WAYNE STATE UNIVERSITY

The Proton Radius Puzzle

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2(q^2)q^\nu\right]u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2\rangle_E^p = 6\,\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

The proton radius puzzle

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]

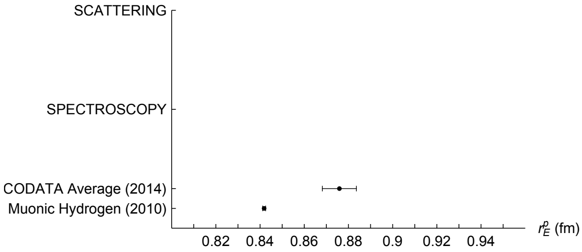
  $r_E^p = 0.84184(67)$ fm

  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]

  $r_E^p = 0.87680(690)$ fm

  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]

  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!** This is the proton radius puzzle



Ways to extract the proton charge radius

- What could be the reason for the discrepancy?
  - Experimental problem?
  - Theoretical problem?
  - **New Physics?**

- Four ways to extract the proton charge radius
  - Muonic hydrogen spectroscopy
  - Muon proton scattering
  - Electron proton scattering
  - Regular hydrogen spectroscopy

- What is the current status of each method?
- Declaimer: I will mostly focus on work I am involved in

Proton charge radius from muonic hydrogen

[Hill, GP, PRD **95**, 094017 (2017)]

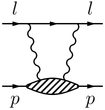
Muonic hydrogen theory

- Is there a problem with muonic hydrogen *theory*? Potentially yes! [Hill, GP PRL **107** 160402 (2011)]

- Muonic hydrogen measures ΔE and translates it to r_E^p
  - [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]

  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

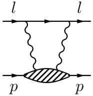
- Apart from r_E^p need two-photon exchange (TPE)



- Imaginary part related to data: form factors and structure functions

- Cannot reproduce TPE from imaginary part: need $W_1(0, Q^2)$ which is not well-constrained

$W_1(0, Q^2)$



- $W_1(0, Q^2)$ is calculable for small Q^2 using NRQED

  The photon sees the proton "almost" like an elementary particle

  [Hill, GP, PRL **107** 160402 (2011)]

- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE)

  The photon "sees" the quarks and gluons inside the proton

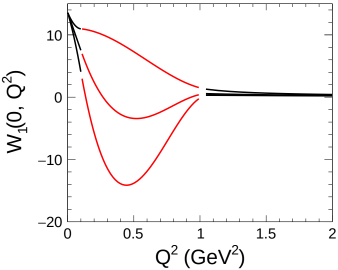
- Spin-0 calculated in

  [J. C. Collins, NPB **149**, 90 (1979)]
- Spin-2 calculated and spin-0 corrected in

  [Hill, GP PRD **95**, 094017 (2017)]

Two Photon Exchange: Modeling

- Small Q^2: NRQED, Large Q^2: OPE, Between: interpolation



- Energy contribution: $\delta E(2S)^{W_1(0,Q^2)} \in [-0.046\,\text{meV}, -0.021\,\text{meV}]$
  To explain the puzzle need this to be ~ -0.3 meV
- Caveats: OPE valid for larger Q^2, W_1 different than interpolation
- How to test? MUSE: new $\mu - p$ scattering experiment at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]
- Need to connect $\mu - p$ scattering and muonic hydrogen
  Using a new effective field theory: QED-NRQED

Proton charge radius from $\mu - p$ scattering

[Dye, Gonderinger, GP, PRD **94** 013006 (2016)]

[Dye, Gonderinger, GP, PRD **100** 054010 (2019)]

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV $\ll m_\mu, m_p$

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:

- Use QED for muon alone
- Use NRQED for proton alone
- Use contact terms for combined muon-proton interaction

  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in

  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- QED-NRQED calculation
  [Dye, Gonderinger, GP, PRD **94** 013006 (2016)]
  reproduces TPE at the lowest order in $1/m_p$
  [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)]

- QED-NRQED allows to calculate $1/m_p$ corrections
  One γ exchange: QED-NRQED $= 1/m_p$ expansion of form factors
  [Dye, Gonderinger, GP, PRD **94** 013006 (2016)]

