Gluon PDF from Lattice QCD Calculation

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Parton distribution function

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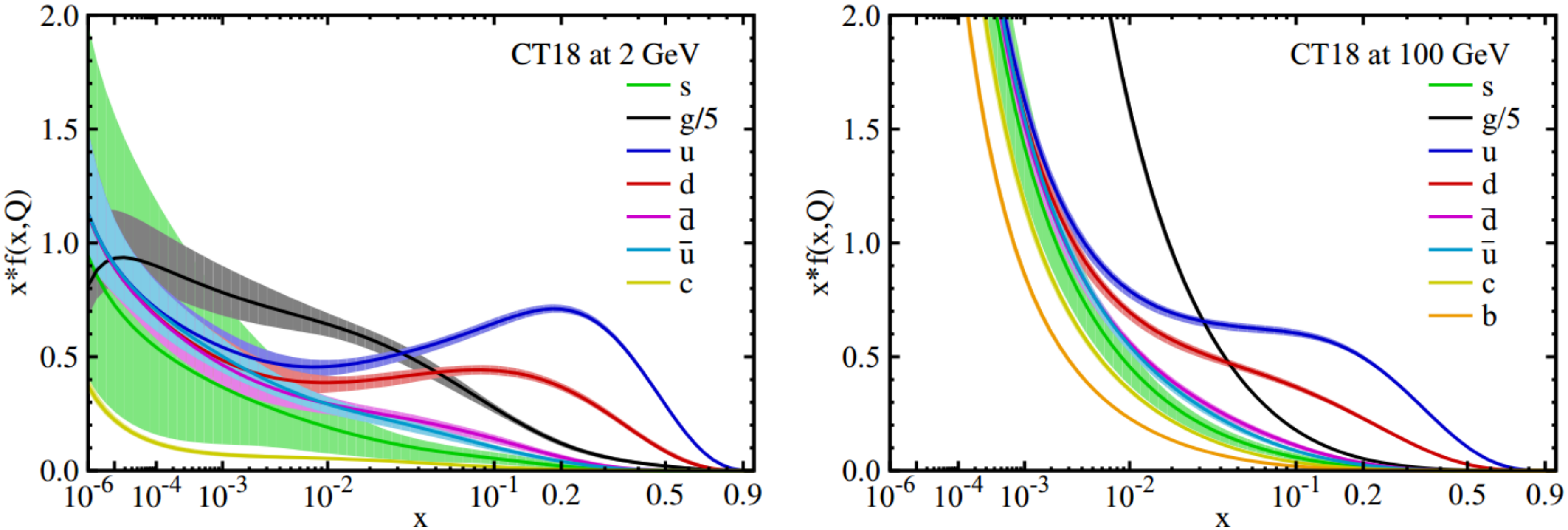
Parton Distribution Functions (PDF) represents the probability densities to find a parton carrying a momentum fraction x at energy scale Q.

