Muonium-Antimuonium Oscillations in Effective Field Theory

Renae Conlin
au9969@wayne.edu

Department of Physics and Astronomy

with: Alexey Petrov

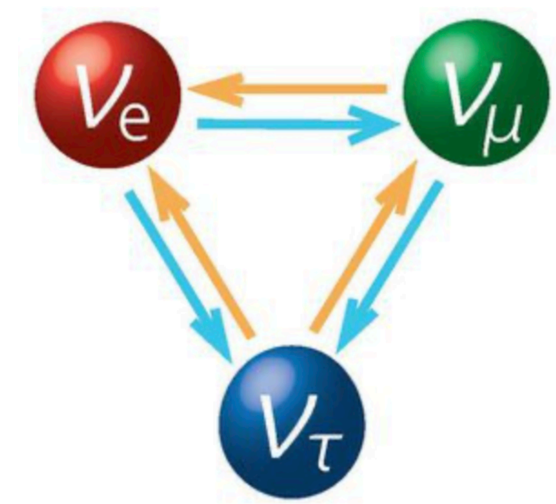
WAYNE STATE UNIVERSITY

Outline

- Motivation
- Muonium Oscillation Formalism
- Calculation of mixing parameters Δm and $\Delta\Gamma$
- Summary

Motivation

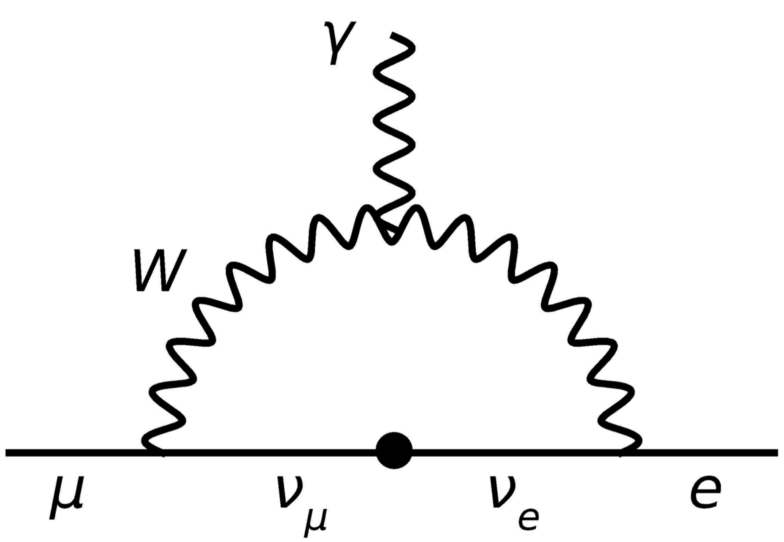
- Neutrino's mix and change flavor - must have non-zero mass
    - This can lead to flavor violating processes in the charged lepton sector
    - Lepton flavor violating (LFV) processes are highly suppressed in the standard model, e.g. $\mu \to e\gamma$ Br$(\mu \to e\gamma)_{SM} \sim 10^{-54}$



https://j-parc.jp/Neutrino/en/intro-t2kexp.html

- LFV processes are highly suppressed in the standard model $\Rightarrow$ no background for NP!

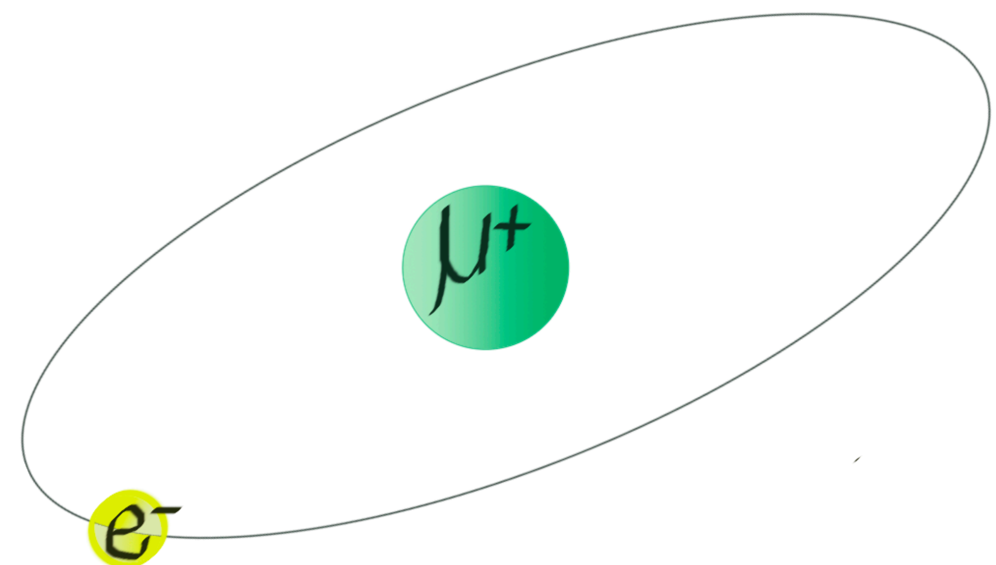
- LFV processes are searched for in various processes:
    - $\Delta L_\mu = 1$: $\mu \to e\gamma$, $\mu \to eee$ and $\mu + N \to e + N$
    - $\Delta L_\mu = 2$: Muonium anti-muonium oscillations
        - In some models we can expect $\Delta L_\mu = 2$ contributions to be the dominant ones, e.g. doubly charged Higgs has a tree level contribution to $\Delta L_\mu = 2$, but not to $\mu \to e\gamma$



