Muonium-Antimuonium Oscillations in Effective Field Theory

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Outline

• Motivation

• Muonium Oscillation Formalism

• Calculation of mixing parameters $\Delta m$ and $\Delta \Gamma$

• Summary
Motivation

- Neutrino’s mix and change flavor - must have non-zero mass
  - This can lead to flavor violating processes in the charged lepton sector
  - Lepton flavor violating (LFV) processes are highly suppressed in the standard model, e.g. $\mu \to e\gamma \quad \text{Br}(\mu \to e\gamma)_{SM} \sim 10^{-54}$

- LFV processes are highly suppressed in the standard model $\Rightarrow$ no background for NP!

- LFV processes are searched for in various processes:
  - $\Delta L_\mu = 1$: $\mu \to e\gamma$, $\mu \to eee$ and $\mu + N \to e + N$
  - $\Delta L_\mu = 2$: Muonium anti-muonium oscillations
    - In some models we can expect $\Delta L_\mu = 2$ contributions to be the dominant ones, e.g. doubly charged Higgs has a tree level contribution to $\Delta L_\mu = 2$, but not to $\mu \to e\gamma$
Muonium Oscillations: $M \rightarrow \overline{M}$

- Muonium is a non-relativistic Coulombic bound state of an anti-muon $\mu^+$ and an electron $e$.
  - Can be Spin-0 (singlet): para-muonium
  - Spin-1 (triplet): ortho-muonium
- An oscillation is the process
  $$M(\mu^+e^-) \rightarrow \overline{M}(\mu^-e^+)$$
  - It violates muon lepton number by two units $\Delta L_\mu = 2$, i.e.: it can probe different types of NP than $\mu \rightarrow e\gamma$ or $\mu + N \rightarrow e + N$.
  - Conversion rate was calculated in several NP models with heavy DOF.
Muonium Oscillation Formalism

- An effective theory approach: all possible heavy NP models

- Most general Lagrangian
  \[ \mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda^2} \sum_i c_i(\mu)Q_i \]
  - \( c_i \)'s are the Wilson coefficients
    - Determined by the (UV) physics at some NP scale \( \Lambda \)

- \( Q_i \)'s are the dimension-six operators
  - Reflect degrees of freedom relevant at the scale in which a process takes place
Muonium Oscillation Formalism

• Similar to meson-antimeson oscillations, but unlike $K\bar{K}$ or $B\bar{B}$ oscillations both spin-0 and spin-1 states can oscillate.

• The time development of $M$ and $\bar{M}$ are coupled and is given by a Schrödinger equation:

$$i \frac{d}{dt} \left( \left| M(t) \right\rangle \right) = \left( m - i \frac{\Gamma}{2} \right) \left( \left| M(t) \right\rangle \right)$$

- matrix Hamiltonian $m$ and $\Gamma$ are 2x2 Hermitian matrices: the mass matrix and the decay matrix.

• Assume CPT invariance, then the diagonal and off diagonal elements are:

$$m_{11} = m_{22}, \quad \Gamma_{11} = \Gamma_{22}, \quad m_{12} = m_{21}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$
Muonium Oscillation Formalism

• The off diagonal element of this matrix is given by

\[
\begin{pmatrix}
m - \frac{i}{2} \Gamma \\
\end{pmatrix}_{12} = \frac{1}{2M_M} \langle M \left| H^{\Delta_{\mu} = 2} \right| M \rangle + \frac{1}{2M_M} \sum_n \frac{\langle M \left| H^{\Delta_{\mu} = 1} \right| n \rangle \langle n \left| H^{\Delta_{\mu} = 1} \right| M \rangle}{M_M - E_n + i\epsilon}
\]

• Since \( \left( m - \frac{i}{2} \Gamma \right) \neq 0 \) then \( M \) and \( \overline{M} \) are not mass eigenstates: diagonalize!

