Generating the Cabibbo Angle in Models of Discrete, Non-Abelian Flavored Gauge Mediation

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Based on LE TG AR [1912.12938], LE TG AR [1812.10811] (PRD '19) and LE TG [1610.09024] (PRD '18)
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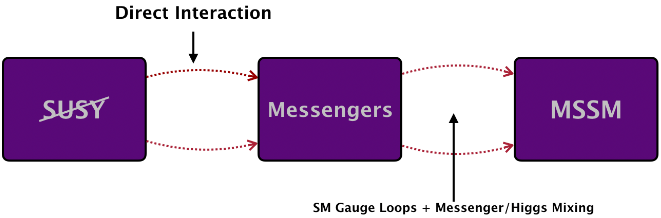
(Flavored) Gauge Mediation and its Discontents

- Minimal Gauge Mediation: elegant but nonviable – vanishing $\tilde{A}$, negligible stop mixing, heavy ($\mathcal{O}(10)$TeV) colored superpartners.
- Modify! $\Rightarrow$ Flavored Gauge Mediation
  - Mix $SU(2)_L$ messenger doublets with $H_{u,d}$.
  - Yukawa superpotential term becomes $Y_u Q \overline{u} \underbrace{\mathcal{H}_u}_{\text{Higgs/messenger admixture}}$.
  - Higgs/messenger mixing governed by imposed flavor symmetry.



FGM Using $\mathcal{S}_3$

Extend PRZ '12 to three families, wherein we have:

- Higgs-Messenger fields in both doublet and singlet reps of $\mathcal{S}_3$
- $\mathcal{S}_3$-charged MSSM fields
- Superpotential (e.g up-sector):

$$W^{(u)} = y_u\Big[Q_{\mathbf{2}}\bar{u}_{\mathbf{2}}\mathcal{H}_u^{(2)} + \beta_1 Q_{\mathbf{2}}\bar{u}_{\mathbf{2}}\mathcal{H}_u^{(1)} + \beta_2 Q_{\mathbf{2}}\bar{u}_{\mathbf{1}}\mathcal{H}_u^{(2)} + \beta_3 Q_{\mathbf{1}}\bar{u}_{\mathbf{2}}\mathcal{H}_u^{(2)} + \beta_4 Q_{\mathbf{1}}\bar{u}_{\mathbf{1}}\mathcal{H}_u^{(1)}\Big]$$

Advantages:

- Can tune $B\mu$ and μ separately to achieve a hierarchy between $H_{u,d}$ and $M_{u,d}$.
- Can achieve nontrivial stop mixing.

Unfortunately, there is no baked-in eigenvalue hierarchy for SM fermion masses for arbitrary β_i.

One Solution

β_4 dominance: $\beta_1 = 1$ and $\beta_4 = \beta_2\beta_3$.

FGM Using $\mathcal{S}_3$

β_4 Dominance (1)

An immediate consequence of β_4 dominance: massless light two generations.

$\Rightarrow$ Free parameter in Yukawa diagonalization matrices.

$\Rightarrow$ CKM tunable?

$$U_{CKM} = \begin{pmatrix} \cos\tilde\theta\cos\tilde\theta_d + \frac{(2+\beta_3\beta_{3d})\sin\tilde\theta\sin\tilde\theta_d}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} & \frac{(2+\beta_3\beta_{3d})\cos\tilde\theta_d\sin\tilde\theta}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} - \cos\tilde\theta\sin\tilde\theta_d & -\frac{\sqrt{2}(\beta_3-\beta_{3d})\sin\tilde\theta}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} \\ -\cos\tilde\theta_d\sin\tilde\theta + \frac{(2+\beta_3\beta_{3d})\cos\tilde\theta\sin\tilde\theta_d}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} & \frac{(2+\beta_3\beta_{3d})\cos\tilde\theta\cos\tilde\theta_d}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} + \sin\tilde\theta\sin\tilde\theta_d & -\frac{\sqrt{2}(\beta_3-\beta_{3d})\cos\tilde\theta}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} \\ \frac{\sqrt{2}(\beta_3-\beta_{3d})\sin\tilde\theta_d}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} & \frac{\sqrt{2}(\beta_3-\beta_{3d})\cos\tilde\theta_d}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} & \frac{2+\beta_3\beta_{3d}}{\sqrt{2+\beta_3^2}\sqrt{2+\beta_{3d}^2}} \end{pmatrix}.$$

Tuning the CKM

$\tilde\theta$ controls the mixing of massless eigenvectors. Viable pheno requires nonzero light masses, fixing $\tilde\theta$

$\Rightarrow$ perturbations to give light generations mass destroy tunability of CKM.