Abdallah, W. 2108 arXiv:1105.1047 [hep-ph]

Muonium Oscillations: $M \to \overline{M}$

- Muonium is a non-relativistic Coulombic bound state of an anti-muon $\mu^+$and an electron e
  - Can be Spin-0 (singlet): para-muonium
    - Spin-1 (triplet): ortho-muonium
- An oscillation is the process $M(\mu^+e^-) \to \overline{M}(\mu^-e^+)$
  - It violates muon lepton number by two units $\Delta L_\mu = 2$, i.e.: it can probe different types of NP than $\mu \to e\gamma$ or $\mu + N \to e + N$
  - Conversion rate was calculated in several NP models with heavy DOF



Muonium (μ^+e^-)

Muonium Oscillation Formalism

- An effective theory approach: all possible heavy NP models

- Most general Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{\Lambda^2}\sum_i c_i(\mu)Q_i$$

  - c_i's are the Wilson coefficients
    - Determined by the (UV) physics at some NP scale Λ
  - Q_i's are the dimension-six operators
    - Reflect degrees of freedom relevant at the scale in which a process takes place

Muonium Oscillation Formalism

- Similar to meson-antimeson oscillations, but unlike $K\overline{K}$ or $B\overline{B}$ oscillations both spin-0 and spin-1 states can oscillate

- The time development of M and $\overline{M}$ are coupled and is given by a Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix}$$

matrix Hamiltonian

m and Γ are 2×2 Hermitian matrices: the *mass* matrix and the *decay* matrix

- Assume CPT invariance, then the diagonal and off diagonal elements are

$$m_{11} = m_{22}, \quad \Gamma_{11} = \Gamma_{22} \qquad\qquad m_{12} = m_{21}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

Muonium Oscillation Formalism

- The off diagonal element of this matrix is given by

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M} \,\middle|\, \mathscr{H}^{\Delta L_\mu = 2} \,\middle|\, M \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M} \,\middle|\, \mathscr{H}^{\Delta L_\mu = 1} \,\middle|\, n \right\rangle \left\langle n \,\middle|\, \mathscr{H}^{\Delta L_\mu = 1} \,\middle|\, M \right\rangle}{M_M - E_n + i\epsilon}$$

- Since $\left(m - i\frac{\Gamma}{2}\right)_{12} \neq 0$ then M and $\overline{M}$ are not mass eigenstates: diagonalize!
    - The CP conserving mass eigenstates, M_1 and M_2, are linear combinations of M and $\overline{M}$

$$\left|M_{1,2}\right\rangle = \frac{1}{\sqrt{2}}\left(|M\rangle \pm |\overline{M}\rangle\right)$$

Muonium Oscillation Time Evolution

- Time development of M and $\overline{M}$

$$\left|M(t)\right\rangle = g_+(t)\left|M\right\rangle + g_-(t)\left|\overline{M}\right\rangle \qquad \left|\overline{M}(t)\right\rangle = g_-(t)\left|M\right\rangle + g_+(t)\left|\overline{M}\right\rangle$$

Where, $g_\pm(t) = \frac{1}{2}e^{-\Gamma_1 t/2}e^{-im_1 t}\left[1 \pm e^{-\Delta\Gamma t/2}e^{i\Delta m t}\right]$

