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Dynamical axion misalignment with small instantons

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BROWN

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University of Pittsburgh

QCD axion

- Solves Strong CP problem

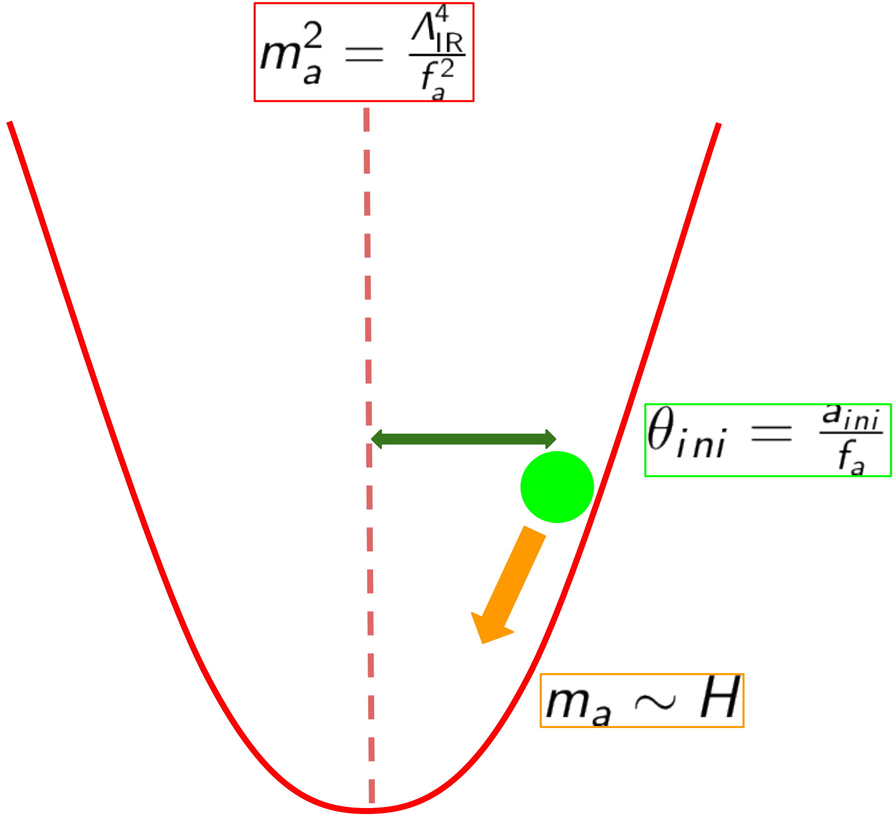
$$V_a = \Lambda_{\mathrm{IR}}^4 \cos\left(\frac{a}{f_a} - \overline{\theta}\right)$$

arXiv:1911.0573

QCD axion

- Solves Strong CP problem

- Can be Dark Matter
    - *misalignment mechanism*



arXiv:1911.0573

Axion DM: misalignment mechanism

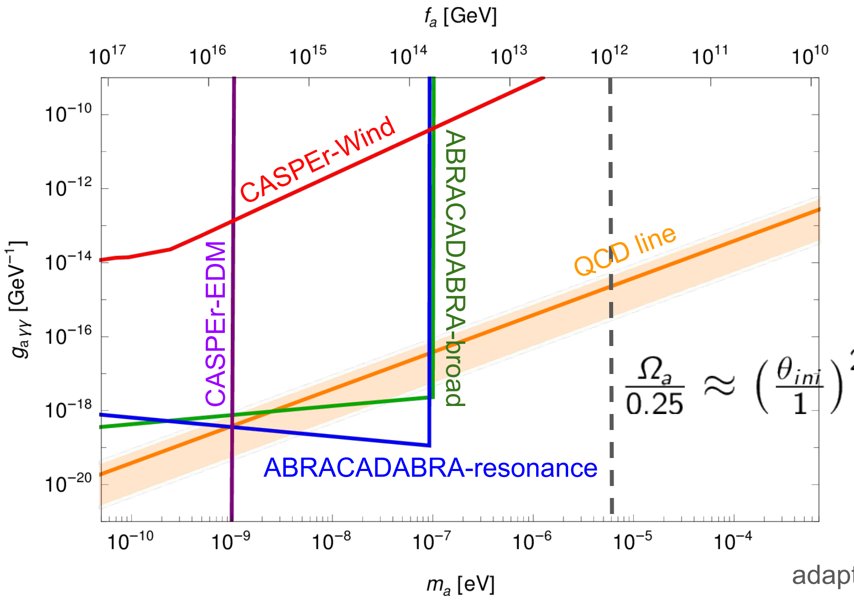
$$\frac{\Omega_a}{0.25} \approx \left(\frac{\theta_{ini}}{1}\right)^2 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)$$

- $f_a < 10^{12}$ **GeV**: not enough DM
- $f_a > 10^{12}$ **GeV**: overclose the Universe!

arXiv:1911.0573

$f_a > 10^{12}$ GeV: why?

- *Theory:* String theories w/ GUTs: $f_a \sim 10^{15}$ **GeV** [Svrcek, Witten '06]
- *Experiments:* ABRACADABRA [Graham, Rajendran], CASPEr [Kahn, Safdi, Thaler]



$$\frac{\Omega_a}{0.25} \approx \left(\frac{\theta_{ini}}{1}\right)^2 \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)$$

arXiv:1911.0573

adapted from Agrawal, Fan, Reece, Wang, '17

QCD axion DM

&

$f_a > 10^{12}$ GeV?

