Axion Coupling Quantization in the Presence of Mixing

Katherine Fraser

Department of Physics
Harvard University

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arXiv: 1910.11349, with M. Reece

kfraser@g.harvard.edu

Outline

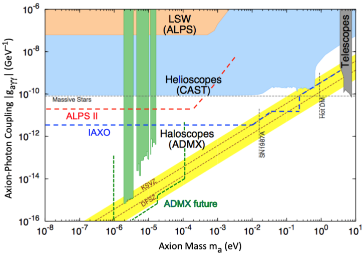
Central Question: Does a massless light axion EFT inherit periodicity when axions mix?

- Motivation - Why consider models of axion mixing?
- Review of θ angles: $S_\theta = \frac{\theta}{32\pi^2} \int d^4x F\tilde{F}$
  - θ is periodic
- Examples
  - Mixing through a periodic potential
  - Mixing with an axion eaten by a spin-1 field
  - Mixing with a non-compact scalar

Motivation

Why consider models of axion mixing?

- Hiearchies between couplings
  - Experimental probes of the QCD axion
- Large field ranges
  - large primordial gravitational wave signals
  - relaxion models
  - Difficult to achieve in string theory
- A combination of the two
  - Preheating
  - Suppress Axion DM abundance
  - Chromonatural inflation



From 1602.00039

Review: θ angles are periodic

- Periodic θ angles couple to gauge fields through $S_\theta = \frac{\theta}{32\pi^2}\int d^4xF\tilde{F}$
- $\frac{1}{32\pi^2}\int d^4xF\tilde{F} \in \mathbb{Z}$ on general topological grounds
- Contribute to partition function as $Z = e^{iS_\theta} \Rightarrow$ Doesn't change when θ shifts by 2π
- For axion coupling $S_{\theta(x)} = k\frac{\theta(x)}{32\pi^2}\int d^4xF\tilde{F}$, $Z = e^{iS_{\theta(x)}}$ only remains unchanged for $\theta(x) \rightarrow \theta(x) + 2\pi$ if $k \in Z$

Central Question

Does a massless light axion EFT inherit periodicity when axions mix?

Answer: Yes.*

-Violations proportional to light axion mass since $\Box a_L \approx -m_L^2 a_L$
-as in the $\pi - \gamma$ case, the $aF\tilde{F}$ coupling isn't quantized when higher order terms in a periodic potential are relevant

Example 1: Periodic Potential

$$\mathcal{L} = \underbrace{\frac{1}{4e^2} FF}_{\substack{\text{Gauge}\\ \text{kinetic term}}} + \underbrace{K_{ij}\partial_\mu \theta_i \partial^\mu \theta_j}_{\substack{\text{Axion}\\ \text{kinetic term}}} - \underbrace{V(j_1\theta_1 + j_2\theta_2)}_{\substack{\text{Peroidic potential for}\\ \text{heavy axion } \theta_H}} + \underbrace{\frac{k_1\theta_1 + k_2\theta_2}{32\pi^2} F\tilde{F}}_{\substack{\text{Axion Coupling}\\ k_1, k_2 \in \mathbb{Z}}}$$

In the lattice basis, axions are periodic and couplings are quantized:

$$\begin{pmatrix} \theta'_L \\ \theta'_H \end{pmatrix} = \underbrace{\begin{pmatrix} j_1 & j_2 \\ l_1 & l_2 \end{pmatrix}}_{GL(2,\mathbb{Z})} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \Rightarrow \frac{1}{32\pi^2}\Big(\underbrace{(l_2 k_1 - l_1 k_2)}_{\text{quantized}} \theta_H + \underbrace{(j_1 k_2 - j_2 k_1)}_{\text{quantized}} \theta_L \Big) F\tilde{F}$$