- Mass eigenstates M_1 and M_2 has mass difference (Δm), and width difference ($\Delta\Gamma$)

$$\Delta m \equiv m_1 - m_2, \quad \Delta\Gamma \equiv \Gamma_2 - \Gamma_1$$

$$x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

but $x, y \ll 1$

- Dependence on x and y!

$$\frac{\Gamma(M \to \bar{f})}{\Gamma(\overline{M} \to \bar{f})} \sim R(x, y), \qquad R(x, y) = \frac{1}{4}(x^2 + y^2)$$

$$\Gamma(M \to \bar{f})(t) = N_f \left|\left\langle \bar{f}\left| S \right| M(t)\right\rangle\right|^2,$$

$$\Gamma(\overline{M} \to \bar{f})(t) = N_f \left|\left\langle \bar{f}\left| S \right| \overline{M}(t)\right\rangle\right|^2$$

- Need to calculate x and y!

Calculation of Matrix Elements

- Our $\Delta L_\mu = 2$ Lagrangian

$$\mathscr{L}_{eff}^{\Delta L_\mu = 2} = -\frac{1}{\Lambda_1^2}\sum_i C_i(\mu) Q_i(\mu)$$

- Most general set of operators, Q_i's

$$Q_1 = \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_L e\right), \qquad Q_2 = \left(\bar{\mu}\gamma_\alpha P_R e\right)\left(\bar{\mu}\gamma^\alpha P_R e\right), \qquad Q_3 = \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_R e\right),$$

$$Q_4 = \left(\bar{\mu} P_R e\right)\left(\bar{\mu} P_R e\right), \qquad Q_5 = \left(\bar{\mu} P_L e\right)\left(\bar{\mu} P_L e\right)$$

- Other possible structures can be Fierz'd into the operators above

Calculation of Matrix Elements

- Matrix element m_{12}

$$m_{12} = \left\langle \overline{M} \left| -\mathscr{L}_{eff}^{\Delta L_\mu = 2} \right| M \right\rangle = \sum_i \frac{C_i}{\Lambda_1^2} \left\langle \overline{M} \left| Q_i \right| M \right\rangle$$

- Muonium is a non-relativistic Coulombic bound state

$$|M(0)\rangle = \sqrt{2E_{\mathbf{q}} 2E_{-\mathbf{q}}} \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}(q) a_{\mathbf{q}}^{(e)\dagger} b_{-\mathbf{q}}^{(\mu)\dagger} |0\rangle$$

$\tilde{\psi}(q)$ is the Fourier transform of the spatial wave function $\psi(x)$

- Perturbative QED bound state: can calculate!

Calculation of Matrix Elements

Example: $$Q_1 = \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_L e\right)$$

$$\left\langle \bar{M} \left| Q_1 \right| M \right\rangle = 4(\bar{u}\gamma^\alpha P_L v)(\bar{v}\gamma_\alpha P_L u)\left|\int \frac{d^3q}{(2\pi)^3}\tilde{\psi}(q)\right|^2$$

Where, $\left|\int \frac{d^3q}{(2\pi)^3}\tilde{\psi}(q)\right|^2 = \frac{1}{4m_\mu m_e}\left|\psi(0)\right|^2$

spatial wavefuntion at the origin

non-relativistic spinors

$$u = \sqrt{m_e}\begin{pmatrix}\xi\\ \xi\end{pmatrix}, \qquad v = \sqrt{m_e}\begin{pmatrix}\eta\\ -\eta\end{pmatrix},$$

$$\bar{u} = \sqrt{m_\mu}\left(\xi^\dagger, \xi^\dagger\right)\gamma^o, \qquad \bar{v} = \sqrt{m_\mu}\left(\eta^\dagger, -\eta^\dagger\right)\gamma^o$$

spinor products: spin-0, spin-1

Spin-1 with 3 possible polarizations

$$\xi\eta^\dagger = \frac{1}{\sqrt{2}}\mathbf{1}_{2\times2} \qquad \eta\xi^\dagger = \frac{1}{\sqrt{2}}\vec{\epsilon}^{\,*}\cdot\vec{\sigma}$$

$$\left\langle \bar{M} \left| Q_1 \right| M \right\rangle_{spin-0} = 2\left|\psi(0)\right|^2$$

$$\left\langle \bar{M} \left| Q_1 \right| M \right\rangle_{spin-1} = -6\left|\psi(0)\right|^2$$

