First Order Electroweak Phase Transitions in the SM with a Singlet Extension

Anthony Hooper

UNIVERSITY *of* NEBRASKA–LINCOLN

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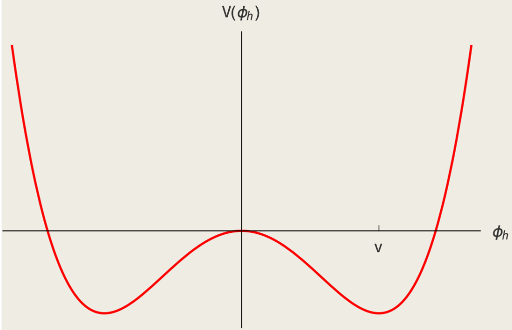
Outline

What we know about the Higgs from measurements



$$V_{Higgs}^{SM} = \frac{1}{4}\lambda_h \left(\phi_h^\dagger \phi_h\right)^2 - \frac{1}{2}\left|\mu^2\right| \left(\phi_h^\dagger \phi_h\right)$$

$\Downarrow$

Vacuum expectation value (vev)

$\Downarrow$

Higgs boson mass

What we don't know from measurements...

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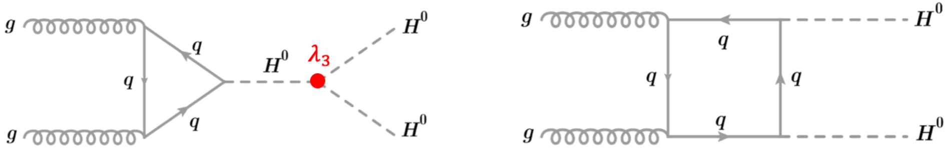
1 

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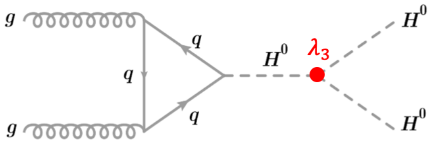
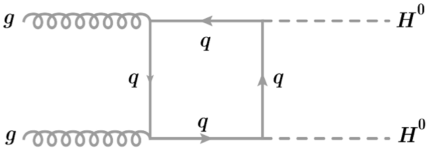
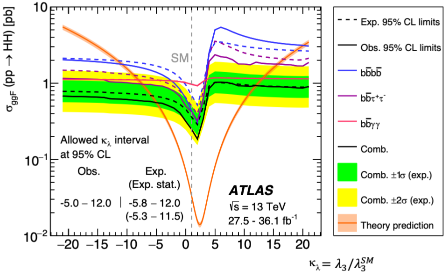


[1]





$$\sigma_{gg\to H^0H^0} \propto \int \mathrm{d}\hat{t} \left| C_\triangle F_\triangle + C_\Box F_\Box \right|^2$$

Confidence interval of 95% CL:

$$-5 < \lambda_3 < 12$$

Room for new physics

[1] arXiv:1906.02025v1

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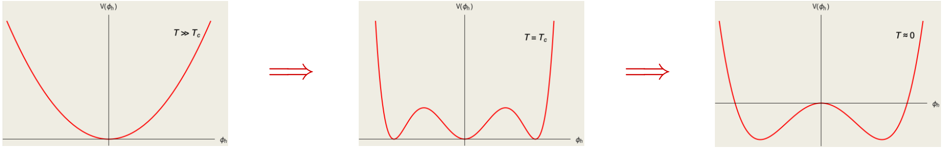
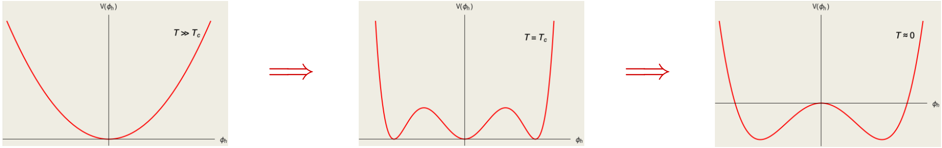
Possible early universe phase transitions

First Order



Possible early universe phase transitions

First Order



Supports electroweak baryogenesis

–sets the energy scale to levels we can measure

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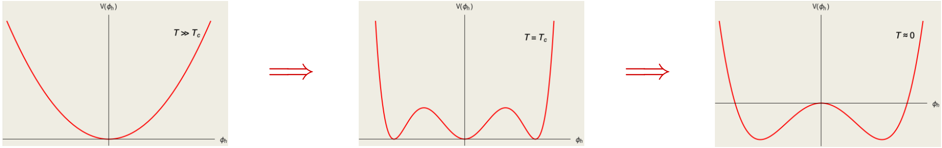
Possible early universe phase transitions

