Charm-meson Triangle Singularity in e^+e^- annihilation into $D^{*0}\bar{D}^0\gamma$

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Charm-meson Triangle Singularity in e^+e^- Annihilation into $D^{*0}\bar{D}^0 + \gamma$

Authors: Eric Braaten, Li-Ping He, Kevin Ingles, Jun Jiang

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Introduction: X(3872) and Triangle Singularity

- First exotic hadron discovered (S. K. Choi, *et al.* [Belle Collaboration], 2003)
- $J^{PC} = 1^{++}$ (R. Aaij, *et al.* [LHCb Collaboration], 2013)
- 7 different decay modes
- The internal structure of X(3872) is under debate.

Compact tetraquark state, $\chi_{c1}(2P)$ charmonium,
$D^{*0}\bar{D}^0$ molecule state, mixed molecule-charmonium state

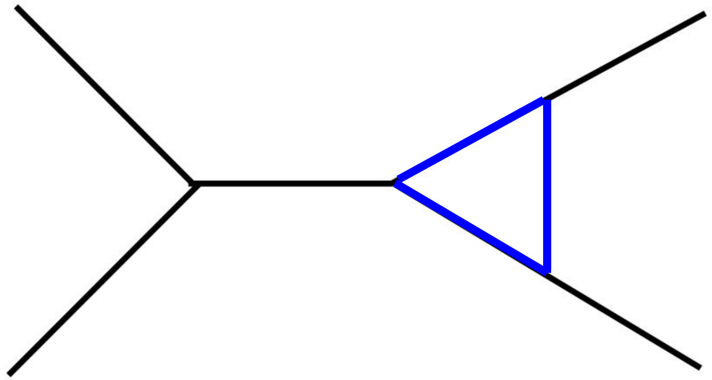
- The radius of tetraquark is similar to that of charmonium state, less than 1 fermi.
- For molecule state, the binding energy is small, resulting in radius as large as several fermi.
- For X(3872), the difference between M_X and mass threshold of $D^{*0}\bar{D}^0$ is

$$E_X \equiv M_X - (M_{*0} + M_0) = (+0.01 \pm 0.18)\ \text{MeV}.$$

(M. Tanabashi *et al.* [Particle Data Group], 2018)

Binding energy of about 0.2 MeV corresponds to radius of 5 fermi.

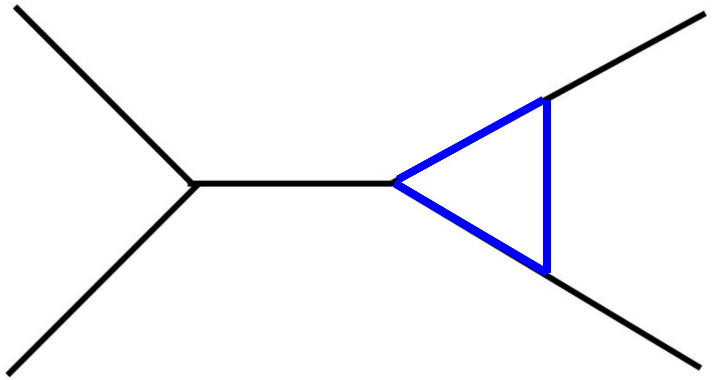
Triangle Singularity



Matrix element has logarithmic branch point
when the 3 particles forming the triangle
are all simultaneously on-shell

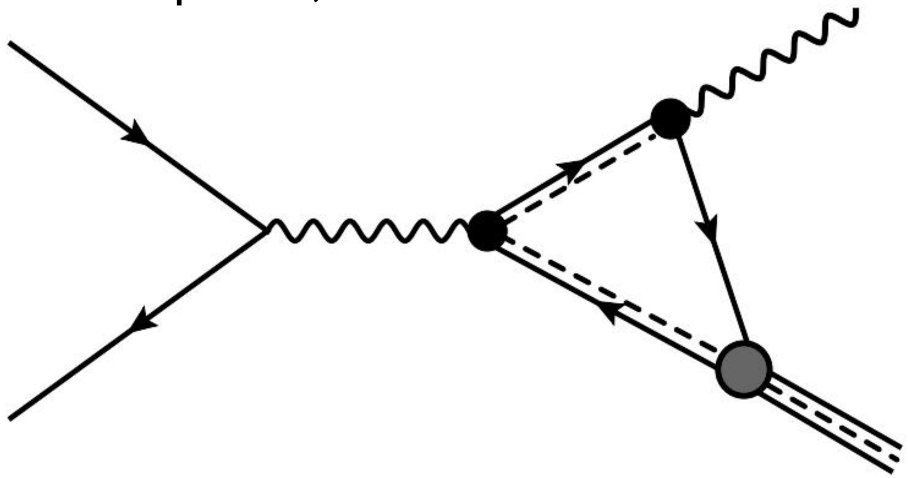
Cross section may diverge as $\log^2$ at center-of mass energy
determined by masses of particle in triangle

Triangle Singularity



Matrix element has logarithmic branch point when the 3 particles forming the triangle are all simultaneously on-shell

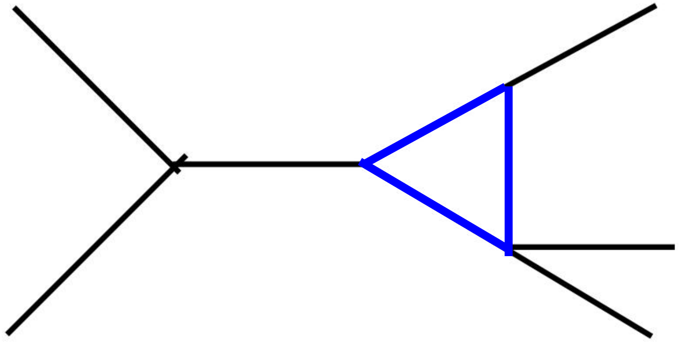
In Chapter 2, we discuss



Feynman diagrams for $e^+e^- \to X + \gamma$ from rescattering of $D^{*0}\bar{D}^{*0}$.

Cross section may diverge as log² at center-of mass energy determined by masses of particle in triangle

Triangle Singularity



If 2 particles in the triangle scatter elastically

differential cross section may diverge as $\log^2$

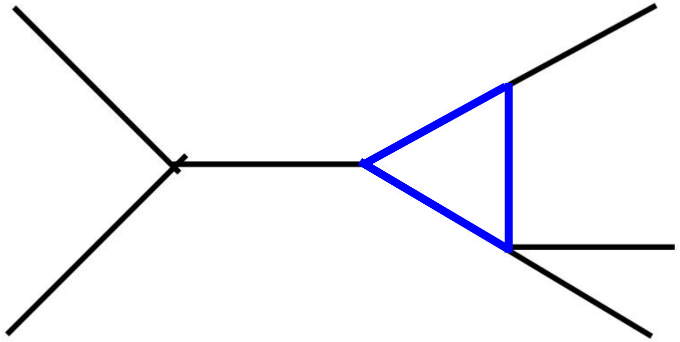
Schmid cancellation

If differential cross section is integrated over t

$\log^2$ divergence is canceled — Schmid 1967

but log divergence remains — Anisovitch & Anisovitch 1995

Triangle Singularity



If 2 particles in the triangle scatter elastically
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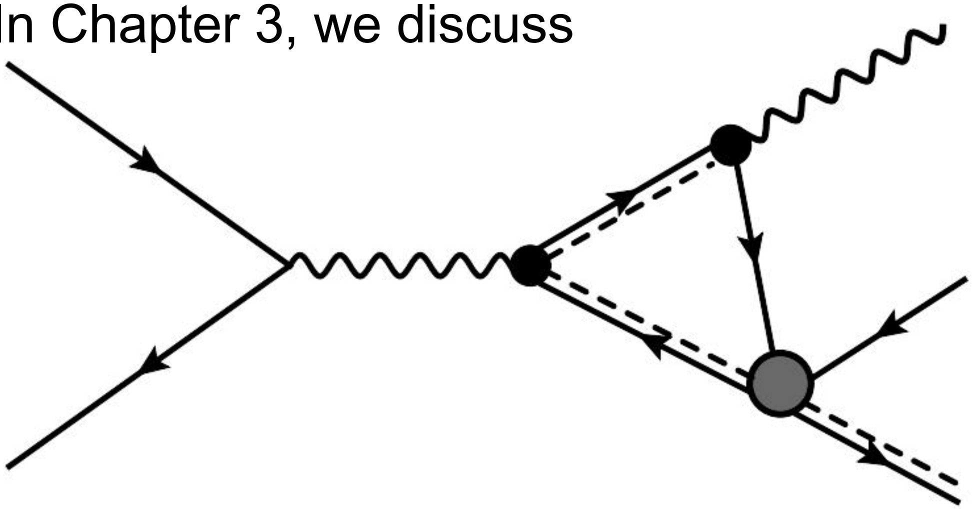
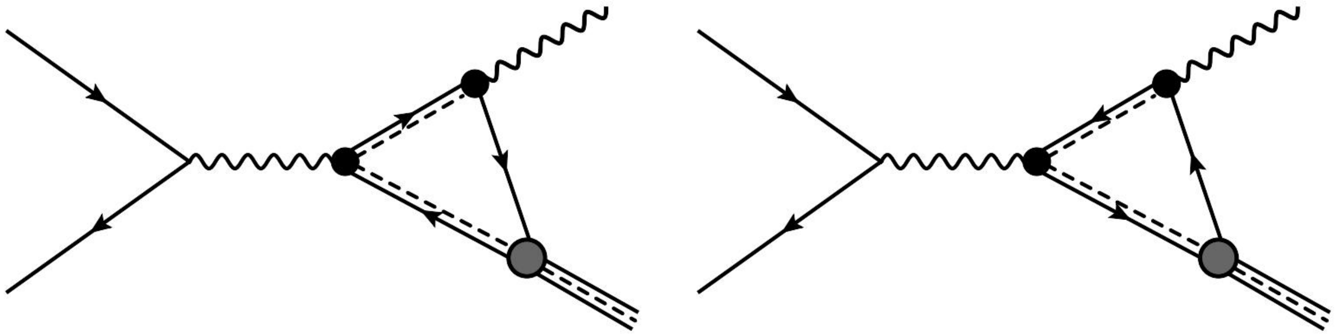


Feynman diagrams for $e^+e^- \to D^{*0}\bar{D}^0 + \gamma$ with a charm-meson loop.

