THE OHIO STATE UNIVERSITY

Doubly Heavy Tetraquarks in the Heavy-Diquark Limit with Error Bars

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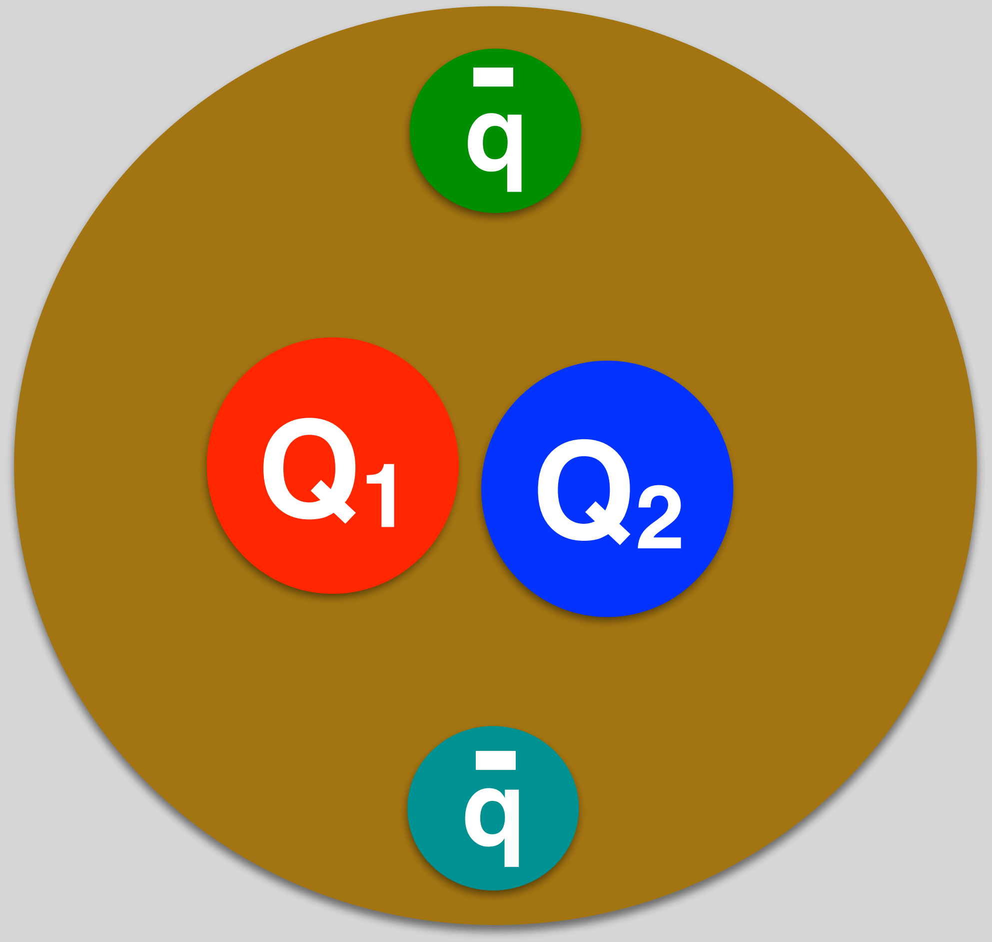
Outline

- Introduction to doubly heavy tetraquarks
- Singly heavy hadron masses in HQET
- Predictions for doubly heavy tetraquark masses in heavy-diquark limit with error bars
- Summary

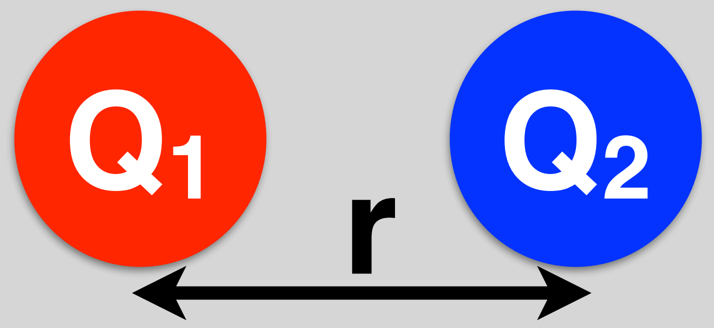
Doubly heavy tetraquarks in heavy-diquark limit