• The CP conserving mass eigenstates, \( M_1 \) and \( M_2 \), are linear combinations of \( M \) and \( \overline{M} \)

\[
\left| M_{1,2} \right> = \frac{1}{\sqrt{2}} (\left| M \right> \pm \left| \overline{M} \right>)
\]
Muonium Oscillation Time Evolution

- Time development of $M$ and $\bar{M}$

$$ |M(t)\rangle = g_+(t) |M\rangle + g_-(t) |\bar{M}\rangle \quad \quad |\bar{M}(t)\rangle = g_-(t) |M\rangle + g_+(t) |\bar{M}\rangle $$

Where, \( g_\pm(t) = \frac{1}{2} e^{-\Gamma_1 t / 2} e^{-i \Delta m t} \left[ 1 \pm e^{-\Delta \Gamma t / 2} e^{i \Delta m t} \right] \)

- Mass eigenstates $M_1$ and $M_2$ has mass difference ($\Delta m$), and width difference ($\Delta \Gamma$)

\[ \Delta m \equiv m_1 - m_2, \quad \Delta \Gamma \equiv \Gamma_2 - \Gamma_1 \]

- Dependence on $x$ and $y$!

\[ \frac{\Gamma(M \to \bar{f})}{\Gamma(\bar{M} \to \bar{f})} \sim R(x, y), \quad \quad R(x, y) = \frac{1}{4} (x^2 + y^2) \]

\[ \Gamma(M \to \bar{f})(t) = N_f \left| \langle \bar{f} | S | M(t) \rangle \right|^2, \]

\[ \Gamma(\bar{M} \to \bar{f})(t) = N_f \left| \langle \bar{f} | S | \bar{M}(t) \rangle \right|^2 \]

- Need to calculate $x$ and $y$!
• Our $\Delta L_\mu = 2$ Lagrangian

$$\mathcal{L}_{\Delta L_{\mu}^{2}} = -\frac{1}{\Lambda_{1}^{2}} \sum_{i} C_{i}(\mu)Q_{i}(\mu)$$

• Most general set of operators, $Q_{i}$’s

$$Q_{1} = (\bar{\mu}\gamma_{\alpha}P_{L}e)(\bar{\mu}\gamma^{\alpha}P_{L}e), \quad Q_{2} = (\bar{\mu}\gamma_{\alpha}P_{R}e)(\bar{\mu}\gamma^{\alpha}P_{R}e), \quad Q_{3} = (\bar{\mu}\gamma_{\alpha}P_{L}e)(\bar{\mu}\gamma^{\alpha}P_{R}e),$$

$$Q_{4} = (\bar{\mu}P_{R}e)(\bar{\mu}P_{R}e), \quad Q_{5} = (\bar{\mu}P_{L}e)(\bar{\mu}P_{L}e)$$

• Other possible structures can be Fierz’d into the operators above
Calculation of Matrix Elements

- **Matrix element** $m_{12}$

$$m_{12} = \left\langle M \left| -L_{\text{eff}}^{\Delta L_\mu=2} \right| M \right\rangle = \sum_i \frac{C_i}{\Lambda_i^2} \left\langle M \left| Q_i \right| M \right\rangle$$

- **Muonium is a non-relativistic Coulombic bound state**

$$|M(0)\rangle = \sqrt{2E_q 2E_{-q}} \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}(q) a^{(e)\dagger}_{q} b^{(\mu)\dagger}_{-q} |0\rangle$$

$\tilde{\psi}(q)$ is the Fourier transform of the spatial wave function $\psi(x)$

- **Perturbative QED bound state: can calculate!**
**Calculation of Matrix Elements**

Example: 

\[ Q_1 = (\bar{\mu}\gamma_\alpha P_L e) (\bar{\mu}\gamma^\alpha P_L e) \]

\[
\langle \bar{M} | Q_1 | M \rangle = 4(\bar{\mu}\gamma^\alpha P_L v)(\bar{\nu}\gamma_\alpha P_L u) \left| \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}(q) \right|^2
\]

where,

\[
\left| \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}(q) \right|^2 = \frac{1}{4m_\mu m_e} |\psi(0)|^2
\]

Spatial wavefunction at the origin

Non-relativistic spinors

\[
u = \sqrt{m_e} \left( \begin{array}{c} \eta \\ -\bar{\eta} \end{array} \right),
\]

\[
u = \sqrt{m_e} \left( \begin{array}{c} \eta \\ -\bar{\eta} \end{array} \right),
\]