Calculation of Matrix Elements

Matrix elements from operators -

Spin-0, para-muonium

$$\left\langle \bar{M} \middle| Q_1 \middle| M \right\rangle = 2\,|\psi(0)|^2, \quad \left\langle \bar{M} \middle| Q_2 \middle| M \right\rangle = 2\,|\psi(0)|^2$$

$$\left\langle \bar{M} \middle| Q_3 \middle| M \right\rangle = -3\,|\psi(0)|^2, \quad \left\langle \bar{M} \middle| Q_4 \middle| M \right\rangle = -\frac{1}{2}\,|\psi(0)|^2$$

$$\left\langle \bar{M} \middle| Q_5 \middle| M \right\rangle = -\frac{1}{2}\,|\psi(0)|^2$$

Spin-1, ortho-muonium

$$\left\langle \bar{M} \middle| Q_1 \middle| M \right\rangle = -6\,|\psi(0)|^2, \quad \left\langle \bar{M} \middle| Q_2 \middle| M \right\rangle = -6\,|\psi(0)|^2$$

$$\left\langle \bar{M} \middle| Q_3 \middle| M \right\rangle = -3\,|\psi(0)|^2, \quad \left\langle \bar{M} \middle| Q_4 \middle| M \right\rangle = -\frac{3}{2}\,|\psi(0)|^2$$

$$\left\langle \bar{M} \middle| Q_5 \middle| M \right\rangle = -\frac{3}{2}\,|\psi(0)|^2$$

Calculation of Δm - Results

- Relation of Δm to m_{12}

$$|\Delta m| = 2\left|\text{Re } m_{12}\right|$$

Using,

$$\psi_{100} = \frac{1}{\sqrt{\pi a_{M\bar{M}}^3}} e^{-r/a_{M\bar{M}}}$$

muonium Bohr radius

- Δm in terms of Wilson coefficients

$$\Delta m_{spin-0} = \left(\frac{(m_{red}\alpha)^3}{\pi \Lambda_1^2}\right)\left(2C_1 + 2C_2 - 3C_3 - \frac{1}{2}C_4 - \frac{1}{2}C_5\right)$$

$$\Delta m_{spin-1} = \left(\frac{(m_{red}\alpha)^3}{\pi \Lambda_1^2}\right)\left(-6C_1 - 6C_2 - 3C_3 - \frac{3}{2}C_4 - \frac{3}{2}C_5\right)$$

Calculation of $\Delta\Gamma$

- Calculated Δm from operators that change the lepton quantum flavor number by two units, $\Delta L_\mu = 2$

- Particles can also oscillate by two insertions of operators that change lepton flavor number by one unit, $\Delta L_\mu = 1$

- Mass eigenstates has mass difference (Δm), but they also have width difference ($\Delta\Gamma$)

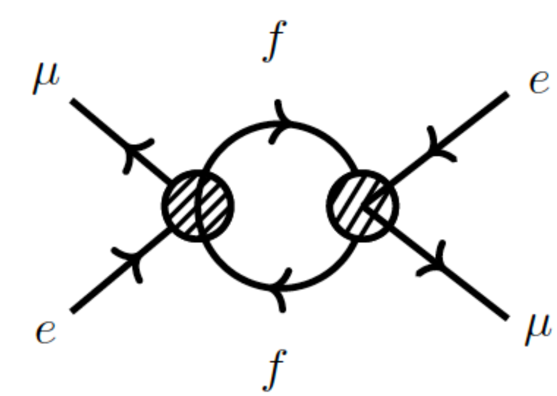
- Here the $\Delta L_\mu = 1$ insertions are the only contribution to $\Delta\Gamma$

- $\Delta\Gamma$ calculated for the first time!