Schmid cancellation

If differential cross section is integrated over t

$\log^2$ divergence is canceled — Schmid 1967
but log divergence remains — Anisovitch & Anisovitch 1995



E. Braaten, L. P. He and K. Ingles studied using nonrelativistic framework in PRD 101, 014021 (2020)

Within relativistic framework:

$$\frac{d\sigma}{d\Omega} = 32\pi^2\alpha^3\nu^2\,|\gamma_X|\,M_X^3 M_{*0}^2\left(\frac{s-M_X^2}{s}\right)^5 \left|F(s, M_X^2 - iM_X\Gamma_X)\right|^2 \times\left[\left|A_0 - \frac{1}{\sqrt{5}}A_2\right|^2(1-\cos^2\theta) + \frac{9}{40}|A_2|^2(1+\cos^2\theta)\right],$$

$\sqrt{s}$: center-of-mass energy
γ_X: binding momentum of X
Γ_X: decay width of X
E_X: binding energy of X

Loop amplitude $F(s, M^2_X - i M_X \Gamma_X)$ has a log branch point at s_Δ with Γ_X and $E_X=0$.

$$s_\triangle = 4M_{*0}^2 + (M_{*0}/M_0)\,(M_{*0} - M_0)^2.$$

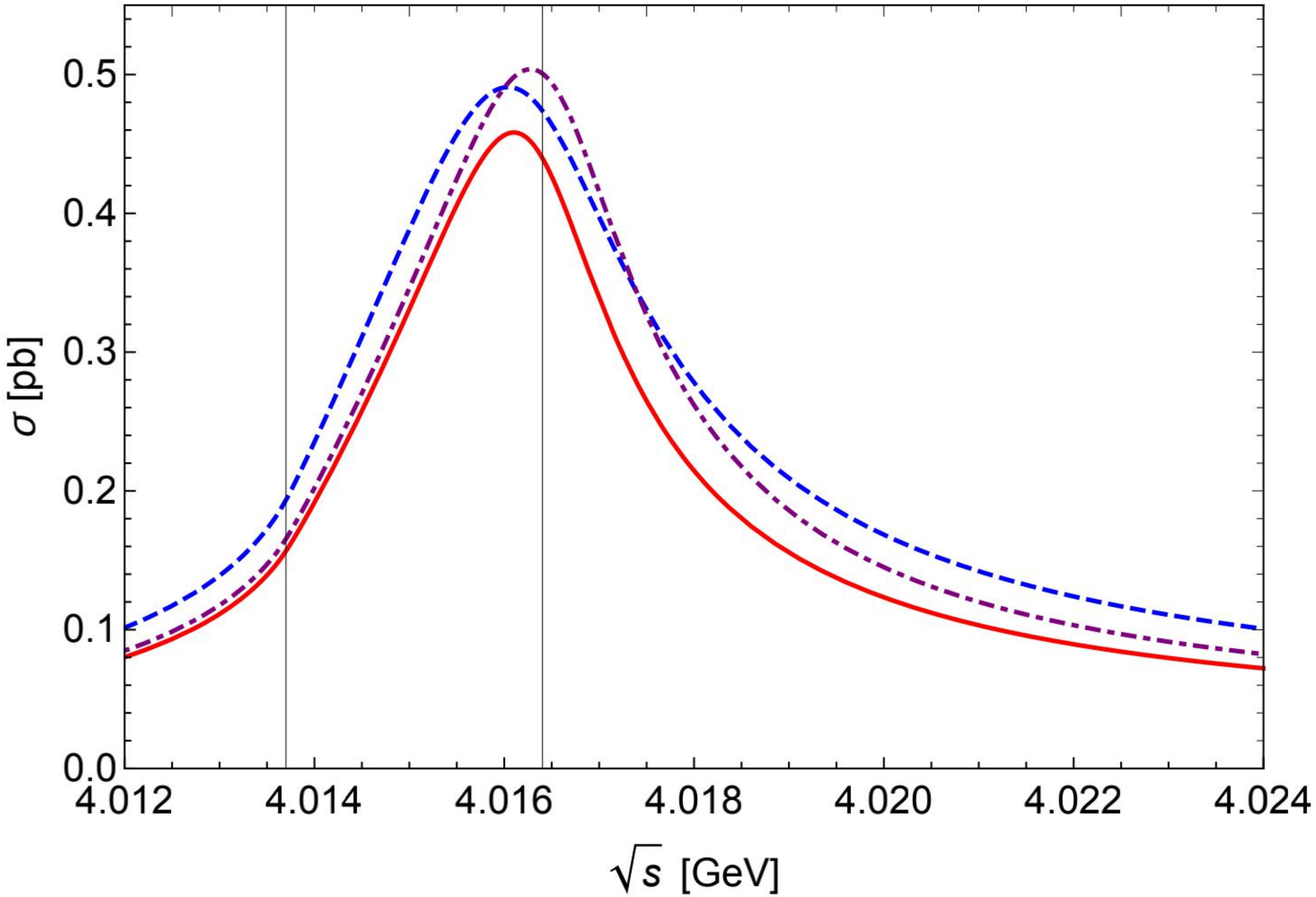
M_{*0}: mass of D^{*0}
M_0: mass of D^0

$\sqrt{s_\triangle} = 4016.4$ MeV, 2.7 MeV above the $D^{*0}\bar{D}^{*0}$ threhold.

$|E_X|$ = 0.05 MeV, $\Gamma_X = \Gamma_{*0}$ (solid red curve),
$|E_X|$ = 0.10 MeV, $\Gamma_X = \Gamma_{*0}$ (dashed blue curve)
$|E_X|$ = 0.05 MeV, $\Gamma_X = 2\Gamma_{*0}$ (dot-dashed purple curve).

$$E_X = -\frac{\mathrm{Re}[\gamma_X]^2 - \mathrm{Im}[\gamma_X]^2}{2\mu},$$

$$\Gamma_X = \Gamma_{*0} + \frac{2\,\mathrm{Re}[\gamma_X]\,\mathrm{Im}[\gamma_X]}{\mu},$$



- Triangle singularity produces narrow peak in cross section at energy near $\sqrt{s_\Delta}$.

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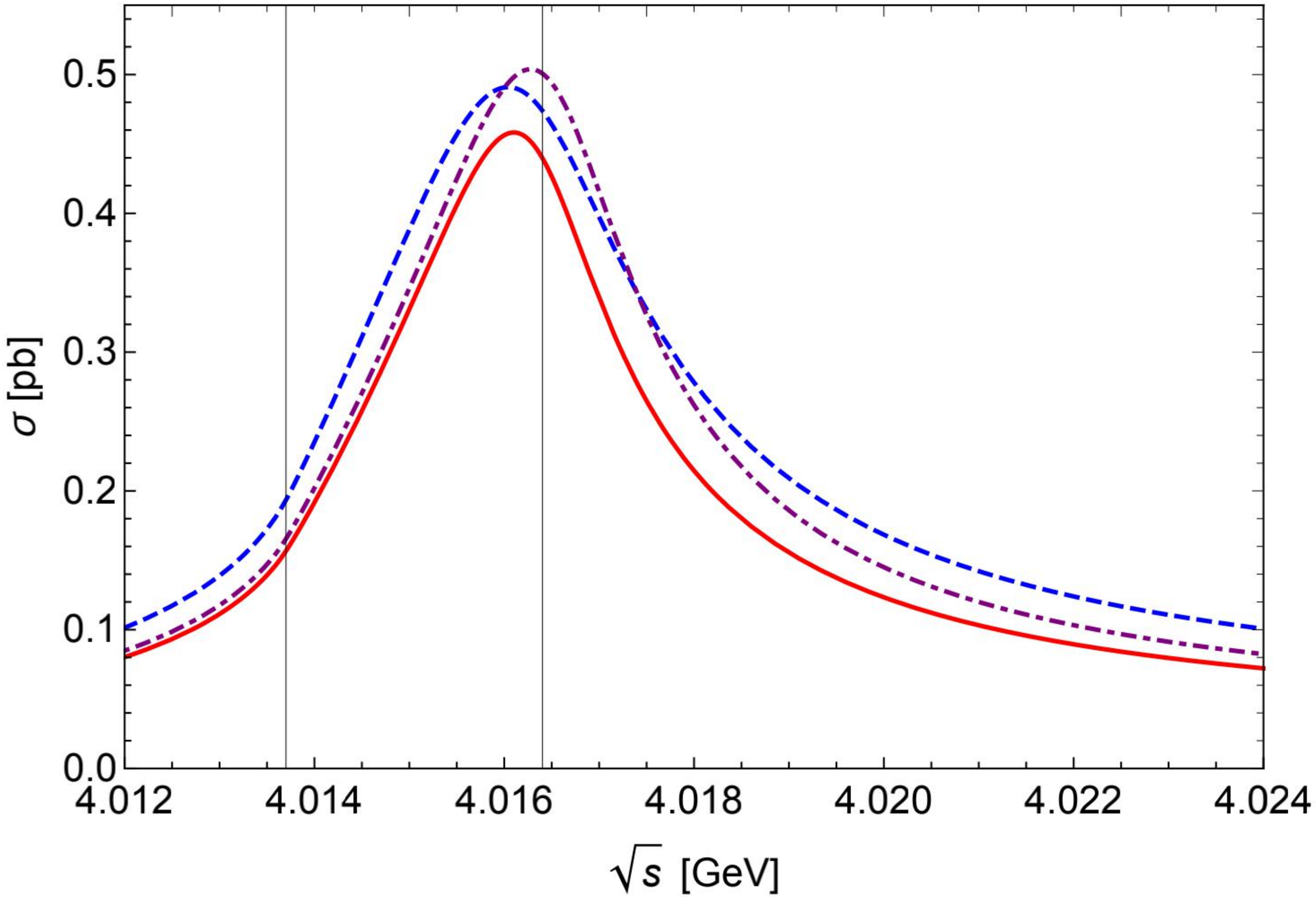

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- The observation of the narrow peak would support the identification of X(3872) as a weakly bound charm-meson molecule.