Example 1: Periodic Potential

- Puzzle: diagonalize the kinetic terms $\Rightarrow$

$$\underbrace{a_L = a_2 + \epsilon a_1}_{\text{not periodic}} \text{ and } \underbrace{a_H = \sqrt{1-\epsilon^2}\theta_H}_{\textit{periodic}} \text{ where } \epsilon = \frac{K_{12}}{\sqrt{K_{11}K_{22}}}$$

$$\mathcal{L} \supset \frac{1}{32\pi^2}\left(\underbrace{k_2 \frac{a_L}{f_L}}_{\text{quantized}} + \underbrace{\left(k_1 - \epsilon k_2 \frac{F_1}{F_2}\right) \frac{a_H}{\sqrt{1-\epsilon^2}}}_{\text{not quantized}} \right) F\tilde{F}$$

- Resolution:
  - When a_H shifts, so does a_L
  - a_L is periodic in the IR.

Example 2: Higgs/Stückelberg Case

$$\mathcal{L} = \underbrace{\sum_{i=1}^{2} \frac{1}{2} F_i^2 (\partial_\mu \theta_i - q_i A_\mu)^2}_{\text{Scalar Kinetic Terms with Stückelberg Couplings}} - \underbrace{\frac{1}{4e^2} FF}_{\text{Stückelberg Gauge Field kinetic term}} - \underbrace{\frac{1}{4g^2} GG}_{\text{Massless Gauge Field Kinetic Term}} + \underbrace{\frac{k_1\theta_1 + k_2\theta_2}{32\pi^2} G\tilde{G}}_{\text{Axion Coupling } k_1, k_2 \in \mathbb{Z}} + \underbrace{\mathcal{L}_{con}}_{\text{addt'l terms required for gauge inv}}$$

3 Gauge Symmetries:

- Discrete shifts of θ_i by 2π
- U(1) gauge symmetry for A_μ: $A_\mu \mapsto A_\mu + \partial_\mu \alpha, \theta_i \mapsto \theta_i + q_i \alpha$
- $\theta G\tilde{G}$ term shifts under the U(1) symmetry: $\delta_\alpha \mathcal{L}_{\theta G\tilde{G}} = \frac{k_1 q_1 + k_2 q_2}{32\pi^2} \alpha G\tilde{G}$

Example 2: Higgs/Stückelberg Case

If we diagonalize the mass terms:

$$\mathcal{L} \supset \frac{1}{32\pi^2}\Bigg(\underbrace{\Big(k_1 q_2 \frac{F_2}{F_1} - k_2 q_1 \frac{F_1}{F_2}\Big)}_{\text{not quantized?}} a_L + \underbrace{\Big(k_1 q_1 + k_2 q_2\Big)}_{\text{vanishes when } \mathcal{L}_{con}=0} a_H\Bigg) G\tilde{G}$$

$$a_H \equiv \frac{1}{m_A^2}(F_1^2 q_1 \theta_1 + F_2^2 q_2 \theta_2),$$

$$a_L \equiv \frac{F_1 F_2}{m_A^2}(q_2\theta_1 - q_1\theta_2), m_A^2 = F_1^2 q_1^2 + F_2^2 q_2^2$$

Model discussed in

- 1503.01015, 1503.02965: Shiu, Staessens, Ye
- 1906.10193: Fonseca, Harling, de Lima, Machado

Example 2: Higgs/Stückelberg Case

a_L coupling is **still quantized**!

- $\mathcal{L}_{con} = 0$:
  U(1) invariance $\rightarrow k_1 q_1 + k_2 q_2 = 0$. The coupling is
  $\frac{1}{32\pi^2}\Big(k_1 q_2 \frac{F_2}{F_1} - k_2 q_1 \frac{F_1}{F_2}\Big) a_L = -\frac{k_2}{q_1} gcd(q_1, q_2) \frac{a_L}{P_L} G\tilde{G}$
- $\mathcal{L}_{con} \neq 0$: ex. KSVZ like model, $\mathcal{L}_{con}$ from heavy fermions
  $\mathcal{L}_{con}$ cancels with irrational parts of a_L coupling.
  (See 1910.11349; 1909.11685: Choi, Shin, Yun)

Light axion period $F_L = \frac{F_1 F_2 \mathrm{gcd}(q_1, q_2)}{m_A}$ is **smaller** than F_1, F_2
Read off from kinetic term $\frac{1}{2} F_L^2 \partial_\mu \theta_L \partial_\mu \theta_L$

Example 3: Non-Compact Scalar

Mix ordinary and monodromy axions, from dimensionally reducing two 5d gauge fields.