Our Idea/Title:

Dynamical axion misalignment with small instantons

1 2

1 $\theta_0 \sim \mathcal{O}(1) \xrightarrow{\text{dynamics}} \theta_{ini} \sim \mathcal{O}(10^{-2})$

$m_i^2 = \frac{\Lambda_i^4}{f_a^2}$

2 $V_a = V_{IR} + V_{UV} \longrightarrow m_a^2 = m_{IR}^2 + m_{UV}^2$

UV/small instantons contribute to axion potential

other ideas: *a)* anthropics [Tegmark et al. '05], *b)* $\theta_{ini} \sim O(1)$: entropy production [Kawasaki et al '95, Banks et al. '96], energy transfer [Agrawal et al. '17; Kitajima et al. '17]; *c)* dynamical $\theta_{ini} \ll 1$: exponentially long inflation [Graham et al. '18; Takahashi et al. '18], stronger QCD [Dvali '95; Choi et al. '97; Co et al. '18]

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Dynamical misalignment θ = dynamical m_{UV}

$$m_a^2 = m_{IR}^2 + m_{UV}^2$$

during inflation

HUGE

dynamics

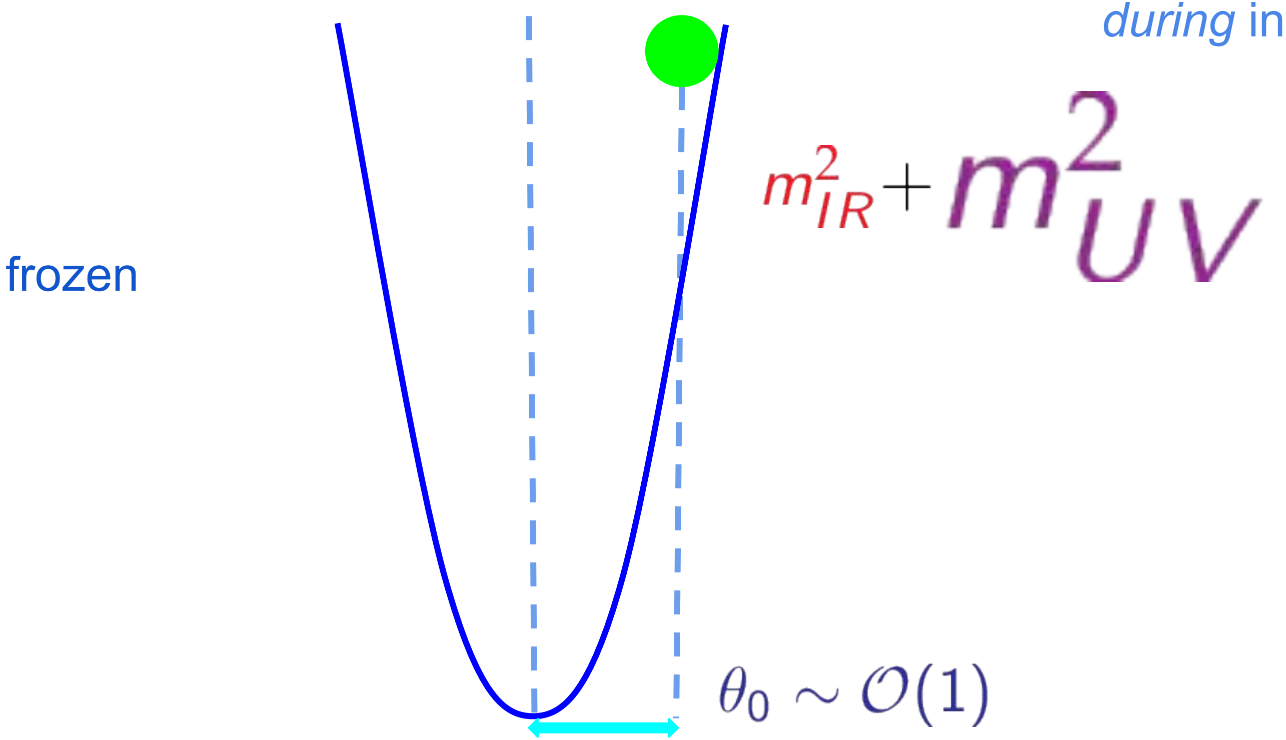
$$m_a^2 = m_{IR}^2 + m_{UV}^2$$

tiny

after inflation

Dynamical misalignment θ = dynamical m_{UV}

during inflation



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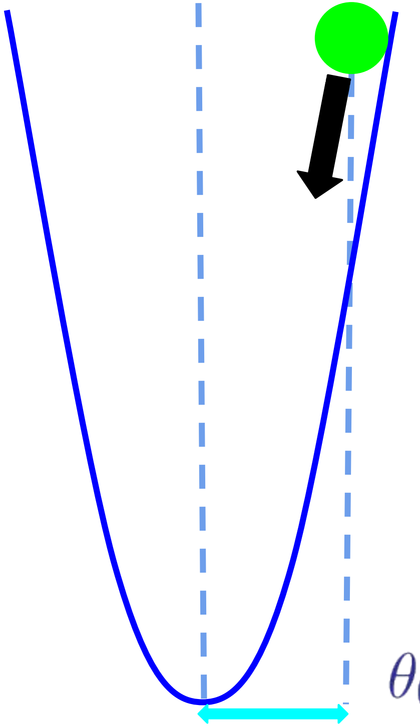
Dynamical misalignment **θ** = dynamical m_{UV}

