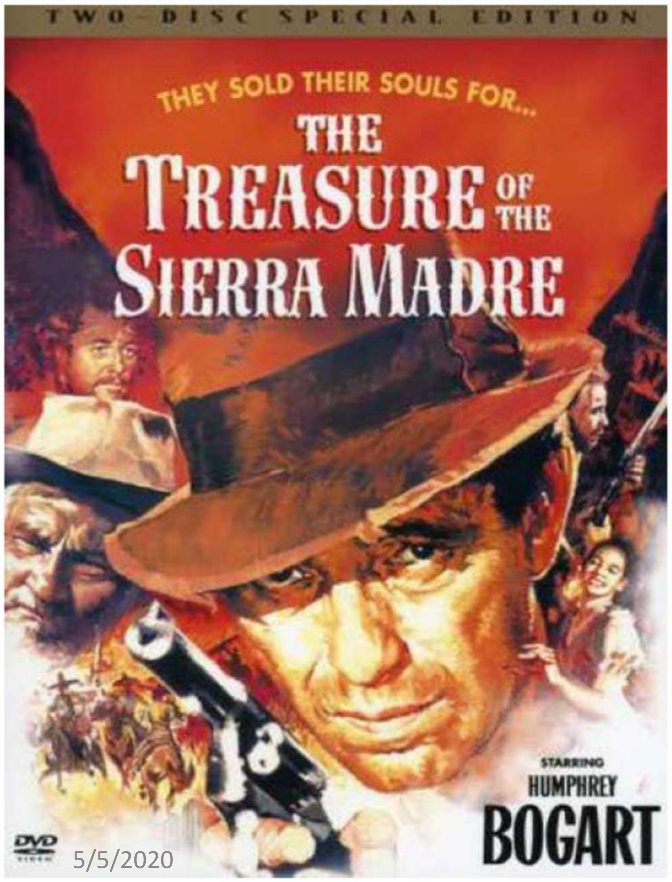
[More] Treasures from Kaons



Amarjit Soni
BNL-HET

Pheno 2020v

Outline

- **Memories**
- **Introduction + Motivation**
- **Basics of ΔI=1/2 enhancement/Rule & Direct CP in K=> ππ**

 i.e. ε'
- **Early attempt(s) , hurdles & resolution**
- **DWQ & Lellouch-Luscher 1st completion ~2015 & indication of difficulty**
- **Improved stats & systematic new result**
- **Summary + Outlook**

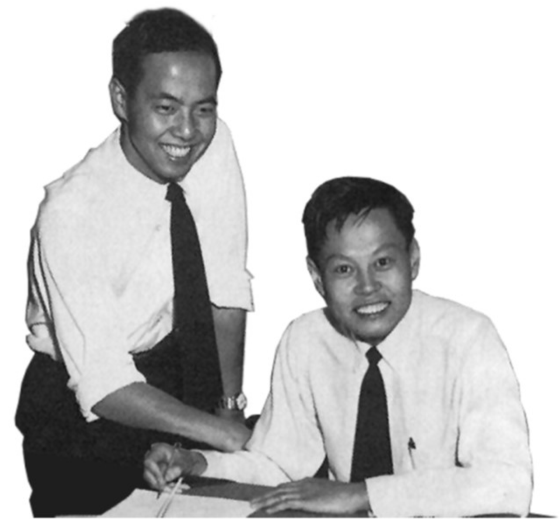
What an exciting history!

? $\to 2\pi;\ P=+$

$\to 3\pi;\ P=-$

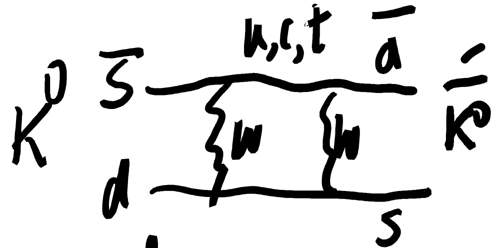
- **Theta – tau puzzle………..Nature does care about L vs R**

- **… the Nobel goes to Kids!**



K decay violate P !

II $K^0-\bar{K}^0$ Mixing, Decay, Indirect CP violation



QM mixing

$\Delta S=2$

$O(G_F^2)$

2 States K_L, K_S If CP is exact

$K_L \equiv \frac{K^0-\bar{K}^0}{\sqrt{2}}$; $K_S \equiv \frac{K^0+\bar{K}^0}{\sqrt{2}}$

CP− | CP+

$\rightarrow 3\pi$ | $\rightarrow 2\pi$

$\not\rightarrow 2\pi$ | $\not\rightarrow 3\pi$

$\frac{\Delta m_K}{m_K} \sim 7\times 10^{-15}$ But $\tau_{K_L}/\tau_{K_S} \sim O(500) \gg 1$

The long lifetime of K_L a very important blessing; led to one of the most important discoveries in Particle Physics ie CP violation

III Indirect CP violation

BNL 1964 Fitch, Cronin, Christensen + Turlay

NOBLE

CRONIN + FITCH

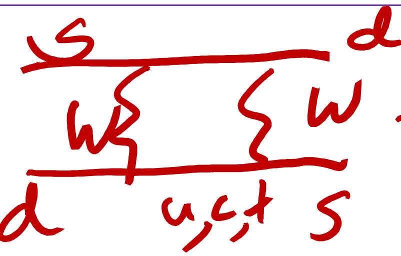
$$\frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} \neq 0 \; !$$

$\epsilon_K \to \sim 2.23 \times 10^{-3}$

K^0 $\bar{K}^0$ CPV in state mixing, $\Delta S=2$ H_{eff}

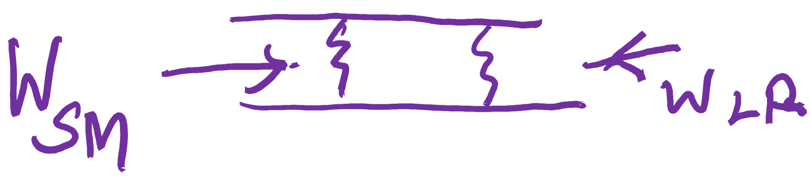
ΔmK : a powerful constraint on BSM

In SM $\Delta S = 2$



$+ \cdots\cdots$

an explicit illustration: LRS Beall + Bander + AS PRL 1982



$\left(\dfrac{\Delta m_K}{m_K}\right)^{expt} \approx 10^{-14}$!!

$\Rightarrow m_R \gtrsim 1.6\ \text{TeV}$

$\Rightarrow$ precision to flavor & CP problem

Delta I=1/2 rule/ puzzle: a challenge for generations

MAIN MODES $\qquad \tau$

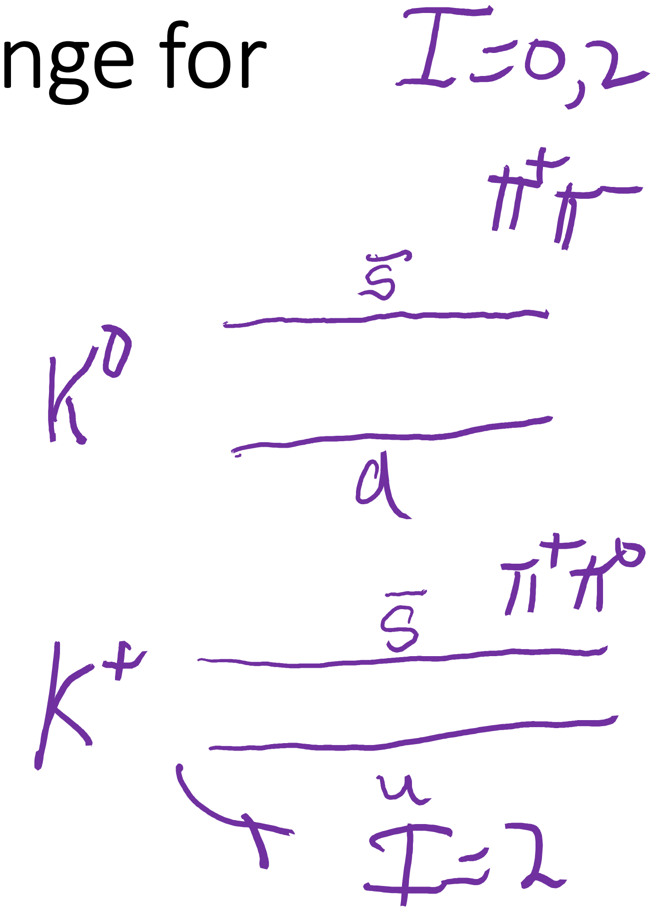
- K_s $\qquad \pi^+\pi^- \qquad \sim 0.9\times10^{-10}\,s$

$\Delta I = 1/2, 3/2$

- K^+ $\qquad \pi^+\pi^0 \quad \Delta I = 3/2 \quad \sim 1.2\times10^{-8}\,s$

$K_L \qquad \pi^+\pi^-\pi^0 \qquad \sim 5\times10^{-8}\,s$

phase space suppressed



IV: ε′ / ε: Direct CPV

EXPERIMENTAL ROUTE

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}|e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' \simeq \frac{1}{3}\left[\eta_{+-} - \eta_{00}\right] \Rightarrow O(10^{-3}) - O(10^{-3})$$

$$\Rightarrow 10^{-6}$$

$$\epsilon = \frac{1}{3}\left[2\eta_{+-} + \eta_{00}\right]$$

Conf on NP@LHC 02/29/16; A. Soni 10

BSM-CP: Theoretical motivation

- **To the extent that SM is not a complete theory, BSM-CP phase(s) are exceedingly likely to exist**
- **Adding fermions, scalars or gauge bosons as a rule entails new phase(s)**
- **Explicit examples: 4G SM: + 2; LRS : at least + 1; 2HDM : neutral scalar sector**

as well as charged sector can have new phases; SUSY or WEXD [see e.g Agashe, Perez & AS, PRD '04; c also Neubert et al'08; Buras et al '08] : tens of new O(1) CP-odd phases arise *naturally*

- **SM cannot account for baryogenesis…..CKM CP not enough**
- **Due to all of the above (and some more), searching for BSM CP-phase(s) is just about the most powerful way to look for NP…..*an early realization & a driving force for past few decades***

MORE LATER

$K \to 2\pi$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$I=2$ $I=0$

Use lattice to calculate 6 quantities:
ReA0, ReA2 known from expt; δ0,δ2 via ChPT etc..So very good checks;
ImA. ImA2 unknown

$\omega \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 \sim 0.045$

Indirect CP

$$|\epsilon| = 2.228(11) \times 10^{-3},$$

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}.$$

DIRECT CP

$\epsilon' << \epsilon$

$\epsilon' \sim 10^{-6}$!