two heavy quarks + two light antiquarks



doubly heavy core

spin: $1/2 \otimes 1/2 = \boxed{0 \oplus 1}$



color: $3 \otimes 3 = 6 \oplus \boxed{3^*}$

one-gluon-exchange

potential: $\boxed{\alpha_s/3r}$ $\boxed{-2\alpha_s/3r}$

repulsive **attractive**

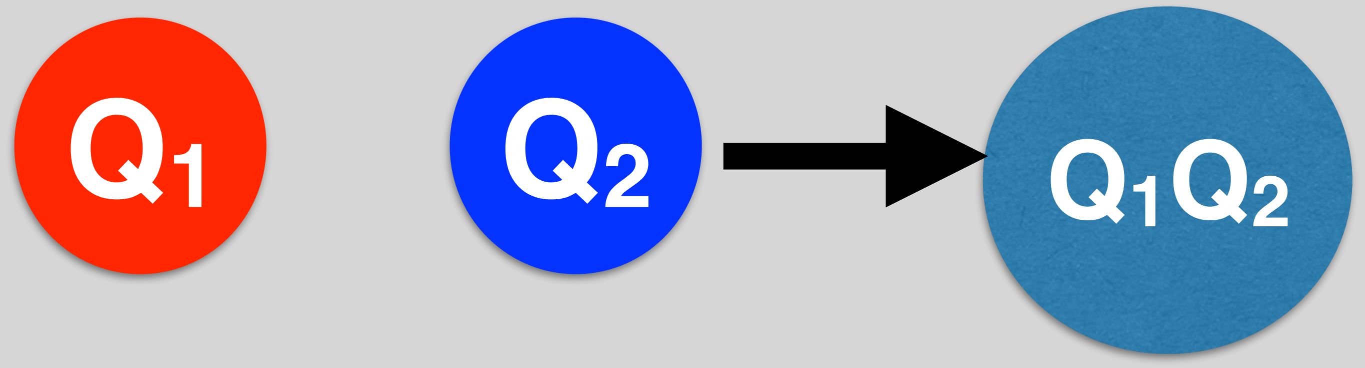
Doubly heavy tetraquarks in heavy-diquark limit

doubly heavy core

spin: $1/2 \otimes 1/2 = 0 \oplus 1$

antisymmetric (0)

symmetric (1)



color: $3 \otimes 3 = 6 \oplus 3^*$

antisymmetric (3^*)

flavor: $\{Q_1Q_2\}$ or $[Q_1Q_2]$

symmetric ($\{Q_1Q_2\}$)

antisymmetric ($[Q_1Q_2]$)

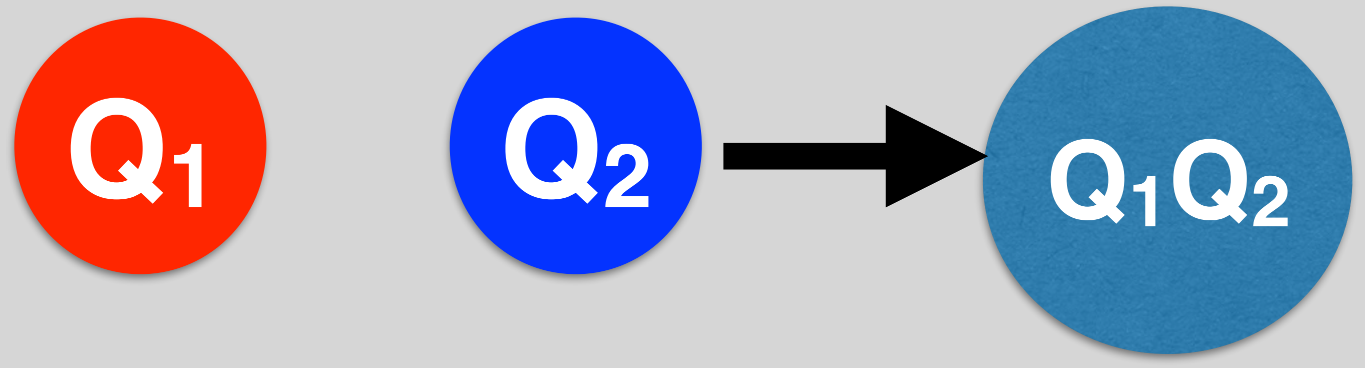
parity: $P = (-1)^L = +1$ (L=0)

