Master Integrals for the mixed EW-QCD corrections to the Drell-Yan production of a massive lepton pair

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In collaboration with

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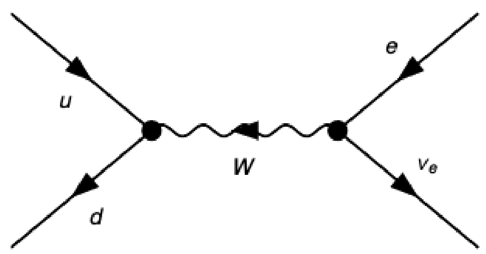
Outline

- Motivation
- Mixed QCD-EW corrections
- Method
- Outlook

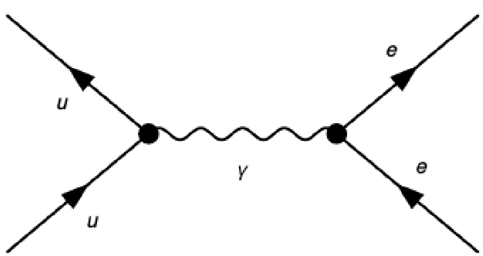
Motivation

W and Z production at the LHC via Drell-Yan Processes

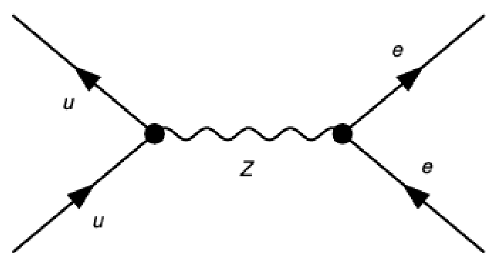


T1 C1 N1

Charged Current



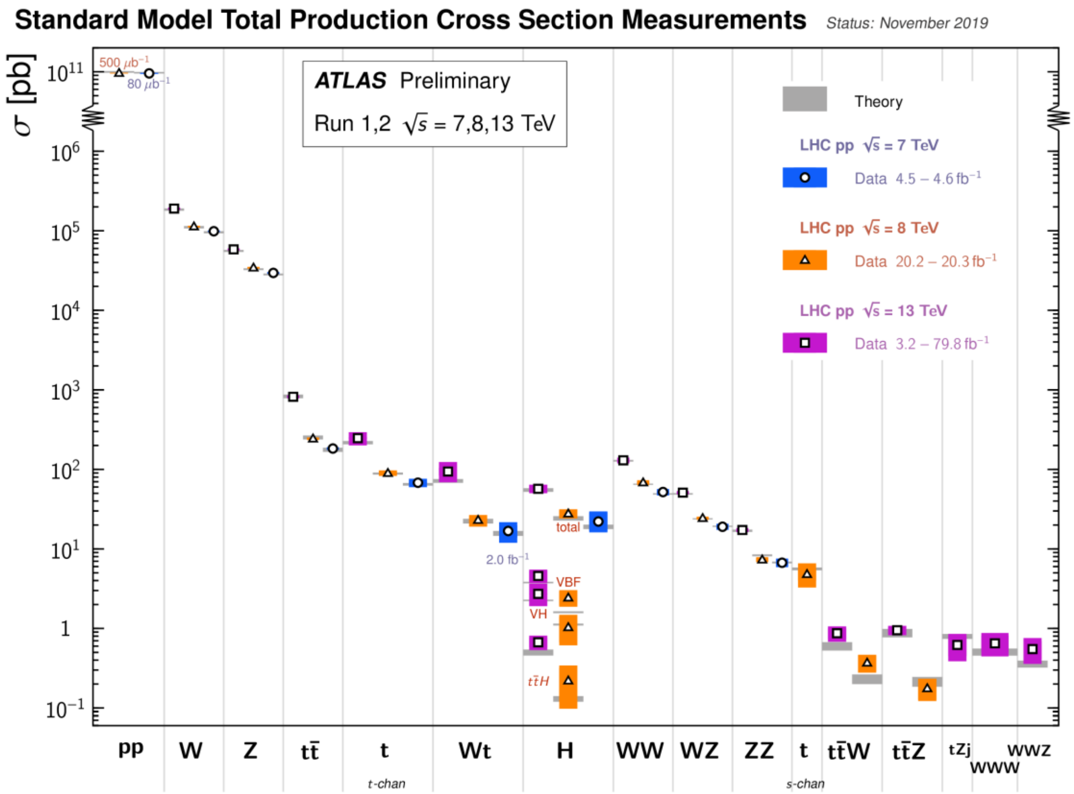
T1 C1 N1



T1 C2 N2

Neutral Current

W and Z production at the LHC



- Big cross section and clean experimental signature.
- W boson mass and $sin^2\theta^l_{eff}$ determination.
- New physics search, eg. W' and Z' resonances.
- Constraining PDF, detector calibration and determination of collider luminosity.

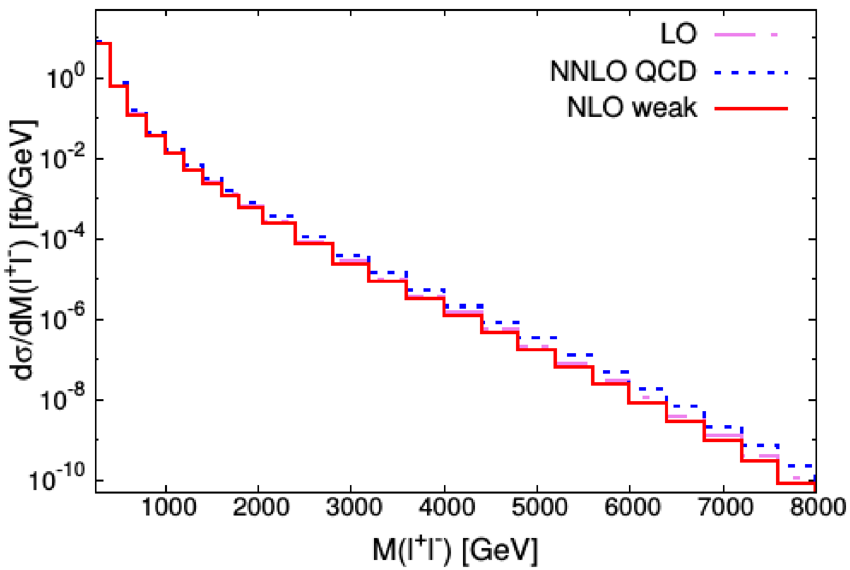
Mixed QCD-EW correction enhancement at higher energy