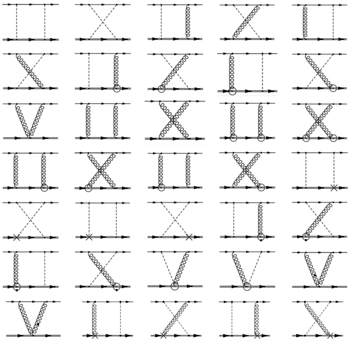
- Connecting to muonic hydrogen requires contact interactions

$$\mathcal{L}_{\ell\psi} = \frac{b_1}{m_p^2}\psi^\dagger\psi\,\bar{\ell}\gamma^0\ell + \frac{b_2}{m_p^2}\psi^\dagger\sigma^i\psi\,\bar{\ell}\gamma^i\gamma^5\ell + \mathcal{O}\left(1/M^3\right)$$

  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Calculation of b_1 and b_2 was done in
  [Dye, Gonderinger, GP, PRD **100** 054010 (2019)]

QED-NRQED calculation



- *Surprisingly* $b_1 = 0$ at $\mathcal{O}(Z^2\alpha^2)$ (see backup slides)
  [Dye, Gonderinger, GP, PRD **100** 054010 (2019)]
- QED-NRQED scattering not sensitive to SI TPE effects from scales above m_p at $\mathcal{O}(Z^2\alpha^2/m_p^2)$
- MUSE experiment is much less sensitive to TPE
  but extraction of the proton charge radius will be more robust

Proton charge radius from $e - p$ scattering

[GP, arXiv:2004.03077 (hep-ph)]

How to extract r_E^p from scattering data?

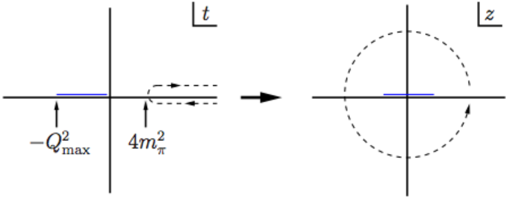
- Main problem: form factors are non-perturbative objects.
- **Nobody** knows the *exact* functional form of G_E^p
- Using models (dipole, polynomial, etc.) can bias the extraction of r_E^p
- Should use model-independent z-expansion
- The method for **meson** form factors, see e.g.
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]
- First applied to **baryon** form factors in
  [Hill, GP PRD **82** 113005 (2010)]

z expansion

- Notation: $q^2 = t = -Q^2$
- $G_E^p(t)$ analytic outside a cut $t \in [4m_\pi^2, \infty]$
- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$

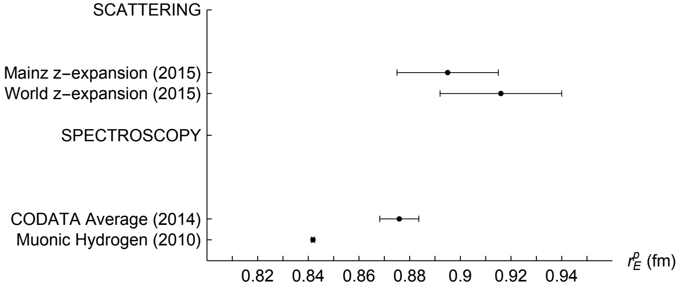


- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

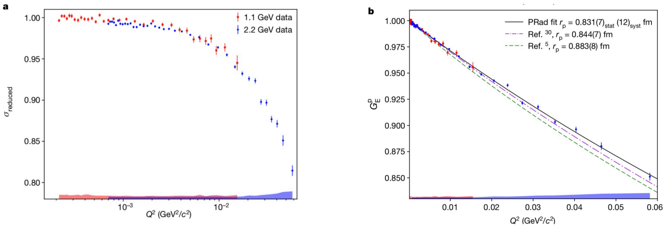
Extracting r_E^p using the z expansion

- First extraction using z expansion [Hill, GP PRD **82** 113005 (2010)]
  - $r_E^p = 0.871 \pm 0.009$ fm
- Most recent extraction using z expansion [Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

  Analyze high-statistics "Mainz" data [Bernauer et al. PRL **105**, 242001 (2010)] and world data (excluding Mainz)
  - World data: $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz data: $r_E^p = 0.895 \pm 0.020$ fm



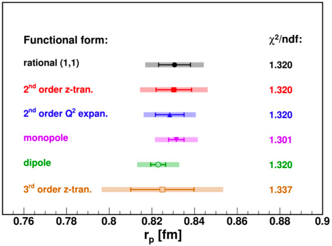
November 2019 new scattering results



- PRad: new low-Q^2 $e - p$ scattering experiment at JLab [Xiong et al., Nature **575**, 147 (2019)]
- PRad reached the lowest Q^2 ever in $e - p$ scattering: 2.1×10^{-4} GeV^2
- Small Q^2 is meant to reduce extrapolation errors
- PRad fitted G_E by "rational (1,1)" $G_E(Q^2) = \dfrac{1 + p_1 Q^2}{1 + p_2 Q^2}$
- The extracted radius is $r_E^p = 0.831 \pm 0.007 \pm 0.012$ fm