spatial wave function is symmetric

Doubly heavy tetraquarks in heavy-diquark limit

doubly heavy core

spin: $1/2 \otimes 1/2 = \boxed{0 \oplus 1}$



color: $3 \otimes 3 = 6 \oplus \boxed{3^*}$

flavor:	$\{Q_1Q_2\}$	$[Q_1Q_2]$
J^P:	1^+	0^+

light antiquarks

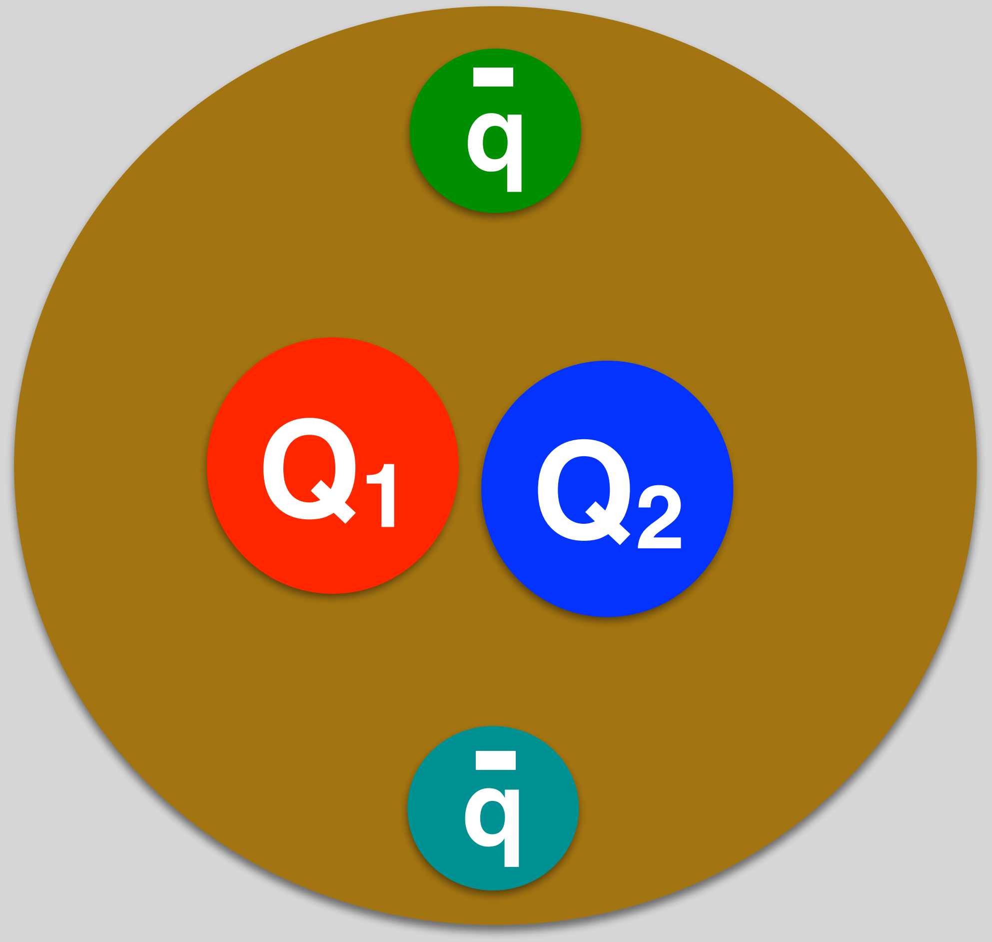
$\{\bar{q}\bar{q}'\}, 1^+$ $[\bar{q}\bar{q}'], 0^+$

doubly heavy tetraquarks

flavor	J^P
$[Q_1Q_2]\ [\bar{q}\bar{q}']$	0^+
$[Q_1Q_2]\ \{\bar{q}\bar{q}'\}$	1^+
$\{Q_1Q_2\}\ [\bar{q}\bar{q}']$	1^+
$\{Q_1Q_2\}\ \{\bar{q}\bar{q}'\}$	$(0, 1, 2)^+$

Doubly heavy tetraquarks in heavy-diquark limit

$QQ\bar{q}\bar{q}$: stable from strong decays?



- **Manohar and Wise** [Nucl.Phys. B399, 17 (1993)]

  There must be stable $QQ\underline{q}\underline{q}$ tetraquarks in the limit $m_Q \to \infty$

- **Karliner and Rosner** [Phys. Rev. Lett 119, 202001(2017)]:

  stable $bb[\underline{u}\underline{d}]$

  near threshold $cc[\bar{u}\bar{d}]$ and $bc[\bar{u}\bar{d}]$

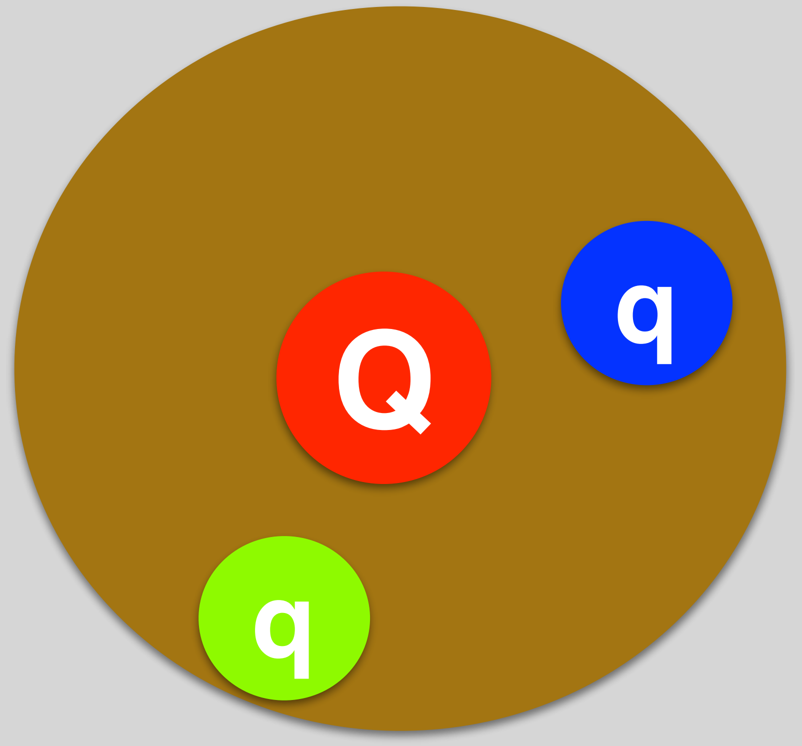
- **Eichten and Quigg** [Phys. Rev. Lett 119, 202002 (2017)]

  stable $bb[\bar{u}\bar{d}]$, $bb[\bar{q}\bar{s}]$, cc and bc tetraquarks are not stable.