during inflation

$$m_{IR}^2 + m_{UV}^2 \sim H_I^2$$

thawed



$$\theta_0 \sim \mathcal{O}(1)$$

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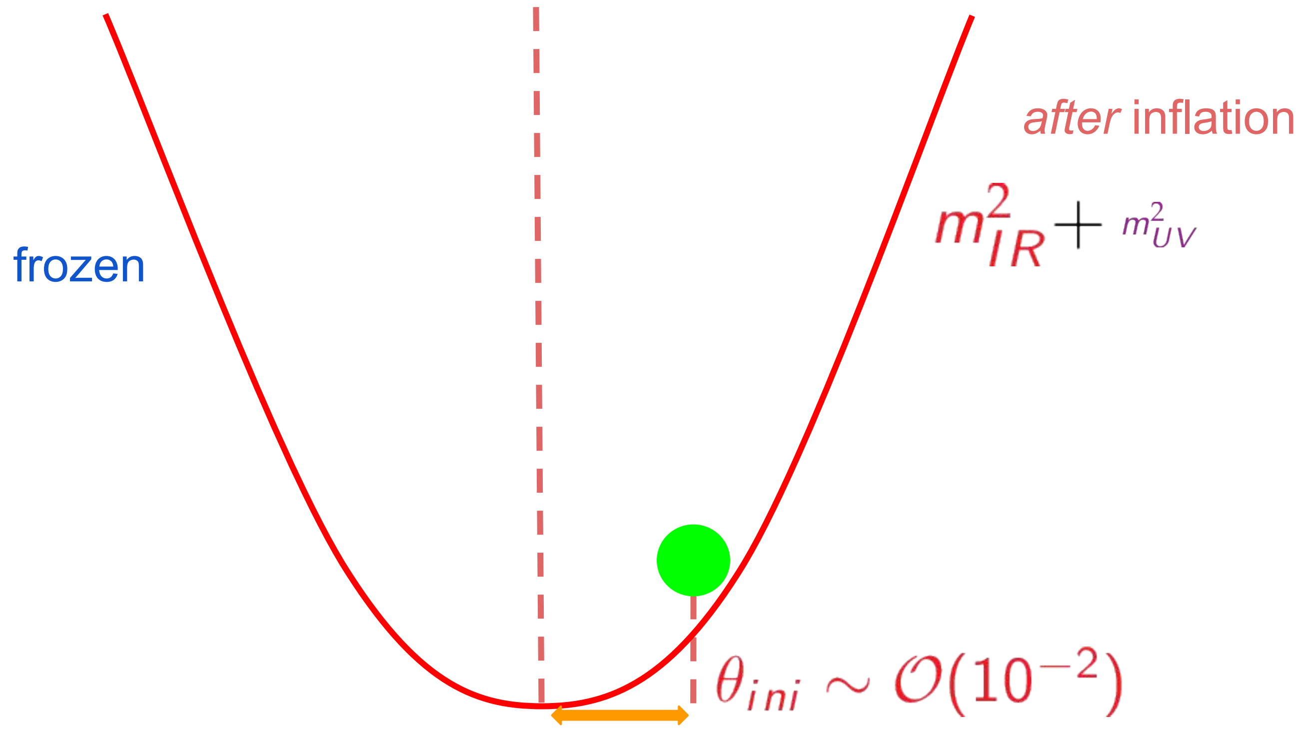
Dynamical misalignment θ = dynamical m_{UV}



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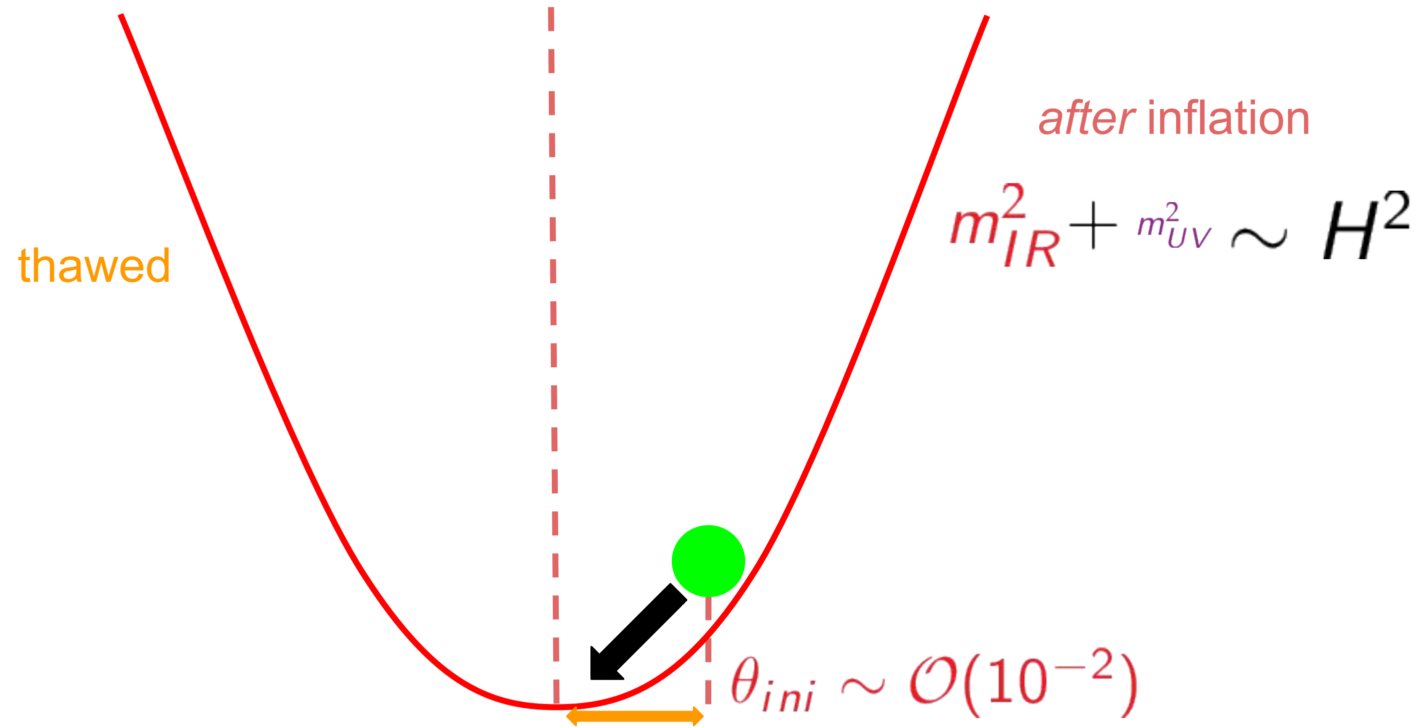