A.S. in Proceedings of Lattice '85 (FSU)..1st Lattice meeting ever attended

The matrix elements of some penguin operators control in the standard model another CP violation parameter, namely ϵ'/ϵ.[6,8] Indeed efforts are now underway for an improved measurement of this important parameter.[10] In the absence of a reliable calculation for these parameters, the experimental measurements, often achieved at tremendous effort, cannot be used effectively for constraining the theory. It is therefore clearly important to see how far one can go with MC techniques in alleviating this old but very difficult

With C. Bernard [UCLA]

Serves as a template for the need of Lattice calculations for more economical use of almost all experimental data From IF

MOTHER of all (lattice) calculations to date: A Personal Perspective

Tianle WANG, Dan Hoying

- Calculation K=> ππ & ε′ were the reasons I went into lattice over 1/3 of a century ago!
- *9 + (3 new) PhD thesis*: Terry Draper (UCLA'84), George Hockney(UCLA'86), Cristian Calin (Columbia=CU'01), Jack Laiho(Princeton'04), Sam Li(CU'06), Matthew Lightman(CU'09), Elaine Goode(Southampton'10), Qi Liu(CU'12), Daiqian Zhang(CU'15)+ [new ones starting from CU, U Conn and Southampton] + many PD's & junior facs.. obstacles & challenges (and of course "mistakes"!) ad infinitum.....

Flavor anomalies; Lyon; A Soni(BNL-HET) 67

A key point to emphasize is that overcoming each major obstacle led to significant application to phenomenology and/or lattice

[necessity is the parent of……]

EXTREMELY valuable inputs from countless:

- **Fred Gilman and Mark Wise**
- **Andrzej Buras et al**
- **Guido Martinelli et al**
- **Yigal Shamir**
- **Laurent Lellouch + Martin Luscher**
- **……**
- **……..**
- **……..**

Basic calculational framework

Δ S=1 H_W

WC to NLO

Buchalla, Buras, Lautenbacher RMP '96; Cirichini et al 95

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu).$$

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}.$$

$M_i = \langle K | Q_i | \pi\pi \rangle$ Needed to all orders in α_s

Tree

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

QCDP

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

I=0 $\Longrightarrow$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$\to 0$

$m_q \to 0$

EWP

$$Q_7 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_R,$$

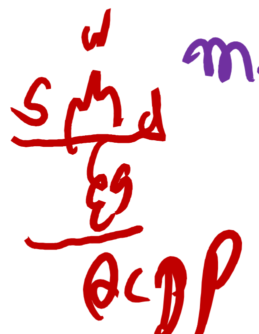
I=2

$$Q_9 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_L,$$

$\longrightarrow$ const

$m \to 0$



EWP

Why EWK cannot be neglected : 3 Reasons

- **Despite $\alpha_{QED,EWK}$ << α_{QCD} , EWK contributions are extremely important and CANNOT be neglected:**
- **EWK are (8,8) and QCD are (8,1), and (8,8) go to constant whereas (8,1) vanish in the chiral limit**
- **EWK, i.e. those due Z exch have Wilson coeff that go as m_t^2/m_W^2**
- **In ε' they enter as**

EWP QCDP

$$\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]$$

small large

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim 22$$

Why EWK cannot be neglected: 3 Reasons

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- EWK are (8,8) and QCD are (8,1), and (8,8) go to constant whereas (8,1) vanish in the chiral limit
- EWK, i.e. those due Z exch have Wilson coeff that go as mt^2 / mW^2
- In ε' they enter as

EWP QCDP

$$\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]$$

small large

$\frac{ReA_0}{ReA_2} \sim 22$

For simplicity: 1st strategy via ChPT

PHYSICAL REVIEW D VOLUME 32, NUMBER 9 1 NOVEMBER 1985

Application of chiral perturbation theory to $K \to 2\pi$ decays

→ LEEFT

BDSPW-85

Claude Bernard, Terrence Draper,* and A. Soni
Department of Physics, University of California, Los Angeles, California 90024

H. David Politzer and Mark B. Wise
Department of Physics, California Institute of Technology, Pasadena, California 91125

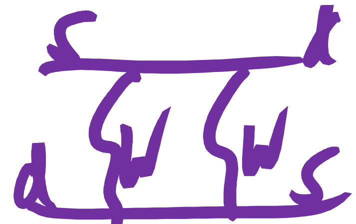
(Received 3 December 1984)

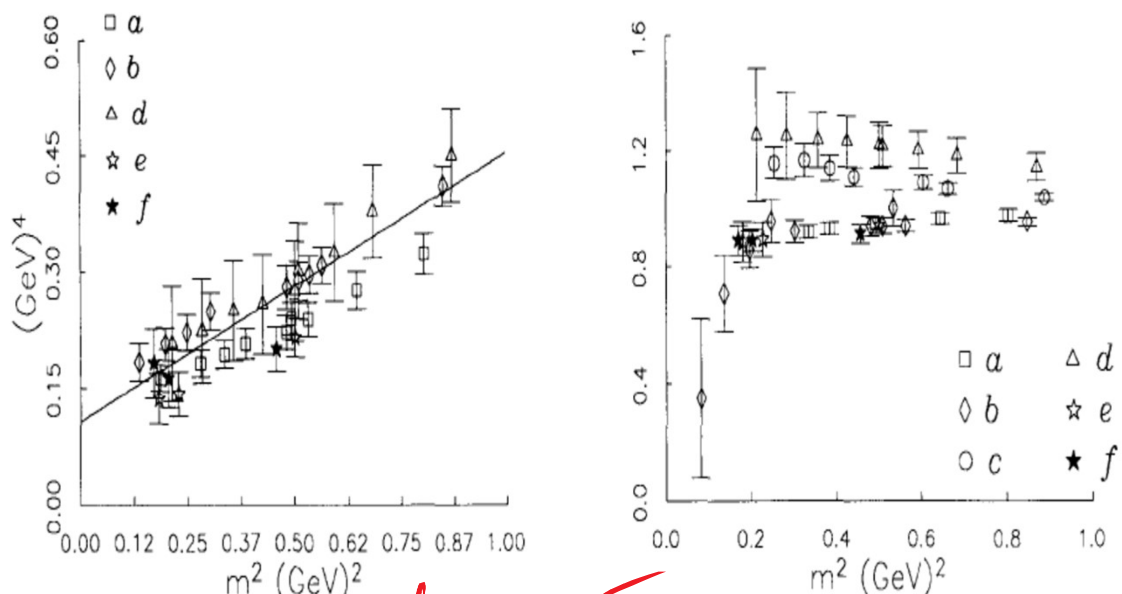
Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

12/20/2017 USED extensively on lattice for ~20 years ⇒ NLO J. LAIHO PhD Thesis ~'03

ODE to YESTERYEARS!

$\langle K | (\bar{s}\gamma_\mu L d)^2 | \bar{K} \rangle$





χS Violation by K-$\bar{K}$ $\Rightarrow$ FINE TUNING PROBLEM

Lattice computation of the decay constants of B and D mesons

Claude W. Bernard
Department of Physics, Washington University, St. Louis, Missouri 63130

James N. Labrenz
Department of Physics FM-15, University of Washington, Seattle, Washington 98195

Amarjit Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 1 July 1993)

Semileptonic decays on the lattice: The exclusive 0^- to 0^- case

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Aida X. El-Khadra
Theory Group, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510

Amarjit Soni
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 and Department of Physics, Brookhaven National Laboratory, Upton, New York 11973[†]
(Received 21 December 1990)

PHYSICAL REVIEW D VOLUME 45, NUMBER 3 1 FEBRUARY 1992

Lattice study of semileptonic decays of charm mesons into vector mesons

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Aida X. El-Khadra
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Amarjit Soni
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(Received 30 September 1991)

We present our lattice calculation of the semileptonic form factors for the decays $D\to K^*$, $D_s\to\phi$, and $D\to\rho$ using Wilson fermions on a $24^3\times39$ lattice at $\beta=6.0$ with 8 quenched configurations. For $D\to K^*$, we find for the ratio of axial form factors $A_2(0)/A_1(0)=0.70\pm0.16^{+0.20}_{-0.15}$. Results for other form factors and ratios are also given.