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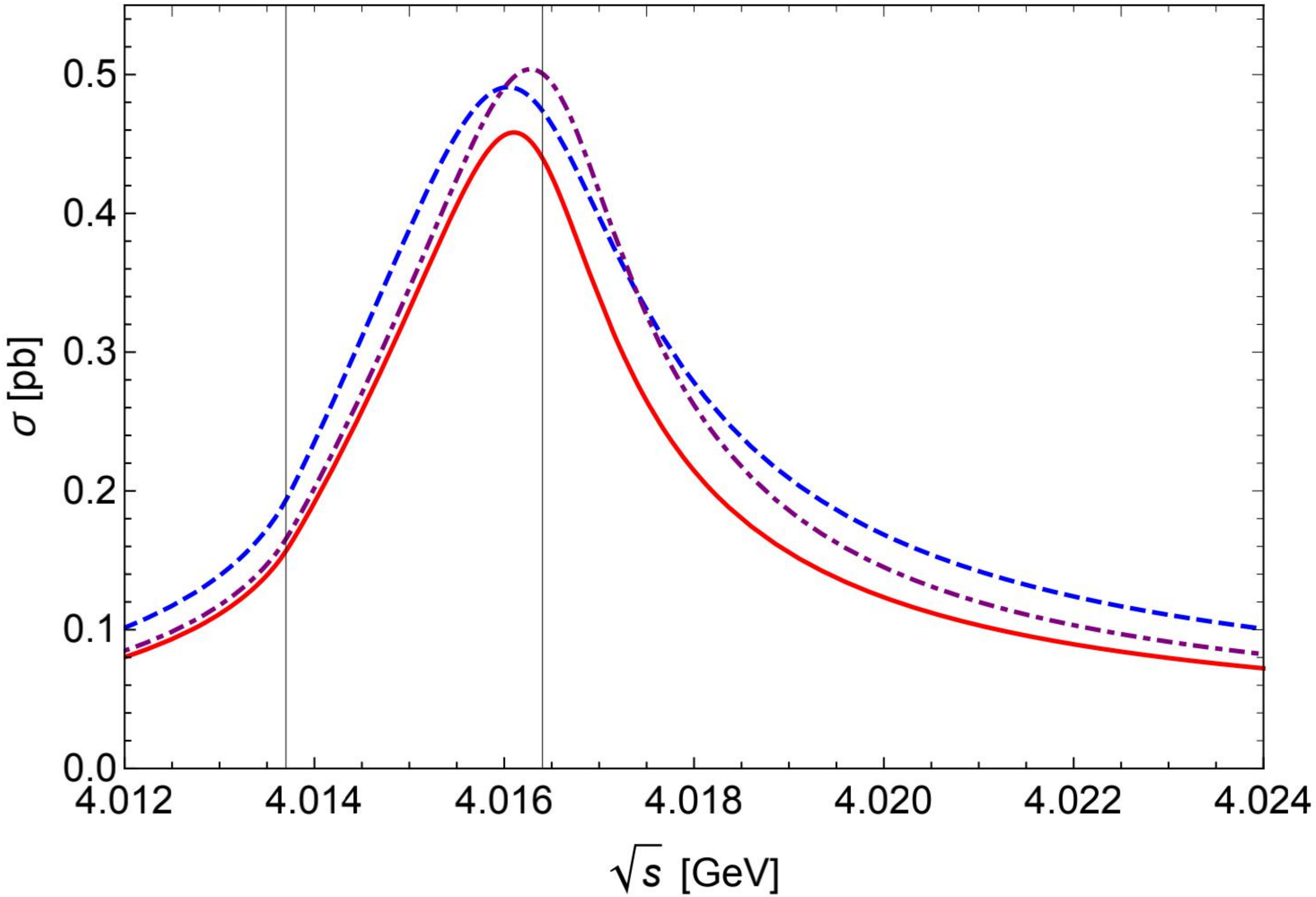
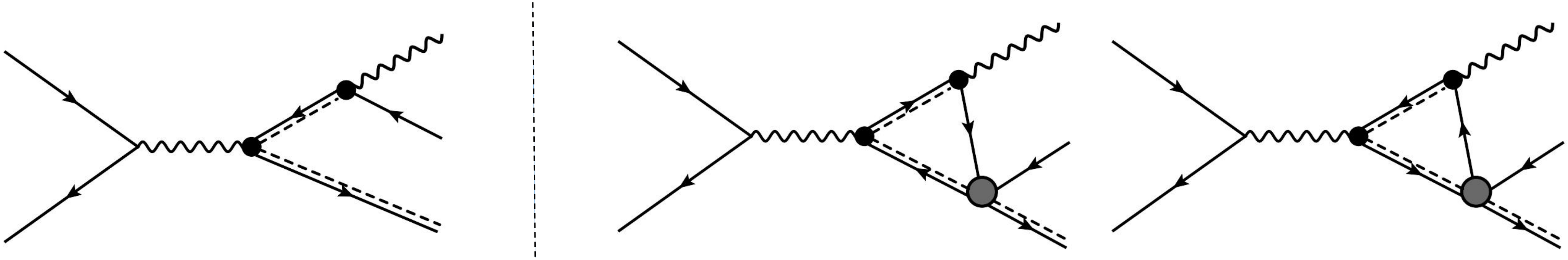


- Triangle singularity produces narrow peak in cross section at energy near $\sqrt{s_\Delta}$.
- The observation of the narrow peak would support the identification of X(3872) as a weakly bound charm-meson molecule.

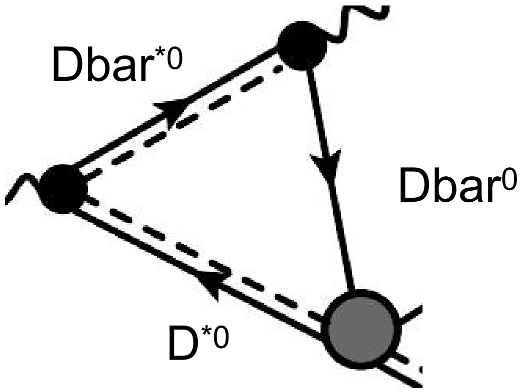
Its energy is in a range not covered by previous measurements of BESIII collaboration in 2014 and 2019.