First Order



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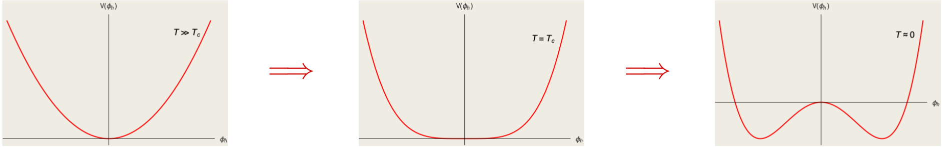
SM Crossover



Higgs+Singlet Potential

General Higgs+Singlet

$$V_o = \frac{1}{2}m_o^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + t_s\phi_s + a_{hs}\phi_h^2\phi_s + \frac{1}{2}m_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \frac{1}{3}a_s\phi_s^3 + \frac{1}{4}\lambda_s\phi_s^4$$

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$\mathbb{Z}_2$ Symmetry Considerations

$$V_o = \frac{1}{2}m_o^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + \cancel{t_s\phi_s} + \cancel{a_{hs}\phi_h^2\phi_s} + \frac{1}{2}m_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \cancel{\frac{1}{3}a_s\phi_s^3} + \frac{1}{4}\lambda_s\phi_s^4$$

Perturbation Considerations

$$V_o = \frac{1}{2}m_o^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + t_s\phi_s + a_{hs}\phi_h^2\phi_s + \frac{1}{2}m_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \cancel{\frac{1}{3}a_s\phi_s^3} + \cancel{\frac{1}{4}\lambda_s\phi_s^4}$$

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Higgs+Singlet Potential

General Considerations

$$V_o = \frac{1}{2}m_o^2\phi_h^2 + \frac{1}{4}\lambda_h\phi_h^4 + \cancel{t_s\phi_s} + a_{hs}\phi_h^2\phi_s + \frac{1}{2}m_s^2\phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \frac{1}{3}a_s\phi_s^3 + \frac{1}{4}\lambda_s\phi_s^4$$

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At finite T, the one-loop thermal potential leading terms in the high temperature expansion

$$V_{1-loop}^{T\neq 0} = \left(\frac{1}{2}c_h\phi_h^2 + \frac{1}{2}c_s\phi_s^2 + m_3\phi_s\right)T^2,$$

where[2]

$$c_h = \frac{1}{48}(9g^2 + 3g'^2 + 2(y_t^2 + 12\lambda_h + 2\lambda_{hs}))$$

$$c_s = \frac{1}{12}(4\lambda_{hs} + 3\lambda_s),$$

$$m_3 = \frac{1}{12}(a_s + 4a_{hs}).$$

$$V = V_o + V_{1-loop}^{T\neq 0}$$

[2] arXiv:1107.5441v1

Reparametrizing

$$V_o = \frac{1}{2} m_o^2 \phi_h^2 + \frac{1}{4}\lambda_h \phi_h^4 + a_{hs}\phi_h^2\phi_s + \frac{1}{2} m_s^2 \phi_s^2 + \frac{1}{2}\lambda_{hs}\phi_h^2\phi_s^2 + \frac{1}{3} a_s \phi_s^3 + \frac{1}{4}\lambda_s \phi_s^4$$

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Minimum Equations:

$$\left.\frac{\mathrm{d}V_o}{\mathrm{d}\phi_h}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} = 0 \qquad \left.\frac{\mathrm{d}V_o}{\mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} = 0$$

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In the basis (ϕ_h, ϕ_s), the mass squared matrix is

$$\mathcal{M}^2 = \begin{pmatrix} \left.\frac{\mathrm{d}^2 V_o}{\mathrm{d}\phi_h^2}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} & \left.\frac{\mathrm{d}^2 V_o}{\mathrm{d}\phi_h \mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} \\ \left.\frac{\mathrm{d}^2 V_o}{\mathrm{d}\phi_h \mathrm{d}\phi_s}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} & \left.\frac{\mathrm{d}^2 V_o}{\mathrm{d}\phi_s^2}\right|_{\substack{\phi_h\to v\\ \phi_s\to u}} \end{pmatrix} = \begin{pmatrix} 2v^2\lambda_h & 2a_{hs}v + 2vu\lambda_{hs} \\ 2vu\lambda_{hs} & m_s^2 + 2a_s u + v^2\lambda_{hs} + 3u^2\lambda_s \end{pmatrix}$$