FGM Using $\mathcal{S}_3$

β_4 Dominance (2)

- Only requiring $\beta_1 = 1$, $\beta_4 = \beta_2\beta_3$ isn't enough; need to guarantee large stop mixing.
- Therefore take $\beta_{2,3} \gg 1$.
  - Equivalent to $\mathcal{H}_u^{(1)}$ (or third gen) dominance.

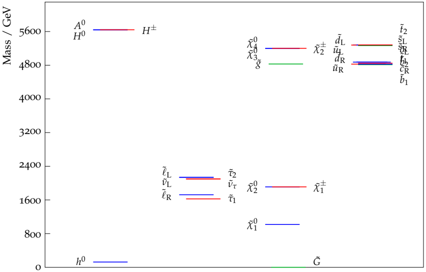


Figure: Superpartner spectrum for $M_{\text{Mess}} = 1 \times 10^{12}$ GeV and $\tan\beta = 10$, from [arXiv:1912.12938]

Breaking Gen 1 & 2 Degeneracy

The next step is breaking the degeneracy between the two massless generations – how?

- Perturb β_i around β_4 dominance constraints.
  - Breaks tunability of $\tilde{\theta}$, fixing it to either 0 or $\pi/2$.
  - $\Rightarrow \lambda \sim \mathcal{O}(0 \text{ or } 1)$.
- Add a new operator to $W^{(u)}$.
  - Must be nonrenormalizable, otherwise just a redefinition of β_i.
  - Necessitates an additionally scalar field (a flavon).

A Possibility

$$W^{(u)}_{NR} = \frac{\epsilon}{\Lambda'} \left(Q_{\mathbf{2}}\phi_{\mathbf{2}}\right) \left(\mathcal{H}^{(2)}_u \overline{u}_{\mathbf{2}}\right)$$

Nonrenormalizable Operator

Operator in question:
$W_{NR}^{(u)} = \frac{\epsilon}{\Lambda'} \left(Q_{\mathbf{2}}\phi_{\mathbf{2}}\right) \left(\mathcal{H}_u^{(2)}\overline{u}_{\mathbf{2}}\right)$

- ϵ: dimensionless coupling strength.
- Λ': scale of new physics responsible for $W_{NR}^{(u)}$.
- $\phi_{\mathbf{2}}$: scalar field in $\mathbf{2}$ of $\mathcal{S}_3$.

Through unknown dynamics $\phi_{\mathbf{2}}$ acquires a vev in its scalar component

$$\langle\phi_{\mathbf{2}}\rangle = v \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$\Rightarrow$ modifies up Yukawas

For convenience, define $\beta_\epsilon = v\epsilon/\Lambda'$

Nonrenormalizable Operator

Yukawas

SM up quark Yukawa

$$Y_u = \begin{pmatrix} \frac{\beta_\epsilon \sin\theta}{\sqrt{3}} & \frac{\beta_\epsilon \sin\theta}{\sqrt{3}} & 0 \\ \frac{\beta_\epsilon \cos\theta}{\sqrt{3}} & \frac{\beta_\epsilon \cos\theta}{\sqrt{3}} & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

Messenger Yukawas

$$Y'_{u1,2} = \begin{pmatrix} \pm\beta_\epsilon \left(\frac{1}{2} \mp \frac{1}{2\sqrt{3}}\right) \sin\theta & \mp\beta_\epsilon \left(\frac{1}{2} \pm \frac{1}{2\sqrt{3}}\right) \sin\theta & 0 \\ \pm\beta_\epsilon \left(\frac{1}{2} \mp \frac{1}{2\sqrt{3}}\right) \cos\theta & \mp\beta_\epsilon \left(\frac{1}{2} \pm \frac{1}{2\sqrt{3}}\right) \cos\theta & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Might expect that θ, a free parameter, appears in Y_u eigenvalues and soft terms. Surprisingly, not the case; θ only appears in CKM.

Nonrenormalizable Operator

Mass eigenbasis

Up quark mass diagonalization:

$$U_{uL}^{\dagger} Y_u U_{uR} = Y_u^{\text{diag}} = \text{Diag}\left(0, \sqrt{\frac{2}{3}}\beta_{\epsilon}, y_t\right) \Rightarrow$$

Mass degeneracy lifted!