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Small instantons

['t Hooft '76; Callan et al. '78; Andre et al. '78, Flynn et al., '87]

$$m_{UV}^2 = \frac{\Lambda_{UV}^4}{f_a} \approx \frac{M^4 D K}{f_a}$$

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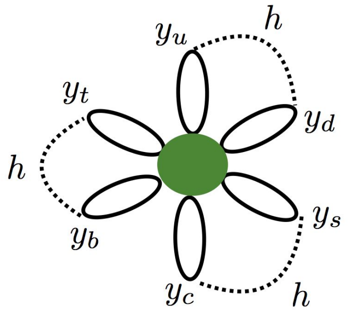

UV scale

$$M^4$$

instanton density

$$D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$$

chiral suppression

$$K = \prod_f \frac{y_f}{4\pi}$$

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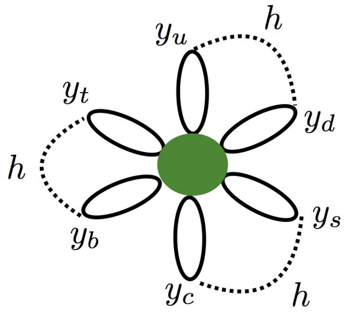

UV scale

M^4

instanton density

$$D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$$

asymptotic freedom

chiral suppression

$$K = \prod_f \frac{y_f}{4\pi}$$

$$\sim 10^{-23}$$

tiny!

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Making small instantons important

$$m_{UV}^2 = \frac{\Lambda_{UV}^4}{f_a} \approx \frac{M^4 D K}{f_a}$$

1 $$D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$$

2 $$K = \prod_f \frac{y_f}{4\pi}$$

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Making small instantons important

[Agrawal, Howe '17]

Agrawal-Howe Model

$$m^2_{UV} = \frac{\Lambda^4_{UV}}{f_a} \approx \frac{M^4 DK}{f_a}$$

1 $$D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$$

2 $$K = \prod_f \frac{y_f}{4\pi}$$

$$SU(3)_1 \times SU(3)_2 \xrightarrow[M]{\text{scalar } \Sigma} SU(3)_c$$

axion ($SU(3)_1$), quarks ($SU(3)_2$)

$$\frac{1}{\alpha_1(M)} + \frac{1}{\alpha_2(M)} = \frac{1}{\alpha_s(M)}$$

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Making small instantons important

[Agrawal, Howe '17]

$$m_{UV}^2 = \frac{\Lambda_{UV}^4}{f_a} \approx \frac{M^4 DK}{f_a}$$

Agrawal-Howe Model

1 $$D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$$ ✓

$$SU(3)_1 \times SU(3)_2 \xrightarrow[M]{\text{scalar } \Sigma} SU(3)_c$$

axion — $SU(3)_1$, quarks — $SU(3)_2$

2 $$K = \prod_f \frac{y_f}{4\pi}$$

$$\frac{1}{\alpha_1(M)} + \frac{1}{\alpha_2(M)} = \frac{1}{\alpha_s(M)}$$

small, small, big

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Making small instantons important

$$m_{UV}^2 = \frac{\Lambda_{UV}^4}{f_a} \approx \frac{M^4 DK}{f_a}$$

[Agrawal, Howe ‘17]

Agrawal-Howe Model

$$\frac{1}{\alpha_1(M)} + \frac{1}{\alpha_2(M)} = \frac{1}{\alpha_s(M)}$$

1 $D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$ ✓

2 $K = \prod_f \frac{y_f}{4\pi}$

[Froggatt, Nielsen ‘79]

Dynamical Froggatt-Nielsen Model

3 $y_f \sim \left(\frac{\langle S\rangle}{\Lambda_s}\right)^{n_f}$

4 $V_s \supset -(\mu^2 + g\phi^2)S^2$

flavon

inflaton

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Making small instantons important

$$m_{UV}^2 = \frac{\Lambda_{UV}^4}{f_a} \approx \frac{M^4 D K}{f_a}$$

[Agrawal, Howe ‘17]

Agrawal-Howe Model

$$\frac{1}{\alpha_1(M)} + \frac{1}{\alpha_2(M)} = \frac{1}{\alpha_s(M)}$$

1 $D \approx 0.1 \left(\frac{2\pi}{\alpha(M)}\right)^6 e^{-\frac{2\pi}{\alpha(M)}}$ ✓

2 $K = \prod_f \frac{y_f}{4\pi}$ ✓

[Froggatt, Nielsen ‘79]

Dynamical Froggatt-Nielsen Model

3 $y_f \sim \left(\frac{\langle S \rangle}{\Lambda_s}\right)^{n_f}$

4 $V_s \supset -(\mu^2 + g\phi^2)S^2$

5 $\langle S \rangle \sim \Lambda_s \Rightarrow y_f \sim \mathcal{O}(1) \Rightarrow K \sim 10^{-7}$ *during* inflation

6 $\langle S \rangle \sim 0.2\Lambda_s \Rightarrow y_f = y_{SM} \Rightarrow K \sim 10^{-23}$ *after* inflation

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Raised f_a

$y_f \sim \mathcal{O}(1)$

$$f_a \lesssim 10^{15}\ \text{GeV}\ \left(\frac{D}{1}\right)^{1/2}\left(\frac{K}{10^{-7}}\right)^{1/2}$$

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Conclusions

- *Proof of concept:* **Agrawal-Howe** + (dynamical) **Froggatt-Nielsen**

  ⇒ small instantons:

  - ***important during*** inflation
  - ***irrelevant after*** inflation
- ⇒ relax $\boldsymbol{\theta_{ini} \sim 10^{-2}}$ & raise $\boldsymbol{f_a \sim 10^{15}}$ **GeV**
- Other scales: $\boldsymbol{M \sim 50}$ **TeV** & $\boldsymbol{H_I \sim}$ **eV**
- No new CP phases