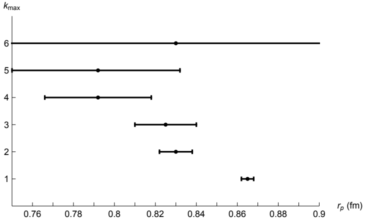
Two parameter fit

- Should we trust a two parameter fit? Error grows for 3rd z-expansion:



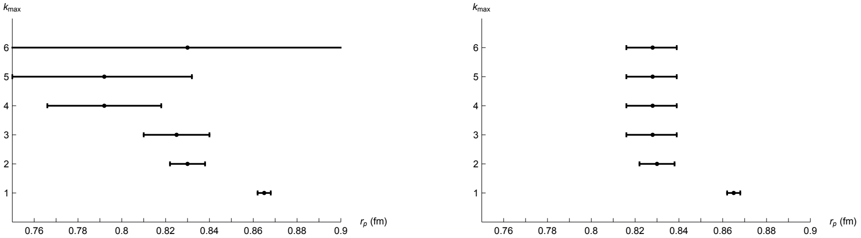
[Xiong et al., Nature **575**, 147 (2019) Supplementary information]

- What happens if we add more powers of z? (statistical errors shown)



[GP, arXiv:2004.03077 (hep-ph)]

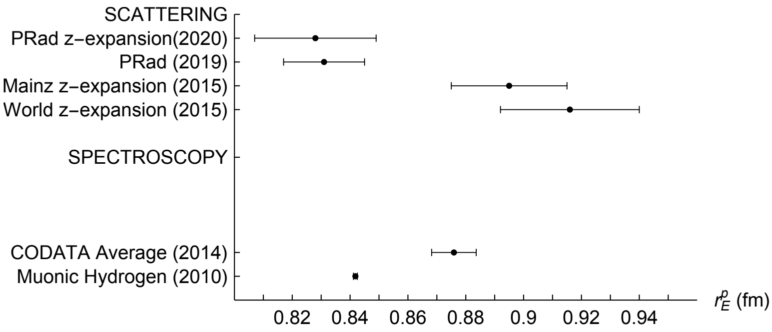
Model independent extraction



$$G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$$

- Need to bound the coefficients for model-independent fit [Hill, GP PRD **82** 113005 (2010)]
- Model-independent fit to PRad data: $r_E^p = 0.828^{+0.011}_{-0.012}$ fm [GP, arXiv:2004.03077 (hep-ph)]
- PRad's two parameter fit : $r_E^{p,\text{rational}} = 0.831 \pm 0.007$ fm
- Almost same central values, uncertainty 50% larger for z-expansion fit

The proton radius puzzle



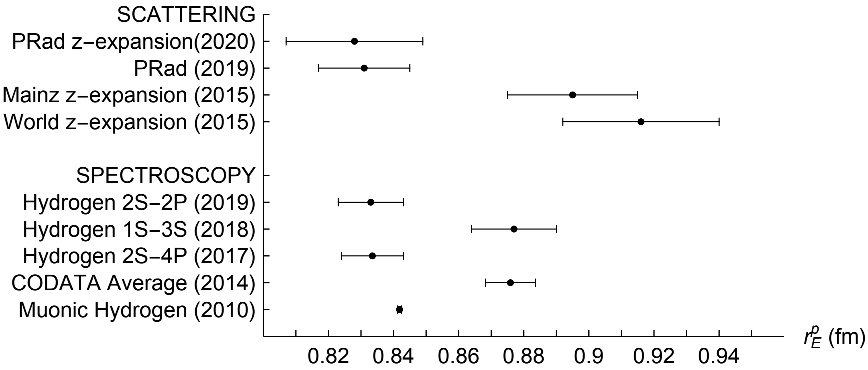
Proton charge radius from regular hydrogen