$\mathrm{Diag}\left[\mathcal{M}^2\right] = \begin{pmatrix} m_h^2 & 0 \\ 0 & m_s^2 \end{pmatrix}$, where m_h is the Higgs mass and m_ϕ is the singlet mass

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Similarity invariance of the trace: $tr\left(\mathcal{M}^2\right) = tr\left(Diag\left[\mathcal{M}^2\right]\right)$

Determinant properties of rotational matrices: $det\left(\mathcal{M}^2\right) = det\left(Diag\left[\mathcal{M}^2\right]\right)$

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Reparametrizing

Solve for m_o^2, m_s^2, a_{hs}, and a_s in terms of m_ϕ, λ_h, λ_{hs}, λ_s, u, m_h, v; yields two sets of solutions:

$$m_o^2 = -v^2\lambda_h \pm \frac{u}{v}\Delta + u^2\lambda_{hs}$$

$$m_s^2 = -m_h^2 - m_\phi^2 + 2v^2\lambda_h \pm \frac{v}{u}\Delta + v^2\lambda_{hs} + u^2\lambda_s$$

$$a_{hs} = \mp\frac{1}{2v}\Delta - u\lambda_{hs}$$

$$a_s = \frac{1}{2u^2}(2m_h^2 u + 2m_\phi^2 u - 4v^2 u\lambda_h \mp v\Delta - 2v^2 u\lambda_{hs} - 4u^3\lambda_s)$$

where $\Delta = \sqrt{(m_h^2 - 2v^2\lambda_h)(2v^2\lambda_h - m_\phi^2)}$

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$$
\begin{aligned}
m_o^2 &= -v^2\lambda_h \pm \frac{u}{v}\Delta + u^2\lambda_{hs} \\
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a_{hs} &= \mp\frac{1}{2v}\Delta - u\lambda_{hs} \\
a_s &= \frac{1}{2u^2}(2m_h^2 u + 2m_\phi^2 u - 4v^2 u\lambda_h \mp v\Delta - 2v^2 u\lambda_{hs} - 4u^3\lambda_s)
\end{aligned}
$$

where $\Delta = \sqrt{(m_h^2 - 2v^2\lambda_h)(2v^2\lambda_h - m_\phi^2)}$

$$V \to V(\phi_h, \phi_s, T, m_\phi, \lambda_h, \lambda_{hs}, \lambda_s, u)$$

Ranges of the new parameters

$$
\begin{aligned}
\text{Stability conditions :} &\quad \lambda_h\lambda_s \in [0, \sqrt{4\pi}], \quad \lambda_{hs} \in [-\sqrt{\lambda_h\lambda_s}, \sqrt{4\pi}] \\
\text{singlet mass :} &\quad m_\phi \sim 500 - 5000\,\text{GeV} \\
\text{singlet vev :} &\quad u\,\text{can be anything?}
\end{aligned}
$$

Higgs Trilinear Coupling

Gauge to mass eigenstate basis: $(\phi_h, \phi_s) \rightarrow (h_1, h_2)$

$$\phi_h = h_1 \cos\theta - h_2 \sin\theta + v$$
$$\phi_s = h_1 \sin\theta + h_2 \cos\theta + u$$

where θ is the mixing angle and is found from $\mathcal{M}^2$ with $\tan 2\theta = \frac{m_{12}+m_{21}}{m_{11}-m_{22}}$

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Higgs Trilinear Coupling: Let $h_1 < h_2$, then

$$\lambda_3 = \frac{\mathrm{d}^3 V_o(h_1, h_2)}{\mathrm{d}h_1^3}$$

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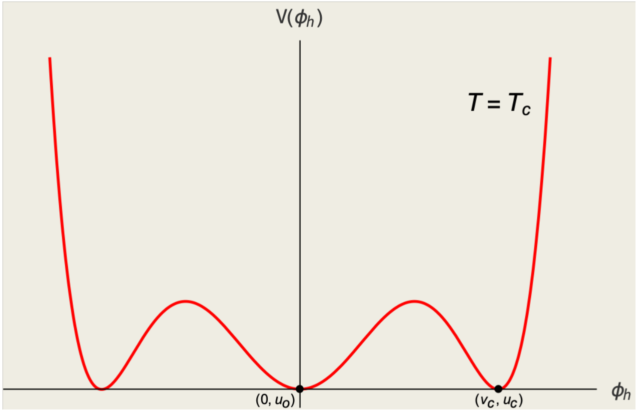
$$\lambda_3 = \frac{\mathrm{d}^3 V_o(h_1, h_2)}{\mathrm{d}h_1^3}$$