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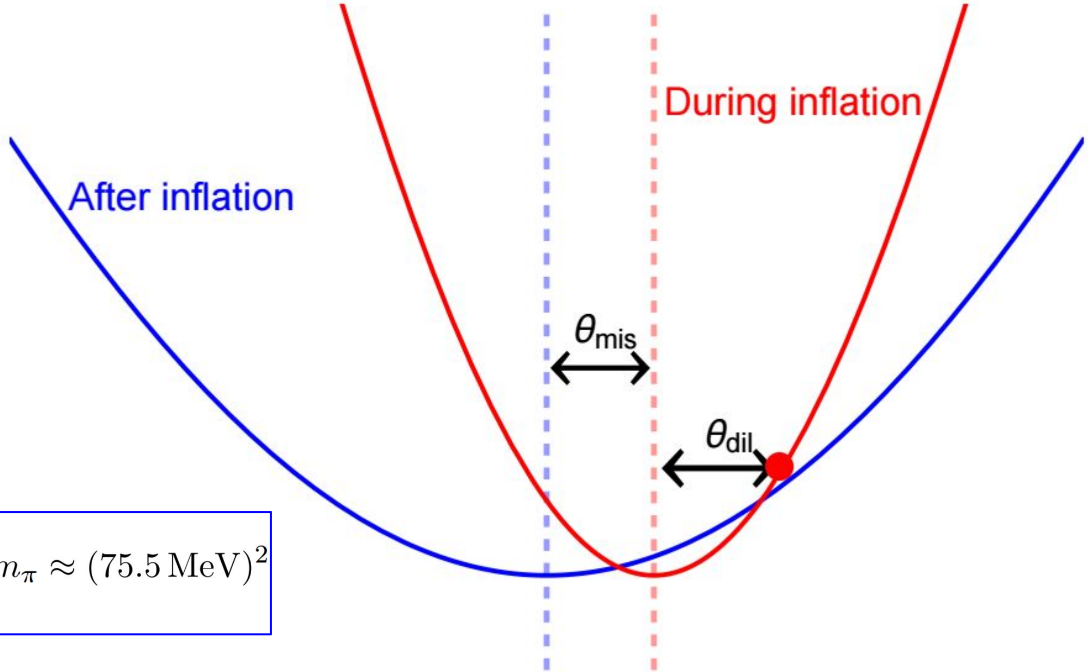
Backup slides

Idea



$$\Lambda_0^2 = \sqrt{\frac{m_u m_d}{(m_u + m_d)^2}} f_\pi m_\pi \approx (75.5\,\mathrm{MeV})^2$$

Raising f_a -- part 1/2

inflation

1 $$\rho_\Sigma < \rho_I \Rightarrow M^4 < H_I^2 M_{Pl}^2$$

dynamical misalignment

2 $$m_{UV}^2 \approx \frac{M^4}{f_a^2} DK \sim H_I^2$$

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Raising f_a -- part 2/2

inflation

1 $$\rho_\Sigma < \rho_I \;\Rightarrow\; M^4 < H_I^2 M_{Pl}^2$$

dynamical misalignment

2 $$m_{UV}^2 \approx \frac{M^4}{f_a^2} DK \sim H_I^2$$

$$y_f \sim \mathcal{O}(1)$$

3 $$\Rightarrow f_a \lesssim 10^{15}\ \text{GeV}\ \left(\frac{D}{1}\right)^{1/2}\left(\frac{K}{10^{-7}}\right)^{1/2}$$

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Details: Agrawal-Howe Model -- part 1/3

$$SU(3)_1 \times SU(3)_2 \to SU(3)_c$$ 1

bi-fundamental

$$\mathcal{V}(\Sigma) = -m_\Sigma^2 \mathrm{Tr}\left(\Sigma\Sigma^\dagger\right) + \frac{\lambda}{2}\left(\mathrm{Tr}\left(\Sigma\Sigma^\dagger\right)\right)^2 + \frac{\kappa}{2}\mathrm{Tr}\left((\Sigma\Sigma^\dagger)^2\right)$$ 2

VEV

$$\langle\Sigma\rangle = \frac{f_\Sigma}{2}\mathbb{1}_3 \qquad f_\Sigma = \frac{m_\Sigma}{\sqrt{\kappa + 3\lambda}}$$ 3

symmetry breaking scale

$$M^2 = (g_1^2 + g_2^2) f_\Sigma^2$$ 4

Details: Agrawal-Howe Model -- part 2/3

1 $$\frac{\alpha_1}{8\pi}\left(\frac{a_1}{f_a}-\bar{\theta}_1\right)G_1\tilde{G}_1+\frac{\alpha_2}{8\pi}\left(\frac{a_2}{f_a}-\bar{\theta}_2\right)G_2\tilde{G}_2,$$

original axions couplings

2 $$\Lambda_1^4\cos\left(\frac{a_1}{f_a}-\bar{\theta}_1\right)+\Lambda_2^4\cos\left(\frac{a_2}{f_a}-\bar{\theta}_2\right)+\frac{\alpha_s}{8\pi}\left(\frac{a_1}{f_a}-\bar{\theta}_1+\frac{a_2}{f_a}-\bar{\theta}_2\right)G\tilde{G},$$

axions couplings after symm. breaking

3 $$\frac{1}{\alpha_s(M)}=\frac{1}{\alpha_1(M)}+\frac{1}{\alpha_2(M)}$$

gauge couplings

4 $$\bar{\theta}_{\text{eff}}=\frac{a_1+a_2}{f_a}-(\bar{\theta}_1+\bar{\theta}_2)$$

QCD strong CP problem solved

Details: Agrawal-Howe Model -- part 3/3

1
$$\frac{\alpha_1}{8\pi}\left(\frac{a_1}{f_a}-\bar{\theta}_1\right)G_1\tilde{G}_1+\frac{\alpha_2}{8\pi}\left(\frac{a_2}{f_a}-\bar{\theta}_2\right)G_2\tilde{G}_2,$$
original axions couplings