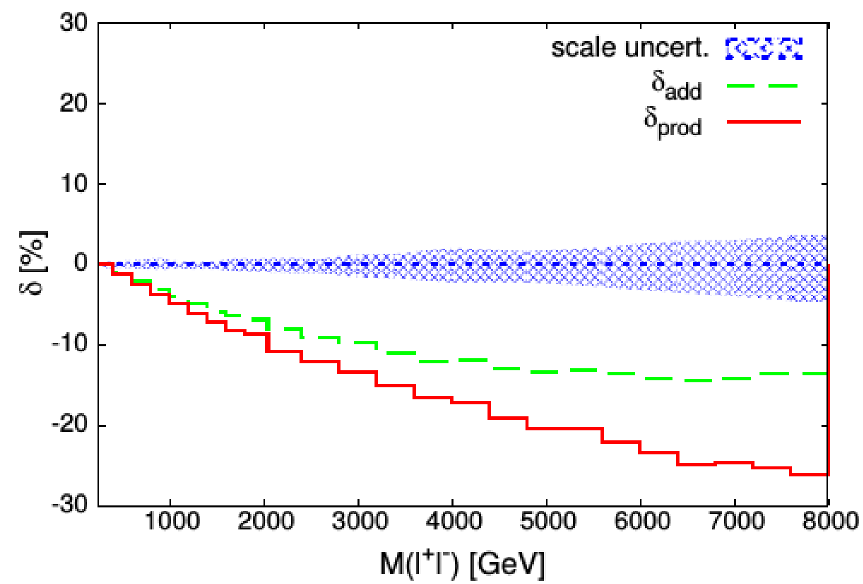
$$\sigma_{QCD+\mathrm{wk}} = \sigma_{(N)NLO\,QCD} + \sigma_{\mathrm{wk}}$$

$$\sigma_{QCD\times\mathrm{wk}} = \sigma_{(N)NLO\,QCD}\left(1 + \frac{\sigma_{\mathrm{wk}}}{\sigma_{LO}}\right)$$

$$\delta_{\mathrm{add}} = \frac{\sigma_{QCD+\mathrm{wk}} - \sigma_{(N)NLO\,QCD}}{\sigma_{(N)NLO\,QCD}} = \frac{\sigma_{\mathrm{wk}}}{\sigma_{(N)NLO\,QCD}}$$

$$\delta_{\mathrm{prod}} = \frac{\sigma_{QCD\times\mathrm{wk}} - \sigma_{(N)NLO\,QCD}}{\sigma_{(N)NLO\,QCD}} = \frac{\sigma_{\mathrm{wk}}}{\sigma_{LO}}$$



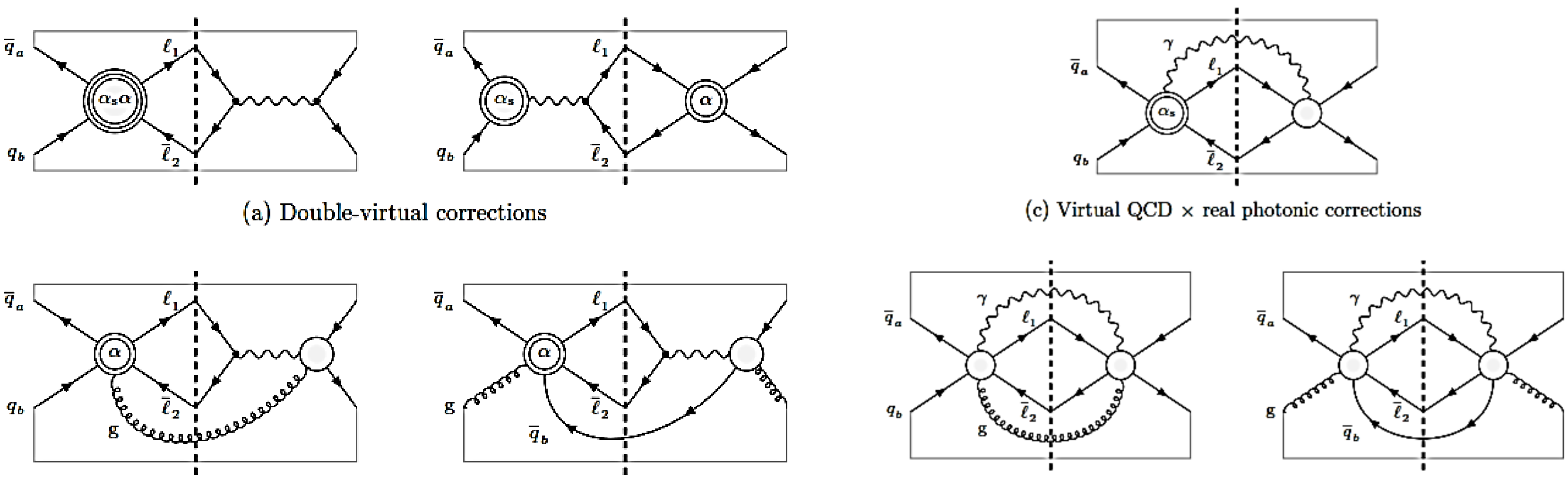


[Campbell, Wackeroth, Zhou ,2016]

Mixed QCD-EW Corrections

Structure of the fixed order prediction

$$d\sigma = d\sigma_{LO} + \alpha d\sigma_{\alpha} + \alpha^2 d\sigma_{\alpha^2} + \ldots + \alpha_s d\sigma_{\alpha_s} + \alpha_s^2 d\sigma_{\alpha_s^2} + \ldots + \alpha\alpha_s d\sigma_{\alpha\alpha_s} + \alpha\alpha_s^2 d\sigma_{\alpha\alpha_s^2} + \ldots$$



(a) Double-virtual corrections

(c) Virtual QCD × real photonic corrections

(b) Real QCD × virtual EW corrections

(d) Double-real corrections

[Alexander Huss '14]