It is important to have predictions of the masses with error bars

Singly heavy hadrons

$\ell = \bar{q}/qq, j^P$



- **Hamiltonian through first order in 1/m$_Q$:**

$$H_\ell^Q = m_Q + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2m_Q} + \frac{\mathcal{S}_\ell}{2m_Q} \vec{S} \cdot \vec{j}_\ell$$

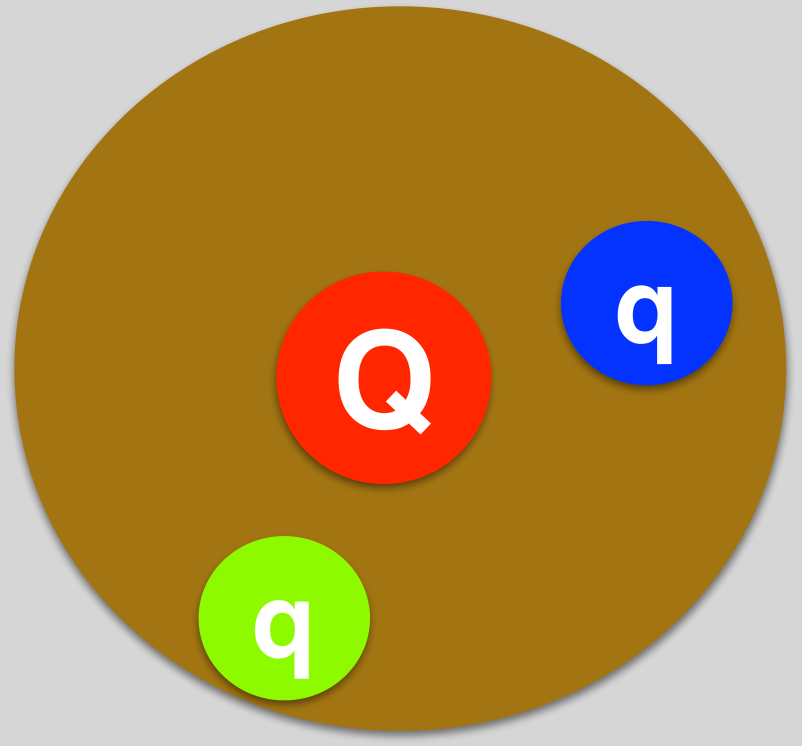
hadron spin:

$J = S + j_l$

- **parameters can be fitted with measured masses in PDG(2018)**

Singly heavy hadrons

$\ell = \bar{q}/qq, j^P$



- **Hamiltonian through first order in $1/m_Q$:**

$$H_\ell^Q = m_Q + \boxed{\mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2m_Q}} + \frac{\mathcal{S}_\ell}{2m_Q}\vec{S}\cdot\vec{j}_\ell$$

$\mathcal{E}_{\ell,Q}$

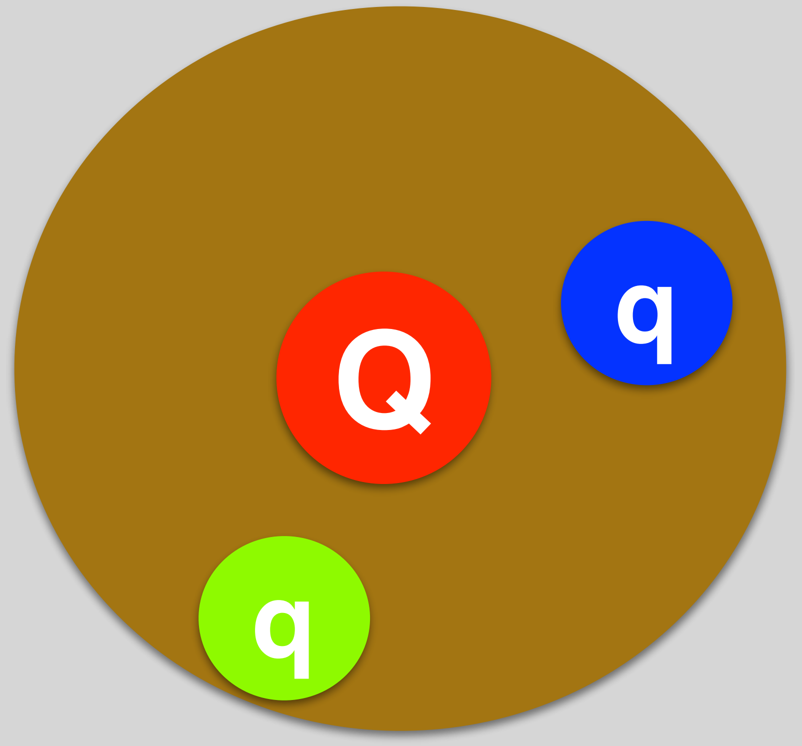
hadron spin:

$J = S + j_l$

- **parameters can be fitted with measured masses in PDG(2018)**
  - **theoretical errors: isospin splittings** [effects of electromagnetism and the u − d mass difference]
  - **correction to $\mathcal{S}_\ell$ of order $1/m_Q$: $\mathcal{S}_\ell \to \mathcal{S}_{\ell,Q}$**
  - **strange quark mass m_s : difference in $\mathcal{E}_{\ell,Q}$ when u or d $\to$ s**

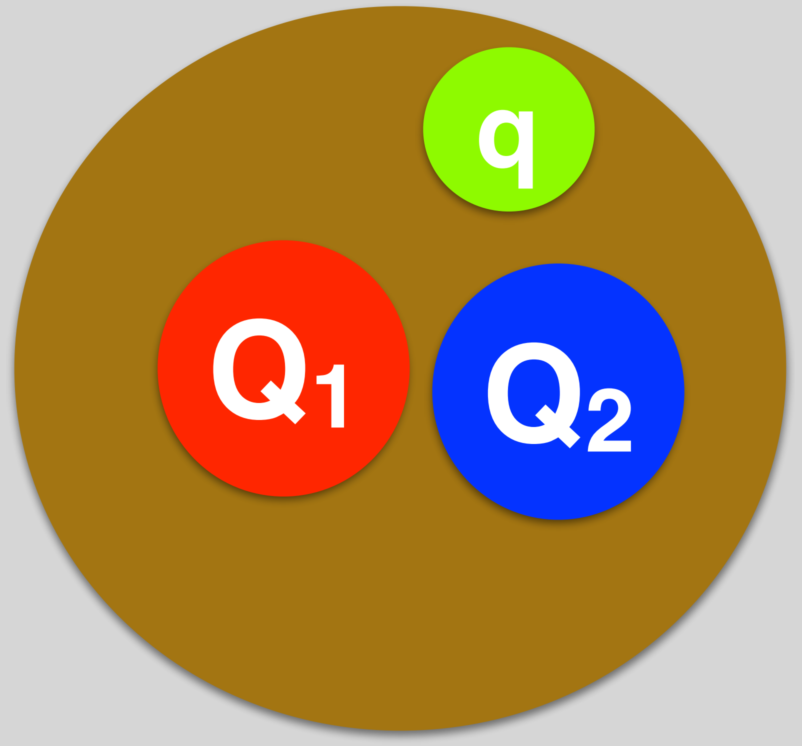
Singly heavy hadrons

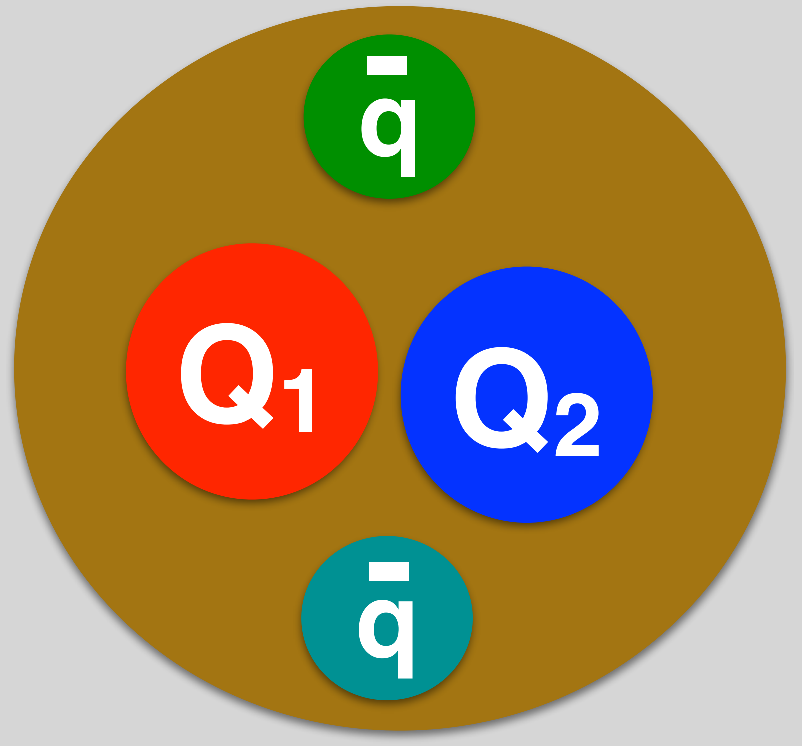
$\ell = \bar{q}/qq, j^P$ **parameters can be fitted with measured masses in PDG(2018)**