Spinor products: spin-0, spin-1

Spin-1 with 3 possible polarizations

\[
\xi \eta^\dagger = \frac{1}{\sqrt{2}} 1_{2\times 2}, \quad \eta \xi^\dagger = \frac{1}{\sqrt{2}} \varepsilon^* \cdot \sigma
\]

\[
\langle \bar{M} | Q_1 | M \rangle_{\text{spin}-0} = 2 |\psi(0)|^2
\]

\[
\langle \bar{M} | Q_1 | M \rangle_{\text{spin}-1} = -6 |\psi(0)|^2
\]
Calculation of Matrix Elements

Matrix elements from operators -

Spin-0, para-muonium

\[
\langle \bar{M} | Q_1 | M \rangle = 2 |\psi(0)|^2, \quad \langle \bar{M} | Q_2 | M \rangle = 2 |\psi(0)|^2
\]

\[
\langle \bar{M} | Q_3 | M \rangle = -3 |\psi(0)|^2, \quad \langle \bar{M} | Q_4 | M \rangle = -\frac{1}{2} |\psi(0)|^2
\]

\[
\langle \bar{M} | Q_5 | M \rangle = -\frac{1}{2} |\psi(0)|^2
\]

Spin-1, ortho-muonium

\[
\langle \bar{M} | Q_1 | M \rangle = -6 |\psi(0)|^2, \quad \langle \bar{M} | Q_2 | M \rangle = -6 |\psi(0)|^2
\]

\[
\langle \bar{M} | Q_3 | M \rangle = -3 |\psi(0)|^2, \quad \langle \bar{M} | Q_4 | M \rangle = -\frac{3}{2} |\psi(0)|^2
\]

\[
\langle \bar{M} | Q_5 | M \rangle = -\frac{3}{2} |\psi(0)|^2
\]
Calculation of $\Delta m$ - Results

- Relation of $\Delta m$ to $m_{12}$

$$ |\Delta m| = 2 \left| \text{Re } m_{12} \right| $$

- $\Delta m$ in terms of Wilson coefficients

$$ \Delta m_{\text{spin}^{-0}} = \left( \frac{(m_{\text{red}}\alpha)^3}{\pi \Lambda_i^2} \right) \left( 2C_1 + 2C_2 - 3C_3 - \frac{1}{2}C_4 - \frac{1}{2}C_5 \right) $$

$$ \Delta m_{\text{spin}^{-1}} = \left( \frac{(m_{\text{red}}\alpha)^3}{\pi \Lambda_i^2} \right) \left( -6C_1 - 6C_2 - 3C_3 - \frac{3}{2}C_4 - \frac{3}{2}C_5 \right) $$

Using,

$$ \psi_{100} = \frac{1}{\sqrt{\pi a_{\text{MM}}^3}} e^{-r/a_{\text{MM}}} $$

muonium Bohr radius
Calculation of $\Delta \Gamma$

- Calculated $\Delta m$ from operators that change the lepton quantum flavor number by two units, $\Delta L_\mu = 2$

- Particles can also oscillate by two insertions of operators that change lepton flavor number by one unit, $\Delta L_\mu = 1$

- Mass eigenstates has mass difference ($\Delta m$), but they also have width difference ($\Delta \Gamma$)

- Here the $\Delta L_\mu = 1$ insertions are the only contribution to $\Delta \Gamma$

- $\Delta \Gamma$ calculated for the first time!

$\Delta m$ calculated for $Q_2$

[Fienberg and Wienberg, (1961)]
Calculation of $\Delta \Gamma$

- $\Delta \Gamma^{ff}$ is generated by on-shell degrees of freedom, $f = e, \nu$
- Two insertions of $\Delta L_\mu = 1$ operators
- Most general dimension-six Lagrangian