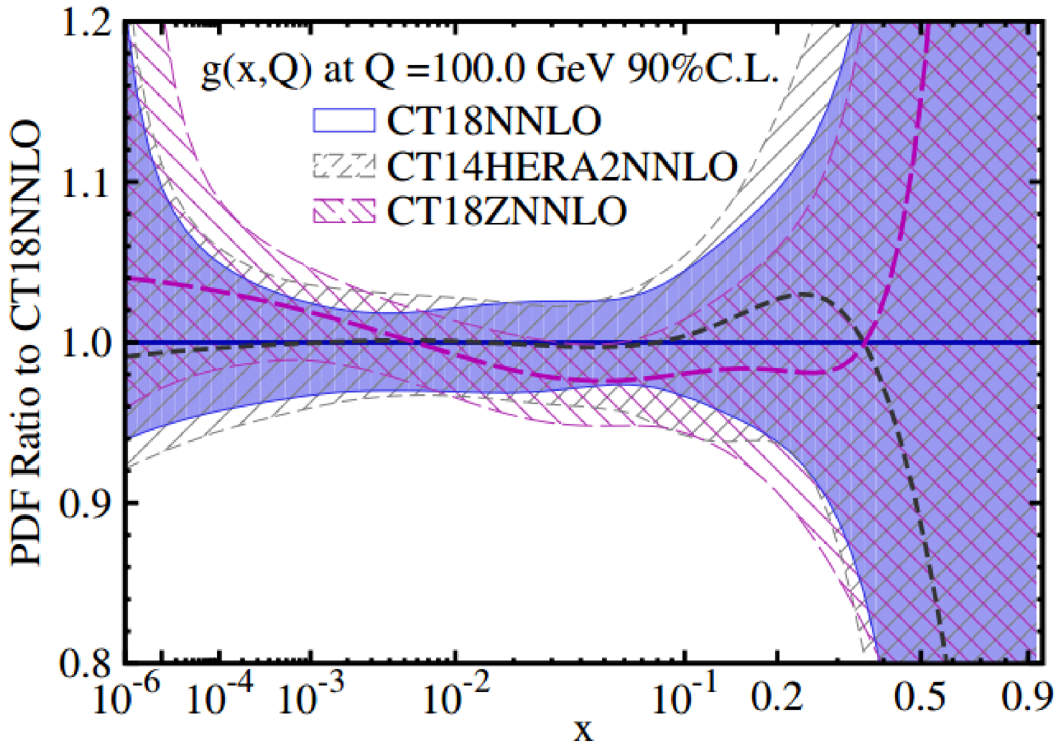


The CT18 parton distribution functions at Q = 2 GeV and Q = 100 GeV for u, $\bar{u}$, d, $\bar{d}$, $s = \bar{s}$, c, b, and g.

arXiv:1912.10053v2

Parton distribution function

The global fit gluon PDF error is improved with more experimental data including in the fit. However, its error at large x is still quite large.



arXiv:1912.10053v2

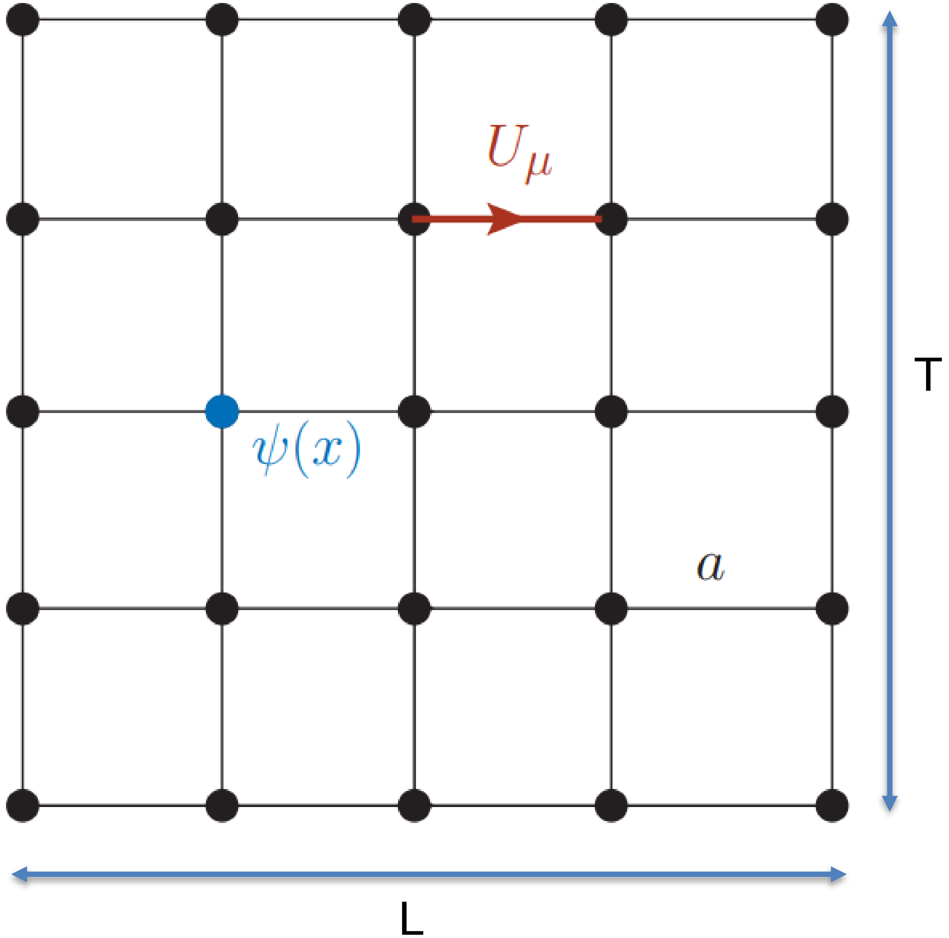
Lattice QCD

Lattice regularization is the only method that can describe non-perturbative QCD.