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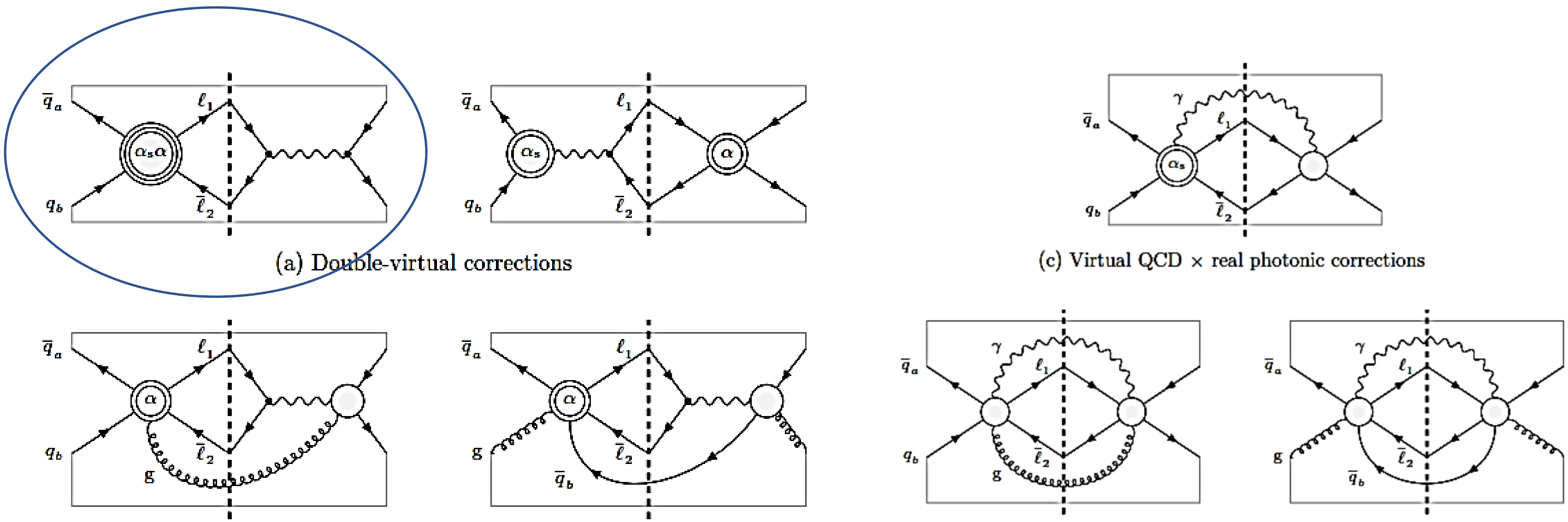

(a) Double-virtual corrections

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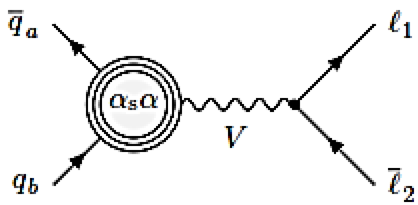
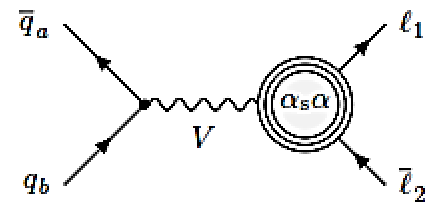
[Alexander Huss '14]

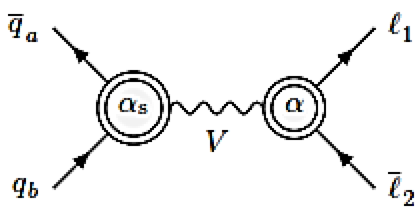
Born interfered double virtual corrections at $O(\alpha\alpha_s)$



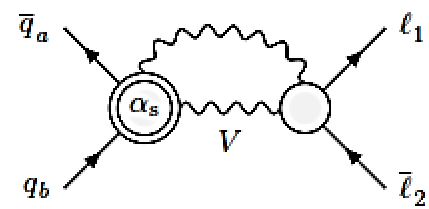
(a) Factorizable “initial–initial” corrections



(b) Factorizable “final–final” corrections



(c) Factorizable “initial–final” corrections



(d) Non-factorizable corrections

[Alexander Huss ‘14]

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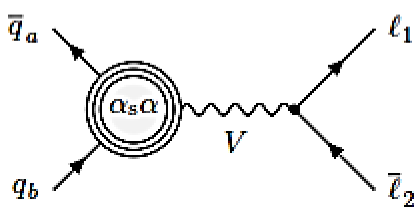

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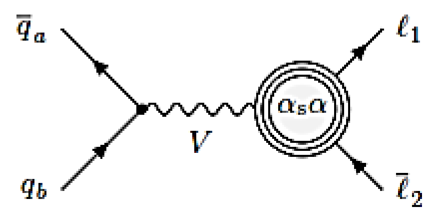

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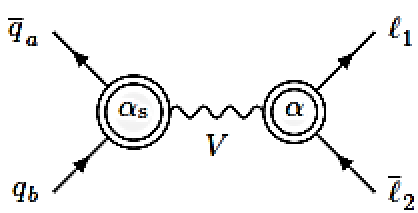

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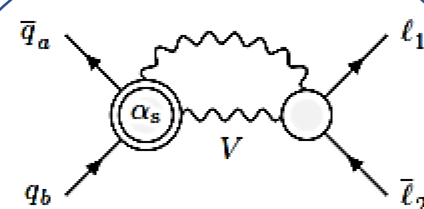
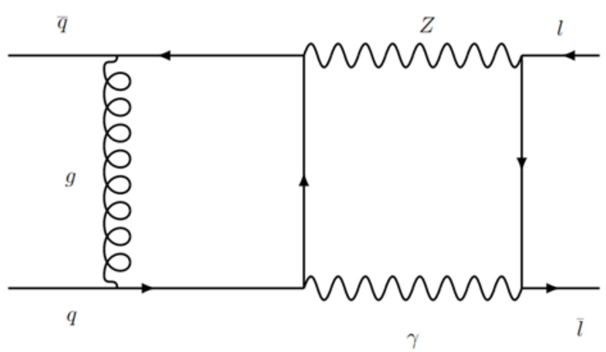
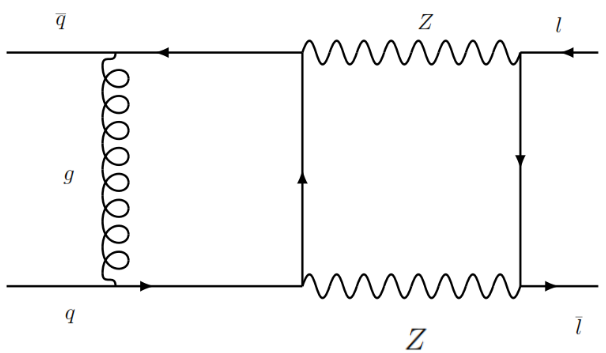