\[
\mathcal{L}_{\text{eff}}^{\Delta L_\mu=1} = -\frac{1}{\Lambda^2} \sum_f \left[ \left( C_{VR}^f \bar{\mu} \gamma^\alpha P_R e + C_{VL}^f \bar{\mu} \gamma^\alpha P_L e \right) \bar{f} \gamma_\alpha f \\
+ \left( C_{AR}^f \bar{\mu} \gamma^\alpha P_R e + C_{AL}^f \bar{\mu} \gamma^\alpha P_L e \right) \bar{f} \gamma_\alpha \gamma_5 f \\
+ m_e m_f G_F \left( C_{SR}^f \bar{\mu} P_L e + C_{SL}^f \bar{\mu} P_R e \right) \bar{f} f \\
+ m_e m_f G_F \left( C_{PR}^f \bar{\mu} P_L e + C_{PL}^f \bar{\mu} P_R e \right) \bar{f} \gamma_5 f \\
+ m_e m_f G_F \left( C_{TR}^f \bar{\mu} \sigma^{\alpha\beta} P_L e + C_{TL}^f \bar{\mu} \sigma^{\alpha\beta} P_R e \right) \bar{f} \sigma_{\alpha\beta} f + h.c. \right]
\]
Calculation of $\Delta \Gamma$ - Matrix Elements

- Neglecting terms proportional to $m_e$, we find the surviving matrix elements

\[
\Gamma_{12}^{VLee} = \left( \frac{C_{VL}}{\Lambda^2} \right)^2 \left( \frac{M_M^2}{12\pi} \right) (\langle Q_1 \rangle + \langle Q_5 \rangle),
\]

\[
\Gamma_{12}^{ALee} = \left( \frac{C_{AL}}{\Lambda^2} \right)^2 \left( \frac{M_M^2}{12\pi} \right) (\langle Q_1 \rangle + \langle Q_5 \rangle),
\]

\[
\Gamma_{12}^{(VL,VR)ee} = \left( \frac{C_{VL}C_{VR}}{\Lambda^4} \right) \left( \frac{M_M^2}{12\pi} \right) (\langle Q_3 \rangle + \langle Q_6 \rangle),
\]

\[
\Gamma_{12}^{(AL,AR)ee} = \left( \frac{C_{AL}C_{AR}}{\Lambda^4} \right) \left( \frac{M_M^2}{12\pi} \right) (\langle Q_3 \rangle + \langle Q_6 \rangle)
\]
Calculation of $\Delta \Gamma$ - Results

- $\langle Q_i \rangle$ is defined by
  $$\langle Q_i \rangle = \langle M | Q_i | M \rangle$$

- After neglecting terms proportional to $m_e$ and using $| \Delta \Gamma | = 2 | \text{Re} \, \Gamma_{12} |$

- We get $\Delta \Gamma^{ee}$ for spin-0:
  $$\Delta \Gamma^{ee}_{\text{spin-0}} = \frac{(m_{\text{red}} \alpha)^3}{4\pi^2} \frac{M^2_M}{\Lambda^4} \left[ C_{VL}^2 + C_{VR}^2 + C_{AL}^2 + C_{AR}^2 - C_{VL} C_{VR} - C_{AL} C_{AR} \right]$$

- And $\Delta \Gamma^{ee}$ for spin-1:
  $$\Delta \Gamma^{ee}_{\text{spin-1}} = -\frac{(m_{\text{red}} \alpha)^3}{3\pi^2} \frac{M^2_M}{\Lambda^4} \left[ 5C_{VL}^2 + 5C_{VR}^2 + 5C_{AL}^2 + 5C_{AR}^2 + C_{VL} C_{VR} + C_{AL} C_{AR} \right]$$

- How large is $\Delta \Gamma^{ee}$: data?
Calculation of $\Delta \Gamma$ - Constraints

- Current upper bound on $\mu \rightarrow eee$ [SINDRUM experiment (1988)]

  \[ BR(\mu \rightarrow eee) \leq 1.0 \times 10^{-12} @ 90\% \text{ C.L.} \]

- Decay widths of $\mu \rightarrow eee$

  \[ \Gamma_{ij} = \frac{m_\mu^5}{768\pi^3} \left( \frac{C_{ij}}{\Lambda^2} \right)^2 \]

  \[ i = V, A \text{ and } j = L, R \]