PIONEERING WORKS Leading to Modern Day UT

PHYSICAL REVIEW D, VOLUME 58, 014501

SU(3) flavor breaking in hadronic matrix elements for B-$\bar{B}$ oscillations

Later ΔM_s CDF, Dϕ

C. Bernard
Department of Physics, Washington University, St. Louis, Missouri 63130

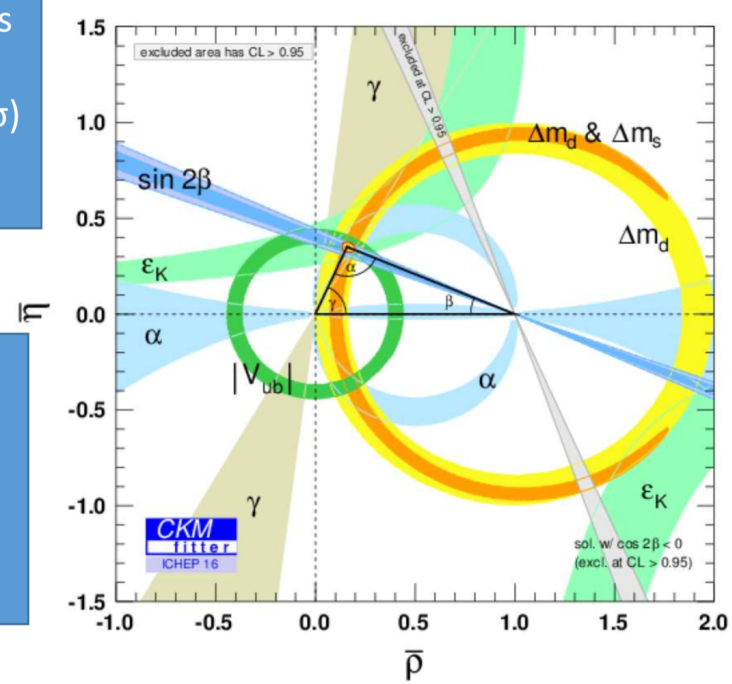
T. Blum and A. Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 28 January 1998; published 5 May 1998)

12/20/2017 IMSC; HEP BNL,Soni

Use exptal data + lattice WME to test SM & search for new physics

http://ckmfitter.in2p3.fr
see also http://www.utfit.org

Looks great; but looks can be deceiving…
In fact at level of O(2σ) tension(s) exist



O(10-15%) new physics is possible and is HUGE!

LAT EXPT

f_B Δm_B

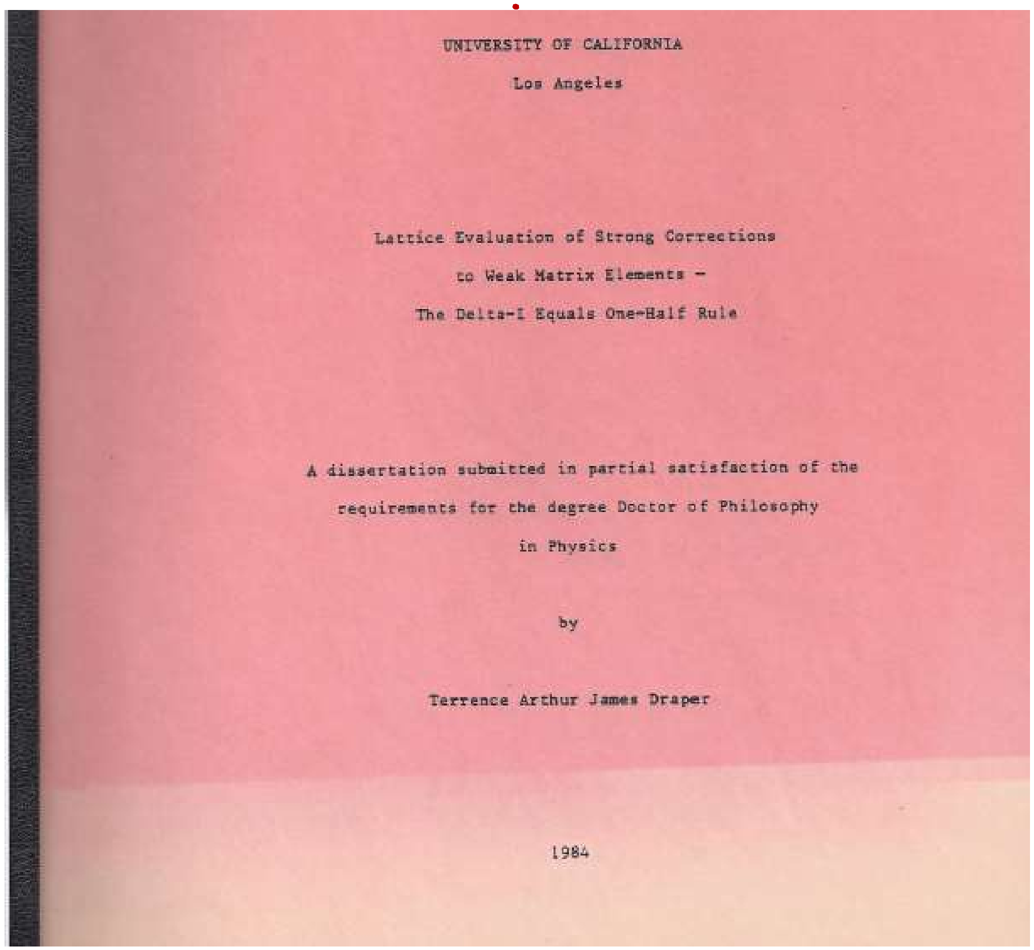
$\frac{f_{B_s}}{f_B}$ … $\frac{\Delta m_{B_s}}{\Delta m_{B_d}}$

$f(q^2), f_0(q^2) \to B \ell \nu \pi$ (D)

$B_K \to \varepsilon_K$

Δm_K NOT used

The 1st PhD Thesis


UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections

to Weak Matrix Elements –

The Delta-I Equals One-Half Rule

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Physics

by

Terrence Arthur James Draper

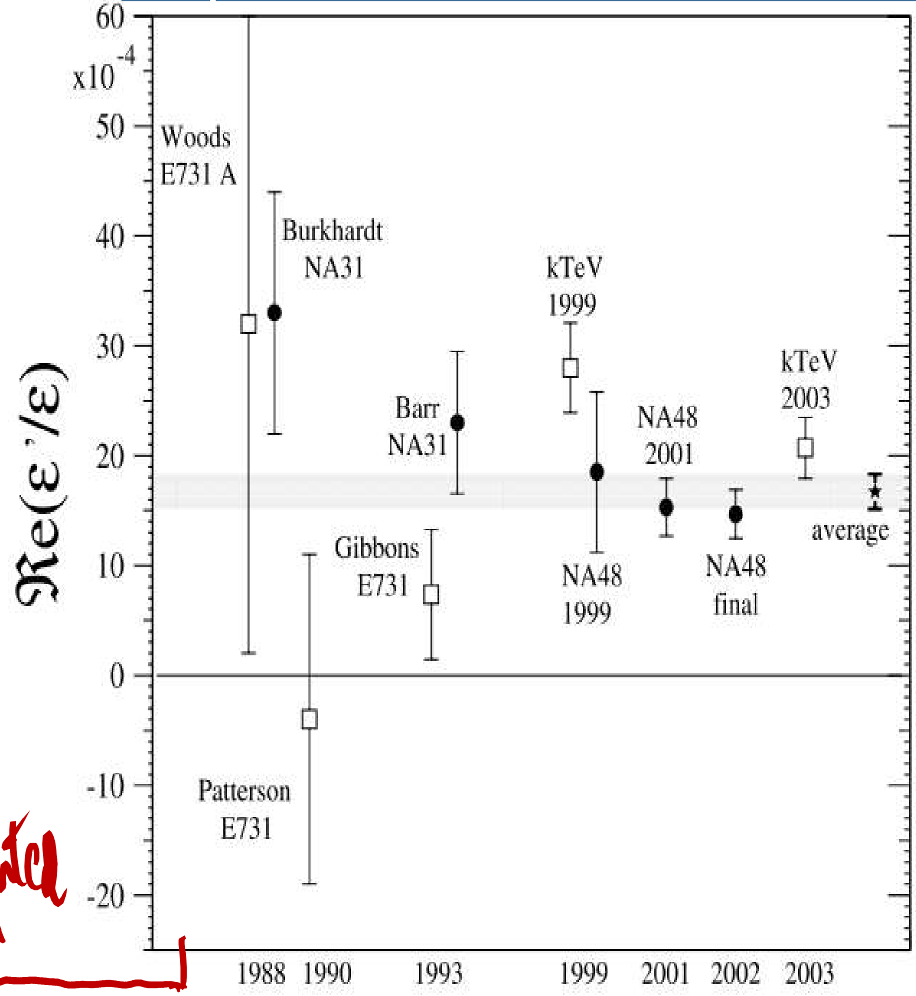
1984

Grew from end of year Beer Party ~June 20, 1982!!

[UCLA]

Beware of End of year Beer Parties!

A monumental experimental achievement!