Define fields on discrete Euclidean spacetime ($L^3 \times T$) with lattice spacing a

Quark fields $\psi(x), \bar{\psi}\ (x)$ on each site. Gauge field $U_\mu(x)$ on links.

Pion mass m_π, usually not physical pion mass



Large Momentum Effective Theory

Large Momentum Effective Theory (LaMET) [**PRL 110.26 (2013): 262002**]. is a direct approach to compute parton physics on a Euclidean lattice through Lorentz boost.

The unpolarized gluon distribution in the nucleon in the light-cone coordinates

$$g(x,\mu^2) = \int \frac{d\xi^-}{2\pi x P^+}\, e^{-ix\xi^- P^+} \left\langle P \left| F_\mu^+(\xi^-)\mathcal{P}\times \exp\left(-ig\int_0^{\xi^-} d\eta^-\, A^+(\eta^-)\right) F^{\mu+}(0) \right| P \right\rangle$$

The unpolarized gluon quasi-PDF is defined as

$$\tilde{g}(x,\mu^2,P^z) = \int \frac{dz}{2\pi x P^z}\, e^{ixzP^z} \left\langle P \left| F_\mu^z(z)\mathcal{P}\times \exp\left(-ig\int_0^{z} d\eta A^z(\eta)\right) F^{\mu z}(0) \right| P \right\rangle$$

The quasi-PDF can be related to the P_z-independent light-front PDF through,

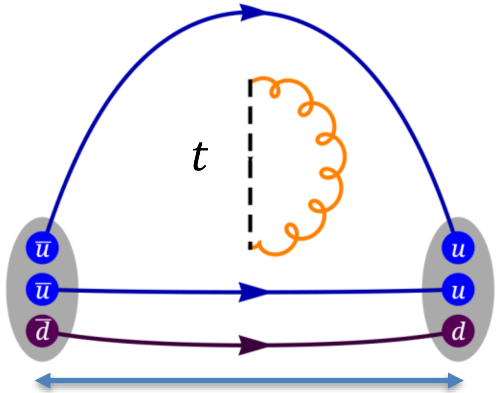
$$\tilde{g}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\frac{\mu}{yP^z}\right) g(y,\mu^2) + \mathcal{O}\left(\left(\frac{M}{P^z}\right)^2, \left(\frac{\Lambda_{QCD}}{P^z}\right)^2\right)$$

Lattice calculation

The matrix elements of unpolarized gluon quasi-PDF on the lattice is defined by

$$h(z, P_z) = \langle P | O_i(z) | P \rangle$$

The multiplicatively renormalizable operators definitions are, PhysRevD.100.074509



source-sink separation: t_{sep}

$$O(F^{\mu\nu}, F^{\rho\sigma}; z) = F^{\mu\nu}(z) U(z, 0) F^{\rho\sigma}(0)$$

$$O_{1JZ}(z) = \sum_{i \neq z,t} O(F^{ti}, F^{ti}; z) \qquad O_{2JZ}(z) = \sum_{i \neq z,t} O(F^{zi}, F^{zi}; z)$$

$$O_{3JZ}(z) = \sum_{i \neq z,t} O(F^{ti}, F^{zi}; z) \qquad O_{4JZ}(z) = \sum_{\mu=0,1,2,3} O(F^{z\mu}, F^{z\mu}; z)$$