Diagonalization matrices:

$$U_{uL} = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_{uR} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Nonrenormalizable Operator

Soft Term Corrections (Modulo $\Lambda/(4\pi)^2$)

$$\left(\delta m^2_{\tilde{Q}}\right)_{22} = \left(-2y_b^2 - 2y_t^2 - \frac{2y_\tau^2}{3} - \frac{16g_1^2}{9} - 8g_2^2 - \frac{128g_3^2}{9}\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{Q}}\right)_{33} = 36y_b^4 + 8y_t^2y_b^2 + 8y_\tau^2y_b^2 - \frac{14g_1^2y_b^2}{15} - 6g_2^2y_b^2 - \frac{32g_3^2y_b^2}{3} + 36y_t^4 - \frac{26g_1^2y_t^2}{15} - 6g_2^2y_t^2 - \frac{32g_3^2y_t^2}{3} + \left(-\frac{8y_b^2}{3} - 2y_t^2\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{u}}\right)_{11} = \left(-\frac{26g_1^2}{15} - 6g_2^2 - \frac{32g_3^2}{3}\right)\beta_\epsilon^2, \qquad \left(\delta m^2_{\tilde{u}}\right)_{22} = \left(-4y_t^2 - \frac{26g_1^2}{45} - 2g_2^2 - \frac{32g_3^2}{9}\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{u}}\right)_{33} = 72y_t^4 + 8y_b^2y_t^2 - \frac{52g_1^2y_t^2}{15} - 12g_2^2y_t^2 - \frac{64g_3^2y_t^2}{3} - 4\beta_\epsilon^2y_t^2$$

$$\left(\delta m^2_{\tilde{d}}\right)_{11} = \left(4y_\tau^2 - \frac{14g_1^2}{15} - 6g_2^2 - \frac{32g_3^2}{3}\right)\beta_\epsilon^2, \quad \left(\delta m^2_{\tilde{d}}\right)_{22} = \left(-4y_b^2 - \frac{14g_1^2}{45} - 2g_2^2 - \frac{32g_3^2}{9}\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{d}}\right)_{33} = 72y_b^4 + 8y_t^2y_b^2 + 24y_\tau^2y_b^2 - \frac{28g_1^2y_b^2}{15} - 12g_2^2y_b^2 - \frac{64g_3^2y_b^2}{3}$$

$$\left(\delta m^2_{\tilde{L}}\right)_{22} = \left(-2y_b^2 - \frac{2y_\tau^2}{3} - \frac{12g_1^2}{5} - 4g_2^2\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{L}}\right)_{33} = 20y_\tau^4 + 24y_b^2y_\tau^2 - \frac{18g_1^2y_\tau^2}{5} - 6g_2^2y_\tau^2 - \frac{8\beta_\epsilon^2y_\tau^2}{3}$$

$$\left(\delta m^2_{\tilde{e}}\right)_{11} = \left(-\frac{18g_1^2}{5} - 6g_2^2\right)\beta_\epsilon^2 \qquad \left(\delta m^2_{\tilde{e}}\right)_{22} = \left(-4y_b^2 - \frac{4y_\tau^2}{3} - \frac{6g_1^2}{5} - 2g_2^2\right)\beta_\epsilon^2$$

$$\left(\delta m^2_{\tilde{e}}\right)_{33} = 40y_\tau^4 + 48y_b^2y_\tau^2 - \frac{36g_1^2y_\tau^2}{5} - 12g_2^2y_\tau^2 - \frac{16\beta_\epsilon^2y_\tau^2}{3}$$

$$\delta m^2_{\tilde{H}_u} = -6y_t^2(y_b^2 + 3y_t^2), \qquad \delta m^2_{\tilde{H}_d} = -6(3y_b^4 + y_b^2y_t^2 + 3y_\tau^4)$$

$$\left(\tilde{A}_u\right)_{33} = -2y_t\left(y_b^2 + 3y_t^2\right), \qquad \left(\tilde{A}_d\right)_{33} = -2y_b\left(3y_b^2 + y_t^2\right), \qquad \left(\tilde{A}_e\right)_{33} = -6y_\tau^3.$$

Nonrenormalizable Operator

CKM

Assuming corresponding structure in down sector ($\theta \leftrightarrow \theta_d$, $y_u \leftrightarrow y_d$, and $\beta_i \leftrightarrow \beta_{di}$), the CKM is given by

$$U_{\text{CKM}} = U_{uL}^{\dagger} U_{dL} = \begin{pmatrix} \cos(\theta - \theta_d) & \sin(\theta - \theta_d) & 0 \\ -\sin(\theta - \theta_d) & \cos(\theta - \theta_d) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Nota Bene:

- Appropriate $1 - 2$ Cabibbo mixing if $\sin(\theta - \theta_d) \simeq \lambda$.
- Non-degenerate quark masses $\Rightarrow$ unambiguous ordering of generations. Can confidently say this CKM describes $1 - 2$ mixing.