Konrad Kleinknecht "Uncovering CPV"

$16.6(2.3) \times 10^{-4}$ PDG 2014

LATTICE work started

PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

Two key papers

QCD with domain wall quarks

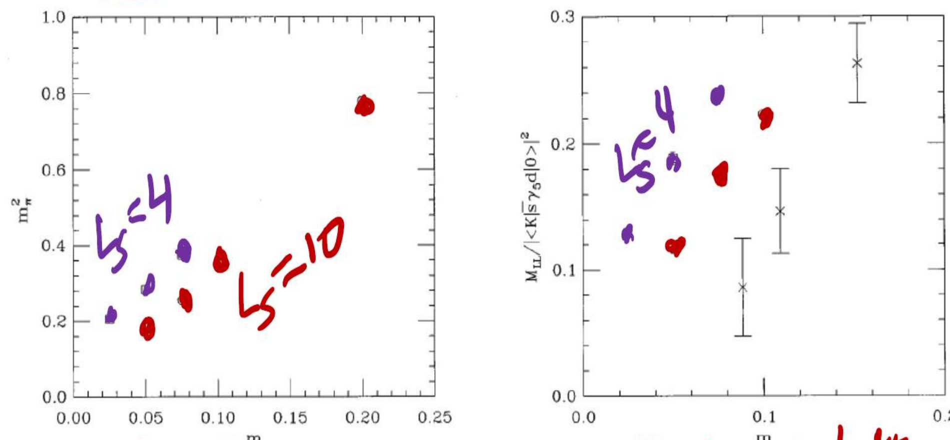
1st Simulation with DWQ

96-'97

T. Blum* and A. Soni†
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 27 November 1996)

We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\overline{K_0}$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s = 10$. [S0556-2821(97)00113-6]



excellent Chiral symmetry with ~10 sites in 5th dim.

12/20/2017

MAJOR BREAKTHROUGH FOR K→ππ Lattice Calculations i.e No ChPT

CMP/01.

a new method

another major development for $K \to \pi\pi$ on lattice

Direct K-> $\pi\pi$ (a la Lellouch-Luscher), using finite volume correlation* functions, [i.e. w/o ChPT] RBC initiates around 2006

CONTINUED BY RBC-UKQCD {(mostly) Edinburgh-Southampton}

* Allows to bypass Maini-Testa theorem

COMMON Interest: use of DWQ for simulations

Relating lattice ME to physical amplitudes

$$A_{2/0} = F\frac{G_F}{\sqrt{2}}V_{ud}V_{us}\sum_{i=1}^{10}\sum_{j=1}^{7}\left[\Big(z_i(\mu)+\tau y_i(\mu)\Big)Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}}M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}}\right]$$

F

F is the Lellouch-Luscher factor which relates finite volume ME to the infinite volume

Phase shift

$$A = \frac{1}{\pi q}\sqrt{\frac{\partial\phi}{\partial q}+\frac{\partial\delta}{\partial q}}\sqrt{m_K}E_{\pi\pi}L^{2/3}M \, ,$$

A/M is LL factor F

$q = \frac{pL}{2\pi}$;

$\propto \frac{\delta}{q}$ for small p

φ is a somewhat complicated function of q and boundary Conditions [See Daiqian Zhang thesis]

RBC-UKQCD PRL2015

Results for ε'

EWP

QCDP

~80% {Central Value}

- Using Re(A_0) and Re(A_2) from experiment and our lattice value for Im(A_0) and Im(A_2) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{= 1.38(5.15)(4.43) \times 10^{-4}, \quad 16.6(2.3) \times 10^{-4}}$$

LARGE CANCELLATION!!

RBC-UKQCD PRL'15 EDITOR'S CHOICE

Bearing in mind the largish errors in this first calculation, we interpret that our result are consistent with experiment at ~2σ level

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \sim 0.045$$

with expt
Computed ReA0 good agreement with expt
Offered an "explanation" of the Delta I=1/2 enhancement

A possible difficulty: strong phases

- **The continuum and our lattice determinations of strong phase**

differ

- $$\phi_{\epsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} = \begin{cases} (42.3 \pm 1.5)^{\circ} & \text{[illegible] [2]} \\ (54.6 \pm 5.8)^{\circ} & \text{~~RBC~~ [47, 48]} \end{cases}$$

Colangelo et al
ChPT etc

RBC-UKQCD

OK

off by $\simeq 2\sigma \Rightarrow$ a Concern

Statistics increase CKelly LAT'18

- Original goal was a 4x increase in statistics over 216 configurations used in 2015 analysis.
- 4x reduction in configuration generation time obtained via algorithmic developments (exact one-flavor implementation) → D Murphy
- Large-scale programme performed involving many machines:

SCs over 3 continents

Source	Determinant computation	Independent configs.
Blue Waters	RHMC	34+18+4+3
KEKSC	RHMC	106
BNL	RHMC	208
DiRAC	RHMC	151
KEKSC	EOFA	275+215
BNL	EOFA	245
		1259 total

- Measurements performed using IBM BG/Q machines at BNL and the Cori computer (Intel KNL) at NERSC largely complete.
- Including original data, now have 6.7x increase in statistics!

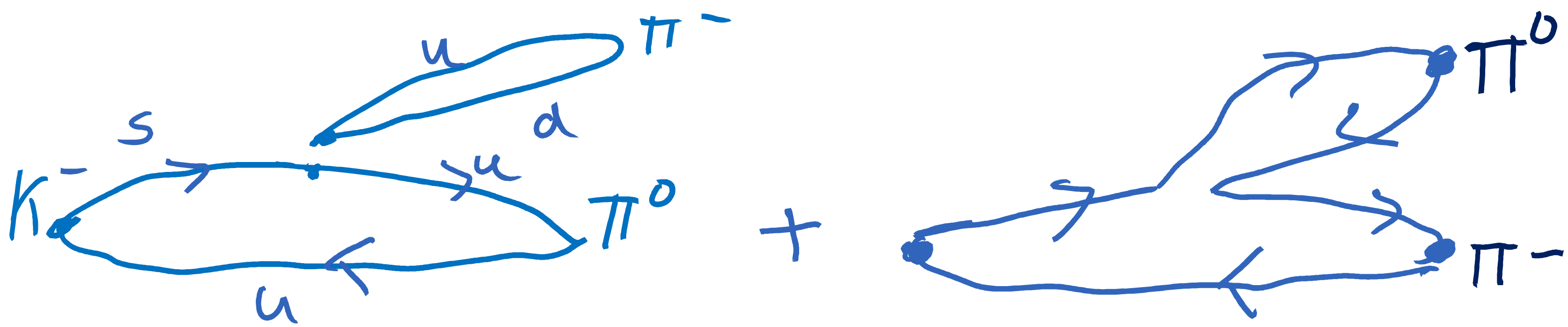
NOW ~1440 g.c

Implications for K→ππ and resolution

- Despite vast increase in statistics, *this second state cannot be resolved from the time dependence using only a single ππ operator.*
- Possibly a significant underestimate of excited state systematic error in K→ππ calculation that can only be resolved by adding additional operators.
- In response we have **expanded the scope of the calculation**:
  - Added K→σ matrix elements
  - Added K→ππ matrix element of new ππ operator with larger relative pion momenta (still p_{CM}=0)
- Result is **3x increase in the number of I=0 ππ operators in K→ππ calc.**
- Also added ππ 2pt functions with non-zero total ππ momenta.
  Calculate phase shift at several (smaller) additional center-of-mass energies.
  - Additional points that can be compared to dispersive result / experiment
  - Improve ~11% systematic on Lellouch-Luscher factor associated with slope of phase shift.
- Currently have 152 measurements with new operators! Adding ~100/month

Unravelling the $\Delta I=1/2$ rule

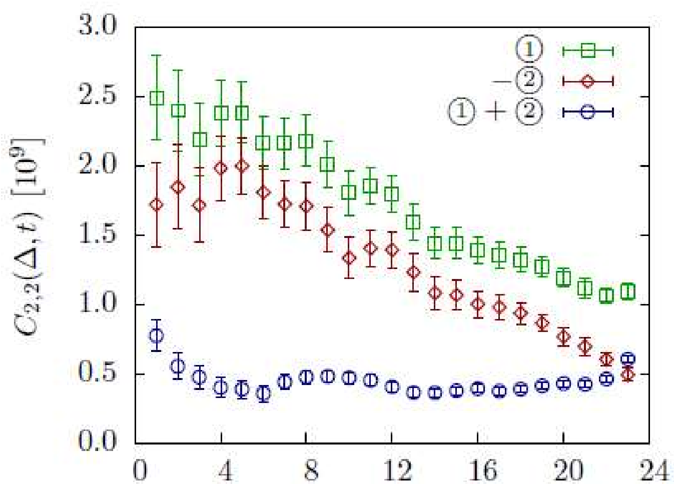
Dissecting (the much easier) ΔI=3/2 [I=2 ππ] Amp on the lattice: 2 contributing topologies only



Tr x Tr
(N^2)

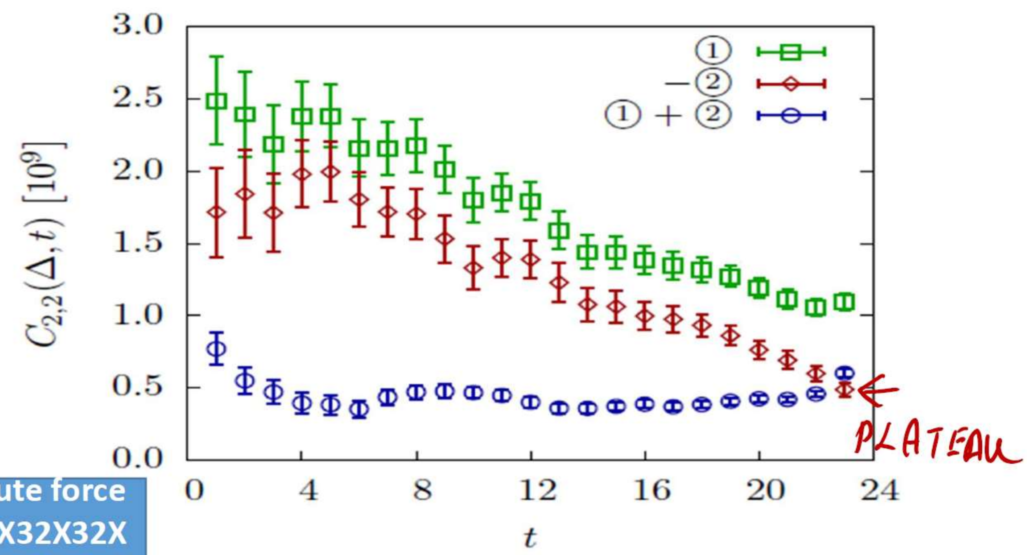
Tr (N)

Simplest basic step is Significantly different from phenomenological Expectations!