(d) Non-factorizable corrections

[Alexander Huss '14]

Example Feynman diagrams for non factorizable double virtual correction at $\boldsymbol{O(\alpha\alpha_s)}$ to DY process

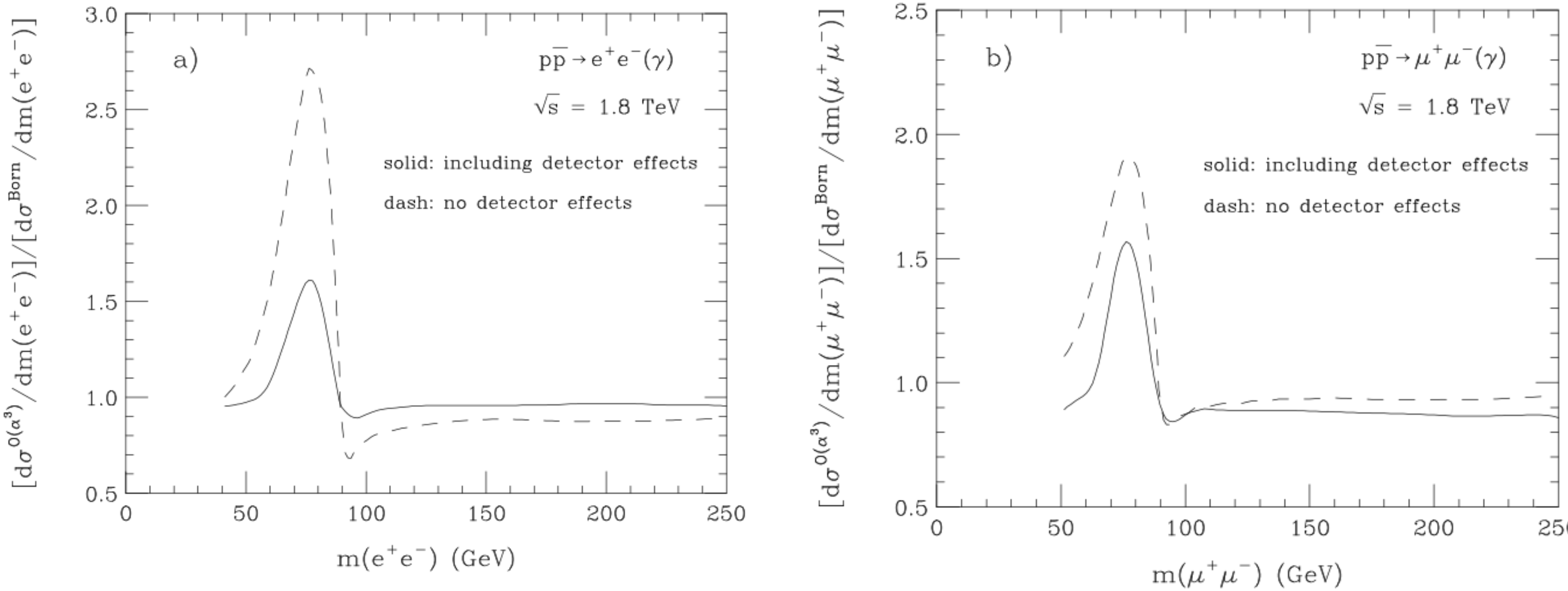




-> All the ingredients, i.e. Master Integrals, for the process $q\bar{q} \to l^+l^-$and $q\bar{q}' \to l\bar{\nu}$ are calculated in the massless final state approximation.

[Bonciani, Di Vita, Mastrolia, Schubert, 2016]
[Manteuffel, Schabinger, 2017]
[Heller, Manteuffel, Schabinger, 2019]

Mixed QCD-EW corrections



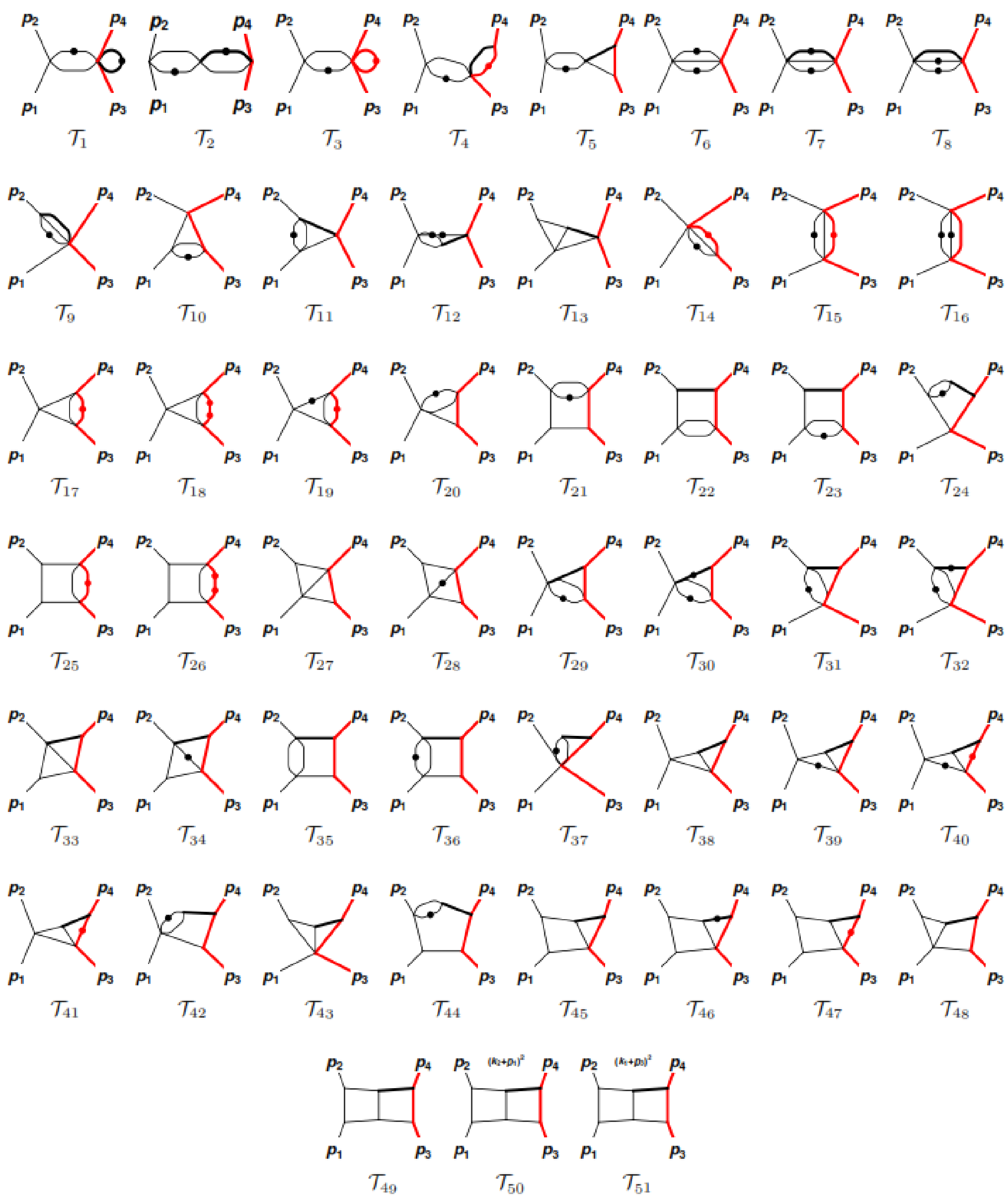
detector effects require control over the inclusiveness