Chapter 3 e^+e^- Annihilation into $D^{*0}\bar{D}^0 + \gamma$



√s: center-of-mass energy
√u: invariant mass of D^{*0}Dbar^0

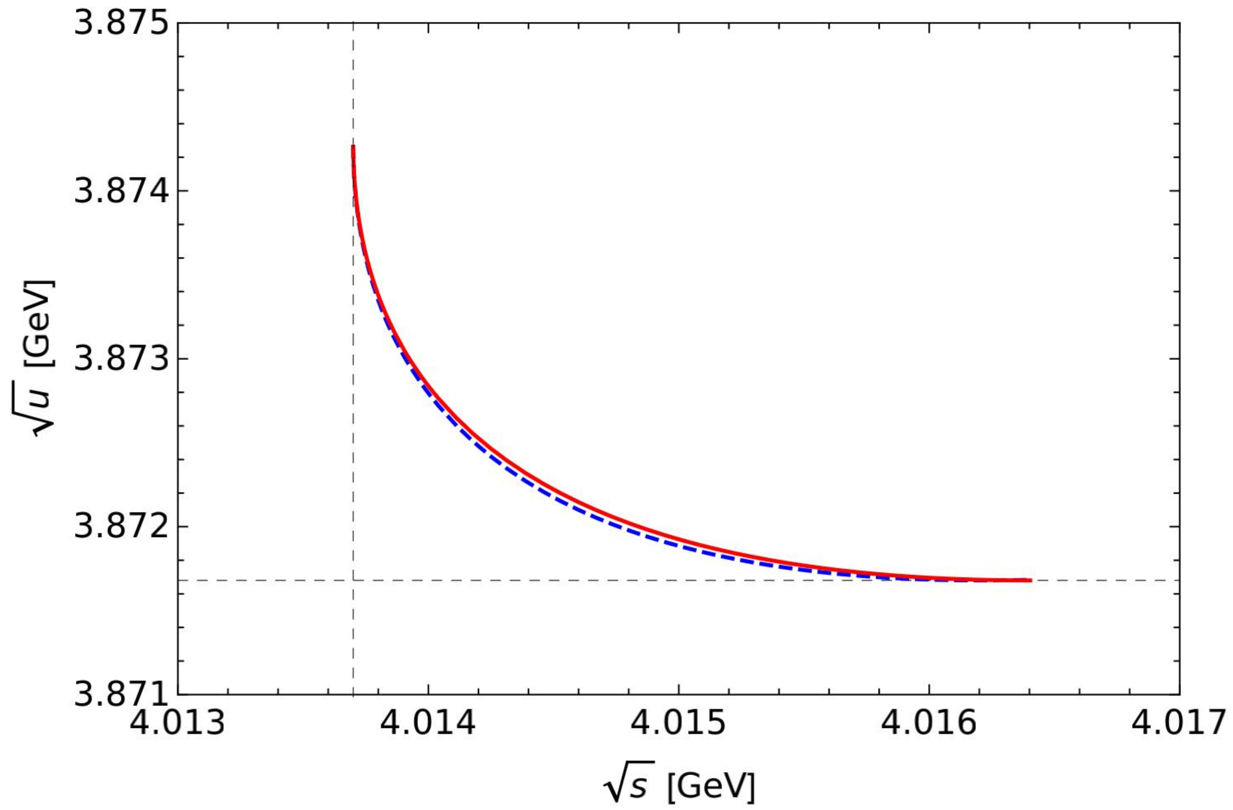
Conditions for triangle singularity



(1) All 3 legs of triangle are on-shell

(2) D^{*0} and Dbar^0 move into the same direection

(3) Velocity of Dbar^0 >= velocity of D^{*0}



√s: center-of-mass energy
√u: invariant mass of $D^{*0}\bar{D}^0$

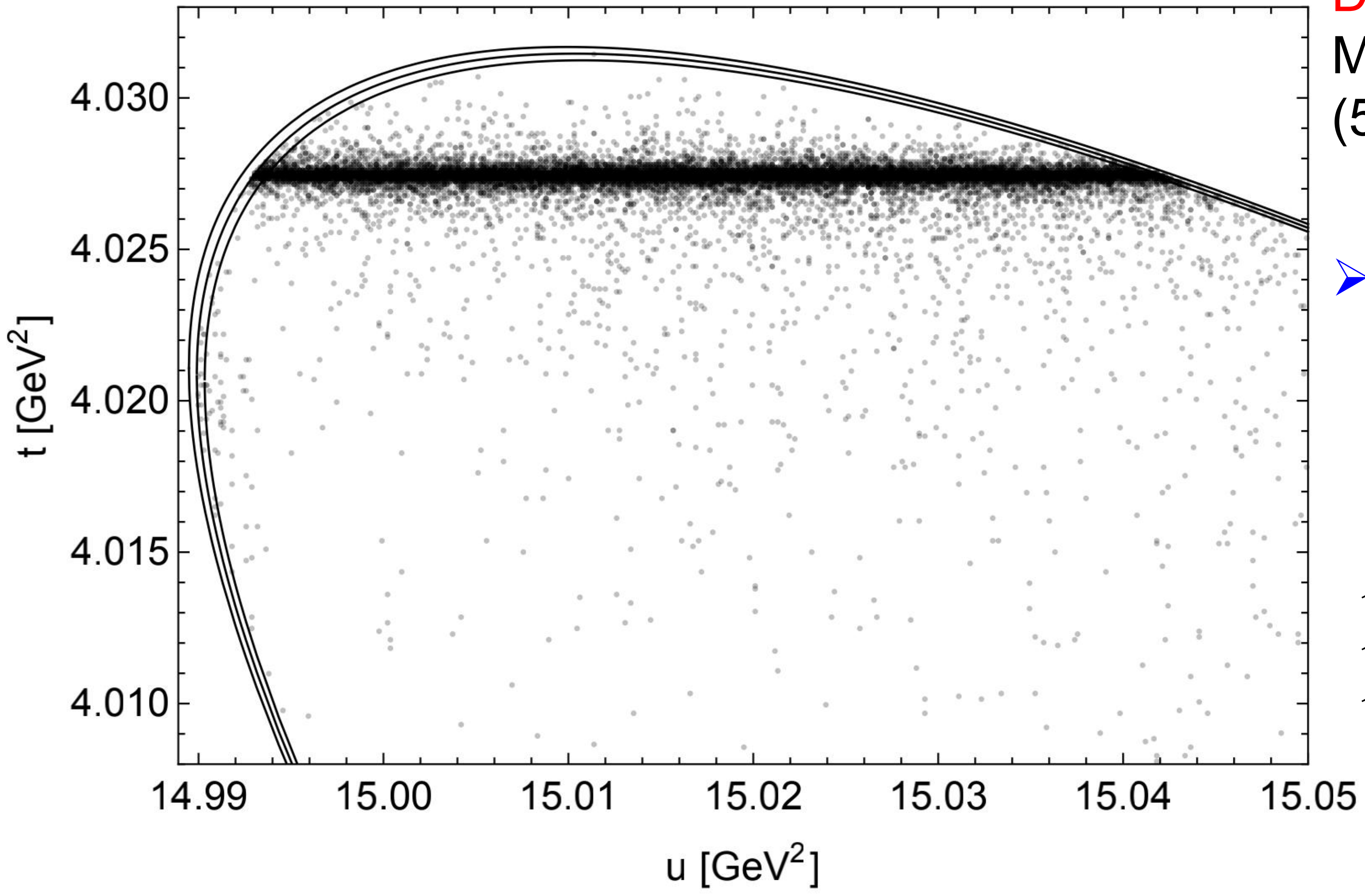
√s: 4013.7~4016.4 MeV

u: 15.010~14.990 GeV2

Triangle Singularity Range

$$4M_{*0}^2 < s \le s_\triangle = 4M_{*0}^2 + (M_{*0}/M_0)\,(M_{*0} - M_0)^2$$

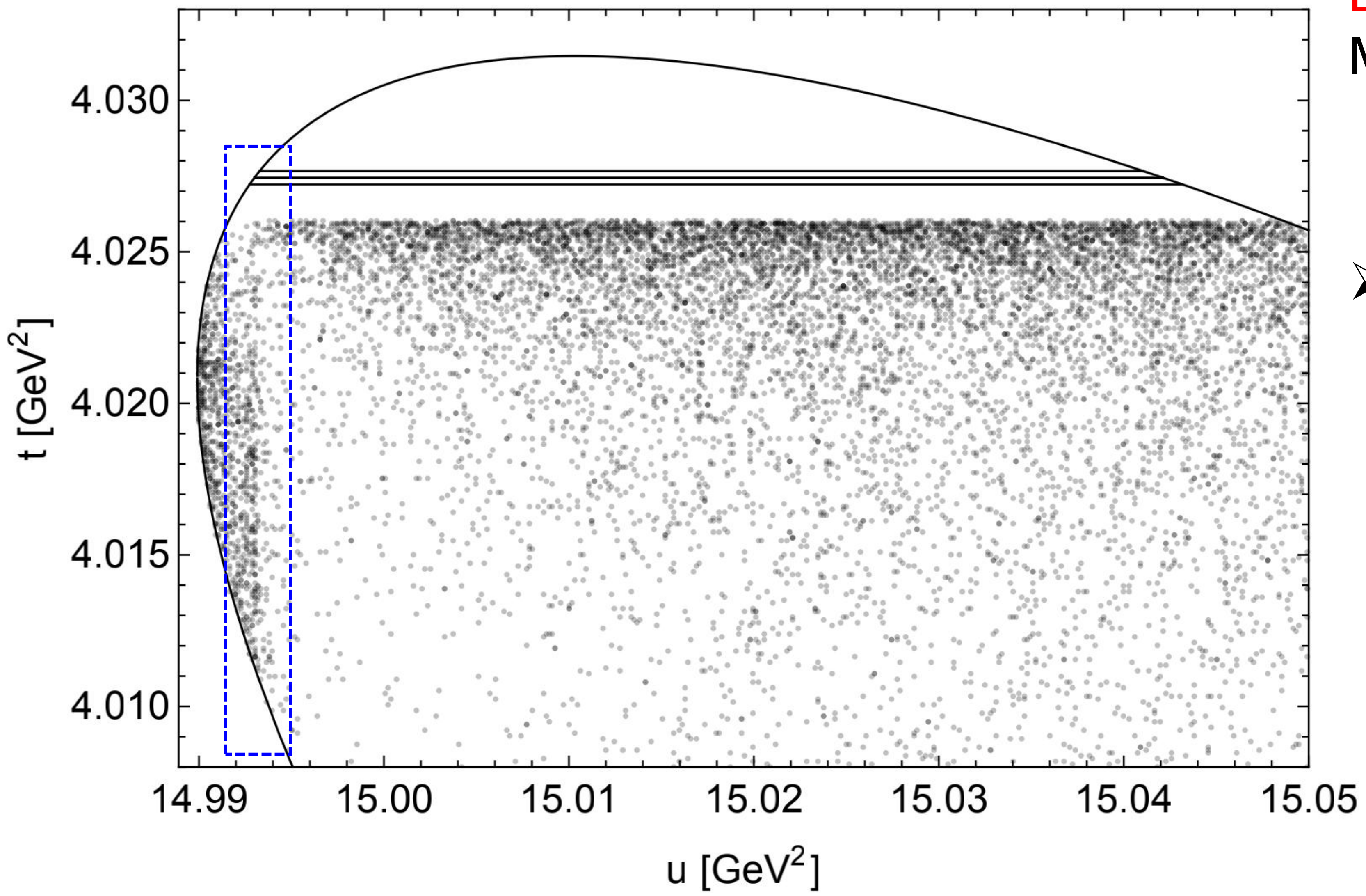
$$u_\triangle(s) = (M_{*0} + M_0)^2 + \frac{\left[(M_{*0} - M_0)\sqrt{s} - (M_{*0} + M_0)\sqrt{s - 4M_{*0}^2}\right]^2}{4M_{*0}^2}$$



Dalitz plot for √s = 4014.7 MeV with **physical D^{*0} width** (55 keV)

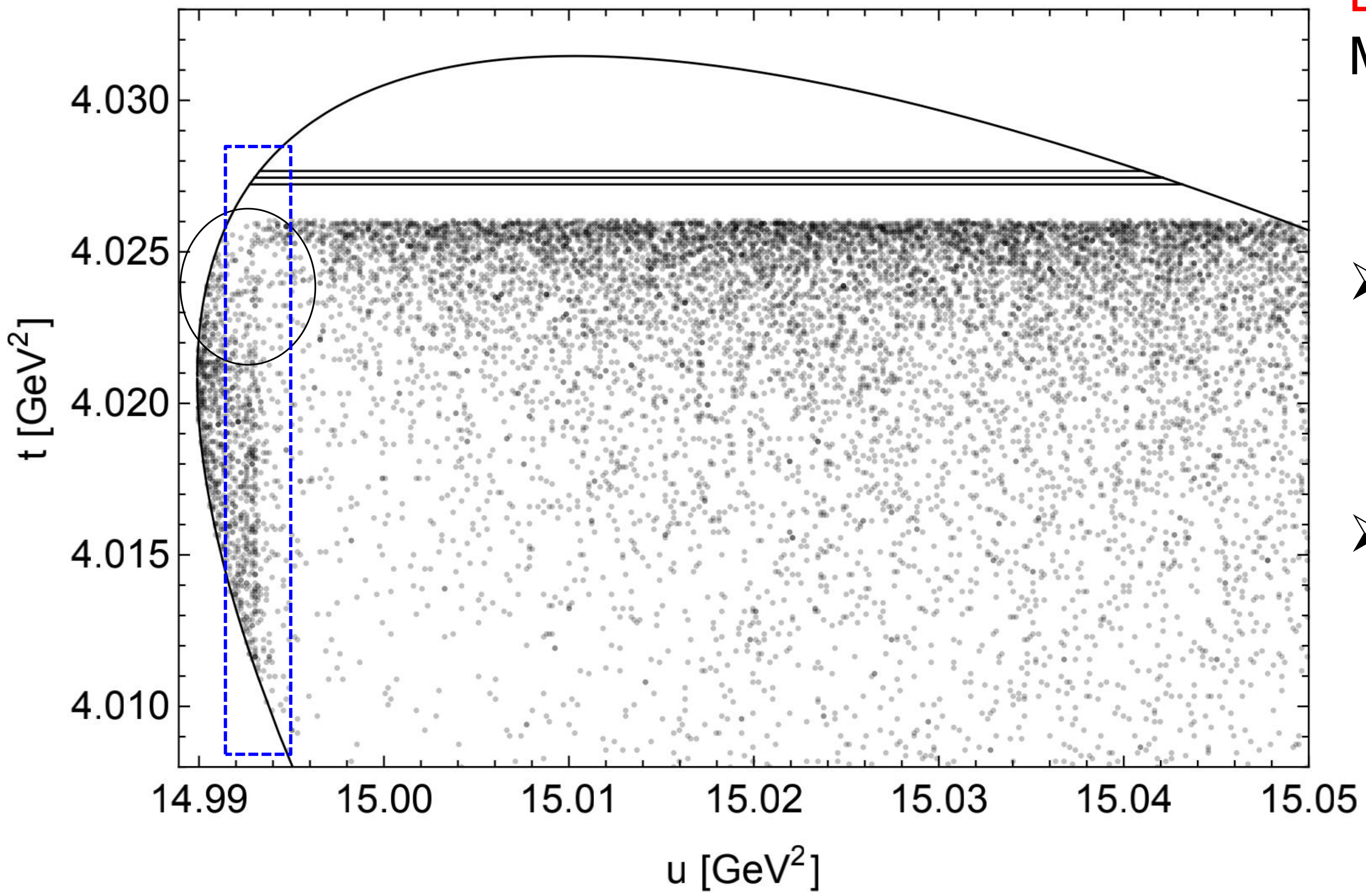
- Obvious Dbar^{*0} resonance band from tree diagram