DRAMATIC CANCELLATION!
($m_\pi \sim 140$ MeV)

RBC-UKQCD PRL 2012: Unravelling the origin of the textbook Delta I=1/2 Puzzle: Unnatural("accidental") suppression of ReA2 at m_pi ~140 MeV



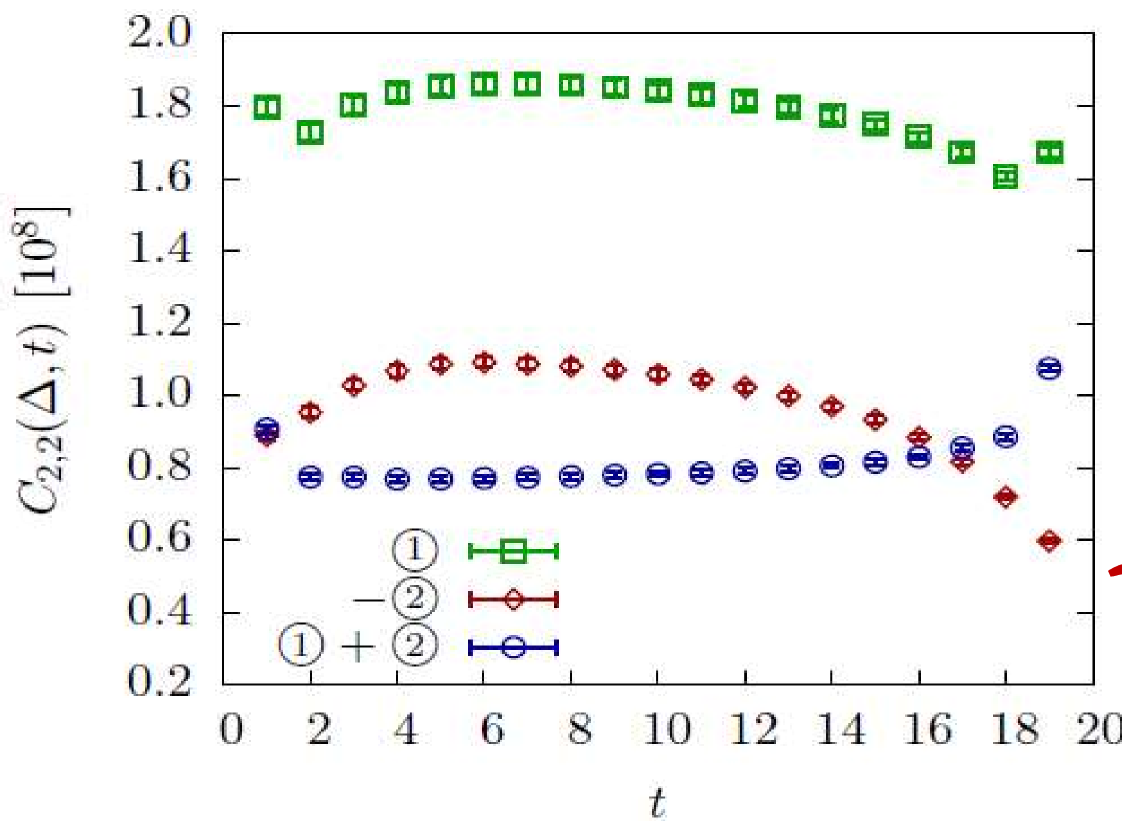
Brute force 32X32X32X 64X16

FIG. 2: Contractions ①, -② and ① + ② as functions of t from the simulation at physical kinematics and with $\Delta = 24$.

QCDOC 10 Tf

12/20/2017 IMSC; HET-BNL;soni 96

UNLIKE WHAT TEXT BOOKS SAY, INFACT NAÏVE FACTORIZATION FAILS IN I=2 K=> 2 pi decays



For heavier π,
$m_\pi \sim 330$ MeV
less cancellation
bet. N^2 & N
Large N begins
to improve!

FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.

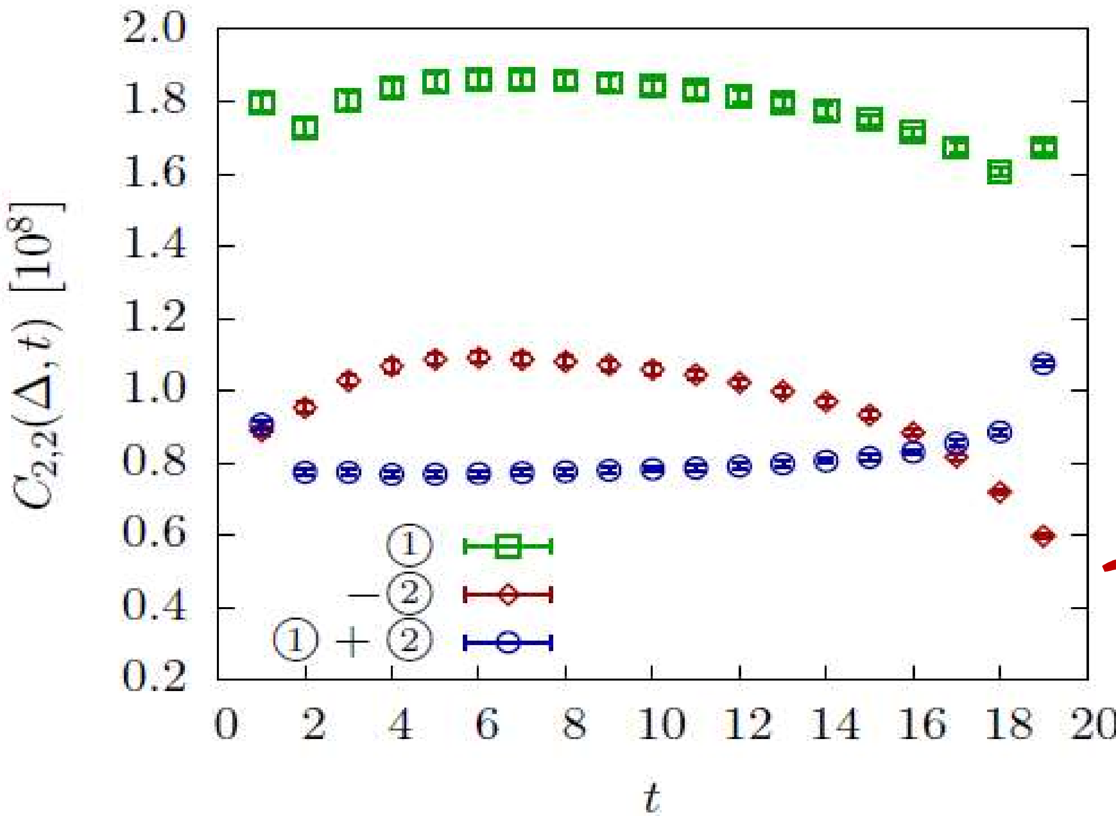
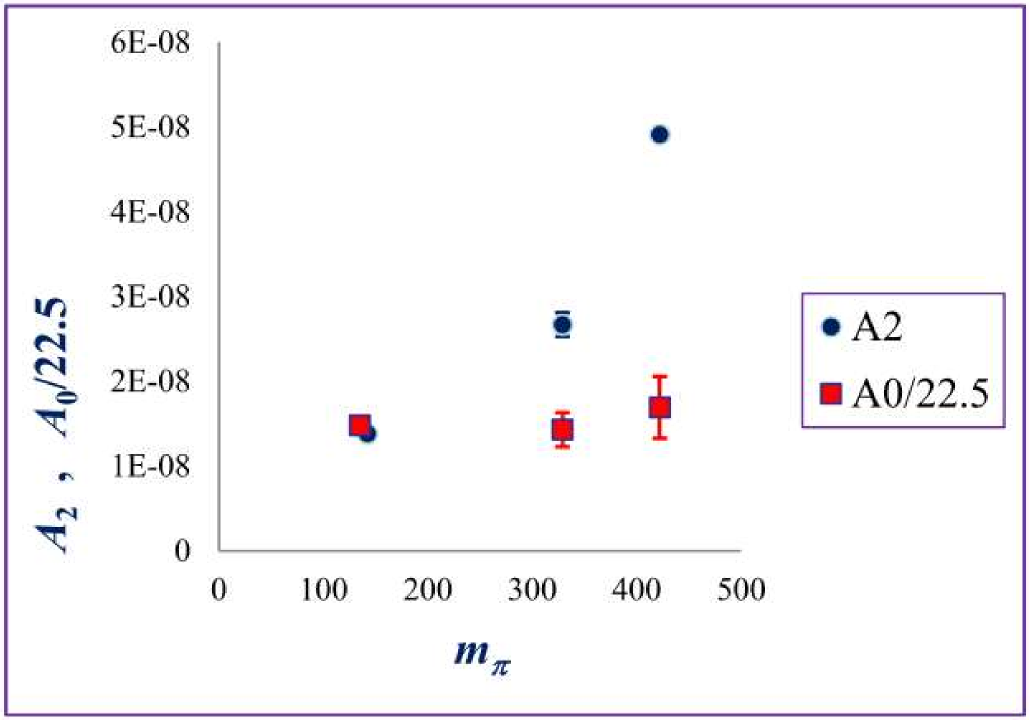


FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.