Ensuing Phenemonology (1)

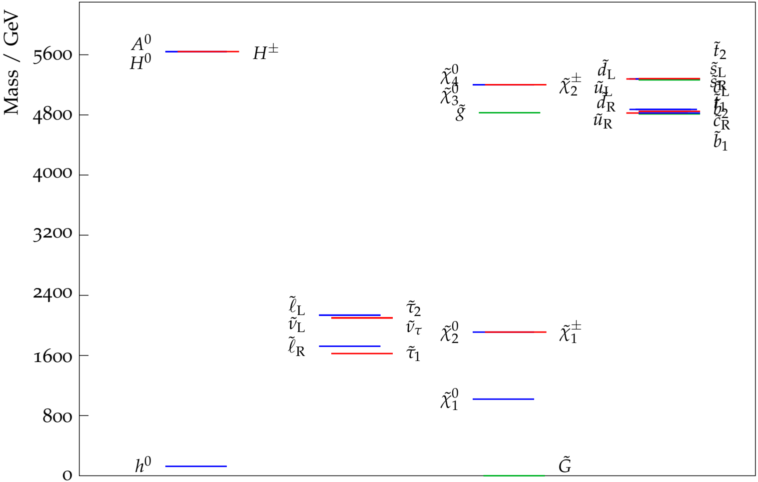


Figure: $M_{\text{mess}} = 10^12$ GeV, $\tan\beta = 10$, $\beta_\epsilon = 0.01$ with Λ being fixed by the Higgs mass. From [arXiv:1912.12938]

Ensuing Phenemonology (2)

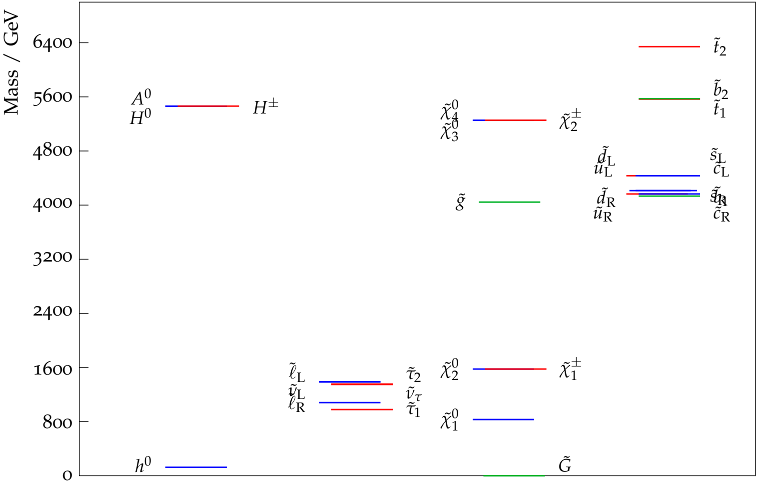


Figure: $M_{\text{mess}} = 10^6$ GeV, $\tan\beta = 10$, $\beta_\epsilon = 0.01$ with Λ being fixed by the Higgs mass. From [arXiv:1912.12938]

Ensuing Phenemonology (3)

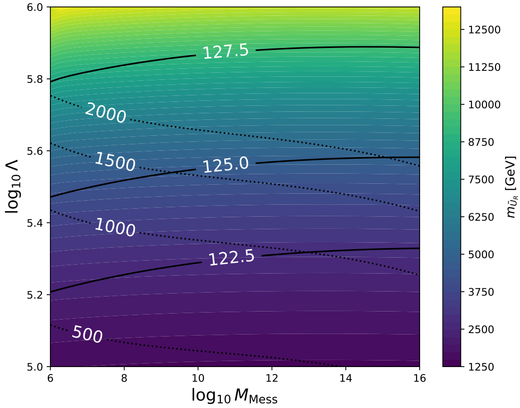


Figure: The Higgs mass (solid contours), right-handed sup mass (color shading) and right-handed selectron masses (dotted contours) in this scenario with $\beta_\epsilon = 0$. From [arXiv:1912.12938]

Ensuing Phenemonology (4)

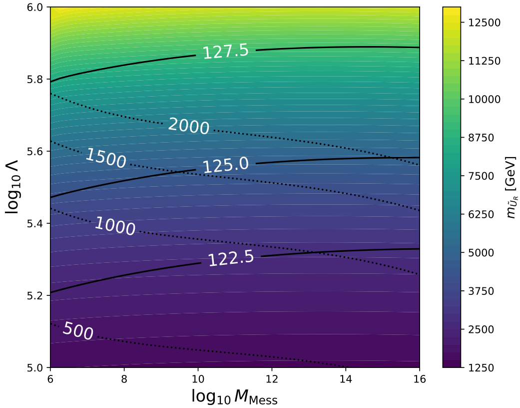


Figure: The Higgs mass (solid contours), right-handed sup mass (color shading) and right-handed selectron masses (dotted contours) in this scenario with $\beta_\epsilon = 0.05$. From [arXiv:1912.12938]