Q	ℓ	$\mathcal{E}_{u/d,Q}$ [MeV]	$m_{s,Q}$ [MeV]	$\mathcal{S}_Q$ [GeV^2]	dof	χ^2/dof
c	$\bar{q}, \frac{1}{2}^-$	313.4 ± 2.0	102.3 ± 3.5	0.472 ± 0.012	3	0.28
b	$\bar{q}, \frac{1}{2}^-$	306.4 ± 0.2	87.6 ± 0.5	0.455 ± 0.004	2	1.20
c	$[q\,q'], 0^+$	626.5 ± 1.1	182.9 ± 1.4		1	1.09
b	$[q\,q'], 0^+$	612.5 ± 3.2	174.7 ± 4.0		1	0.40
c	$\{q\,q'\}, 1^+$	837.7 ± 0.7	124.0 ± 0.8	0.147 ± 0.003	9	0.85
b	$\{q\,q'\}, 1^+$	820.8 ± 2.2	117.7 ± 2.3	0.136 ± 0.028	5	0.30

Doubly heavy hadrons





- **Hamiltonian through first order in $1/m_Q$:**

pNRQCD: [Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005)]

✦ kinetic term: $\propto 1/\mu_{Q1Q2}$

✦ spin dependent term: $\propto (\mathbf{S}_1/m_{Q1} + \mathbf{S}_2/m_{Q2})$

Wigner-Eckart theorem: $\frac{1}{2}\left\langle\left(\frac{\mathcal{S}_\ell}{2m_{Q_1}}S_1+\frac{\mathcal{S}_\ell}{2m_{Q_2}}S_2\right)\cdot j_\ell\right\rangle = \frac{\mathcal{S}_\ell}{2S^2}\left(\frac{S_1\cdot S}{2m_{Q_1}}+\frac{S_2\cdot S}{2m_{Q_2}}\right)\langle S\cdot j_\ell\rangle$

✦ orbital angular momentum dependent term: $\propto \mathbf{L}\cdot\mathbf{B} \to 0$ when L=0

$$H_\ell^{Q_1Q_2} = (m_{Q_1}+m_{Q_2}) + \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\mu_{Q_1Q_2}} + \frac{\mathcal{S}_\ell}{8\mu_{Q_1Q_2}}\vec{S}\cdot\vec{j}_\ell$$

hadron spin: $J = S + j_l$

Fleming and Mehen [Phys. Rev. D73, 034502 (2005)]: QQ system

Doubly heavy hadrons

The kinetic and spin splitting terms are different from those by Eichten and Quigg [Phys. Rev. Lett 119, 202002 (2017)]

$$\frac{\mathcal{K}_\ell}{2(m_{Q_1}+m_{Q_2})} \qquad \frac{\mathcal{S}_\ell}{2(m_{Q_1}+m_{Q_2})} S\cdot j_\ell$$

$$H_\ell^{Q_1Q_2} = (m_{Q_1}+m_{Q_2}) + \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\mu_{Q_1Q_2}} + \frac{\mathcal{S}_\ell}{8\mu_{Q_1Q_2}} S\cdot j_\ell$$

$$\mathcal{E}_{\ell,Q_1Q_2}$$

parameters for Q_1Q_2q fitted with lattice QCD calculations

Q_1Q_2	ℓ	$\mathcal{E}_{u/d,Q_1Q_2}$ [MeV]	m_{s,Q_1Q_2} [MeV]	$\mathcal{S}_{Q_1Q_2}$ [GeV^2]	dof	χ^2/dof
$c\,c$	$q,\ \frac{1}{2}^+$	339.1 ± 4.9	106.4 ± 6.2	0.337 ± 0.013	28	0.31
$[b\,c]$	$q,\ \frac{1}{2}^+$	275.9 ± 37.2	55.0 ± 47.0		0	
$\{b\,c\}$	$q,\ \frac{1}{2}^+$	309.4 ± 27.3	73.5 ± 34.3	0.181 ± 0.046	2	8×10^{-5}
$b\,b$	$q,\ \frac{1}{2}^+$	152.2 ± 25.1	130.0 ± 33.6	0.472 ± 0.075	2	2×10^{-5}

Doubly heavy hadrons

- Parameters of $Q_1Q_2\bar{q}\bar{q}'$ from those of Q_1Q_2q, $\bar{Q}q$, and $\bar{Q}\bar{q}\bar{q}$

QQ:

$$\mathcal{E}_{\ell,QQ} - \mathcal{E}_{\ell',QQ} = \mathcal{E}_{\ell,\bar{Q}} - \mathcal{E}_{\ell',\bar{Q}} + \frac{1}{2m_Q}(\mathcal{K}_\ell - \mathcal{K}_{\ell'})$$

$$m_{s,\ell,QQ} = m_{s,\ell,\bar{Q}} + \frac{1}{2m_Q}\frac{2m_b m_c}{m_b - m_c}\left(m_{s,\ell,\bar{c}} - m_{s,\ell,\bar{b}}\right)$$