√s: center-of-mass energy
√u: invariant mass of D^{*0}Dbar^0
√t : invariant mass of Dbar$^0\gamma$



Dalitz plot for $\sqrt{s}$ = 4014.7 MeV with **zero D^{*0} width**

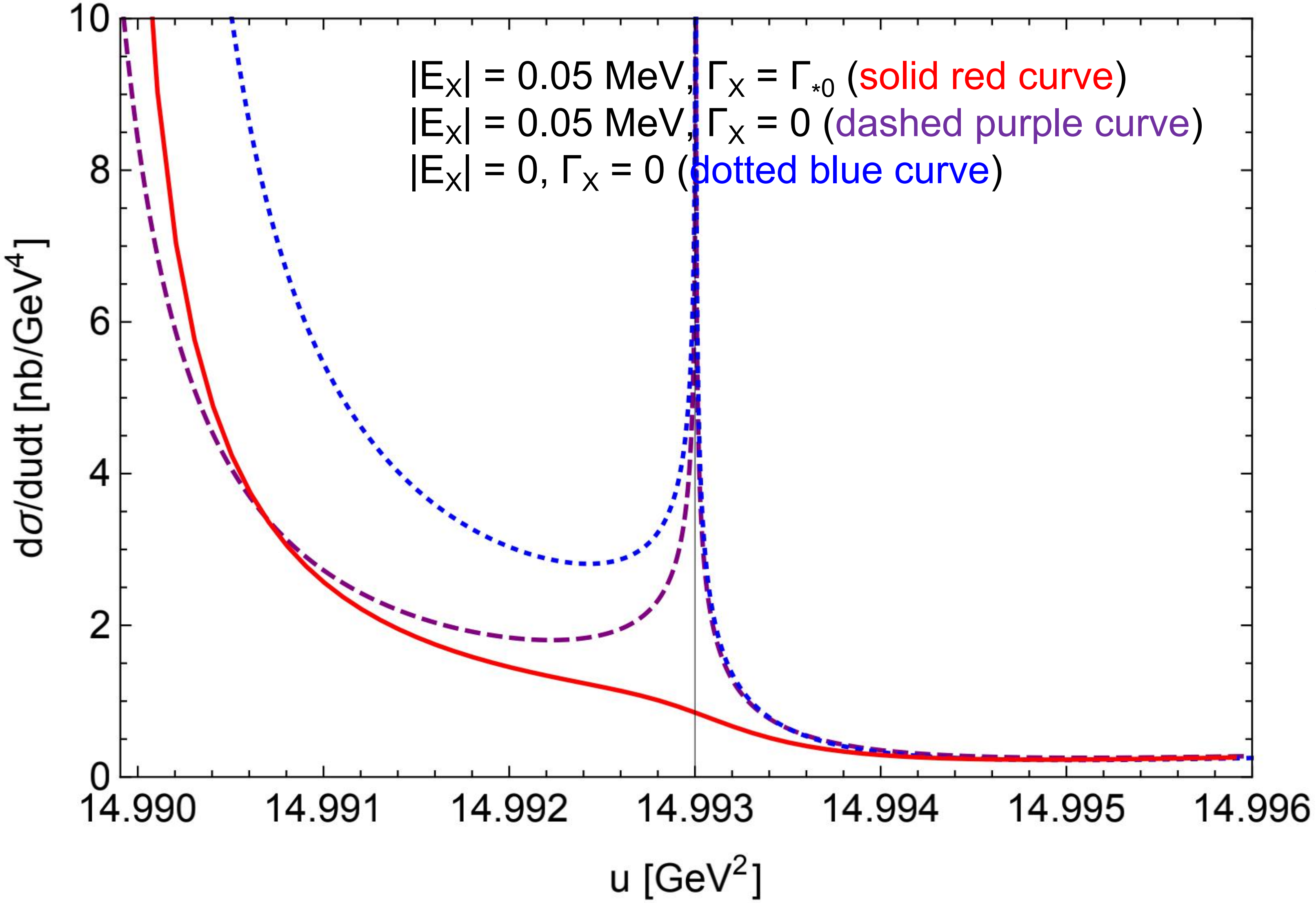
- Besides the Dbar^{*0} band, we have the **narrow triangle singularity band at u_Δ=14.993 GeV^2**



Dalitz plot for $\sqrt{s}$ = 4014.7 MeV with **zero D^{*0} width**

- Besides the $\bar{D}^{*0}$ band, we have the **narrow triangle singularity band at u_Δ=14.993 GeV²**.
- **Smaller density** when t approaches to the $\bar{D}^{*0}$ band along the triangle singularity line.

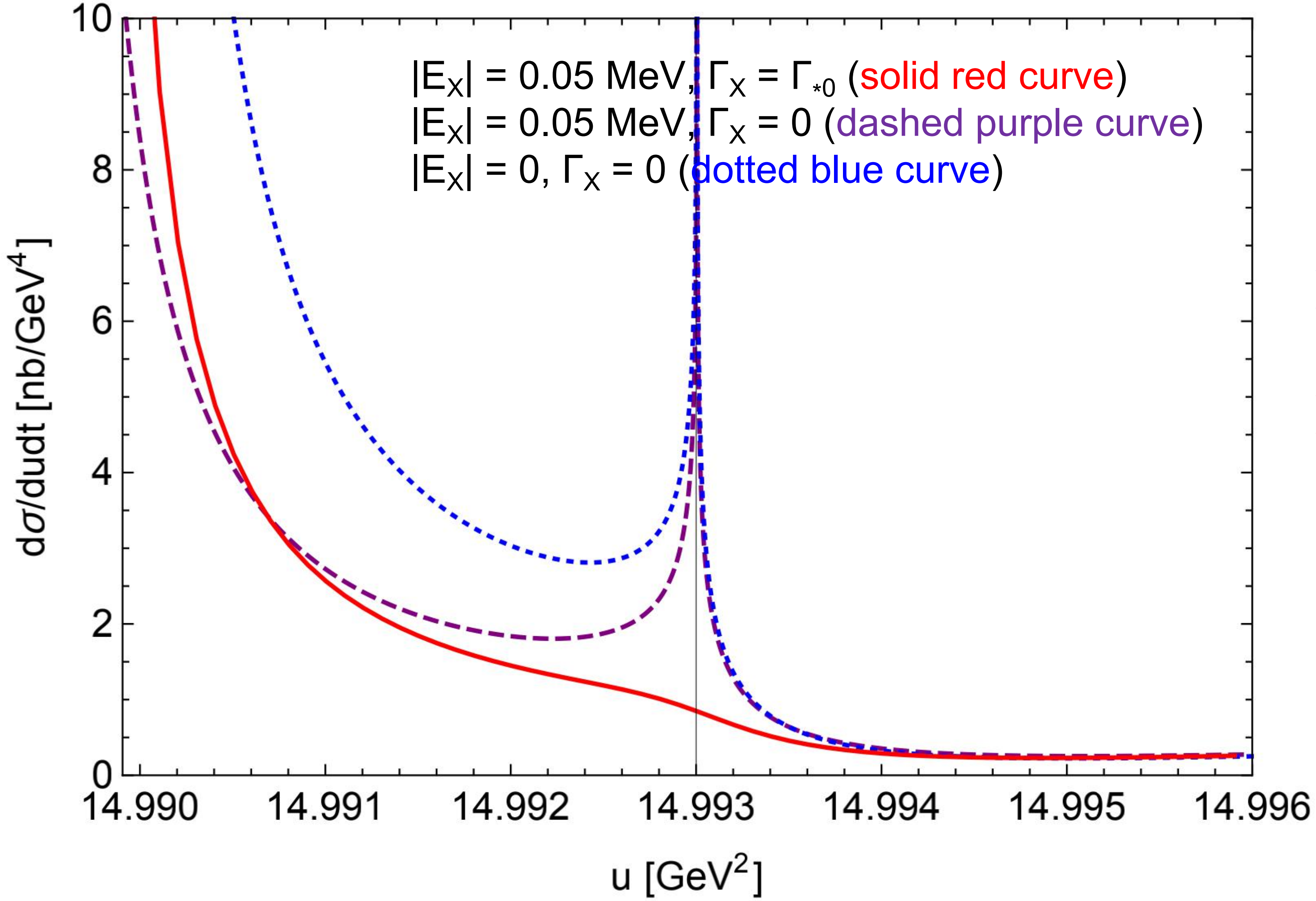
Differential cross section **dσ/dudt** for $\sqrt{s}$ = 4014.7 MeV, t = 4.0209 GeV²