The three-point and two-point correlators are defined as,

$$C_{3pt}(z, P_z; t_{sep}, t) \equiv \langle 0 | \Gamma^e \int d^3y e^{-iyP} \chi(\vec{y}, t_{sep})\, O(z, t)\, \chi(\vec{0}, 0) | 0 \rangle$$

$$C_{2pt}(z, P_z; t_{sep}) \equiv \langle 0 | \Gamma^e \int d^3y e^{-iyP} \chi(\vec{y}, t_{sep})\, \chi(\vec{0}, 0) | 0 \rangle$$

Numerical setup

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The lattice calculation are carried out with Clover valence quarks on the MILC $N_f = 2 + 1 + 1$ HISQ fermion gauge configurations with

Symbol	$L^3 \times T$	a	m_π^{sea}	N_{cfg}	Measurements
a12m310	$24^3 \times 64$	$0.1207(11)$ fm	306.9(5) MeV	898	57472
a15m310	$16^3 \times 48$	$0.1510(20)$ fm	305.3(4) MeV	900	43200

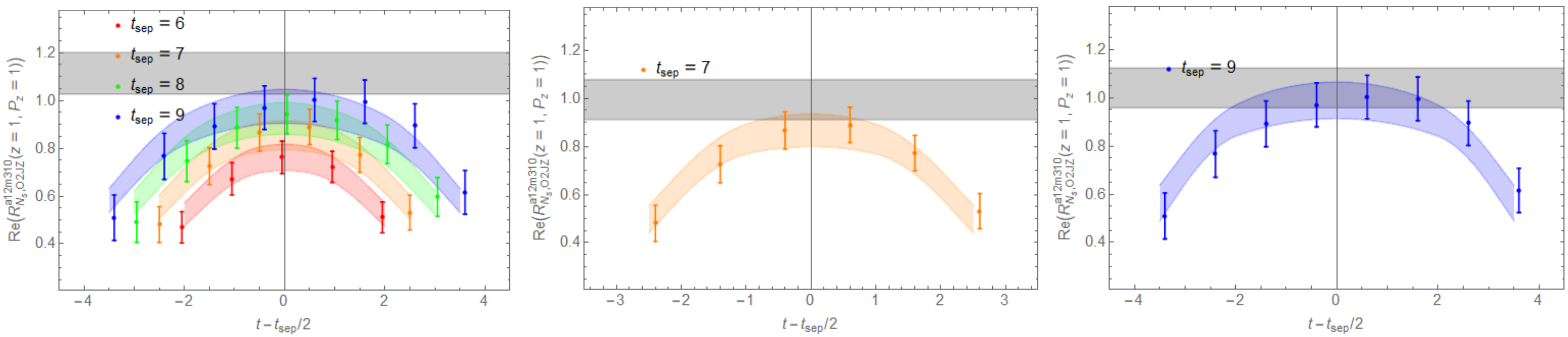
Ground state matrix element

Following the work [arXiv:1306.5435v3], the correlators C_{3pt} and C_{2pt} can be decomposed as,

$$C_{3pt}(z, P_z; t_{sep}, t) = |A_0|^2 \langle 0|O|0\rangle e^{-E_0 t_{sep}} + |A_1|^2 \langle 1|O|1\rangle e^{-E_1 t_{sep}}$$
$$+ \mathrm{A}_1 \mathrm{A}_0^* \langle 1|O|0\rangle e^{-E_1(t_{sep}-t)} e^{-E_0 t}$$
$$+ \mathrm{A}_0 \mathrm{A}_1^* \langle 0|O|1\rangle e^{-E_0(t_{sep}-t)} e^{-E_1 t} + \cdots$$

$$C_{2pt}(z, P_z; t_{sep}) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t} + \cdots$$

where the ground state matrix element is $\langle 0|O|0\rangle$.



The ground state matrix element extracted from the fit (grey band) from different fit choices are consistent within one sigma error.