For heavier π, $m_\pi \sim 330$ MeV less cancellation bet. N^2 & N Large N begins to improve!

Because of mass dependent cancellation
ReA_2 changes with m_π dramatically.
ReA_0 is mild

Compare A_2 and $A_0/22.5$



NHCC
KITP
Aug '15

Net effect in A2

- **This large cancellation between N^2 and N [N=3,for QCD] leads to a reduction in ReA2 compared to “naïve expectations” by a factor of about 4 to 5 in the original effect of around 22.5**
- **Then there is a factor of 2 to 3 from renorm…=> bringing the total to [8 to 15] of the needed 22.5**
- **Still needed is factor of ~ [1.5 to 2.8] …can of course come from ReA0 over “naïve expectations”**

More on A0

$Q_2 \equiv$

- **Another important fact about Re A0 is that at a scale of ~1.3 GeV or more,**

the contribution from penguin operators, Q3,Q4,Q5,Q6,is negligibly small.

- **Indeed, ~85% of ReA0 originates at these scales from Q2 which is just the original**
- **Weak interaction 4-q operator: [s-bar gamma_muL u]X[d-bar gamma_uL u],**

which originates from integrating out the W-boson.

- **The essential moral is that if you take the original weak interaction 4q operraor**

and non-pertubatively compute its matrix element between K to pi pi in the I=0 channel then it accounts for most (~85%) of Re A0…..

- **Lastly, but equally importantly, it should be stressed that the SVZ-penguin operator Q6 is in fact the dominant contributor to Im A0**.

Im A0 & ε’

Parameter	Value	
	2-state fit	3-state fit
Fit range	6-15	4-15
$A^0_{\pi\pi(111)}$	0.3682(31)	0.3718(22)
$A^0_{\pi\pi(311)}$	0.00380(32)	0.00333(27)
A^0_{σ}	−0.0004309(41)	−0.0004318(42)
E_0	0.3479(11)	0.35030(70)
$A^1_{\pi\pi(111)}$	0.1712(91)	0.1748(67)
$A^1_{\pi\pi(311)}$	−0.0513(27)	−0.0528(30)
A^1_{σ}	0.000314(17)	0.000358(13)
E_1	0.568(13)	0.5879(65)
$A^2_{\pi\pi(111)}$	—	0.116(29)
$A^2_{\pi\pi(311)}$	—	0.063(10)
A^2_{σ}	—	0.000377(94)
E_2	—	0.94(10)
p-value	0.314	0.092

TABLE III: Fit parameters in lattice units and the p-values for multi-operator fits to the $I = 0$ $\pi\pi$ two-point functions. Here E_i are the energies of the states and A^i_α represents the matrix element of the operator α between the state i and the vacuum, given in units of $\sqrt{1\times 10^{13}}$. The second column gives the parameters for our primary fit which uses two-states and three operators. The third column shows a fit with the same three operators and one additional state that is used to probe the systematic effects of this third state on the $K \to \pi\pi$ matrix element fits.

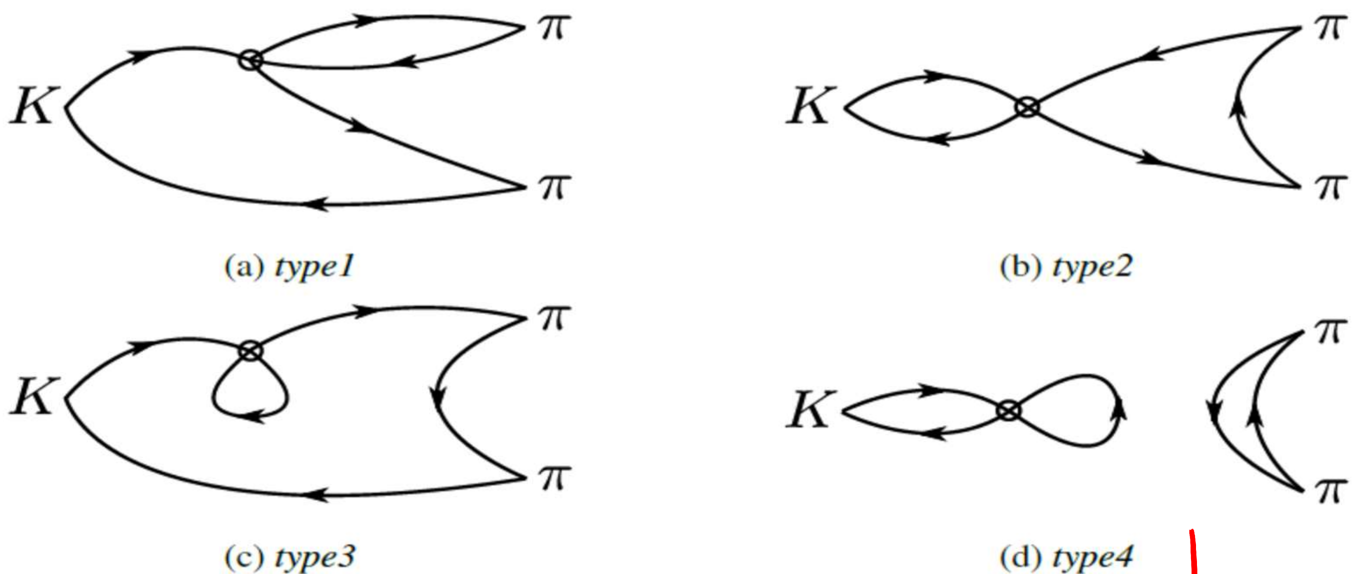
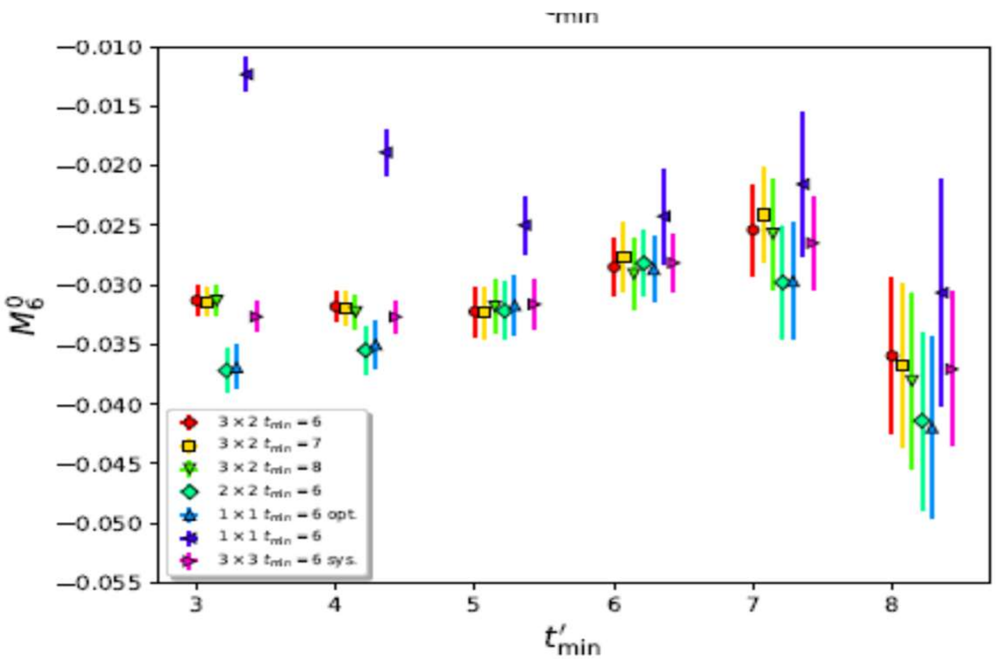


FIG. 2: The four classes of $K \to \pi\pi$ Wick contractions.

"DISCONNECTED"

Very difficult



−0.010
−0.015
−0.020
−0.025
−0.030
−0.035
−0.040
−0.045
−0.050
−0.055
M_6^0
3
4
5
6
7
8
t'_{min}
$3\times2\ t_{\mathrm{min}}=6$
$3\times2\ t_{\mathrm{min}}=7$
$3\times2\ t_{\mathrm{min}}=8$
$2\times2\ t_{\mathrm{min}}=6$
$1\times1\ t_{\mathrm{min}}=6$ opt.
$1\times1\ t_{\mathrm{min}}=6$
$3\times3\ t_{\mathrm{min}}=6$ sys.