Triangle singularity:
u_Δ=14.993 GeV²

- Log² divergence for $\Gamma_X = 0$

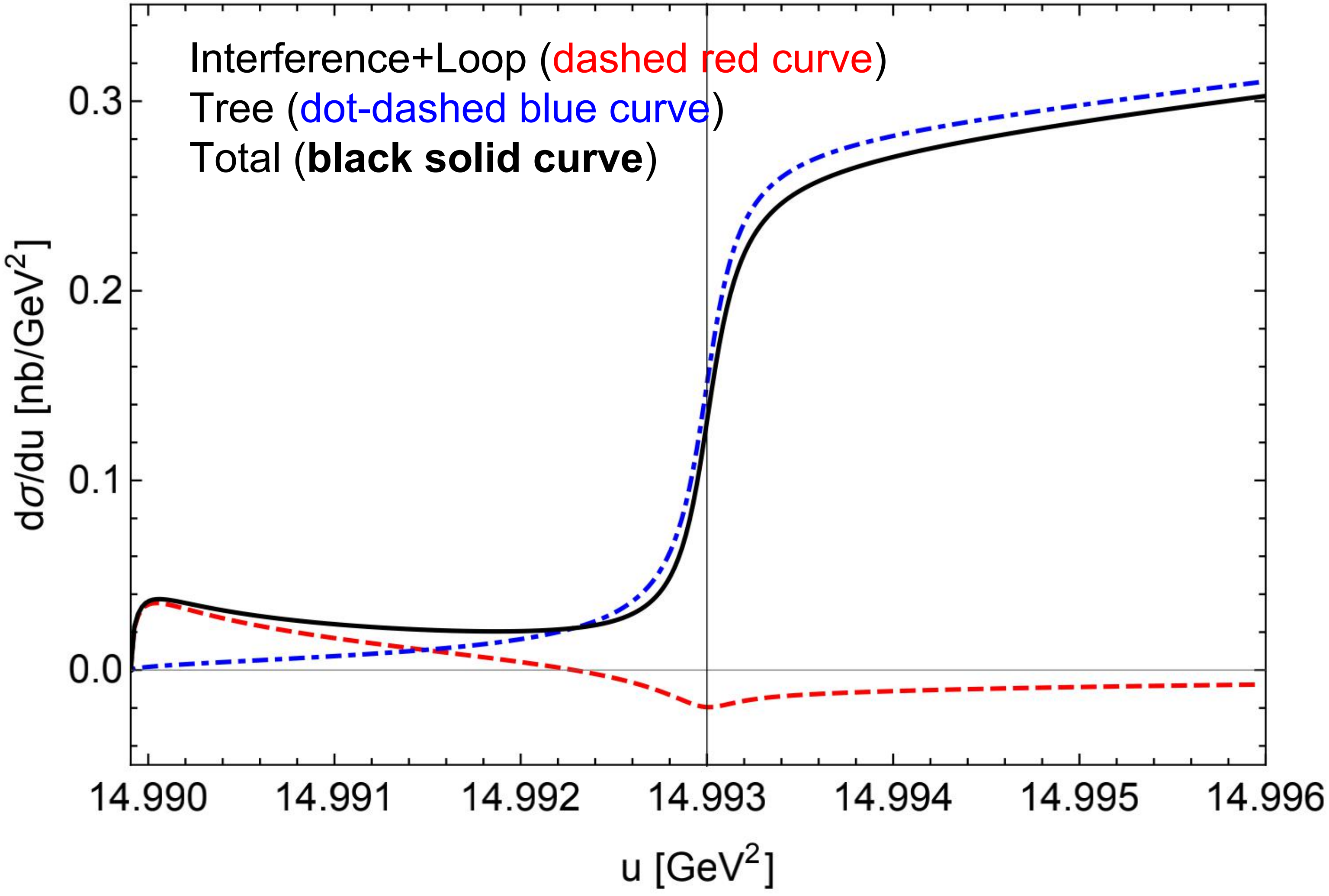
Differential cross section **dσ/dudt** for $\sqrt{s}$ = 4014.7 MeV, t = 4.0209 GeV^2



Triangle singularity:
u_Δ=14.993 GeV^2

- Log² divergence for $\Gamma_X = 0$
- For $|E_X|$ = 0.05 MeV, $\Gamma_X = \Gamma_{*0}$, no narrow peak from triangle singularity.

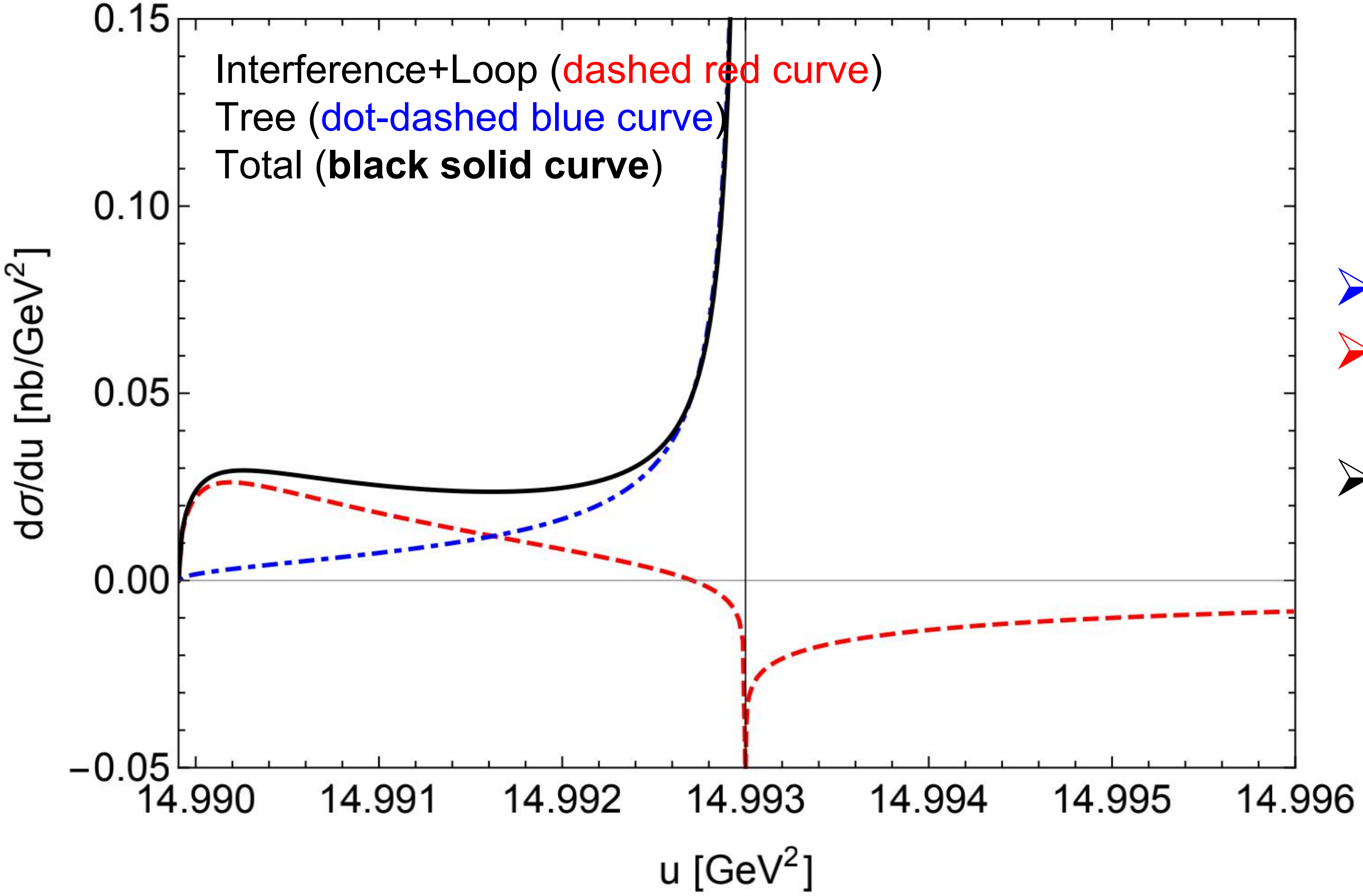
Differential cross section **$d\sigma/du$** for $\sqrt{s}$ = 4014.7 MeV with **physical D^{*0} width**



Triangle singularity:
u_Δ=14.993 GeV²

- Tree dominates, no peak from triangle singularity.

Differential cross section **dσ/du** for $\sqrt{s}$ = 4014.7 MeV with **zero D^{*0} width**



Triangle singularity:
u_Δ=14.993 GeV^2

- Tree diverges as $1/\Gamma_{*0}$
- Interference+Loop diverges as Log.
- No peak from triangle singularity too.

Schmid Cancellation

- **dσ/dudt, Log^2 divergences**
- **dσ/du, Log^2 cancellation (γ_X, Γ_{*0} go to 0),**

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\approx\frac{\pi\alpha^3\nu^2\delta^4}{6M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}\left|A_2\right|^2\right)$$
$$\times{\rm Re}\left[\frac{M_X F(s,u)}{-\gamma_X-i\sqrt{u_-+2iM_{*0}\Gamma_{*0}/2}}\left(\frac{32\pi\delta\, M_X F(s,u)^*\sqrt{u_-}}{-\gamma_X+i\sqrt{u_--2iM_{*0}\Gamma_{*0}/2}}\right.\right.$$
$$\left.\left.+\frac{\delta^2+s_--u_--2iM_{*0}\Gamma_{*0}}{\delta^2}\log\frac{(\delta+\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}{(\delta-\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}+\frac{4\sqrt{u_-}}{\delta}\right)\right].$$

Cancellation

s_-: $s-4M_{*0}^2$
u_-: $u-(M_{*0}+M_0)^2$
t_- : $t-M_{*0}^2$
δ : $M_{*0}-M_0$
x : s_-/δ

Schmid Cancelation

- **dσ/dudt, Log^2 divergences**

- **dσ/du, Log^2 cancellation (γ_X , Γ_{*0} go to 0),**
Log divergence survives

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\approx\frac{\pi\alpha^3\nu^2\delta^4}{6M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$

$$\times{\rm Re}\left[\frac{M_XF(s,u)}{-\gamma_X-i\sqrt{u_-+2iM_{*0}\Gamma_{*0}/2}}\left(\frac{32\pi\delta\,M_XF(s,u)^*\sqrt{u_-}}{-\gamma_X+i\sqrt{u_--2iM_{*0}\Gamma_{*0}/2}}\right.\right.$$
$$\left.\left.+\frac{\delta^2+s_--u_--2iM_{*0}\Gamma_{*0}}{\delta^2}\log\frac{(\delta+\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}{(\delta-\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}+\frac{4\sqrt{u_-}}{\delta}\right)\right].$$