$$V_o(\phi_h, \phi_s) \rightarrow V_o(h_1, h_2)$$

$$\Longrightarrow \boxed{\lambda_3 = 6v\lambda_h \cos^3\theta \left(1 + \frac{\lambda_{hs}u + a_{hs}}{\lambda_h v} \tan\theta + \frac{\lambda_{hs}}{\lambda_h} \tan^2\theta + \frac{3\lambda_s u + a_s}{3\lambda_h v} \tan^3\theta\right)}$$

Conditions Imposed at Critical Temperature



degenerate requirement:

$$V(0, u_o, T_c) = V(v_c, u_c, T_c)$$

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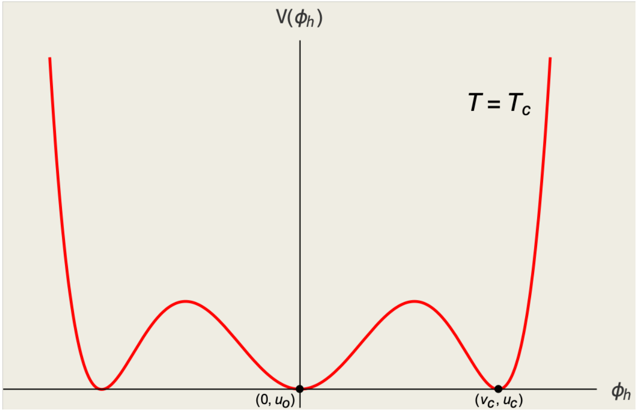

degenerate requirement:

$$V(0, u_o, T_c) = V(v_c, u_c, T_c)$$

minimization requirement:

$$\phi_h = 0 : \quad \frac{\mathrm{d}V(0, u_o, T_c)}{\mathrm{d}\phi_s} = 0$$

$$\phi_h = v_c : \quad \frac{\mathrm{d}V(v_c, u_c, T_c)}{\mathrm{d}\phi_h} = 0$$

$$\frac{\mathrm{d}V(v_c, u_c, T_c)}{\mathrm{d}\phi_s} = 0$$

and $\frac{\mathrm{d}^2V}{\mathrm{d}\phi_h^2} > 0$ at critical points

Equations to Solve

degenerate requirement:

$$0 = 12 m_3 T_c^2 (u_c - u_o) + 6(m_s^2 + c_s T_c^2)(u_c^2 - u_o^2) + 4a_s(u_c^3 - u_o^3) + 3(u_c^4 - u_o^4)\lambda_s$$
$$+ 6 m_o^2 v_c^2 + 6 c_h T_c^2 v_c^2 + 12 a_{hs} u_c v_c^2 + 3 v_c^4 \lambda_h + 6 u_c^2 v_c^2 \lambda_{hs}$$

minimization requirements:

$$0 = m_3 T_c^2 + u_o(m_s^2 + c_s T_c^2 + u_o(a_s + u_o \lambda_s))$$

$$0 = v_c(m_o^2 + c_h T_c^2 + 2 a_{hs} u_c + v_c^2 \lambda_h + u_c^2 \lambda_{hs})$$

$$0 = m_3 T_c^2 + m_s^2 u_c + c_s T_c^2 u_c + a_s u_c^2 + a_{hs} v_c^2 + u_c v_c^2 \lambda_{hs} + u_c^3 \lambda_s$$