-> It is necessary to keep lepton mass up to logarithmic term

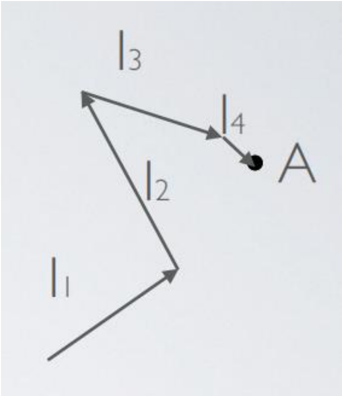
[Baur, Keller, Sakumuto, 1997]

Mixed QCD-EW corrections

MIs for the NC DY process in case of single massive gauge boson exchange



Method



-> Amplitude given by Feynman diagrams

$$A = \sum_i a_i I_i$$

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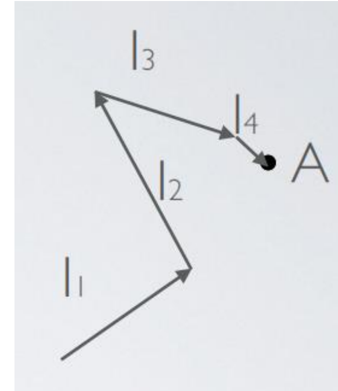
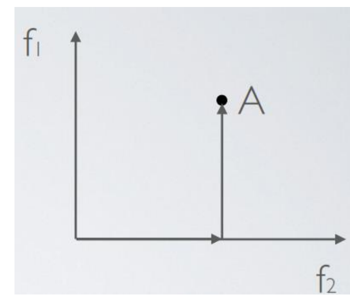


$$A = \sum_i a_i I_i$$

-> Project onto basis using Integration by Parts identities

(Tkachov; Chetyrkin, Tkachov)



$$A = \sum_i c_i f_i$$

Implemented in public codes

REDUZE	(Studerus, von Manteuffel)
Fire	(Smirnov)
Air	(Anastasiou, Lazopolus)
Kira	(Maierhoefer, Usovitsch, Uwer)

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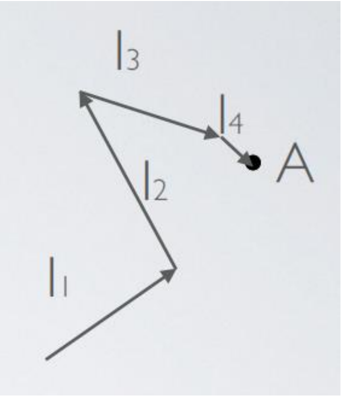

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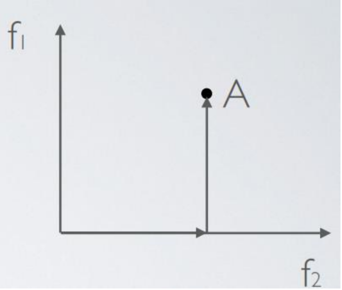

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-> Calculate basis elements via differential equations

Differential Equation

-> Kinematic derivative in space spanned by MIs

$$\partial_x \bar{f} = A_x \bar{f}$$

Differential Equation

-> Kinematic derivative in space spanned by MIs

$$\partial_x \bar{f} = A_x \bar{f}$$

-> Conjecture: There is a basis such that:

$$\partial_x \bar{g} = \epsilon \tilde{A}_x \bar{g}$$ (Henn)

->There are many strategies to get the epsilon factorized form

- Magnus Theorem (Ageri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, US)
- Unit leading Singularity (Henn)
- Reduction to fuchsian form and Eigenvalue normalization (Lee, Smirnov)
- Factorization of Picard-Fuchs operator (Adams, Chaubey, Weinzierl)

Solving Canonical Differential Equation

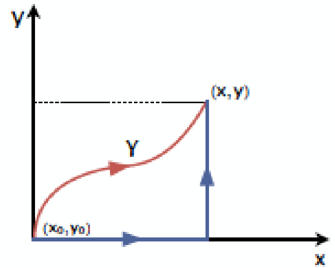
Canonical form

$$\partial_x \vec{g}(x,\epsilon) = \epsilon \tilde{A}_x(x) \vec{g}(x,\epsilon) \qquad d\vec{g}(x,\epsilon) = \epsilon \sum_i M_i dlog(\eta_i) \vec{g}(x,\epsilon)$$

- Kinematic dependence encoded in η
- η's form the alphabet

Solution given by

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_\gamma dA \dots dA \right] \vec{g}(x_0,\epsilon)$$



Algebraic ηs : Chen Iterated Integrals (Chen)

$$C(\vec{\eta_n}; x) = \int_\gamma dlog(\eta_1) \dots dlog(\eta_n)$$

Rational ηs : Generalized Polylogarithms (Goncharov)

$$G(\vec{0}_n; x) = \frac{1}{n!} Log(x)^n$$

$$G(\vec{w}_n; x) = \int_0^x \frac{dt}{t - w_1} G(\vec{w}_{n-1}; t)$$

Boundary Conditions

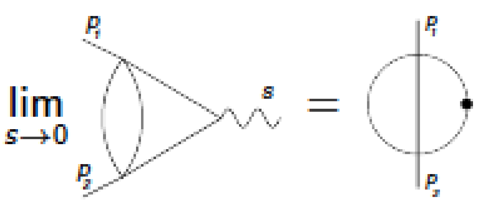
-> Solution given by

$$\vec{g}(x,\epsilon) = \left[1 + \sum_{i=1}^{\infty} \int_{\gamma} dA \dots dA\right] \vec{g}(x_0,\epsilon)$$