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}}$$

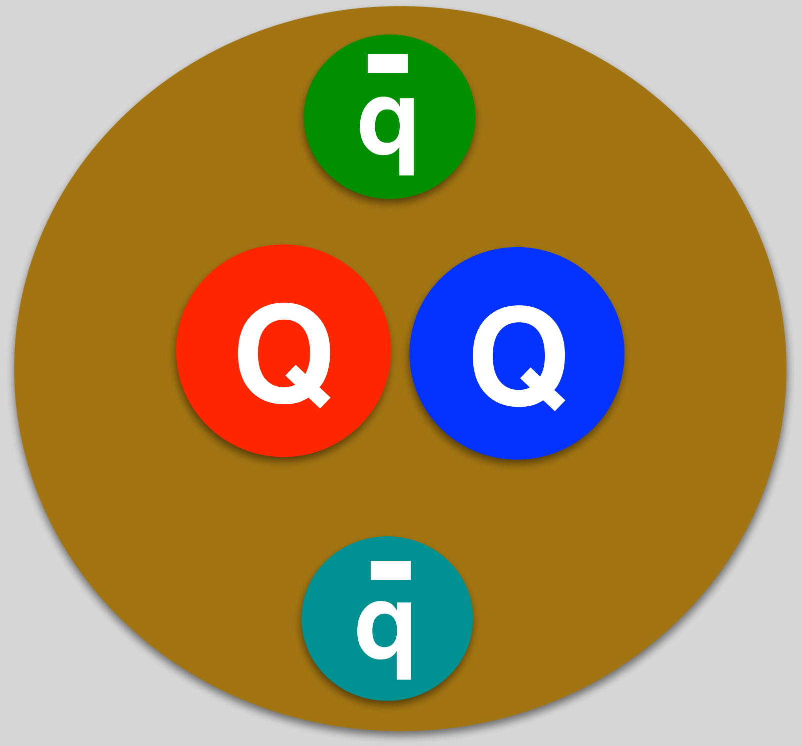
bc:

$$\mathcal{E}_{\ell,bc} - \mathcal{E}_{\ell',bc} = \frac{m_b}{m_b - m_c}\left(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}\right) - \frac{m_c}{m_b - m_c}\left(\mathcal{E}_{l,\bar{b}} - \mathcal{E}_{\ell',\bar{b}}\right)$$

$$m_{s,\ell,bc} = \frac{m_b}{m_b - m_c} m_{s,\ell,c} - \frac{m_c}{m_b - m_c} m_{s,\ell,b}$$

$$\mathcal{S}_{\ell,bc} = \frac{\mu_{bc}}{m_b}\mathcal{S}_{\ell,\bar{b}} + \frac{\mu_{bc}}{m_c}\mathcal{S}_{\ell,\bar{c}}$$

Predictions for doubly heavy tetraquarks



two heavy mesons

flavor	J^P	Eichten-Quigg	this work [MeV]	threshold [MeV]
$cc[\bar{u}\bar{d}]$	1^+	3978	3983 ± 9	3875
$cc[\bar{u}\bar{s}]$	1^+	4156	4178 ± 11	3975
$cc\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$4146 + (0, 21, 64)$	$4154 + (0, 22, 66) \pm 8$	$3734 + (0, 141, 0)$
$cc\{\bar{u}\bar{s}\}$	$0^+, 1^+, 2^+$		$4287 + (0, 22, 66) \pm 9$	$3833 + (0, 142, 0)$
$cc\,\bar{s}\bar{s}$	$0^+, 1^+, 2^+$		$4421 + (0, 22, 66) \pm 11$	$3937 + (0, 144, 0)$
$bb[\bar{u}\bar{d}]$	1^+	**10482**	$\mathbf{10476 \pm 25}$	10604
$bb[\bar{u}\bar{s}]$	1^+	**10643**	$\mathbf{10655 \pm 25}$	10692
$bb\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$10674 + (0, 7, 21)$	$10672 + (0, 7, 20) \pm 25$	$10559 + (0, 45, 0)$
$bb\{\bar{u}\bar{s}\}$	$0^+, 1^+, 2^+$		$10793 + (0, 7, 20) \pm 25$	$10646 + (0, 45, 0)$
$bb\,\{\bar{s}\bar{s}\}$	$0^+, 1^+, 2^+$		$10914 + (0, 7, 20) \pm 25$	$10734 + (0, 48, 0)$