Δm calculated for Q_2
[Fienberg and Wienberg, (1961)]

Calculation of $\Delta\Gamma$

- $\Delta\Gamma^{ff}$ is generated by on-shell degrees of freedom, $f = e, \nu$
- Two insertions of $\Delta L_\mu = 1$ operators
- Most general dimension-six Lagrangian



$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=1} = -\frac{1}{\Lambda^2}\sum_f \Big[\left(C_{VR}^f \, \overline{\mu}\gamma^\alpha P_R e + C_{VL}^f \, \overline{\mu}\gamma^\alpha P_L e \right) \, \overline{f}\gamma_\alpha f$$
$$+ \left(C_{AR}^f \, \overline{\mu}\gamma^\alpha P_R e + C_{AL}^q \, \overline{\mu}\gamma^\alpha P_L e \right) \, \overline{f}\gamma_\alpha\gamma_5 f$$
$$+ m_e m_f G_F \left(C_{SR}^f \, \overline{\mu} P_L e + C_{SL}^f \, \overline{\mu} P_R e \right) \, \overline{f} f$$
$$+ m_e m_f G_F \left(C_{PR}^f \, \overline{\mu} P_L \, e + C_{PL}^f \, \overline{\mu} P_R e \right) \, \overline{f}\gamma_5 f$$
$$+ m_e m_f G_F \left(C_{TR}^f \, \overline{\mu}\sigma^{\alpha\beta} P_L e + C_{TL}^f \, \overline{\mu}\sigma^{\alpha\beta} P_R e \right) \, \overline{f}\sigma_{\alpha\beta} f \; + \; h.c. \Big]$$

Calculation of $\Delta\Gamma$ - Matrix Elements

- Neglecting terms proportional to m_e, we find the surviving matrix elements

Proportional to $\frac{1}{\Lambda^4}$

$$\Gamma_{12}^{VLee} = \left(\frac{C_{VL}}{\Lambda^2}\right)^2 \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_1\rangle + \langle Q_5\rangle\right),$$

$$\Gamma_{12}^{VRee} = \left(\frac{C_{VR}}{\Lambda^2}\right)^2 \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_2\rangle + \langle Q_4\rangle\right),$$

$$\Gamma_{12}^{ALee} = \left(\frac{C_{AL}}{\Lambda^2}\right)^2 \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_1\rangle + \langle Q_5\rangle\right),$$

$$\Gamma_{12}^{ARee} = \left(\frac{C_{AR}}{\Lambda^2}\right)^2 \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_2\rangle + \langle Q_4\rangle\right),$$

$$\Gamma_{12}^{(VL,VR)ee} = \left(\frac{C_{VL}C_{VR}}{\Lambda^4}\right) \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_3\rangle + \langle Q_6\rangle\right),$$

$$\Gamma_{12}^{(AL,AR)ee} = \left(\frac{C_{AL}C_{AR}}{\Lambda^4}\right) \left(\frac{M_M^2}{12\pi}\right)\left(\langle Q_3\rangle + \langle Q_6\rangle\right)$$

Calculation of $\Delta\Gamma$ - Results

- $\langle Q_i \rangle$ is defined by

$$\langle Q_i \rangle = \left\langle \overline{M} \left| Q_i \right| M \right\rangle$$

- After neglecting terms proportional to m_e and using

$$|\Delta\Gamma| = 2 \left| \text{Re}\ \Gamma_{12} \right|$$

- We get $\Delta\Gamma^{ee}$ for spin-0:

$$\Delta\Gamma^{ee}_{\text{spin}-0} = \frac{(m_{red}\alpha)^3}{4\pi^2} \frac{M_M^2}{\Lambda^4} \left[C_{VL}^2 + C_{VR}^2 + C_{AL}^2 + C_{AR}^2 - C_{VL}C_{VR} - C_{AL}C_{AR} \right]$$

- And $\Delta\Gamma^{ee}$ for spin-1:

$$\Delta\Gamma^{ee}_{\text{spin}-1} = -\frac{(m_{red}\alpha)^3}{3\pi^2} \frac{M_M^2}{\Lambda^4} \left[5C_{VL}^2 + 5C_{VR}^2 + 5C_{AL}^2 + 5C_{AR}^2 + C_{VL}C_{VR} + C_{AL}C_{AR} \right]$$

- How large is $\Delta\Gamma^{ee}$: data?

Calculation of $\Delta\Gamma$ - Constraints

- Current upper bound on $\mu \to eee$ [SINDRUM experiment (1988)]

$$BR(\mu \to eee) \leq 1.0 \times 10^{-12} @ 90\,\% \text{ C.L.}$$

- Decay widths of $\mu \to eee$

$$\Gamma_{ij} = \frac{m_\mu^5}{768\pi^3}\left(\frac{C_{ij}}{\Lambda^2}\right)^2$$

$i = V, A$ and $j = L, R$

- Bounds on the Wilson coefficients

$$C_{VL}, C_{VR}, C_{AL}, C_{AR}/\Lambda^2 \leq 2.3 \times 10^{-11}\text{GeV}^{-2}$$