-> Two general ways to fix the boundary

Known Limit

- Taking the limit x to x0
- Fix boundary constant by matching the solution to known function



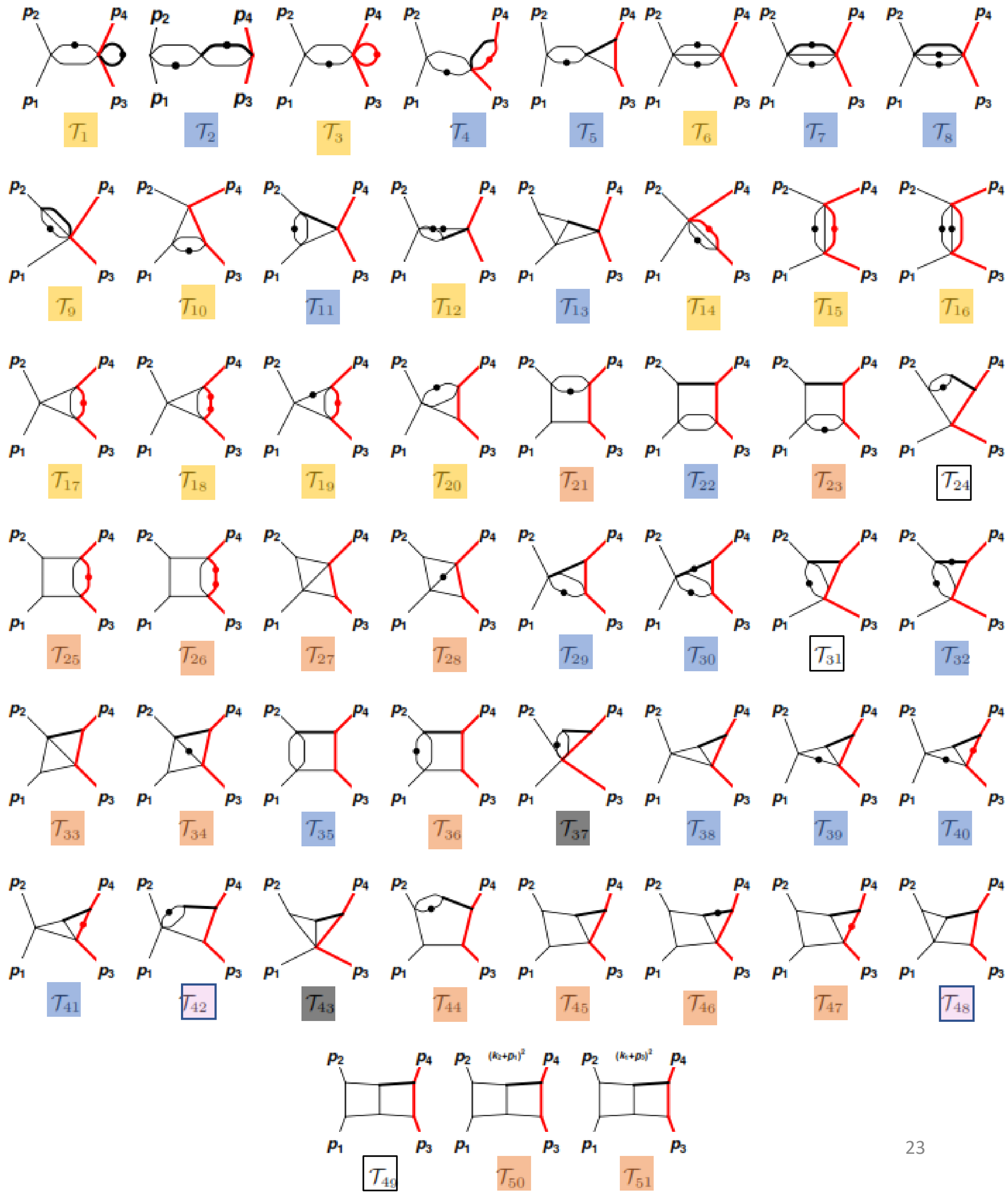
Pseudo-thresholds

- Solution has unphysical divergences
- Demanding absence of unphysical divergences gives relations between boundary constant
- Leftover constants must be provided

Method

Boundary Fixing:

- input
- $m_l^2 \to 0$
- $s \to 0$
- $s \to -t$
- $t \to -m_z^2$
- $t \to -\frac{m_z^2 s}{m_z^2 - s}$



Outlook

- The newly calculated master integrals can be of interest to study the logarithmic structure of the lepton mass in the amplitude.
- With our available ingredients, the amplitude can be written in terms of GPLs which is easy to incorporate into Monte Carlo to simulate differential distributions.

- Thank You

EXTRA SLIDES

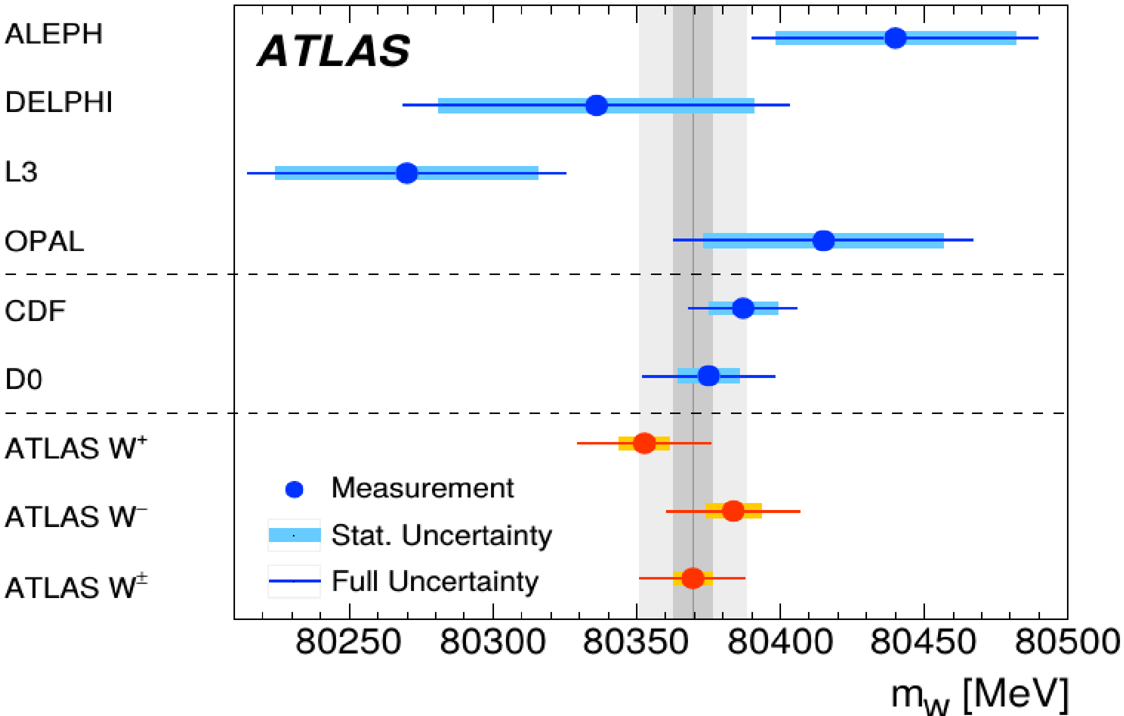
Available fixed order calculations for W and Z production

