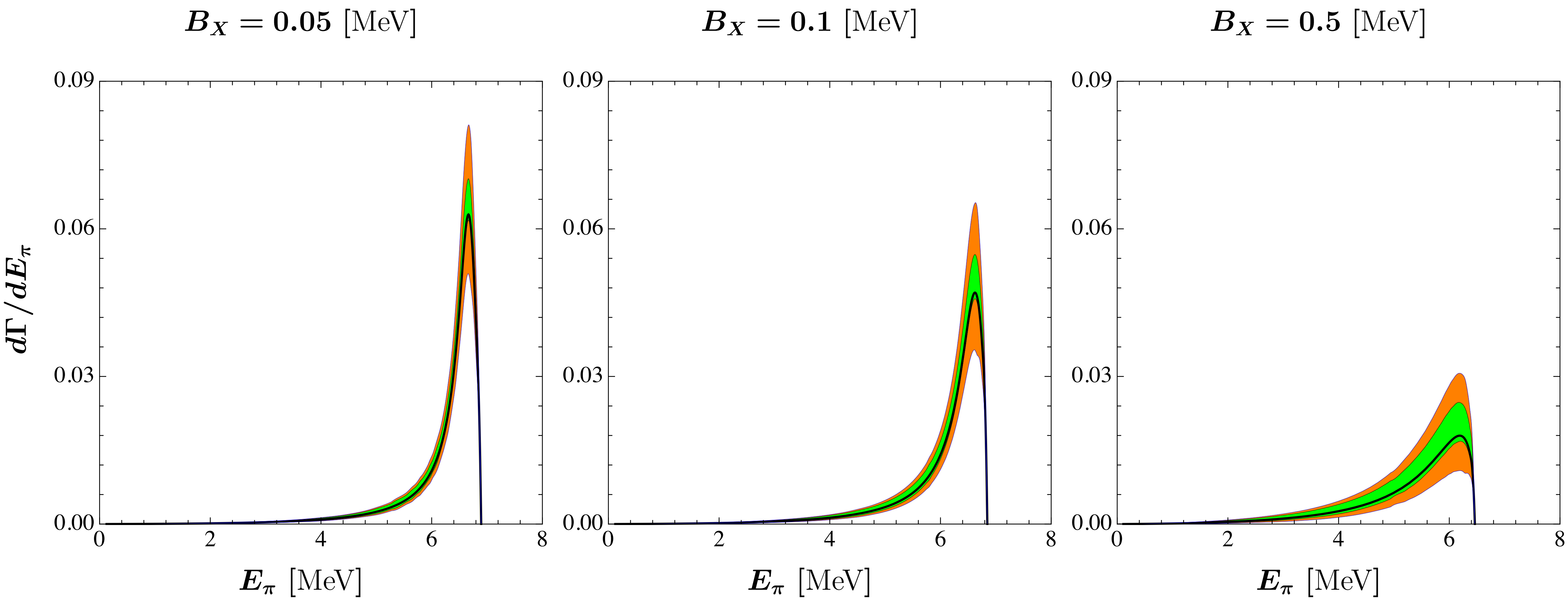
- ✦ Final State Rescattering





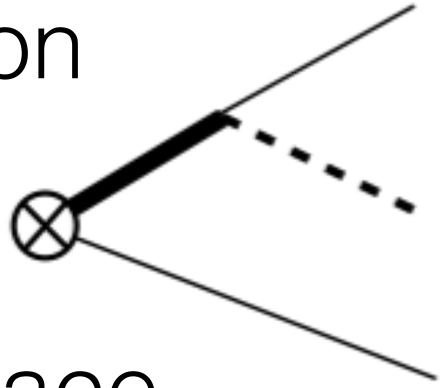
L. Dai, FK. Guo, T. Mehen,
PhysRevD.101.054024 (2020)

# π distribution



Location of peaks less sensitive to NLO correction

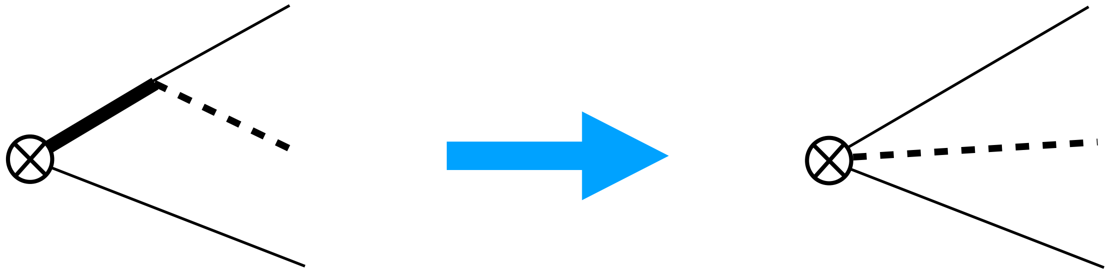
$$M_{D*} - M_D - M_\pi \sim 7\text{MeV}$$