Survival

Cancellation

s_-: $s-4M_{*0}^2$
u_-: $u-(M_{*0}+M_0)^2$
t_- : $t-M_{*0}^2$
δ : $M_{*0}-M_0$
x : s_-/δ

Schmid Cancelation

- **dσ/dudt, Log^2 divergences**
- **dσ/du, Log^2 cancellation (γ_X , Γ_{*0} go to 0), Log divergence survives**

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\approx\frac{\pi\alpha^3\nu^2\delta^4}{6M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
$$\times\mathrm{Re}\left[\frac{M_X F(s,u)}{-\gamma_X-i\sqrt{u_-+2iM_{*0}\Gamma_{*0}}/2}\left(\frac{32\pi\delta\, M_X F(s,u)^*\sqrt{u_-}}{-\gamma_X+i\sqrt{u_--2iM_{*0}\Gamma_{*0}}/2}+\frac{\delta^2+s_--u_--2iM_{*0}\Gamma_{*0}}{\delta^2}\log\frac{(\delta+\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}{(\delta-\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}+\frac{4\sqrt{u_-}}{\delta}\right)\right].$$

Survival

Cancellation

⇩

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\longrightarrow\frac{\alpha^3\nu^2\delta^2}{48M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
$$\times\left(-\frac{x^2\log((1-x)/x)+x}{1-x}\right)\log\frac{2\delta}{\left|\sqrt{u_-}+\sqrt{s_-}-\delta\right|},$$

s_-: $s-4M_{*0}^2$
u_-: $u-(M_{*0}+M_0)^2$
t_- : $t-M_{*0}^2$
δ : $M_{*0}-M_0$
x : s_-/δ

Schmid Cancelation

- $d\sigma/dudt$, Log^2 divergences
- $d\sigma/du$, Log^2 cancellation (γ_X, Γ_{*0} go to 0), Log divergence survives

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\approx\frac{\pi\alpha^3\nu^2\delta^4}{6M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
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Survival

Cancellation

$$\frac{d\sigma_{\rm loop}}{du}+\frac{d\sigma_{\rm int}}{du}\longrightarrow\frac{\alpha^3\nu^2\delta^2}{48M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
$$\times\left(-\frac{x^2\log\left((1-x)/x\right)+x}{1-x}\right)\log\frac{2\delta}{|\sqrt{u_-}+\sqrt{s_-}-\delta|},$$

- In $d\sigma/du$, Log divegences of both Interference+Loop and Tree terms

$$\frac{d\sigma_{\rm tree}}{du}\longrightarrow\frac{\alpha^3\nu^2\delta^2}{48M_{*0}^2}\left[\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)\frac{\pi\, s_-}{2M_{*0}\Gamma_{*0}}\theta\left(\sqrt{u_-}+\sqrt{s_-}-\delta\right)\right.$$
$$+\frac{3}{2}\left((1-x)|A_0|^2-\frac{1-3x}{3}\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{17-11x}{20}|A_2|^2\right)$$
$$\left.\times\log\frac{2\delta}{|\sqrt{u_-}+\sqrt{s_-}-\delta|}\right].$$

s_-: $s-4M_{*0}^2$
u_-: $u-(M_{*0}+M_0)^2$
t_- : $t-M_{*0}^2$
δ : $M_{*0}-M_0$
x : s_-/δ

Schmid Cancelation

- $d\sigma/dudt$, Log^2 divergences
- $d\sigma/du$, Log^2 cancellation (γ_X, Γ_{*0} go to 0), **Log divergence survives**

$$\frac{d\sigma_{\text{loop}}}{du}+\frac{d\sigma_{\text{int}}}{du}\approx\frac{\pi\alpha^3\nu^2\delta^4}{6M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
$$\times\text{Re}\left[\frac{M_X F(s,u)}{-\gamma_X-i\sqrt{u_-+2iM_{*0}\Gamma_{*0}/2}}\left(\frac{32\pi\delta\, M_X F(s,u)^*\sqrt{u_-}}{-\gamma_X+i\sqrt{u_--2iM_{*0}\Gamma_{*0}/2}}+\frac{\delta^2+s_--u_--2iM_{*0}\Gamma_{*0}}{\delta^2}\log\frac{(\delta+\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}{(\delta-\sqrt{u_-})^2-s_-+2iM_{*0}\Gamma_{*0}}+\frac{4\sqrt{u_-}}{\delta}\right)\right].$$

Survival

Cancellation

$$\frac{d\sigma_{\text{loop}}}{du}+\frac{d\sigma_{\text{int}}}{du}\longrightarrow\frac{\alpha^3\nu^2\delta^2}{48M_{*0}^2}\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)$$
$$\times\left(-\frac{x^2\log\left((1-x)/x\right)+x}{1-x}\right)\log\frac{2\delta}{\left|\sqrt{u_-}+\sqrt{s_-}-\delta\right|},$$

- In $d\sigma/du$, **Log divegences of both Interference+Loop and Tree terms** are completely **overwhelmed by** the rapidly increasing contribution from the **opening up of the Dbar^{*0} resonance from Tree diagram**.

$$\frac{d\sigma_{\text{tree}}}{du}\longrightarrow\frac{\alpha^3\nu^2\delta^2}{48M_{*0}^2}\left[\left(\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{9}{20}|A_2|^2\right)\frac{\pi\, s_-}{2M_{*0}\Gamma_{*0}}\theta\left(\sqrt{u_-}+\sqrt{s_-}-\delta\right)\right.$$
$$+\frac{3}{2}\left((1-x)|A_0|^2-\frac{1-3x}{3}\left|A_0-\frac{A_2}{\sqrt{5}}\right|^2+\frac{17-11x}{20}|A_2|^2\right)$$
$$\left.\times\log\frac{2\delta}{\left|\sqrt{u_-}+\sqrt{s_-}-\delta\right|}\right].$$

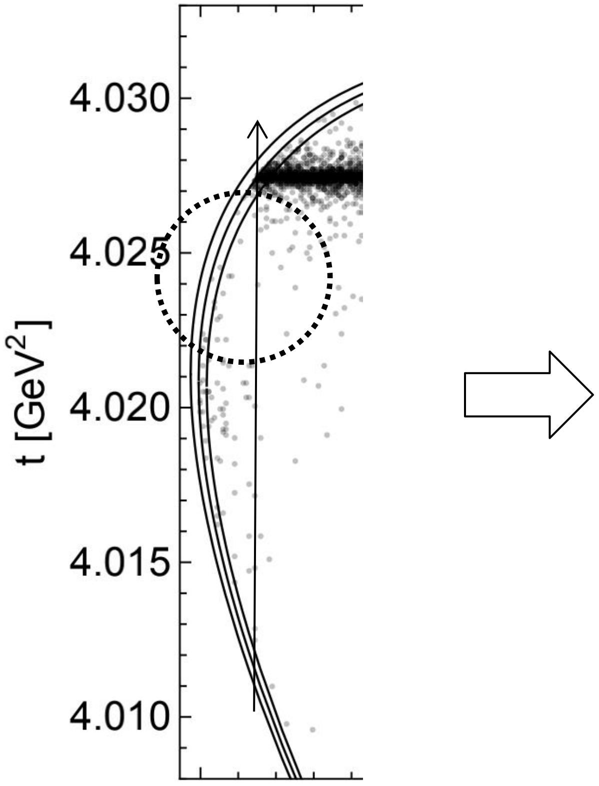
s_- : $s-4M_{*0}^2$
u_- : $u-(M_{*0}+M_0)^2$
t_- : $t-M_{*0}^2$
δ : $M_{*0}-M_0$
x : s_-/δ

No peak in $d\sigma/dudt$ for physical Γ_{*0} width
No peak in $d\sigma/du$ for physical/zero Γ_{*0} width

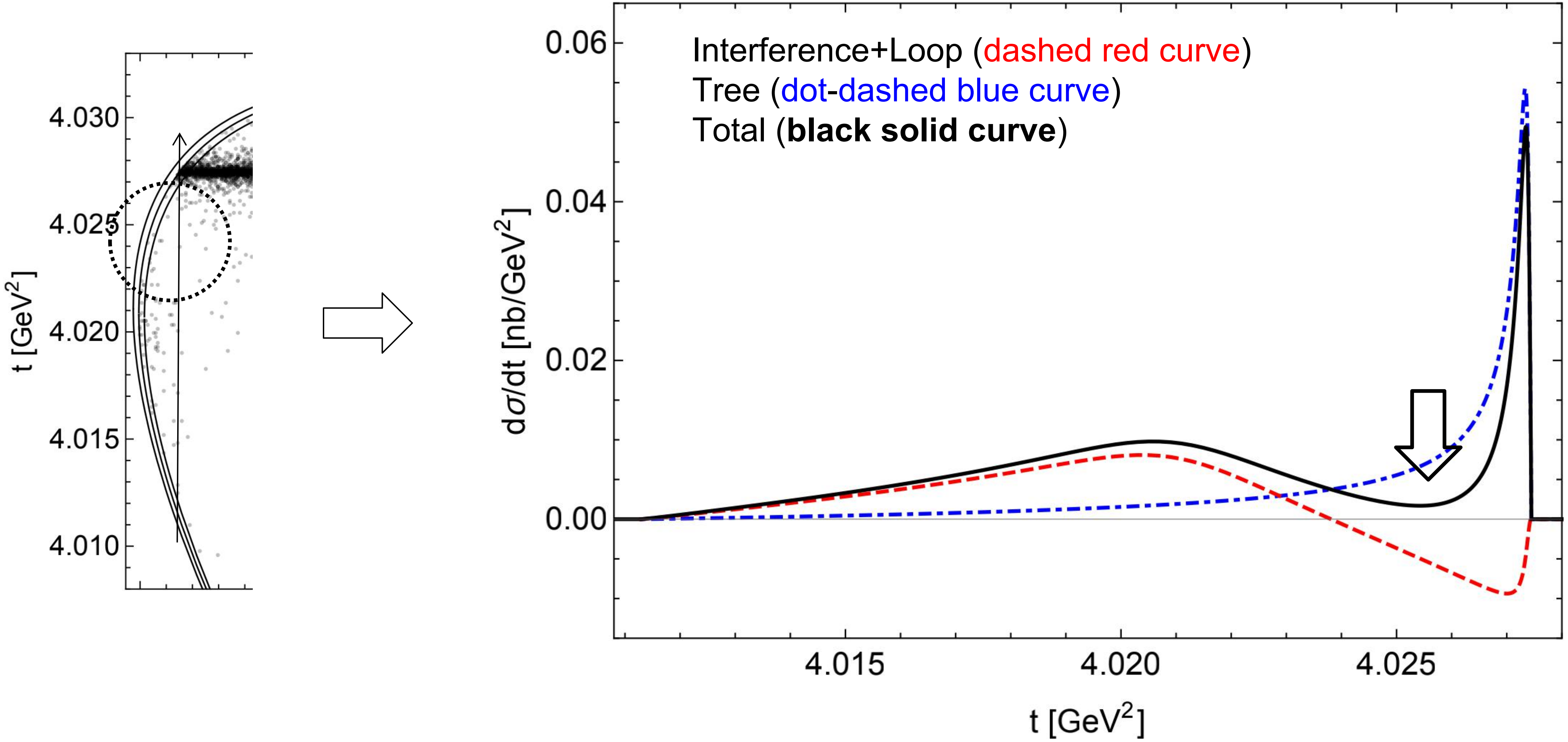
Can we identify the charm-meson triangle singularity?