- NLO QED and QCD corrections. (known for a long time)
- NLO EW corrections. [Wackeroth et al ’97,’98,’04],[Baur et al ‘98,’02,’04],[Dittmaier et al ‘02,’10]
- NNLO QCD and QED corrections [Hamberg et al ’91],[Anastasiou et al ‘03,’04],[Melnikov,Petriello ‘06],[Stefano et al ’07]
- NNLO Mixed QCD-EW corrections to decay of W & Z boson. [Kuhn et al ‘96],[Kara ‘13]
- NNLO Mixed QCD-EW corrections to Z production form factors [Kotikov et al ‘08]
- NNLO QCD-QED virtual corrections to lepton pair production. [Kilgore et al ‘12]
- NNLO Mixed QCD-EW virtual corrections to DY production of W and Z bosons [Bonciani ‘11]
- Double real contribution to total cross section for on-shell single gauge boson production. [Bonciani et al 2016]
- NNLO Mixed QCD-EW corrections adopting pole approximation. [Dittmaier, Huss, Schwinn ‘14,’16]
- QCD×QED [O($\alpha\alpha_s$)] mixed and QED2 [O(α^2)] corrections to the production of an on-shell Z boson [Florian, Ignacio 2018]
- NNLO QCD×EW corrections to Z production in the $q\bar{q}$ channel [Bonciani et al ‘2019]
- To do : Complete NNLO Mixed QCD-EW corrections for W and Z production in a fully flexible Monte Carlo program.

ATLAS Report on W mass (January 2017)

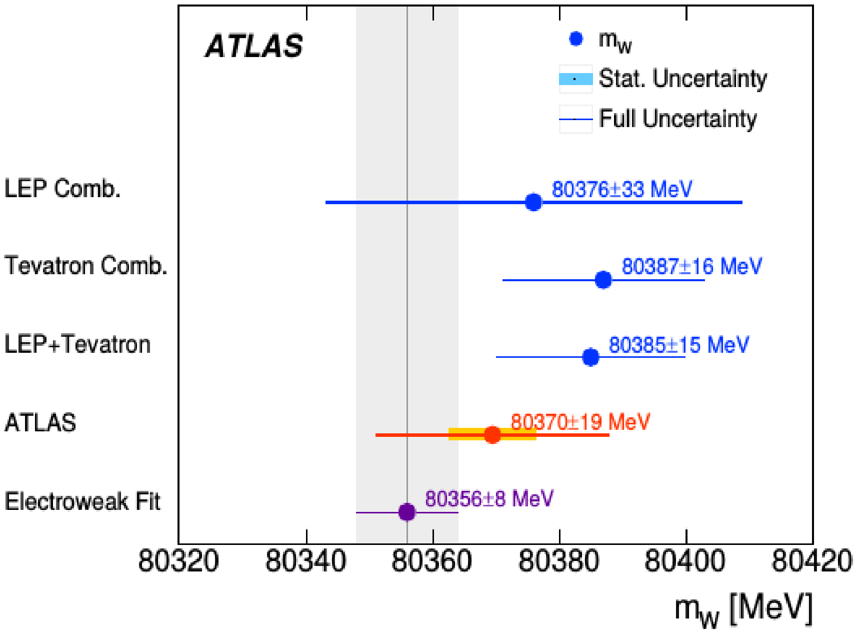
Currently at the LHC M_W is extracted from M_T and P_T of the $l\nu$ in W boson production.

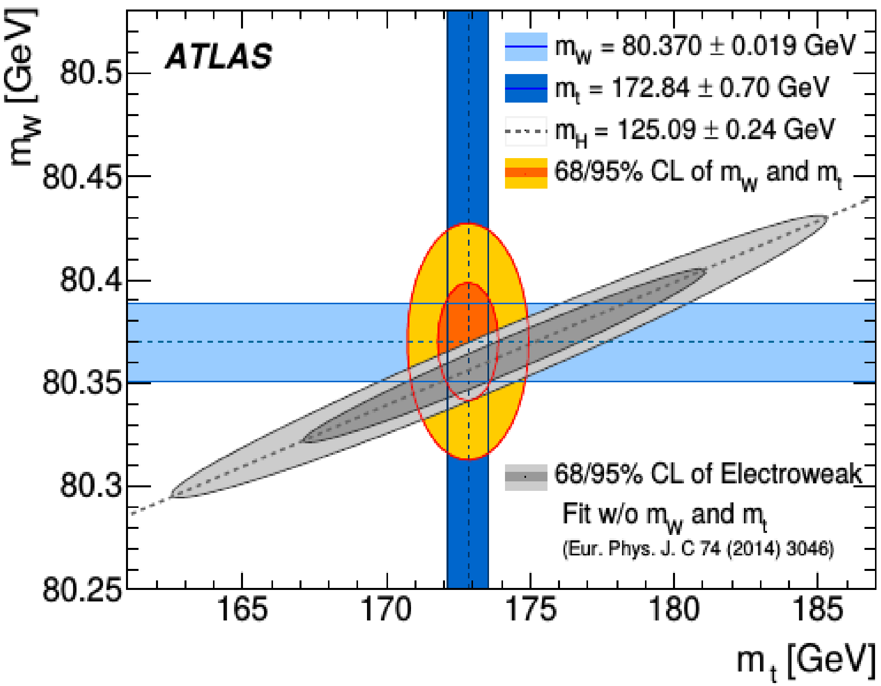
$$\begin{aligned} m_W &= 80370 \pm 7\,(\text{stat.}) \pm 11\,(\text{exp. syst.}) \pm 14\,(\text{mod. syst.})\ \text{MeV} \\ &= 80370 \pm 19\ \text{MeV}, \end{aligned}$$