Calculation of $\Delta\Gamma$ - Results

- From bounds on the Wilson coefficients we can put a constraint on $\Delta\Gamma^{ee}$

$$\Delta\Gamma^{ee}_{\text{spin}-0} \leq 1.5 \times 10^{-41}\text{GeV} \qquad \Delta\Gamma^{ee}_{\text{spin}-1} \leq 1.7 \times 10^{-40}\text{GeV}$$

- Like B^0 and D^0 mixing we can constraint a parameter y_{M_e} for para and ortho-muonium

$$y_{M_e,spin-0,1} \equiv \frac{\Delta\Gamma^{ee}_{spin-0,1}}{2\Gamma_{avg}}$$

$$y_{M_e,\text{spin}-0} \leq 2.5 \times 10^{-23} \qquad y_{M_e,\text{spin}-1} \leq 2.8 \times 10^{-22}$$

Compare $D\overline{D}$: $y_D = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$

[BABAR collaboration, 2007]

Summary

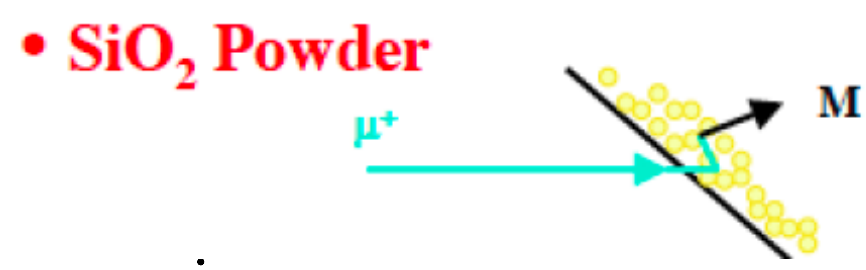
- Calculated Δm with the most general basis of dimension-six operators
- Calculated $\Delta\Gamma^{ee}$ for spin-0 and spin-1 for the first time
- Using the current experimental upper bound on $\mu \to eee$ we found constraints on $\Delta\Gamma^{ee}$ and the experimental observable y_{M_e}

Questions?

Renae Conlin
au9969@wayne.edu

Searches for Muonium Oscillations

- Most recent search for muonium oscillations was at the Paul Scherrer Institute (PSI)
  - Probability of $P_{M\overline{M}} \leq 8.2 \times 10^{-11} @ 90\,\%$ C.L.
  - $P(M \rightarrow \bar{M}) = \int_0^\infty \frac{dt}{\tau} e^{-t/\tau} |\langle \bar{M} | M(t) \rangle|^2$



- Beam of μ^+ at SIO_2 powder target, electron capture to form muonium
- If $M(\mu^+ e^-) \rightarrow \overline{M}(\mu^- e^+)$ then an energetic e^- and an e^+ would be detected, with the e^- from the $\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu$ decay
- Background
  - Bhabha scattering of the e^- and a e^+ from the μ^+ decay
  - $\mu^+ \rightarrow e^- e^+ e^+ \nu_e \overline{\nu}_\mu$

Muonium Oscillation Formalism

- In the Schrödinger picture start with a flavor eigenstate

$$|\psi\rangle = |\psi, t=0\rangle \qquad\qquad |\psi, t\rangle = U(t,0)|\psi\rangle = e^{-iHt}|\psi\rangle$$

- We get the Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

- Generalizing to a two state system describing muonium oscillations

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle\\|\overline{M}(t)\rangle\end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix}|M(t)\rangle\\|\overline{M}(t)\rangle\end{pmatrix}$$

- With matrix Hamiltonian $H = \left(m - i\frac{\Gamma}{2}\right)$

Calculation of $\Delta\Gamma$ - Constraints

- Want constraints on Wilson coefficients - C_i's
- For some operators you can relate the decay $\mu \to eee$ to $\Delta\Gamma$ for a single operator insertion
- Use $\mu \to eee$ experimental data to contain coefficients
- Current upper bound measured by Sindrum

$$BR(\mu \to eee) \leq 1.0 \times 10^{-12} @ 90\,\% \text{ C.L.}$$

- Using the muons average decay width and the branching ratio from $\mu \to eee$, we can find the upper bound on the decay width for $\mu \to eee$

$$\Gamma(\mu \to eee) \leq 3.0 \times 10^{-31} \text{GeV}$$