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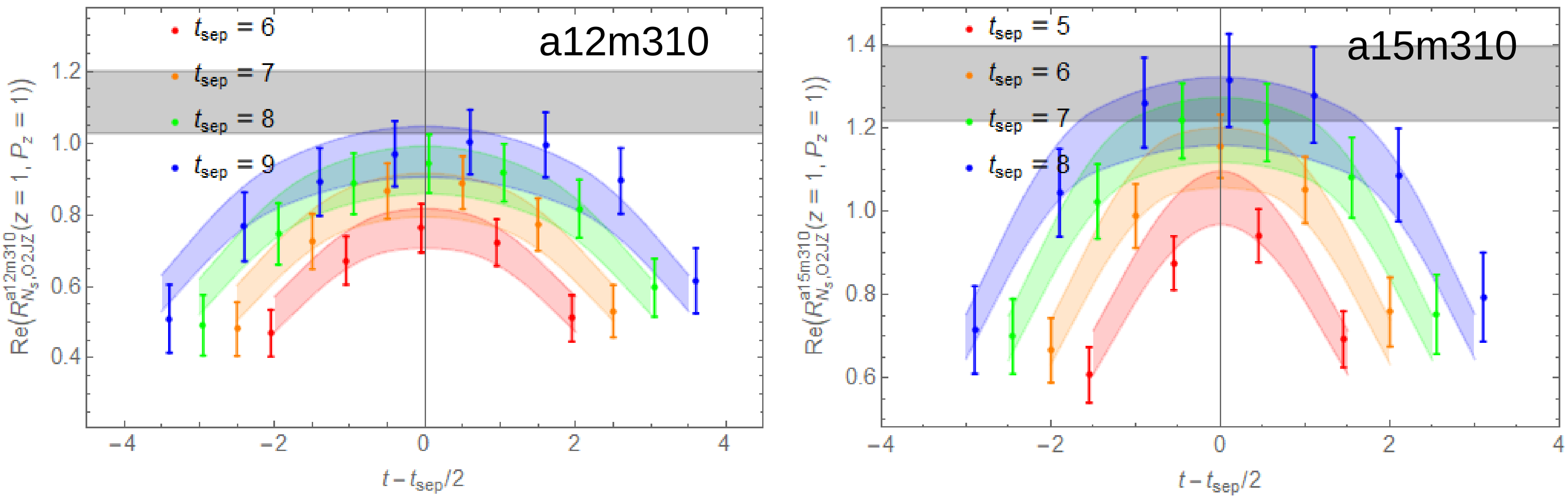
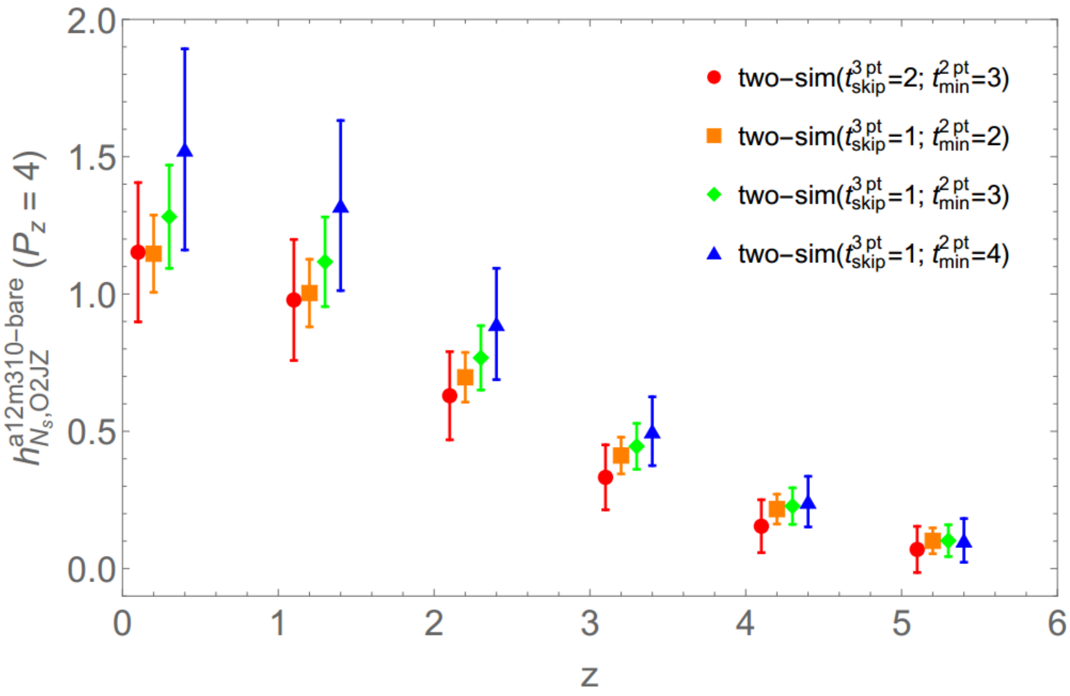