Finale

So far we have:

- Achieved sizable stop mixing in a theory of flavored gauge mediation with $\mathcal{S}_3$ as the family symmetry.
- Shown how the beginnings of embedding SM flavor structure can be achieved.
  - Cabibbo mixing
  - Light quark masses
- Question standard folklore that $\not{SUSY}$ and flavor symmetry breaking must be independent and never the twain shall meet.

Still to do:

- Still have one massless quark.
- No source of 3rd generation CKM mixing.
- Can the model support a full theory of flavor?
- Are we plagued by FCNC constraints?

Thank You!

Backup: $\mathcal{S}_3$ Charges

Higgs-messenger ($\mathcal{H}_{u,d}$) and ~~SUSY~~ (X_H) charges under $\mathcal{S}_3$:

	$\mathcal{H}_u^{(2)}$	$\mathcal{H}_u^{(1)}$	$\mathcal{H}_d^{(2)}$	$\mathcal{H}_d^{(1)}$	X_H
$\mathcal{S}_3$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$

MSSM charges:

	$Q_{\mathbf{2}}$	$Q_{\mathbf{1}}$	$\bar{u}_{\mathbf{2}}$	$\bar{u}_{\mathbf{1}}$	$\bar{d}_{\mathbf{2}}$	$\bar{d}_{\mathbf{1}}$	$L_{\mathbf{2}}$	$L_{\mathbf{1}}$	$\bar{e}_{\mathbf{2}}$	$\bar{e}_{\mathbf{1}}$
$\mathcal{S}_3$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$

Backup: Higgs/Messenger Mass Basis

Higgs-messenger fields are rotated into the electroweak Higgs doublets $H_{u,d}$ and messenger doublets $M_{u,d}$ by

$$\begin{pmatrix} \mathcal{H}^{(2)}_{u1,d1} \\ \mathcal{H}^{(2)}_{u2,d2} \\ \mathcal{H}^{(1)}_{u,d} \end{pmatrix} = \mathcal{R}_{u,d} \begin{pmatrix} H_{u,d} \\ M_{u1,d1} \\ M_{u2,d2} \end{pmatrix}$$

$$\mathcal{R}_{u,d} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \mp\frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right) & \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm\frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right) & -\frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right) \\ \frac{1}{\sqrt{3}} & \pm\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Backup: Generic Yukawas

$$Y_u = \frac{y_u}{\sqrt{3}} \begin{pmatrix} 1 & \beta_1 & \beta_2 \\ \beta_1 & 1 & \beta_2 \\ \beta_3 & \beta_3 & \beta_4 \end{pmatrix},$$

$$Y'_{u1} = y_u \begin{pmatrix} -\frac{1}{2} - \frac{1}{2\sqrt{3}} & \frac{\beta_1}{\sqrt{3}} & \frac{\beta_2}{2} - \frac{\beta_2}{2\sqrt{3}} \\ \frac{\beta_1}{\sqrt{3}} & \frac{1}{2} - \frac{1}{2\sqrt{3}} & -\frac{\beta_2}{2} - \frac{\beta_2}{2\sqrt{3}} \\ \frac{\beta_3}{2} - \frac{\beta_3}{2\sqrt{3}} & -\frac{\beta_3}{2} - \frac{\beta_3}{2\sqrt{3}} & \frac{\beta_4}{\sqrt{3}} \end{pmatrix}$$

$$Y'_{u2} = y_u \begin{pmatrix} \frac{1}{2} - \frac{1}{2\sqrt{3}} & \frac{\beta_1}{\sqrt{3}} & -\frac{\beta_2}{2} - \frac{\beta_2}{2\sqrt{3}} \\ \frac{\beta_1}{\sqrt{3}} & -\frac{1}{2} - \frac{1}{2\sqrt{3}} & \frac{\beta_2}{2} - \frac{\beta_2}{2\sqrt{3}} \\ -\frac{\beta_3}{2} - \frac{\beta_3}{2\sqrt{3}} & \frac{\beta_3}{2} - \frac{\beta_3}{2\sqrt{3}} & \frac{\beta_4}{\sqrt{3}} \end{pmatrix}.$$