7-ops. 10-ops.

i	SMOM($\not{q},\not{q}$) (GeV^3)	SMOM(γ^μ,γ^μ) (GeV^3)	$\overline{\text{MS}}$ via SMOM($\not{q},\not{q}$) (GeV^3)	$\overline{\text{MS}}$ via SMOM(γ^μ,γ^μ) (GeV^3)
1	0.060(39)	0.059(38)	−0.107(22)	−0.093(18)
2	−0.125(19)	−0.106(16)	0.147(15)	0.143(14)
3	0.142(17)	0.128(14)	−0.086(61)	−0.053(44)
4	-	-	0.185(53)	0.200(40)
5	−0.351(62)	−0.313(48)	−0.348(62)	−0.311(48)
6	−1.306(90)	−1.214(82)	−1.308(90)	−1.272(86)
7	0.775(23)	0.790(23)	0.769(23)	0.784(23)
8	3.312(63)	3.092(58)	3.389(64)	3.308(63)
9	-	-	−0.117(20)	−0.114(19)
10	-	-	0.137(22)	0.123(19)

TABLE XIV: Physical, infinite-volume matrix elements in the SMOM($\not{q},\not{q}$) and SMOM(γ^μ,γ^μ) schemes at $\mu = 4.006$ GeV given in the 7-operator chiral basis, as well as those converted perturbatively into the $\overline{\text{MS}}$ scheme at the same scale in the 10-operator basis. The errors are statistical only.

2 schemes

Q_i ReAo ImAo

59

	$\mathrm{Re}(A_0)$		$\mathrm{Im}(A_0)$	
i	$(\not q, \not q)$ $(\times 10^{-7}$ GeV)	(γ^μ, γ^μ) $(\times 10^{-7}$ GeV)	$(\not q, \not q)$ $(\times 10^{-11}$ GeV)	(γ^μ, γ^μ) $(\times 10^{-11}$ GeV)
1	0.383(77)	0.335(64)	0	0
2	2.89(30)	2.81(28)	0	0
3	0.0081(58)	0.0050(42)	0.20(14)	0.12(10)
4	0.081(23)	0.088(17)	1.24(35)	1.34(27)
5	0.0380(68)	0.0339(53)	0.552(99)	0.492(77)
6	−0.410(28)	−0.398(27)	−8.78(60)	−8.54(57)
7	0.001863(56)	0.001900(56)	0.02491(75)	0.02540(75)
8	−0.00726(14)	−0.00708(13)	−0.2111(40)	−0.2060(39)
9	$-8.7(1.5) \times 10^{-5}$	$-8.5(1.4) \times 10^{-5}$	−0.133(22)	−0.128(21)
10	$2.37(38) \times 10^{-4}$	$2.13(32) \times 10^{-4}$	−0.0304(49)	−0.0273(41)
Total	2.99(32)	2.86(31)	−7.15(66)	−6.93(64)

TABLE XVIII: The contributions of each of the ten four-quark operators to $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$ for the two different RI-SMOM intermediate schemes. The scheme and units are listed in the column headers. The errors are statistical, only.

Systematic errors

Error source	Value
Excited state	-
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

TABLE XXV: Relative systematic errors on the infinite-volume matrix elements of $\overline{\mathrm{MS}}$-renormalized four-quark operators Q'_j.

Re A_0

~20%

Error source	Value	
	Re(A_0)	Im(A_0)
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

Im A_0

~21%

TABLE XXVI: Relative systematic errors on Re(A_0) and Im(A_0).

Quantity	Value
$\mathrm{Re}(A_0)$	$2.99(0.32)(0.59)\times10^{-7}$ GeV
$\mathrm{Im}(A_0)$	$-6.98(0.62)(1.44)\times10^{-11}$ GeV
$\mathrm{Re}(A_0)/\mathrm{Re}(A_2)$	19.9(2.3)(4.4)
$\mathrm{Re}(\varepsilon'/\varepsilon)$	0.00217(26)(62)(50)

due IB
see foll
pages

TABLE I: A summary of the primary results of this work. The values in parentheses give the statistical and systematic errors, respectively. For the last entry the systematic error associated with electromagnetism and isospin breaking is listed separately as a third error contribution.

IB+EM effects.....not yet from lattice

We use

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

→ isospin sym ~~formula~~.

$\sim (17 \pm 9.1) \times 10^{-2}$

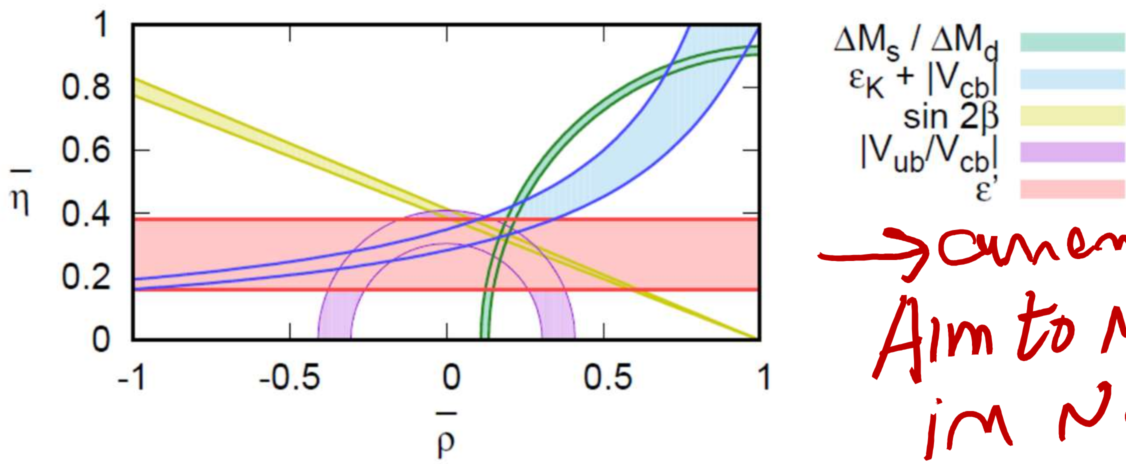
IB + EM eff

$$\Rightarrow \quad \frac{\varepsilon'}{\varepsilon} = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2^{\mathrm{emp}})}{\mathrm{Re}(A_2^{(0)})} - \frac{\mathrm{Im}(A_0^{(0)})}{\mathrm{Re}(A_0^{(0)})}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\right].$$

See Cirigliano et al 1911.01359

THIS IS NOT our work

WE CHOOSE To include THIS in our syst error



→ current systematic ~35%
Aim to reduce this
in ~2 yrs to ≈15%

FIG. 12: The horizontal-band constraint on the CKM matrix unitarity triangle in the $\bar{\rho} - \bar{\eta}$ plane obtained from our calculation of ε', along with constraints obtained from other inputs [6, 70, 71]. The error bands represent the statistical and systematic errors combined in quadrature. Note that the band labeled ε' is historically (e.g. in Ref. [72]) labeled as ε'/ε, where ε is taken from experiment.

Summary + Outlook 1 of 2 pages

- **After decades of effort, overcoming major hurdles, using DWQ with essentially continuum-like fermions along with improved renormalization methodology, cutting edge statistical analysis and algorithmic advances RBC-UKQCD is presenting an updated result on SM-eps' ~ 21.7(26)(62)(50)X10^{-4} which is in good agreement with the measured value 16.6(2.3)X10^{-4}**
- **Bearing in mind that this is an extremely treacherous calculation loaded with numerous avenues of errors and oversights, an independent calculation has been in process for about ~3 years within RBC-UKQCD. This effort is led by Tom Blum with (g.s.) Dan Hoying U Conn-BNL, Taku Izubuchi et al. This path uses PBC unlike the currently finished result which used GPBC…we hope to have 1st results from PBC in ~ 2 years.**
- **Also GPBC effort will be continued at other lattice spacing(s)**

Summary + Outlook

- **Lattice efforts to incorporate IB + EM effects are being studied but have some ways to go before they can tackle K=> pi pi and eps’**
- **With physical pions, kaons and such first glance at lattice ChPT is quite encouraging, see RBC-UKQCD, David Murphy et al 2015 and DM, PhD thesis, Columbia Univ**
- **This begs the question that much simpler path could now be used via BDSPW [LO ChPT] and/or L+S [NLOChPT] to address eps’…This could be tens of times simpler though at some cost in accuracy……….all this needs to be studied…Mattia Bruno, Christoph Lehner + AS et al**
- **Hope to have an improved result on eps’ with O(15%) errors in ~2 years**

EXTRAS

RBC
RIKEN-BNL-Research Center
B=BNL-HET
C=Columbia

UKQCD
Subgroup that
of UKQCD
that uses
DWQ

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

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Norman Christ
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Christopher Kelly
Bob Mawhinney
Masaaki Tomii
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Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
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Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

UAM Madrid

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

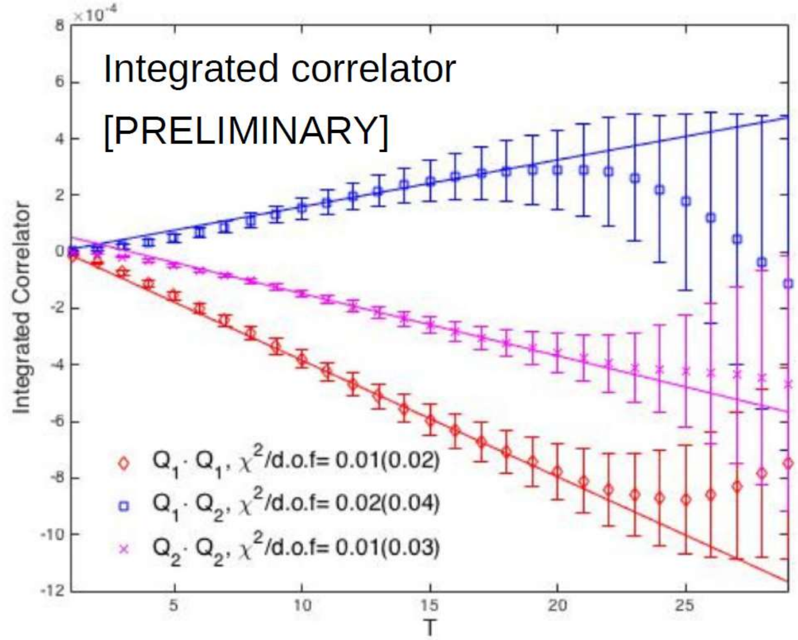
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Southampton
Edinburgh

- Total error on Re(ε'/ε) is ~3x the experimental error
- Find reasonable (2.1σ) consistency with Standard Model
- "This is now a quantity accessible to lattice QCD"!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.