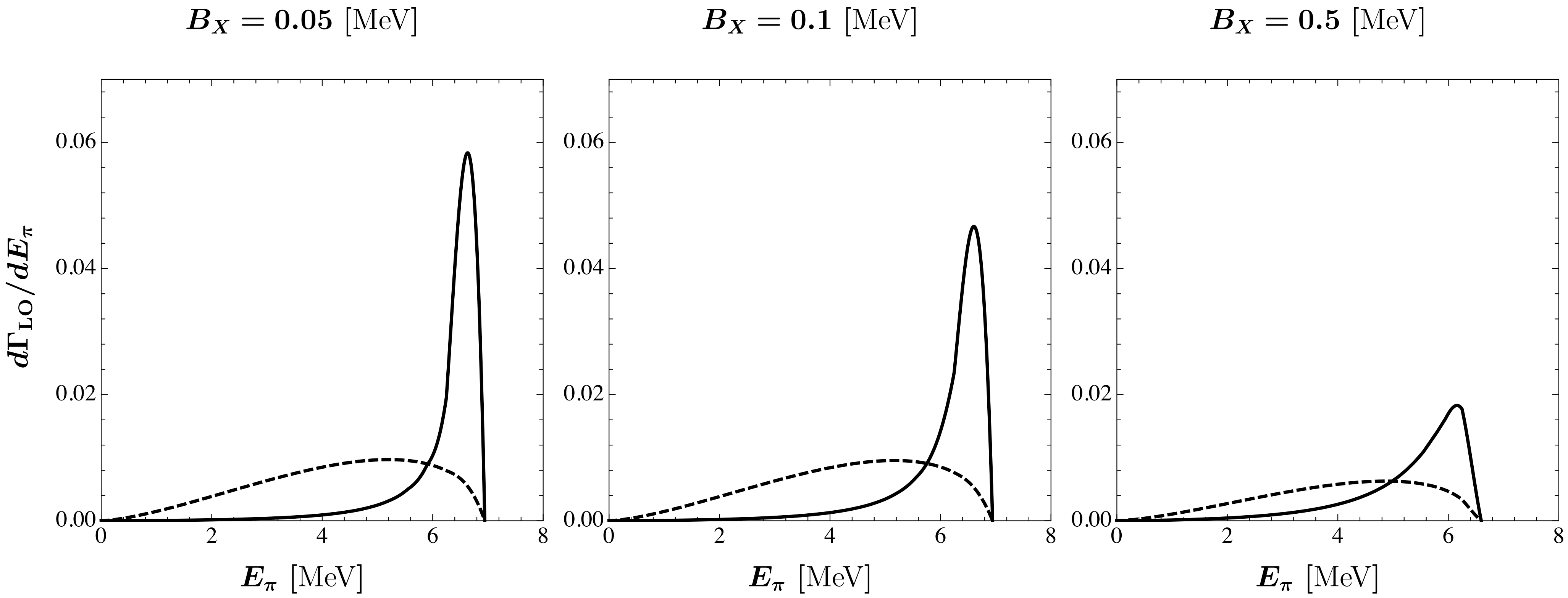
Locations mostly controlled by 3-body phase space, however…

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

# D* propagator



$$i\mathcal{A}_{\rm LO} = \frac{\bar{g}\mu_0}{F_\pi\sqrt{M_{\pi^0}}}\vec{p}_\pi\cdot\vec{\epsilon}_X\left(\frac{1}{\vec{p}_D^{\,2}+\gamma^2}+\frac{1}{\vec{p}_{\bar{D}}^{\,2}+\gamma^2}\right) \longrightarrow \vec{p}_\pi\cdot\vec{\epsilon}_X$$



D* propagator effects, reflection of Molecular nature?

L. Dai, FK. Guo, T. Mehen, PhysRevD.101.054024 (2020)

# Summary

- Added $\pi$D and DD scattering terms, which are needed by symmetry and power counting

- Did NLO calculations, $C_\pi$ correction is small, $C_{0D}$ can be very large

- $\pi$ distribution: extracting X properties, reflecting molecular nature of X