Fit range choice

The fit range for 2pt and 3pt correlator are checked for the bare matrix elements extracted from the fit.



$$O_{2JZ}(z) = \sum_{i\neq z,t} O\left(F^{zi}, F^{zi}; z\right)$$

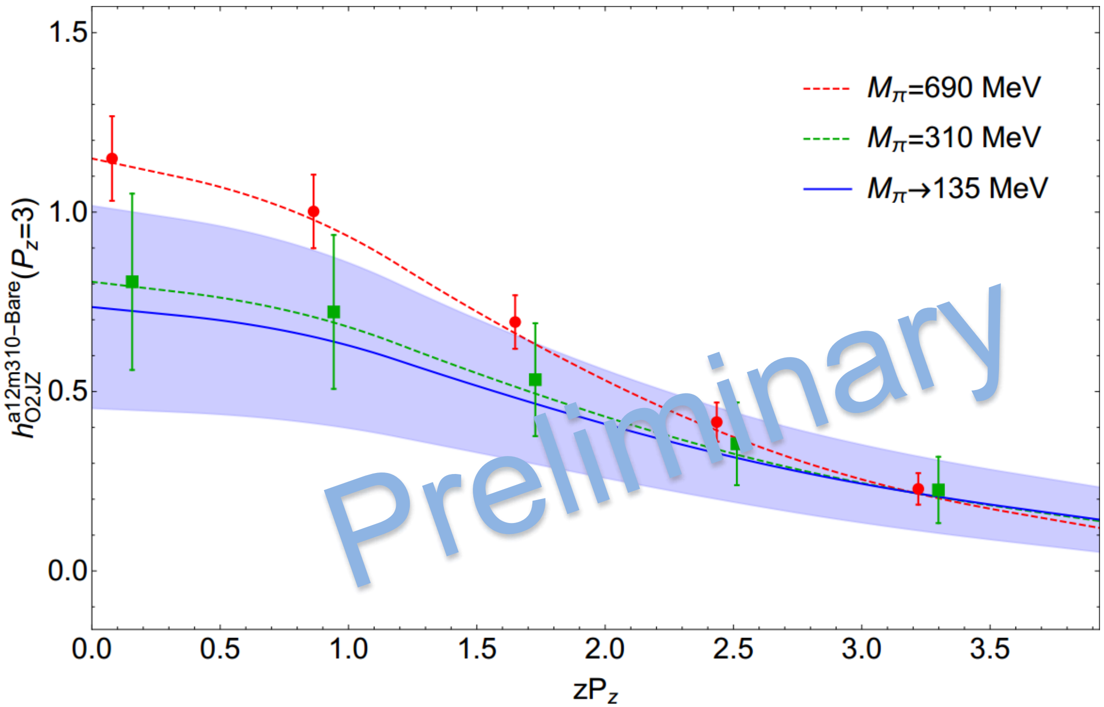
a12m310 ensemble

$M_\pi = 690$ MeV nucleon

Nucleon momentum $P_z = 4 * \frac{2\pi}{L}$

We choose $t_{skip}^{3pt} = 1, t_{min}^{2pt} = 3$ in our following fits considering the error of the fit results and χ^2/dof of the fits.

Extrapolation in the pion mass



$$O_{2JZ}(z) = \sum_{i\neq z,t} O\left(F^{zi}, F^{zi}; z\right)$$

a12m310 ensemble

Nucleon momentum $P_z = 3 * \frac{2\pi}{L}$

The blue band show the uncertainties of the results after extrapolation to the physical pion mass.