Generates 4d potentials:

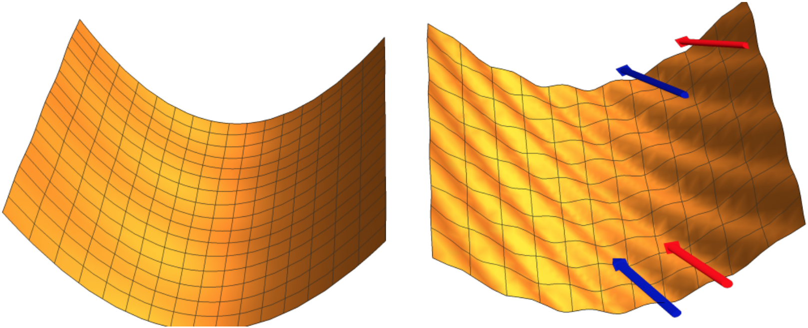
- Monodromy potential:

$$V_{mon}(\theta_H) = \frac{1}{2} m^2 F_H^2 (\theta_H - 2\pi w)^2 \tag{1}$$

- Periodic Potential:

$$V_{per}(\theta_A, \theta_H) = -\frac{3}{64\pi^6 R^4} \sum_{n=1}^{\infty} \frac{\cos(nq_A\theta_A + nq_H\theta_H)}{n^5}$$

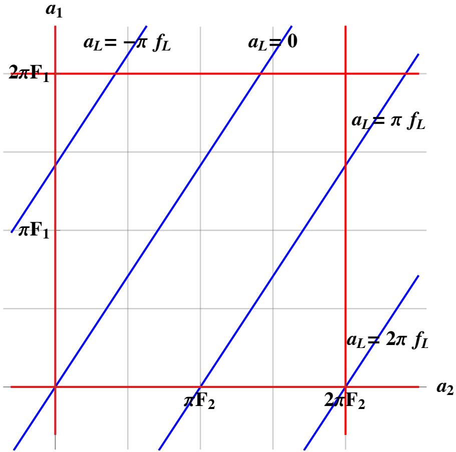
Example 3: Non-Compact Scalar



Summary

- Axion mixing cannot evade coupling quantization constraints. This limits options for generating large axion field ranges.
- These constraints apply to many phenomologically interesting models
- Can also be shown more formally in specific cases

Back Up: Massless Two Axion Space (Example 1)



Back Up: (Example 3)

$$\mathcal{L}_{5D} = -\sum_{F=H,A} -\frac{1}{4g_{5F}^2} F_{MN}(x) F^{MN}(x) - \frac{m^2}{2g_{5H}^2} \mathcal{H}_\mu \mathcal{H}^\mu + D_M \chi^\dagger(x) D^M \chi(x)$$

Field Content:

- $H_M(x)$, $A_M(x)$ are 5d gauge fields
- $H_M(x)$ Higgsed: $\mathcal{H}(x) \equiv H_M(x) - i \exp^{i\theta(x)} \partial_M e^{-i\theta(x)}$
- $\theta(x)$ a periodic scalar
- axions are Wilson loop phases: $\theta_i \equiv \oint dx^5 G_{5i}$

Axion Gauge Field couplings from Chern-Simons terms:

- $\mathcal{L}_{CS} = \frac{C_A}{16\pi^2} \epsilon^{MNPQR} F_M \text{Tr}[G_{NP} G_{QR}]$
- Quantized by gauge invariance