2
$$\Lambda_1^4\cos\left(\frac{a_1}{f_a}-\bar{\theta}_1\right)+\Lambda_2^4\cos\left(\frac{a_2}{f_a}-\bar{\theta}_2\right)+\frac{\alpha_s}{8\pi}\left(\frac{a_1}{f_a}-\bar{\theta}_1+\frac{a_2}{f_a}-\bar{\theta}_2\right)G\tilde{G},$$
axions couplings after symm. breaking

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$$\frac{1}{\alpha_s(M)}=\frac{1}{\alpha_1(M)}+\frac{1}{\alpha_2(M)}$$
gauge couplings

4
$$\bar{\theta}_{\text{eff}}=\frac{a_1+a_2}{f_a}-(\bar{\theta}_1+\bar{\theta}_2)$$
QCD strong CP problem solved

5
$$\Lambda_i^2\approx\sqrt{\frac{2}{b_i-4}K_iD[\alpha_i(M)]M^2},\quad i=1,2,$$
$$D[\alpha_i]\approx 0.1\left(\frac{2\pi}{\alpha_i}\right)^6 e^{-\frac{2\pi}{\alpha_i}}\equiv D_i,$$
$$K_i=\prod_j\left(\frac{y_j}{4\pi}\right).$$
small instantons

6
$$m_i^2=\frac{\Lambda_i^2}{f_a},$$
contributions to axion masses from small instantons

Requirements & Scales -- part 1/2

1

$$\mathcal{V}(\Sigma) \sim M^4 < H_I^2 M_{\rm Pl}^2 \Rightarrow M^2 < H_I M_{\rm Pl},$$

slow-roll inflation

2

$$\begin{aligned} \tilde{m}_1^{\rm UV} &\approx 3\times 10^{-4}\,\frac{M^2}{f_a}\sqrt{D_1}\,\sqrt{\frac{\tilde{K}_1}{10^{-7}}} \\ &\sim H_I\ , \\ \tilde{m}_i^{\rm UV} &\approx \frac{M^2}{f_a}\sqrt{D_i} \\ &> H_I, \qquad\qquad i = 2,\cdots n, \end{aligned}$$

diluting misalignment

3

$$\begin{aligned} m_1^{\rm UV} &= \sqrt{y_u y_d y_c y_s y_t y_b}\ \tilde{m}_1^{\rm UV} \approx 5.6\times 10^{-9}\ \tilde{m}_1^{\rm UV} < m^{\rm IR} \\ &\Rightarrow 2\times 10^{-12}\ M^2\sqrt{D_1}\sqrt{\frac{\tilde{K}_1}{10^{-7}}} < \Lambda_0^2 \approx (75.5\ {\rm MeV})^2, \end{aligned}$$

QCD axion *after* inflation: small instanton < large instantons

combining sets the scales

Requirements & Scales -- part 2/2

$$M \lesssim 50~\text{TeV}~D_1^{-1/4} \left(\frac{\tilde{K}_1}{10^{-7}}\right)^{-1/4},$$

$$f_a \lesssim 10^{15}~\text{GeV}~\sqrt{D_1} \left(\frac{\tilde{K}_1}{10^{-7}}\right)^{1/2},$$

$$\text{eV} \left(\frac{M}{50~\text{TeV}}\right)^2 \lesssim H_I \lesssim 2.5 \times 10^3 \text{eV} \left(\frac{M}{50~\text{TeV}}\right)^2 \left(\frac{10^{15}~\text{GeV}}{f_a}\right)$$

Details: Froggatt-Nielsen Model -- part 1/3

1

$$\mathcal{V}(S) = (-\mu_s^2 - g\phi^2)S^\dagger S + \lambda_s(S^\dagger S)^2 - A^2(SS + S^\dagger S^\dagger)$$

flavon potential

2

$$2\langle S\rangle^2 \equiv v_s^2 = \frac{\mu_s^2 + g\phi^2 + 2A^2}{\lambda_s}.$$

flavon VEV

3

$$\mathcal{L} = y_{ij}^u \left(\frac{S}{\Lambda_s}\right)^{m_{ij}} \bar{Q}_i \tilde{H} u_j + y_{ij}^d \left(\frac{S}{\Lambda_s}\right)^{n_{ij}} \bar{Q}_i H d_j$$

Yukawas from FN

Details: Froggatt-Nielsen Model -- part 2/3

1

$$\mathcal{V}(S) = (-\mu_s^2 - g\phi^2)S^\dagger S + \lambda_s (S^\dagger S)^2 - A^2(SS + S^\dagger S^\dagger)$$

flavon potential

$$2\langle S\rangle^2 \equiv v_s^2 = \frac{\mu_s^2 + g\phi^2 + 2A^2}{\lambda_s}.$$

2

flavon VEV

3

$$\mathcal{L} = y_{ij}^u \left(\frac{S}{\Lambda_s}\right)^{m_{ij}} \bar{Q}_i \tilde{H} u_j + y_{ij}^d \left(\frac{S}{\Lambda_s}\right)^{n_{ij}} \bar{Q}_i H d_j$$

Yukawas from FN

4

$$S : -1,\ \bar{Q}_3 : 0,\ \bar{Q}_2 : 2,\ \bar{Q}_1 : 3,$$

$$u_3 : 0,\ u_2 : 1,\ u_1 : 4,\ d_3 : 2,\ d_2 : 2,\ d_1 : 3.$$

FN charges

$$y_u = \begin{pmatrix} y_{11}^u \epsilon^7 & y_{12}^u \epsilon^4 & y_{13}^u \epsilon^3 \\ y_{21}^u \epsilon^6 & y_{22}^u \epsilon^3 & y_{23}^u \epsilon^2 \\ y_{31}^u \epsilon^4 & y_{32}^u \epsilon & y_{33}^u \end{pmatrix}$$

$$y_d = \begin{pmatrix} y_{11}^d \epsilon^6 & y_{12}^d \epsilon^5 & y_{13}^d \epsilon^5 \\ y_{21}^d \epsilon^5 & y_{22}^d \epsilon^4 & y_{23}^d \epsilon^4 \\ y_{31}^d \epsilon^3 & y_{32}^d \epsilon^2 & y_{33}^d \epsilon^2 \end{pmatrix}$$