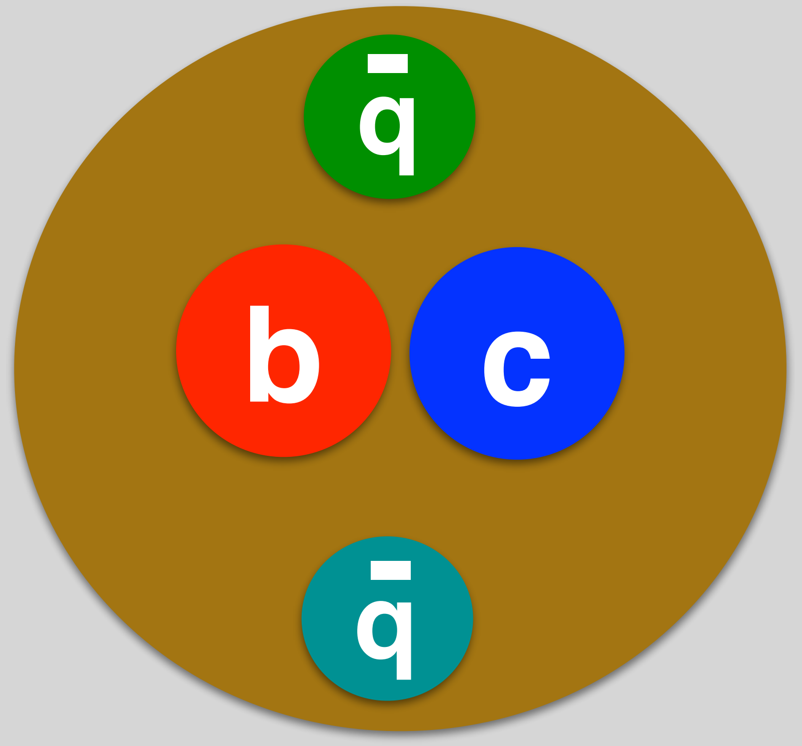
stable tetraquarks

no stable cc tetraquarks

Karliner and Rosner [Phys. Rev. Lett 119, 202001(2017)]

$m(cc[\bar{u}\bar{d}]) = 3882 \pm 12$ MeV, $m(bb[\bar{u}\bar{d}]) = 10398 \pm 12$ MeV

Predictions for doubly heavy tetraquarks



two heavy mesons

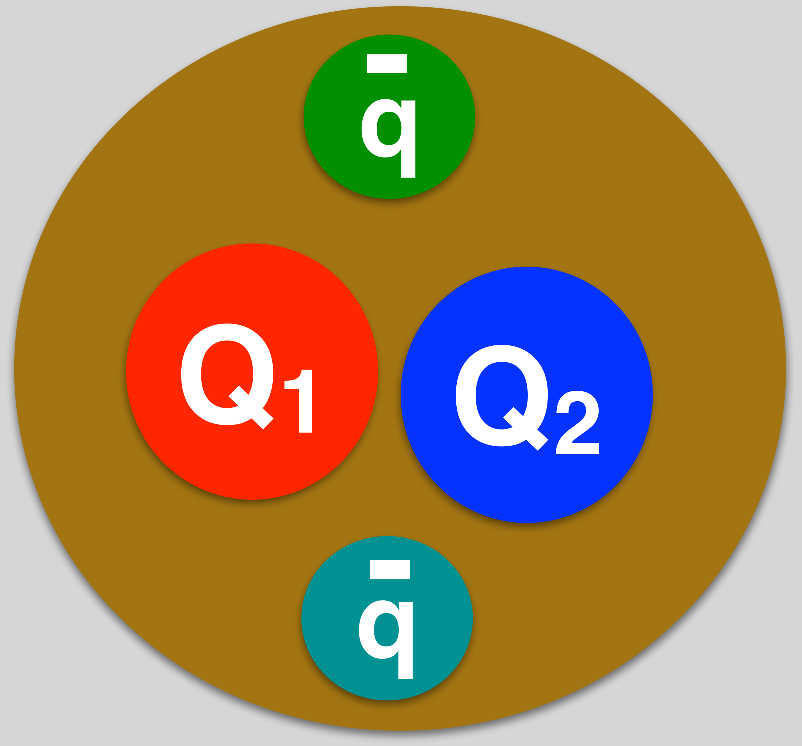
flavor	J^P	Eichten-Quigg	this work [MeV]	threshold [MeV]
$[bc][\bar{u}\bar{d}]$	0^+	7229	7259 ± 37	7144
$[bc][\bar{u}\bar{s}]$	0^+	7406	7446 ± 38	7232
$[bc]\{\bar{u}\bar{d}\}$	1^+	7439	7472 ± 37	7189
$[bc]\{\bar{u}\bar{s}\}$	1^+		7599 ± 37	7280
$[bc]\,\bar{s}\bar{s}$	1^+		7726 ± 37	7384
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	7293 ± 28	7189
$\{bc\}[\bar{u}\bar{s}]$	1^+	7445	7480 ± 28	7280
$\{bc\}\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$7461 + (0, 11, 32)$	$7477 + (0, 14, 43) \pm 28$	$7144 + (0, 45, 0)$
$\{bc\}\{\bar{u}\bar{s}\}$	$0^+, 1^+, 2^+$		$7604 + (0, 14, 43) \pm 28$	$7232 + (0, 49, 0)$
$\{bc\}\,\bar{s}\bar{s}$	$0^+, 1^+, 2^+$		$7731 + (0, 14, 43) \pm 28$	$7355 + (0, 49, 0)$

no stable bc tetraquarks

Karliner and Rosner [Phys. Rev. Lett 119, 202001(2017)]

$m([bc][\bar{u}\bar{d}])$ = 7134 ± 13 MeV, 10MeV below 7144 MeV

Summary



- Predict masses with error bars for doubly heavy tetraquarks
- Verify the two stable tetraquarks:

  $bb[\bar{u}\bar{d}]$ (10476 ± 25 MeV) ≈ 130 MeV below strong-decay threshold

  $bb[\bar{q}\bar{s}]$ (10655 ± 25 MeV) ≈ 40 MeV below strong-decay threshold

- There are no stable cc or bc tetraquarks