→ & use much larger stats

See update by Bigeng Wang @ Lattice 2018



CKL
CKM'18

$$\Delta M_K = 7.0(1.7)_{\mathrm{stat}} \times 10^{-12}\ \mathrm{MeV}$$ (syst?)

- η-state gives significant stat. err. contrib as divergent op. subtraction coeff noisy
- Charm discretization error estimate from naive $(m_c a)^2 \sim 25\%$
- However only 3-10% observed errors in f_D and dispersion relation of η_c
- Aim to continue measurements on ORNL Summit computer and ultimately a second lattice spacing to understand disc. effects.

1. Expect Delta mK with total error < ~ 25% in < 1 yr
2. 2. Calculation with next gen. supercomputer being started now , expect improved answer with error ~15-20% in ~1-2 years. [contrast with pert theory @~40%]

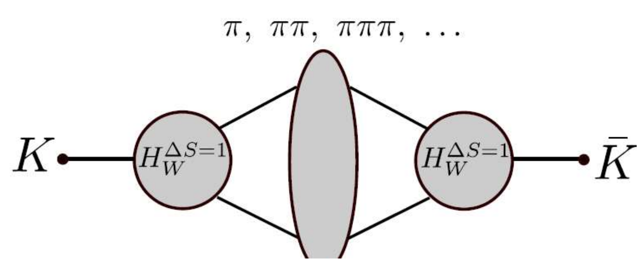
CKeCKM'18

- Neutral kaon mixing induced by 2nd order weak processes gives rise to mass difference between K_L and K_S

B. Wang e
LAT'18

$$\Delta M_K = 2\sum_n \frac{\langle \overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K - E_n}$$

- FCNC → highly suppressed in SM due to GIM mechanism: $\Delta m_K = 3.483(6)\text{x}10^{-12}$ MeV small and highly sensitive to new BSM FCNC.
- PT calc using weak EFT with ΔS=2 eff. Hamiltonian (charm integrated out) dominated by p~m_c: poor PT convergence at charm scale → ~36% PT sys error.
- PT calc neglects long-distance effects arising when 2 weak operators separated by distance ~$1/\Lambda_{QCD}$.
- Use lattice to evaluate matrix element of product of $H_W^{\Delta S=1,\,\text{eff}}$ directly:

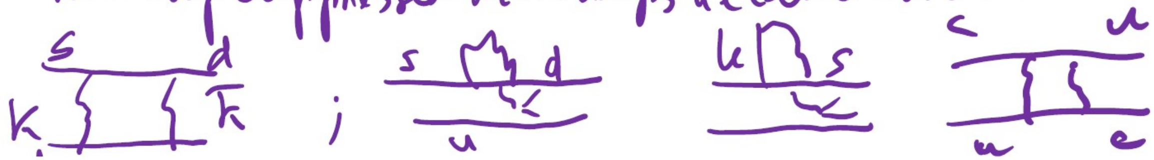


Remarks

- **In the past ~6 years, RBC-UKQCD developed methods for extended applications of Lellouch-Lusher method to 2 insertions of the weak operator for tackling non-local matrix elements [NLME]**
- **ALL loop suppressed transitions in the SM receive some**

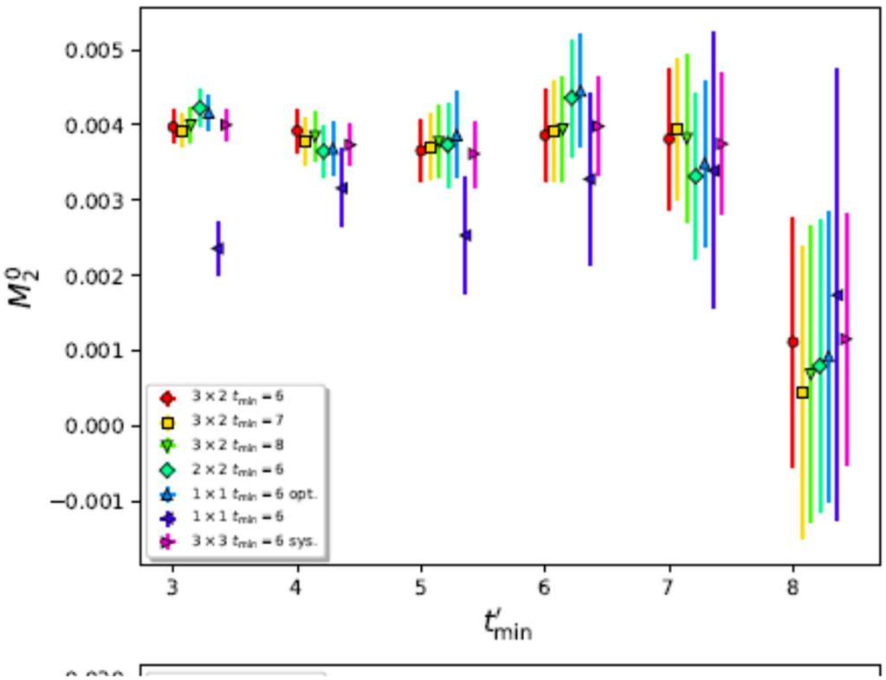
no

ALL Loop suppressed weak amps receive some LD



- **ΔmK extremely sensitive to BSM ‘cause as a rule they contain [unlike SM] non-$(V\text{-}A)^2$; see Beall, Bander, AS PRL’82 => 1st target of our effort for NLME has been therefore ΔmK**
- **Pert. Theory @ NNLO [see Brod + Gorbahn, PRL 2012] estimates ~40% LD contamination; not reliable as NLO estimates [Herrlich + Nierste] were about the same…may well be indicating poor convergence of pert. Theory.**

Non-local ME [1^{st} ex. Kl-Ks mass diff]



0.005
0.004
0.003
0.002
0.001
0.000
−0.001
M_2^0
$t'_{\rm min}$
3
4
5
6
7
8
3×2 $t_{\rm min}=6$
3×2 $t_{\rm min}=7$
3×2 $t_{\rm min}=8$
2×2 $t_{\rm min}=6$
1×1 $t_{\rm min}=6$ opt.
1×1 $t_{\rm min}=6$
3×3 $t_{\rm min}=6$ sys.

Quantity	Value
$\mathrm{Re}(A_0)$	$2.99(0.32)(0.59)\times 10^{-7}$ GeV
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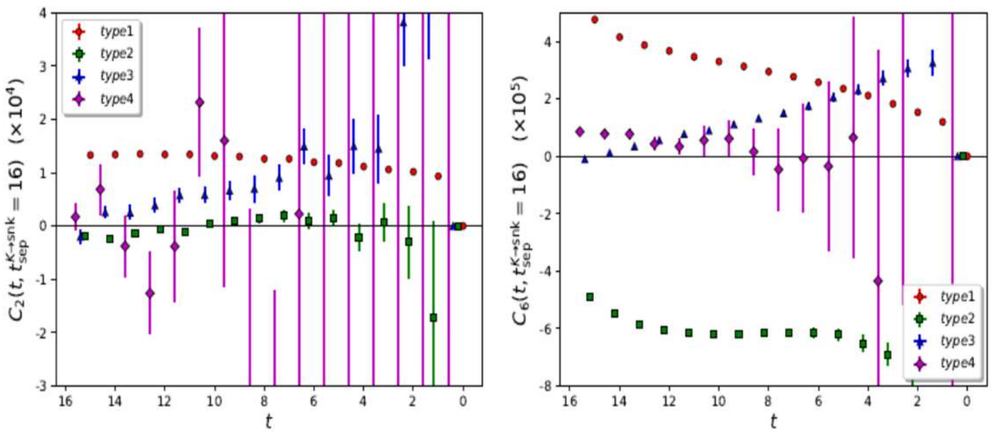

FIG. 3: The contributions of the four Wick contraction topologies *type1-type4* to the C_2 (left) and C_6 (right) three-point functions with the $\pi\pi(111)$ sink operator, plotted as a function of the time separation between the kaon and the four-quark operator, t, at fixed $t_{\mathrm{sep}}^{K\to\mathrm{snk}} = 16$. For clarity we plot with an inverted x-axis such that the $\pi\pi$ sink operator is on the left-hand side. These correlation functions include the subtraction of the pseudoscalar operator.

Mass depends of ReA2, A0

PRL 2013

	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8} GeV]	ReA_0 [10^{-8} GeV]	$\frac{\text{Re}A_0}{\text{Re}A_2}$	notes
16^3 Iwasaki	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold calculation
24^3 Iwasaki	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold calculation
IDSDR	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical kinematics
Experiment	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

Due to the cancellation, 3/2 amplitude decreases significantly as the pion mass is lowered towards its physical value