Monte-Carlo Scan Code Structure

- Generates N lists of random free parameters: $\left\{\{m_\phi, \lambda_h, \lambda_{hs}, \lambda_s, u\}, \dots\right\}$
- Requires parameters to satisfy[3]

$$\sin^2\theta < 0.14$$

- Range for free parameters

$$m_\phi \in [0.5, 5]\ \mathrm{TeV}$$

$$\lambda_h, \lambda_s \in \left[0, \sqrt{4\pi}\right]$$

$$\lambda_{hs} \in \left[-\sqrt{\lambda_h\lambda_s}, \sqrt{4\pi}\right]$$

$$u \in [-10, 10]\ \mathrm{TeV}$$

[3] arXiv:1909.02845

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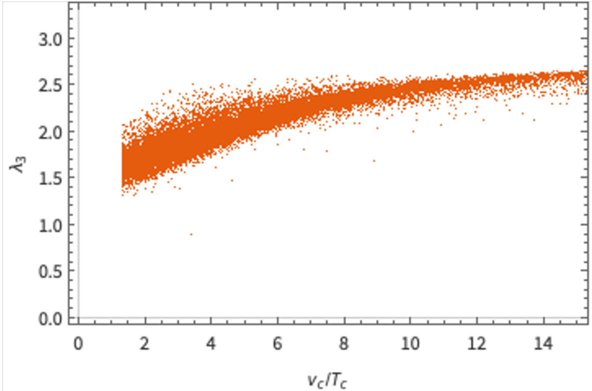
$$u \in [-10, 10]\ \mathrm{TeV}$$

- Takes generated parameters, imposes constraints, solves four equations simultaneously.
- Checks if $0 < v_c < v$.
- Checks if solutions are global minimums at $V(v, u, 0,\ V(0, u_o, T_c)$, and $V(v_c, u_c, T_c)$.
- Checks if phase transition is strong by requiring[4]

$$\frac{v_c}{T_c} > 1.3$$

[3] arXiv:1909.02845
[4] arXiv:1711.11541

Results



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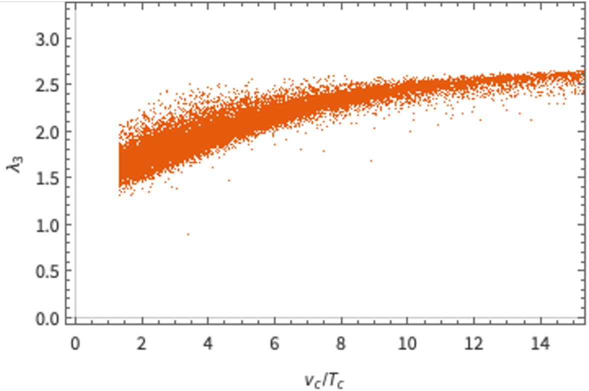
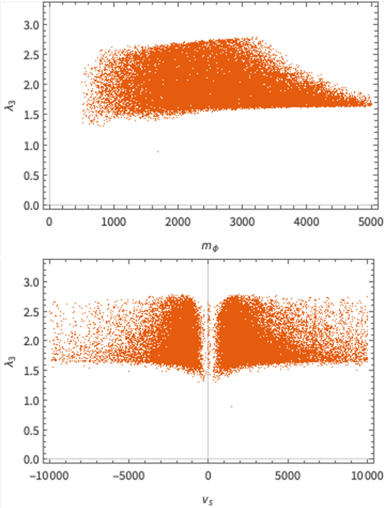
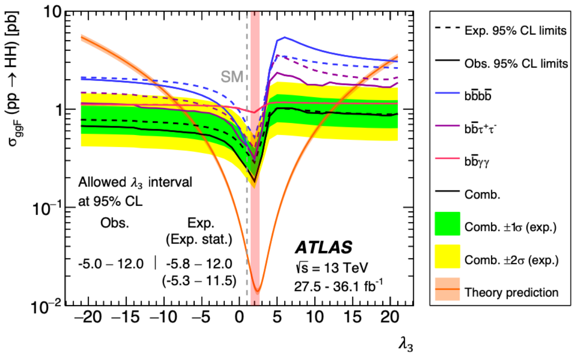
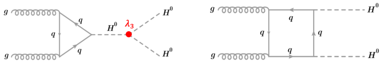

3.0
2.5
2.0
1.5
1.0
0.5
0.0
λ_3
0
2
4
6
8
10
12
14
v_c/T_c

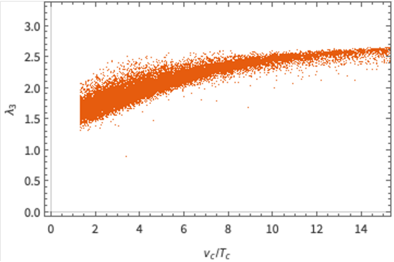

3.0
2.5
2.0
1.5
1.0
0.5
0.0
λ_3
0
1000
2000
3000
4000
5000
m_ϕ
−10000
−5000
0
5000
10000
v_s

Conclusions





$$\sigma_{gg\to H^0H^0} \propto \int d\hat{t}\left|C_\triangle F_\triangle + C_\square F_\square\right|^2$$



Thank you!