New Regular hydrogen spectroscopy results

- The puzzle motivated new regular hydrogen measurements
- Goal: single measurement with precision close to CODATA 2014 average of regular hydrogen spectroscopy ~ 0.01 fm
- Published October 2017: $2S-4P$ Germany
  [Beyer et al., Science **358**, 79 (2017)] $r_E^p = 0.83(1)$ fm
- Published May 2018: $1S-3S$ France
  [Fleurbaey et al., PRL **120**, 183001 (2018)] $r_E^p = 0.88(1)$ fm
- Published September 2019: $2S-2P$ Canada
  [Bezginov et al., Science **365**, 1007 (2019)] $r_E^p = 0.83(1)$ fm
- Expected sometime in 2020: $1S-3S$ Germany with $r_E^p \sim 0.84$ fm
  Two measurements of *same* $1S-3S$ transition extracting *different* r_E^p

Conclusions

April 2020 summary of published results



- PRad reanalysis (2020): GP, arXiv:2004.03077 (hep-ph)
- PRad (2019): Xiong et al., Nature **575**, 147 (2019)
- Mainz z-expansion (2015): Lee, Arrington, Hill, PRD **92**, 013013 (2015)
- World z-expansion (2015): Lee, Arrington, Hill, PRD **92**, 013013 (2015)
- Hydrogen 2S-2P (2019): Bezginov et al., Science **365**, 1007 (2019)
- Hydrogen 1S-3S (2018): Fleurbaey et al., PRL **120**, 183001 (2018)
- Hydrogen 2S-4P (2017: Beyer et al., Science **358**, 79 (2017)
- CODATA Average (2014): Mohr et al. RMP **88**, 035009 (2016)
- Muonic Hydrogen (2010): Pohl et al. Nature **466**, 213 (2010)

Conclusions

- Proton radius puzzle: $> 5\sigma$ discrepancy between
  - r_E^p from muonic hydrogen
  - r_E^p from hydrogen and $e - p$ scattering

- Current Status:
  - <u>$e - p$ scattering</u>: conflicting r_E^p extractions between new low Q^2 and previous higher Q^2
  - <u>Regular hydrogen spectroscopy</u>: conflicting r_E^p extractions between recent experiments (2:1 in favor of smaller r_E^p)
  - <u>$\mu - p$ scattering</u>: MUSE is running in 2019 and 2020

- The puzzle motivates reevaluation of our understanding of the proton

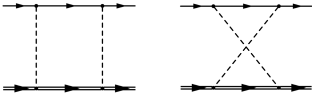
- The proton radius puzzle is still puzzling...

- Thank you!

Backup

Why is $b_1 = 0$? EFT

- Surprisingly, *no* contribution to b_1. Why?
- EFT side:



$$\frac{I^{m,0}_{D,C}}{-i(4\pi)^2} = \int \frac{d^4 l}{(2\pi)^4} \frac{\{m, m - l^0\}}{(l^2 - 2ml^0 + i\epsilon)(l^2 - \lambda^2 + i\epsilon)^2(\pm l^0 - \frac{\vec{l}^{\,2}}{2M} + i\epsilon)}$$

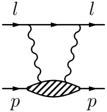
- Usually direct and crossed with even powers of M have opposite signs:

$$\left(\pm l^0 - \frac{\vec{l}^{\,2}}{2M} + i\epsilon\right)^{-1} = \pm\frac{1}{l^0} + \frac{\vec{l}^{\,2}}{2(l^0)^2 M} \pm \frac{(\vec{l}^{\,2})^2}{4(l^0)^3 m_p^2} + \mathcal{O}\left(\frac{1}{M^3}\right)$$

- Direct and crossed diagrams usually appear as a sum for spin-independent terms and cancel each other

Why is $b_1 = 0$? Full theory

- Surprisingly, *no* contribution to b_1. Why?
- Full theory side:



$$i\mathcal{M}_{\text{Full}} = -Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu(\not{k} - \not{l} + m)\gamma_\nu u}{(k-l)^2 - m^2} \left(\frac{1}{l^2 - \lambda^2}\right)^2 W^{\mu\nu}(p,l).$$

where $k = (m, \vec{0})$

- In the limit $m \to 0 \Rightarrow k \to 0$

$$i\mathcal{M}_{\text{Full}}\Big|_{m\to 0} = -Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}\gamma_\mu(-\not{l})\gamma_\nu u}{l^2} \left(\frac{1}{l^2 - \lambda^2}\right)^2 W^{\mu\nu}(p,l).$$

- Translation invariance implies $W^{\mu\nu}(p,l) = W^{\nu\mu}(p,-l)$
- Full spin-independent amplitude vanishes for $m \to 0$