This matters, because it supports for the identification of X as a weakly bound charm-meson molecule.

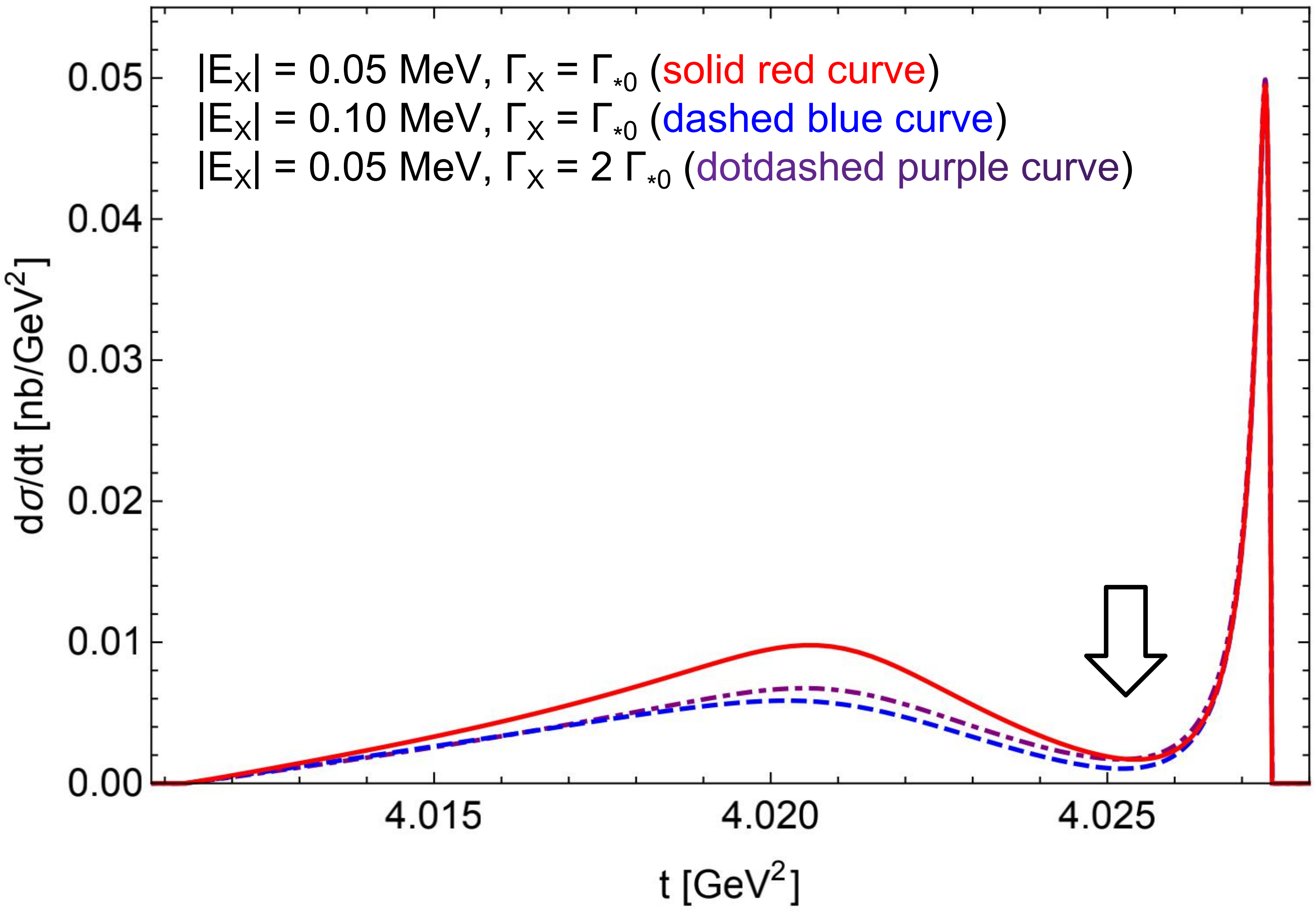
Differential cross section $d\sigma/dt$ for $\sqrt{s}$ = 4014.7 MeV with $u<u_\Delta$



Differential cross section **dσ/dt** for $\sqrt{s}$ = 4014.7 MeV with $u<u_\Delta$



Differential cross section **$d\sigma/dt$** for $\sqrt{s}$ = 4014.7 MeV with **$u<u_\Delta$**



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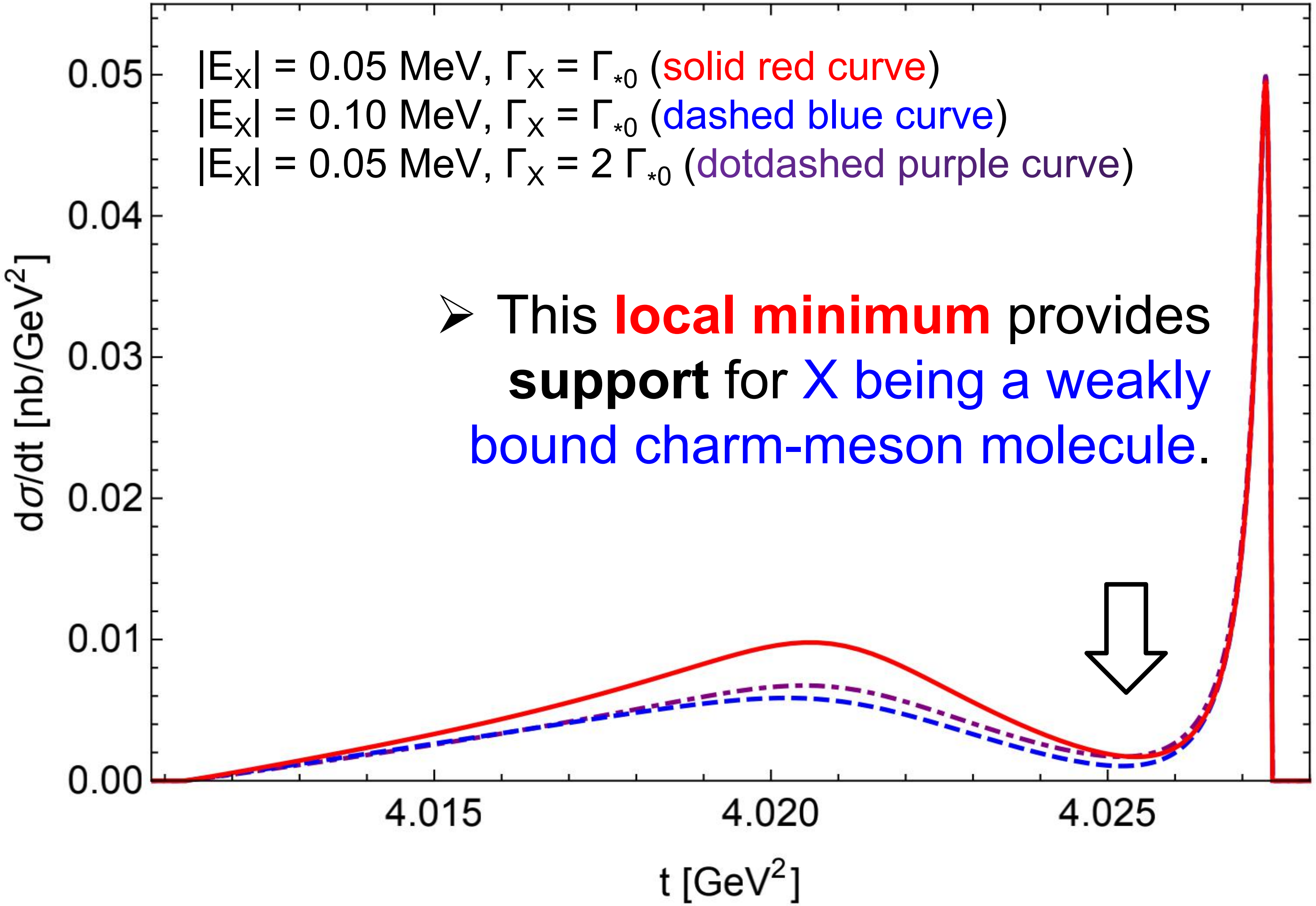

- This **local minimum** provides **support** for X being a weakly bound charm-meson molecule.

Observation of Triangle Singularity ➡ X(3872) being Molecule State

➢ The observation of the narrow peak in the cross section of $e^+e^- \to X + \gamma$ would support the identification of X as a weakly bound charm-meson molecule.

✓ The charm-meson triangle singularity in $e^+e^- \to D^{*0}\bar{D}^0 + \gamma$ cannot be observed as a peak in either dσ/dudt or dσ/du directly .

✓ Charm-meson triangle singularity can be observed indirectly in dσ/dt with $u<u_\Delta$ as a local minimum.

✓ The observation of this minimum provide additional support for the identification of X as a weakly bound charm-meson molecule.

$$|X(3872)\rangle = \frac{1}{\sqrt{2}}\left(|D^{*0}\bar{D}^0\rangle + |D^0\bar{D}^{*0}\rangle\right)$$

Thank You!

arXiv:2004.12841 [pdf, other] hep-ph

Charm-meson Triangle Singularity in e^+e^- Annihilation into $D^{*0}\bar{D}^0 + \gamma$

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