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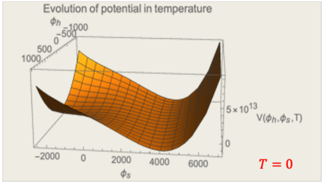
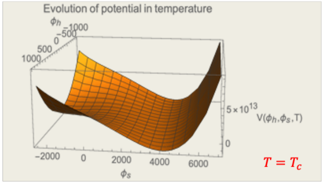
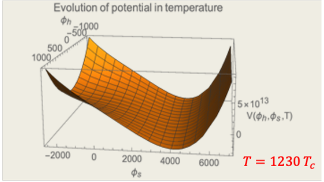
Potential Evolution in Temperature

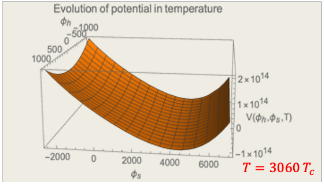
$m_\phi = 3285.4$, $\lambda_h = 3.09441$, $\lambda_{hs} = 2.23889$, $\lambda_s = 0.468243$, $v_s = 3853.98$

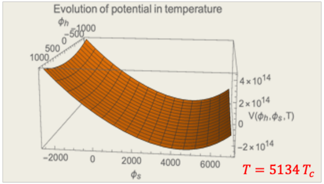
$u_o = 3830.9$, $u_c = 3839.65$, $v_c = 149.589$, $T_c = 51.344$, $\lambda_3 = 1.80276$











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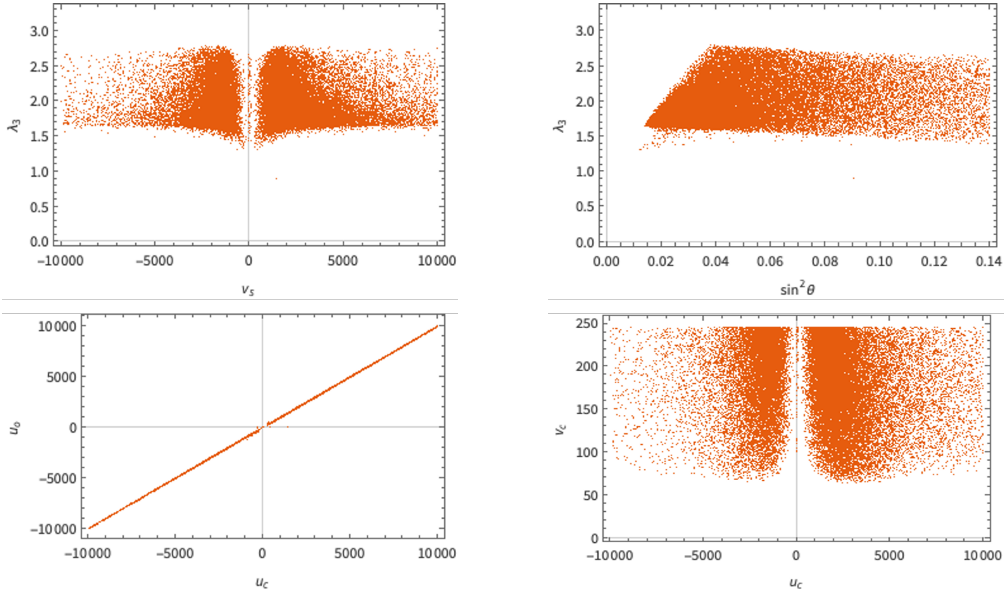
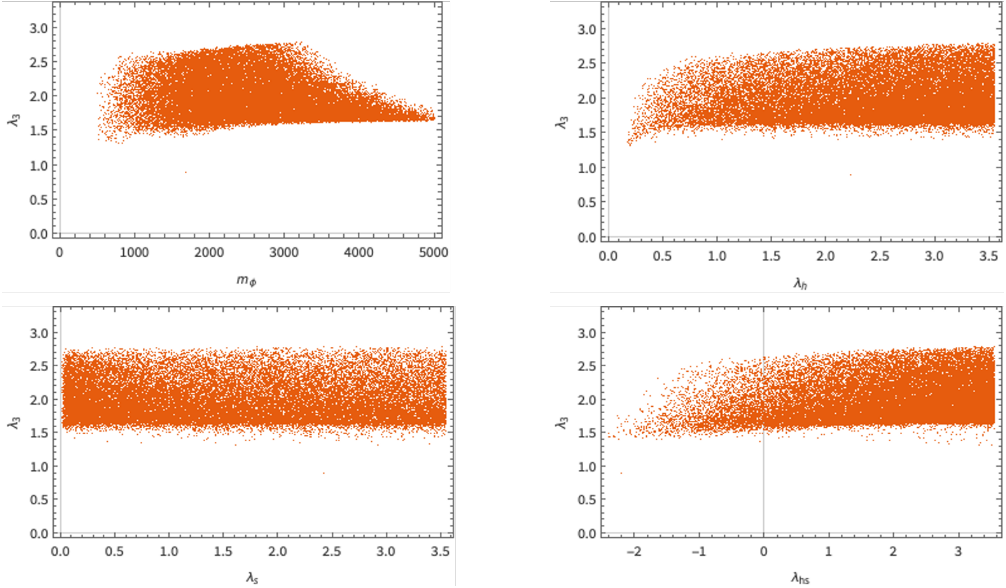
Parameter Space