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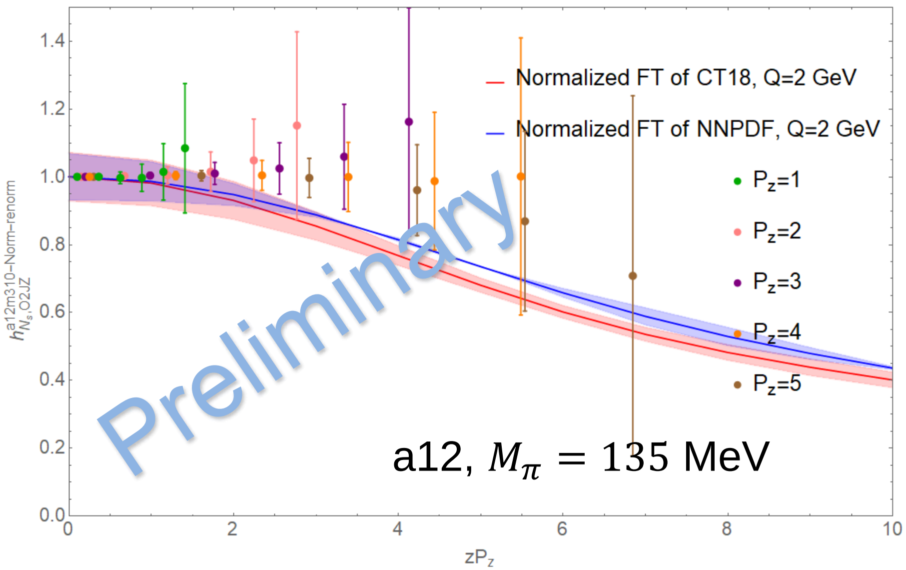
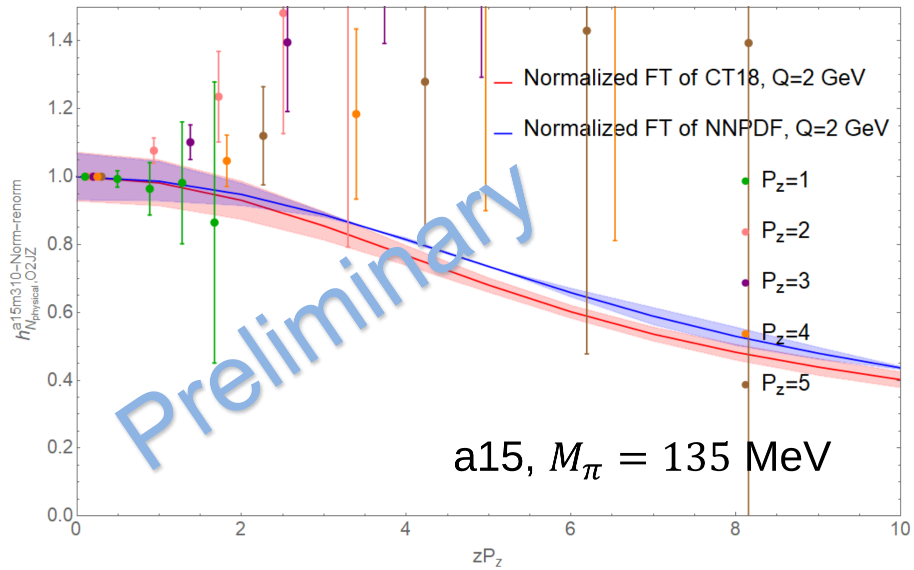
Comparing with global fit gluon PDF

The gluon normalized matrix elements $(\frac{h(z,P_z)}{h(z,0)})/(\frac{h(0,P_z)}{h(0,0)})$ as function of zP_z compare with the fourier transformation of the gluon global fit PDF from CT18.





Summary

- The two ensemble a12m310 and a15m310 with measurements 57472 and 43200 are studied. More ensembles with different lattice spacing a and pion mass are needed to extrapolate results to $a \to 0$ and physical pion mass.

- The matrix element results are comparing with the fourier transformation of the gluon global fit PDF from CT18 in coordinate space. The matching needs to perform and the comparison in the momentum space is needed as well.

- The error at small zP_z region is relatively smaller. Hopefully, we could provide constrain at the large x region where the experimental data is lacked.