$$\epsilon = \frac{v_s}{\sqrt{2}\Lambda_s}.$$

5

resulting Yukawas

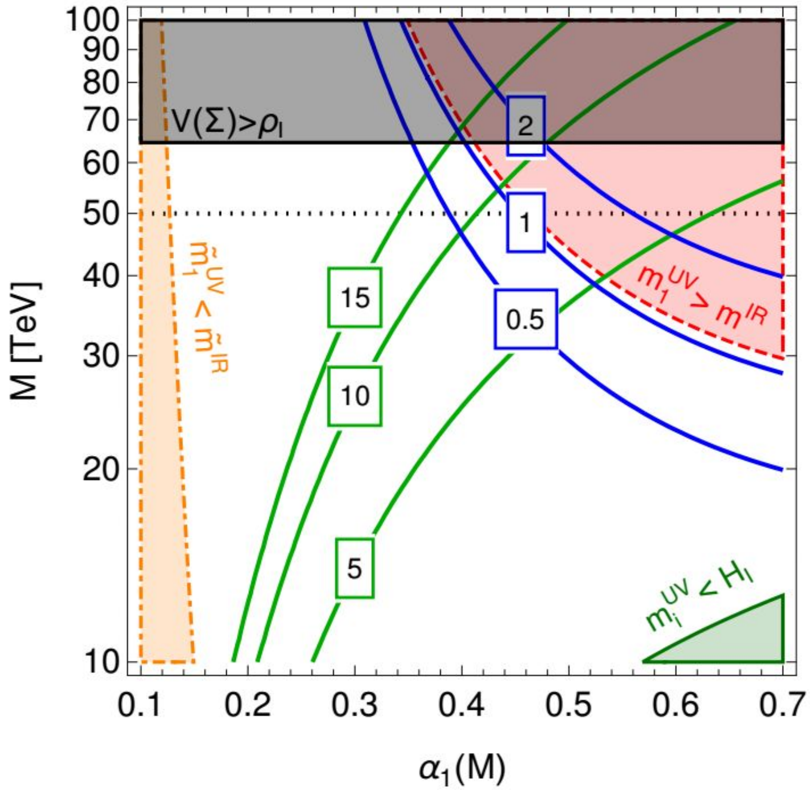
Details: Froggatt-Nielsen Model -- part 3/3

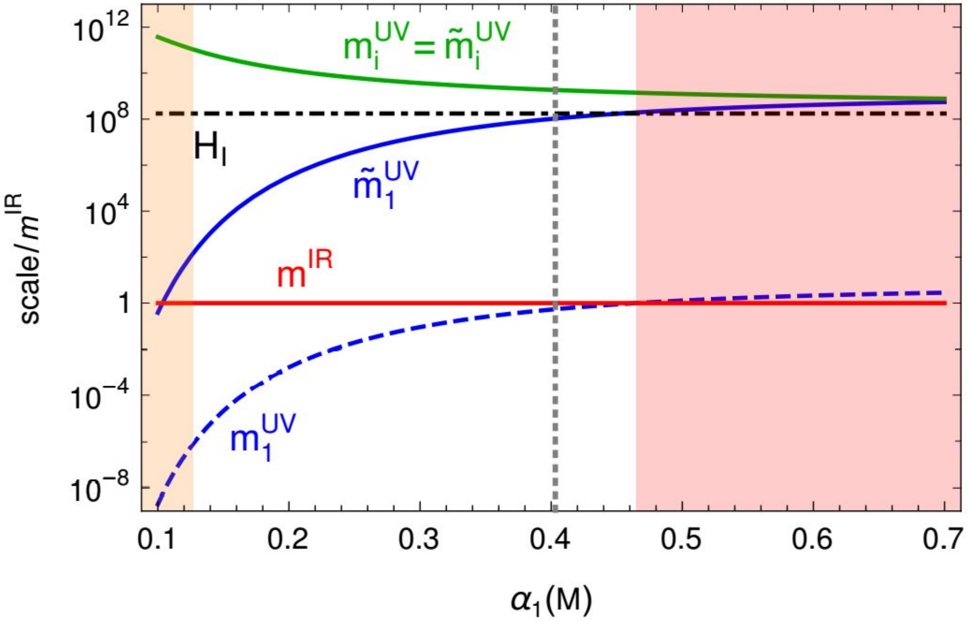
1
$$\mathcal{V}(S) \sim \frac{\lambda_s v_s^4}{4} = \lambda_s \epsilon^4 \Lambda_s^4 < 3 M_{\rm Pl}^2 H_I^2$$
slow-roll inflation

2
$$\Lambda_s < 60 \text{ TeV} \left(\frac{H_I}{\text{eV}}\right)^{1/4}$$
new flavor physics