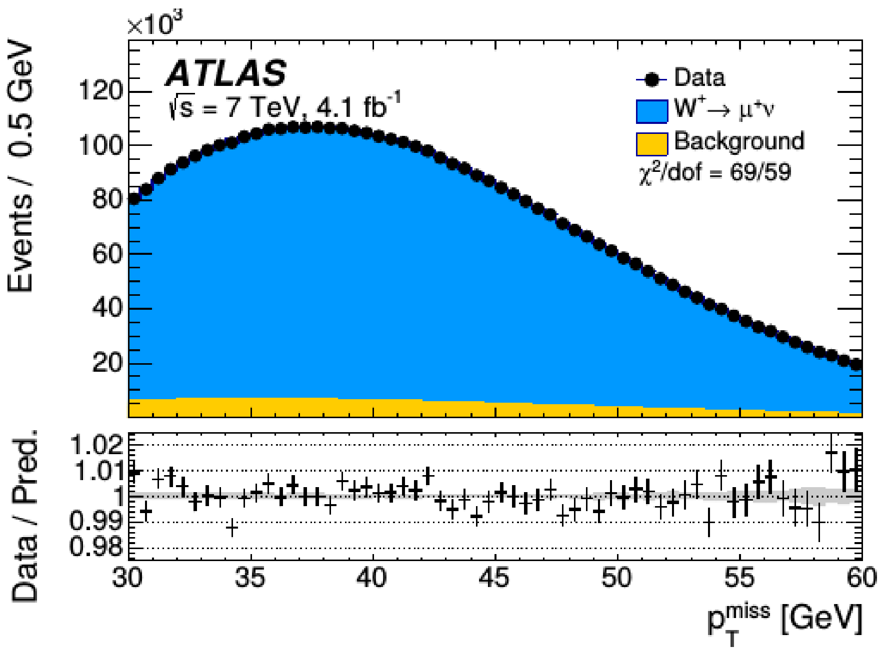
[ATLAS Collaboration. *arXiv preprint arXiv:1701.07240* (2017)]

ATLAS Report on W mass

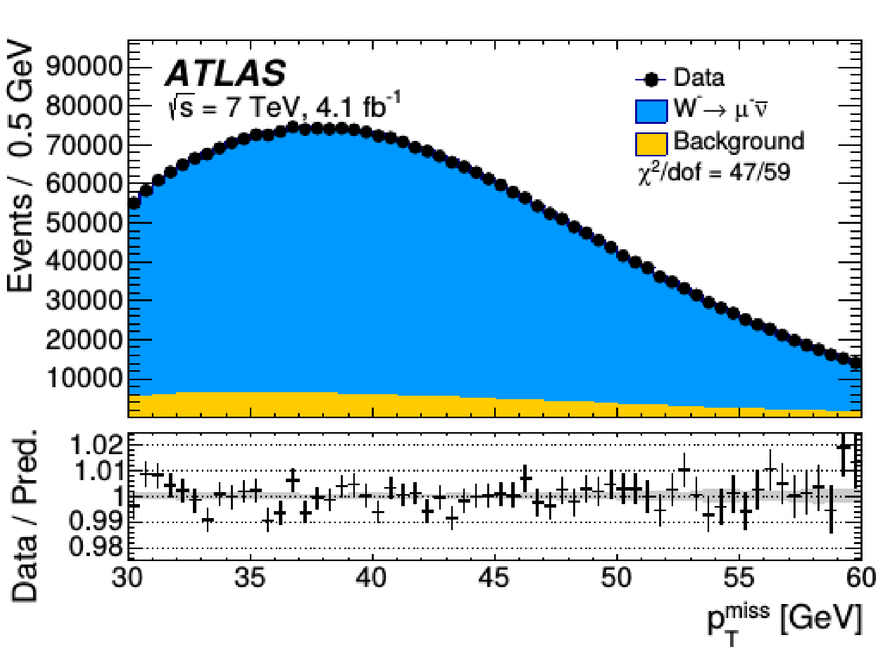




Missing transverse momenta distribution



(e)



(f)

EXTRA SLIDES

Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	yes	125.14 ± 0.24	93^{+25}_{-21}	93^{+24}_{-20}
M_W [GeV]	80.385 ± 0.015	–	80.364 ± 0.007	80.358 ± 0.008	80.358 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1880 ± 0.0021	91.200 ± 0.011	91.2000 ± 0.010
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4950 ± 0.0014	2.4946 ± 0.0016	2.4945 ± 0.0016
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.484 ± 0.015	41.475 ± 0.016	41.474 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.743 ± 0.017	20.722 ± 0.026	20.721 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01626 ± 0.0001	0.01625 ± 0.0001	0.01625 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0005	0.1472 ± 0.0005	0.1472 ± 0.0004
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	0.23150 ± 0.00006	0.23149 ± 0.00007	0.23150 ± 0.00005
A_c	0.670 ± 0.027	–	0.6680 ± 0.00022	0.6680 ± 0.00022	0.6680 ± 0.00016
A_b	0.923 ± 0.020	–	0.93463 ± 0.00004	0.93463 ± 0.00004	0.93463 ± 0.00003
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	0.0738 ± 0.0003	0.0738 ± 0.0003	0.0738 ± 0.0002
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1032 ± 0.0004	0.1034 ± 0.0004	0.1033 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	$0.17226^{+0.00009}_{-0.00008}$	0.17226 ± 0.00008	0.17226 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21578 ± 0.00011	0.21577 ± 0.00011	0.21577 ± 0.00004
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV]	173.34 ± 0.76	yes	$173.81 \pm 0.85^{(\bigtriangledown)}$	$177.0^{+2.3}_{-2.4}{}^{(\bigtriangledown)}$	177.0 ± 2.3
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{(\dagger\triangle)}$	2757 ± 10	yes	2756 ± 10	2723 ± 44	2722 ± 42
$\alpha_s(M_Z^2)$	–	yes	0.1196 ± 0.0030	0.1196 ± 0.0030	0.1196 ± 0.0028