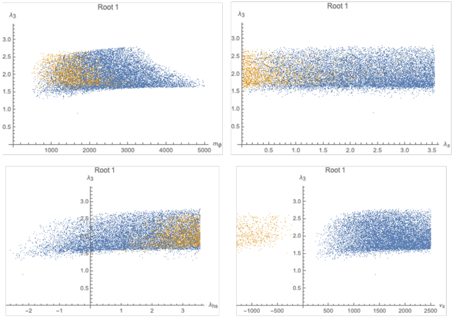


Parameter Space

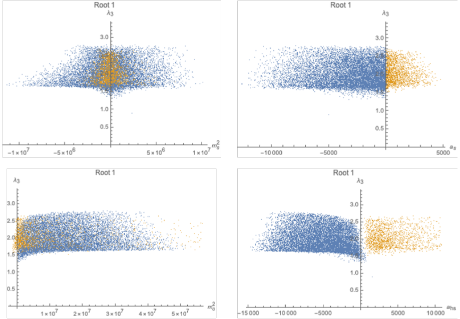


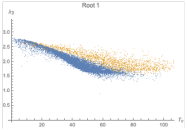
Fixed Parameter Space

Free Parameters



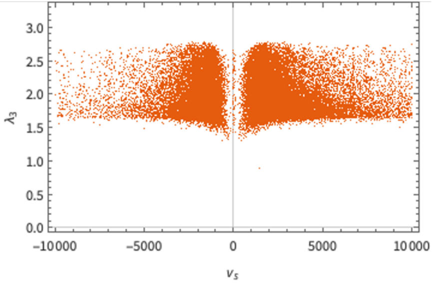
Fixed Parameters

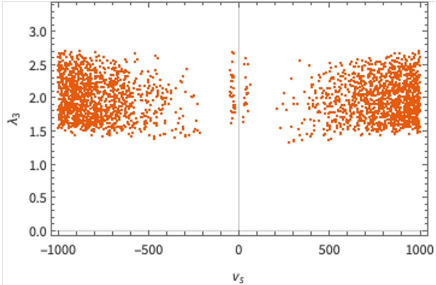




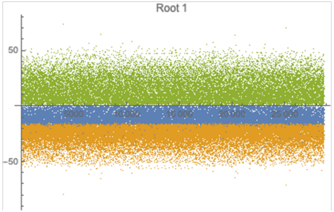
- Preferred
- Not-Preferred

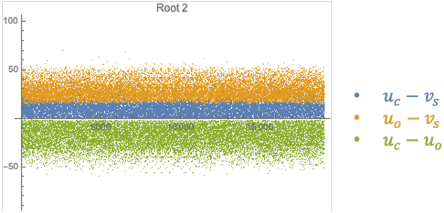
Singlet vev Regimes



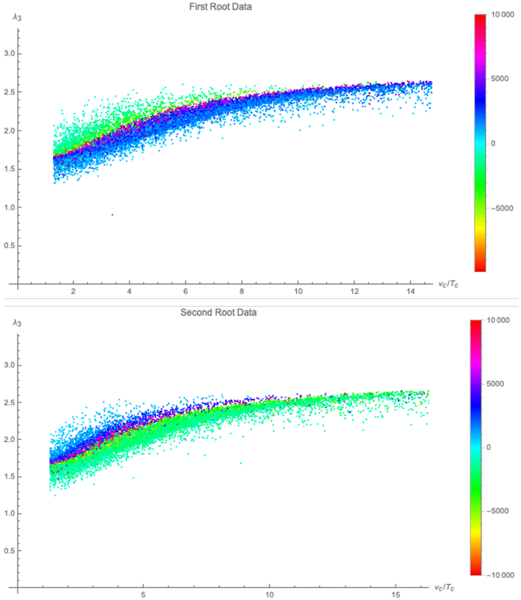


$$(a_s + 4_{hs})T_c^2(u_c - u_o) + 6\left(m_s^2 + \frac{1}{12}(4\lambda_{hs} + 3\lambda_s)T_c^2\right)(u_c^2 - u_o^2) + 4a_s(u_c^3 - u_o^3) + 3(u_c^4 - u_o^4)\lambda_s - 3v_c^4\lambda_h = 0$$