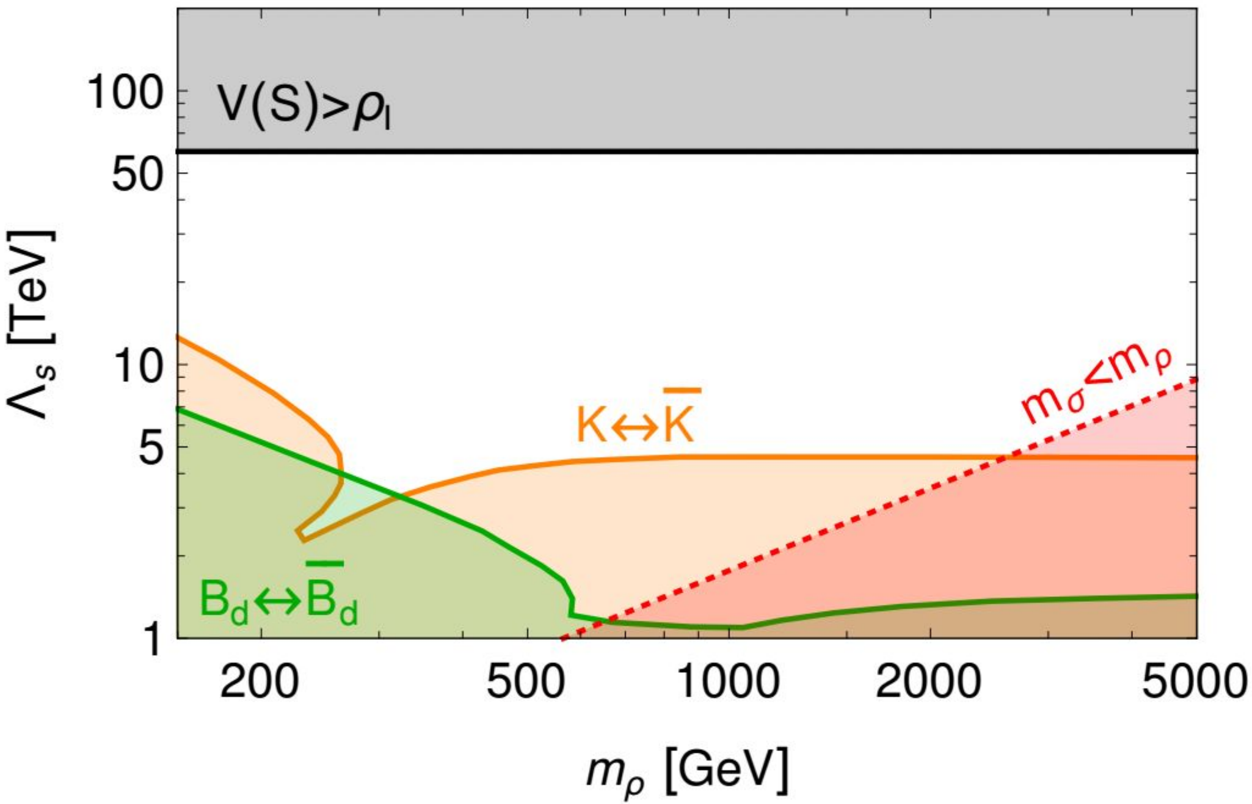
3
$$m_\sigma^2 = 2\lambda_s v_s^2 = 4\epsilon^2 \lambda_s \Lambda_s^2 = 2\mu_s^2 + 4A^2 + 2g\phi^2$$
$$m_\rho^2 = 4A^2 < m_\sigma^2 \ .$$
masses of flavon components

Parameter Space -- part 1/2





Parameter Space -- part 2/2



CP phases and mismatch -- part 1/2

$$\arg(\mathrm{Det}\, M) \;=\; \arg\left[(\mathrm{Det}\, y_u)\,(\mathrm{Det}\, y_d)\right]$$

$$(\mathrm{Det}\, y_u)\,(\mathrm{Det}\, y_d) \propto \epsilon^{\alpha}\epsilon^{\beta} = \epsilon^{22},$$

$$\alpha = \sum_{i=1}^{3}\left[q(Q_i) + q(u_i)\right] = 10, \quad \beta = \sum_{i=1}^{3}\left[q(Q_i) + q(d_i)\right] = 12,$$

as long as the flavon is **real**, the contribution to the strong CP phase from the quark mass matrix ***remains the same*** as its VEV and thus **ε** evolve

CP phases and mismatch -- part 2/2

1

$$\bar{\theta}_{\rm SM}^{\rm finite} = -\frac{7}{9}\frac{\alpha_s}{4\pi}\left(\frac{\alpha_W}{4\pi}\right)^2 \frac{m_s^2 m_c^2}{m_W^4} J \ln\frac{m_t^2}{m_b^2} \ln^2\frac{m_b^2}{m_c^2}\left(\ln\frac{m_c^2}{m_s^2} + \frac{2}{3}\ln\frac{m_b^2}{m_c^2}\right) \sim 10^{-19}$$

Strong CP from Weak CP (4-loop Cheburashka diagrams)

2

$$J \equiv \mathrm{Im}\left(V_{us}V_{cb}V_{ub}^*V_{cs}^*\right) = s_{12}s_{13}s_{23}c_{12}c_{13}^2 c_{23}\sin\delta$$

Jarlskog invariant

3

$$|V_{us}| \sim |V_{cd}| \sim \epsilon$$

$$|V_{cb}| \sim |V_{ts}| \sim \epsilon^2$$

$$|V_{ub}| \sim |V_{td}| \sim \epsilon^3$$

CKM

4

$$\theta_{\rm mis} \sim \left(\frac{\alpha_W}{4\pi}\right)^2 \sim 10^{-5}$$

mismatch during inflation

Dark Matter & Misalignment Mechanism

1
$$\frac{d^2 a_i}{dN^2} + 3\frac{da_i}{dN} + \frac{\tilde{m}_i^2}{H_I^2} a_i = 0.$$

axion evolution
($dN = H_I\, dt$)

2
$$\tilde{m}_1^{\mathrm{UV}}/H_I = 0.59\,, \quad \tilde{m}_i^{\mathrm{UV}}/H_I = 11$$
$$\Rightarrow \theta_{\mathrm{dil};1} = 0.017\,\theta_{0;1}\,, \quad \theta_{\mathrm{dil};i} = 10^{-17}\theta_{0;i}\,, \quad i = 2,3,4.$$

3
$$\Omega_1 h^2 \approx 0.12 \left(\frac{\theta_1}{0.017}\right)^2 \left(\frac{f_a}{10^{15}\,\mathrm{GeV}}\right)^{\frac{n+6}{n+4}} (3.8)^{\frac{2}{4+n}} (10)^{\frac{4-n}{4+n}} \left(\frac{\chi_0(1.5)}{3.7\times 10^{-14}\,\mathrm{GeV}^4}\right)^{-\frac{1}{4+n}}$$