- Bounds on the Wilson coefficients

  \[ C_{VL}, C_{VR}, C_{AL}, C_{AR}/\Lambda^2 \leq 2.3 \times 10^{-11}\text{GeV}^{-2} \]
Calculation of $\Delta \Gamma$ - Results

- From bounds on the Wilson coefficients we can put a constraint on $\Delta \Gamma^{ee}$
  \[ \Delta \Gamma^{ee}_{\text{spin}-0} \leq 1.5 \times 10^{-41}\text{GeV} \quad \Delta \Gamma^{ee}_{\text{spin}-1} \leq 1.7 \times 10^{-40}\text{GeV} \]

- Like $B^0$ and $D^0$ mixing we can constraint a parameter $y_{M_e}$ for para and ortho-muonium
  \[ y_{M_e,\text{spin}-0,1} \equiv \frac{\Delta \Gamma^{ee}_{\text{spin}-0,1}}{2\Gamma_{\text{avg}}} \]

  \[ y_{M_e,\text{spin}-0} \leq 2.5 \times 10^{-23} \quad y_{M_e,\text{spin}-1} \leq 2.8 \times 10^{-22} \]

Compare $D \bar{D}$: $y_D = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$

[BABAR collaboration, 2007]
Summary

• Calculated $\Delta m$ with the most general basis of dimension-six operators

• Calculated $\Delta \Gamma_{ee}$ for spin-0 and spin-1 for the first time

• Using the current experimental upper bound on $\mu \rightarrow eee$ we found constraints on $\Delta \Gamma_{ee}$ and the experimental observable $y_{Me}$
Questions?

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Searches for Muonium Oscillations

- Most recent search for muonium oscillations was at the Paul Scherrer Institute (PSI)

  - Probability of $P_{\mu\mu} \leq 8.2 \times 10^{-11} @ 90\%$ C.L.

  $$P(M \rightarrow \bar{M}) = \int_0^{\infty} dt \frac{e^{-t/\tau}}{\tau} |\langle \bar{M} | M(t) \rangle|^2$$

- Beam of $\mu^+$ at SiO$_2$ powder target, electron capture to form muonium

- If $M(\mu^+e^-) \rightarrow \bar{M}(\mu^-e^+)$ then an energetic $e^-$ and an $e^+$ would be detected, with the $e^-$ from the $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ decay

- Background

  - Bhabha scattering of the $e^-$ and a $e^+$ from the $\mu^+$ decay

  - $\mu^+ \rightarrow e^-e^+\nu_e\bar{\nu}_\mu$
Muonium Oscillation Formalism

• In the Schrödinger picture start with a flavor eigenstate

\[ |\psi\rangle = |\psi, t = 0\rangle \]

\[ |\psi, t\rangle = U(t,0) |\psi\rangle = e^{-iHt} |\psi\rangle \]

• We get the Schrödinger equation

\[ i \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \]

• Generalizing to a two state system describing muonium oscillations

\[ i \frac{d}{dt} \begin{pmatrix} |M(t)\rangle \\ |\bar{M}(t)\rangle \end{pmatrix} = \begin{pmatrix} m - i \frac{\Gamma}{2} \end{pmatrix} \begin{pmatrix} |M(t)\rangle \\ |\bar{M}(t)\rangle \end{pmatrix} \]

• With matrix Hamiltonian

\[ H = \begin{pmatrix} m - i \frac{\Gamma}{2} \end{pmatrix} \]
Calculation of $\Delta \Gamma$ - Constraints

- Want constraints on Wilson coefficients - $C_i$'s
- For some operators you can relate the decay $\mu \rightarrow eee$ to $\Delta \Gamma$ for a single operator insertion
- Use $\mu \rightarrow eee$ experimental data to contain coefficients
- Current upper bound measured by Sindrum

$$BR(\mu \rightarrow eee) \leq 1.0 \times 10^{-12} @ 90 \% \text{ C.L.}$$

- Using the muons average decay width and the branching ratio from $\mu \rightarrow eee$, we can find the upper bound on the decay width for $\mu \rightarrow eee$

$$\Gamma(\mu \rightarrow eee) \leq 3.0 \times 10^{-31}\text{GeV}$$