Singlet vev Density Plots



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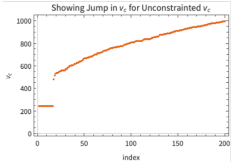
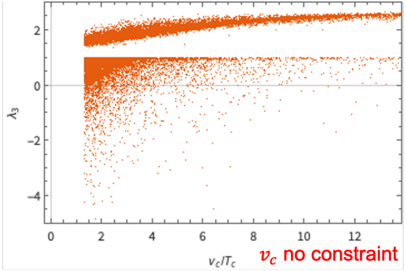
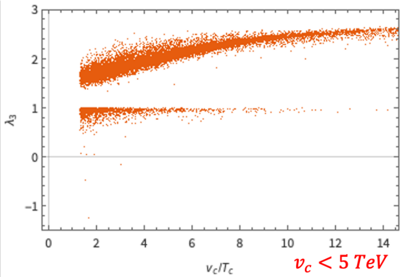
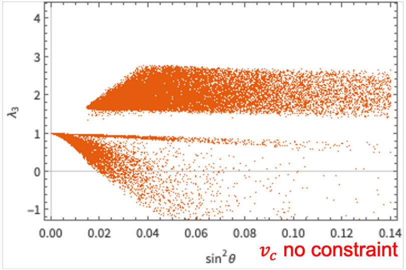
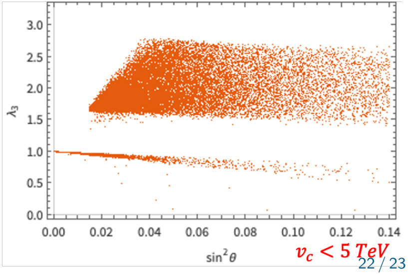
Critical vev Constraints



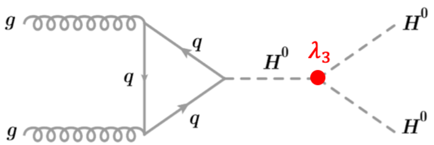


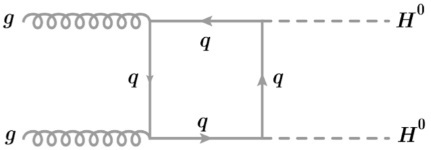


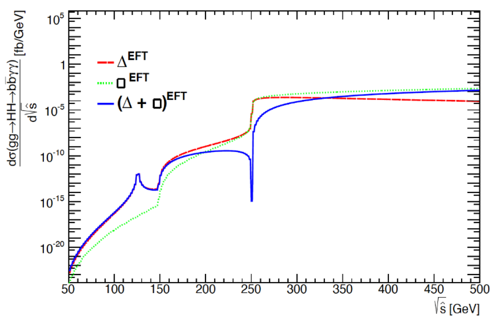


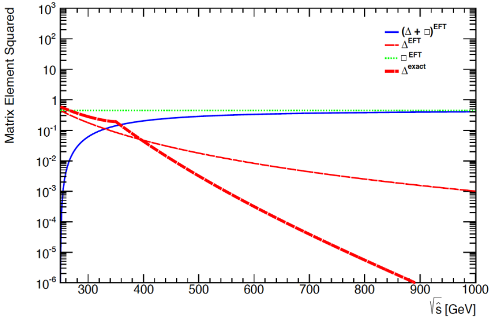


Interference Between Box and Triangle Diagrams









[5] arXiv:1408.5010v2