Multiphonon excitations from dark matter scattering in crystals

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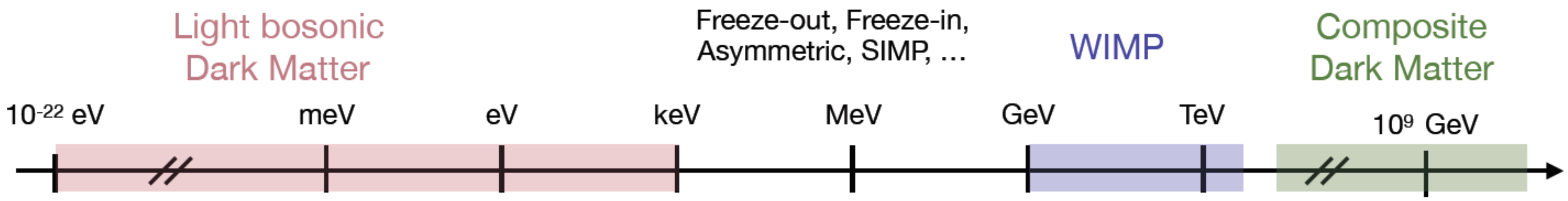
(BASED ON ARXIV:1911.03482 WITH PETER COX, SIMON KNAPEN, TONGYAN LIN, AND TOM MELIA)

Background and Motivation



Wide range of possible DM masses

- Sensitivity of detectors to different ranges
- keV-MeV candidates (“light DM”)

Quantize vibrations in a crystal lattice—phonons

- Acoustic and optical; transverse and longitudinal polarizations
- ~gram size experiment at very low temperatures (mK)

Background and Motivation, cont.

Crystal detectors are a good fit for light DM

- Typical momentum means the wavelength is much larger than the lattice spacing
- Typical energy transfer is meV, as are typical phonon energies
- Complementarity via crystal properties

Single phonon excitation is strong, but highly dependent on detection threshold

- Multiphonon excitations will be higher order, but larger phase space and less threshold sensitivity
- For larger DM masses multiphonon contributions become more important as we transfer into a regime of nuclear recoil

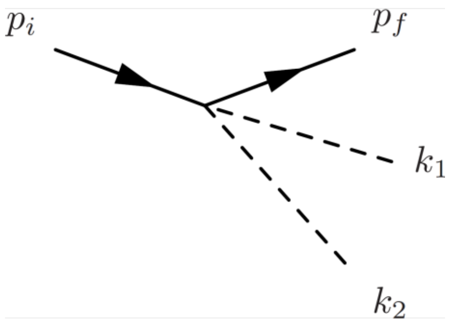
Phonon Scattering Formalism

Our goal: calculate the **structure factor** for the multiphonon processes

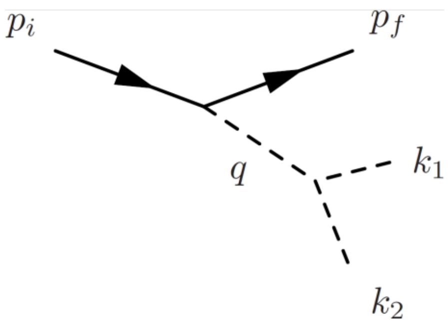
$$S(\mathbf{q},\omega) \equiv \frac{1}{N}\sum_f \left| \sum_{J=1}^{N\times\mathfrak{n}} A_J \langle \Phi_f | e^{i\mathbf{q}\cdot\mathbf{r}_J} | 0 \rangle \right|^2 \delta(E_f - \omega)$$

- connected to the differential cross section

Contact



Anharmonic



Express positions with displacement operators, expand in small *q*

- Leading contribution is single phonon excitation; next two are the leading multiphonon terms corresponding to ***contact*** and ***anharmonic*** interactions

Contact and Anharmonic contributions

$$\mathbf{u}_{\ell,d} = \sum_{\nu}^{3n} \sum_{\mathbf{k}} \sqrt{\frac{1}{2N m_d \omega_{\nu,\mathbf{k}}}} \left(\mathbf{e}_{\nu,d,\mathbf{k}} \hat{a}_{\nu,\mathbf{k}} e^{i\mathbf{k}\cdot(\boldsymbol{\ell}+\mathbf{r}_d^0)} + \mathbf{e}^*_{\nu,d,\mathbf{k}} \hat{a}^\dagger_{\nu,\mathbf{k}} e^{-i\mathbf{k}\cdot(\boldsymbol{\ell}+\mathbf{r}_d^0)} \right)$$

Cartoon for expansion and appearance of anharmonic and contact pieces:

$$\rightsquigarrow e^{i(\mathbf{q}\cdot\mathbf{e})\mathbf{a}^\dagger} \approx 1 + i\,(\mathbf{q}\cdot\mathbf{e})\,\mathbf{a}^\dagger - \frac{\mathbf{1}}{\mathbf{2}}\,(\mathbf{q}\cdot\mathbf{e})^{\mathbf{2}}\,\mathbf{a}^\dagger\mathbf{a}^\dagger$$

- First term in ***q*** combined with self-interactions leads to anharmonic piece; cancellations → $|\mathcal{M}|^2 \sim q^4$
- Second term in ***q*** leads to contact piece; $|\mathcal{M}|^2 \sim q^4$
- Don’t necessarily know a priori if one contribution dominates due to same scaling

Anharmonic Interaction

Anharmonic interaction Hamiltonian has direct connection to macroscopic crystal physics; coefficients are ***elastic constants***

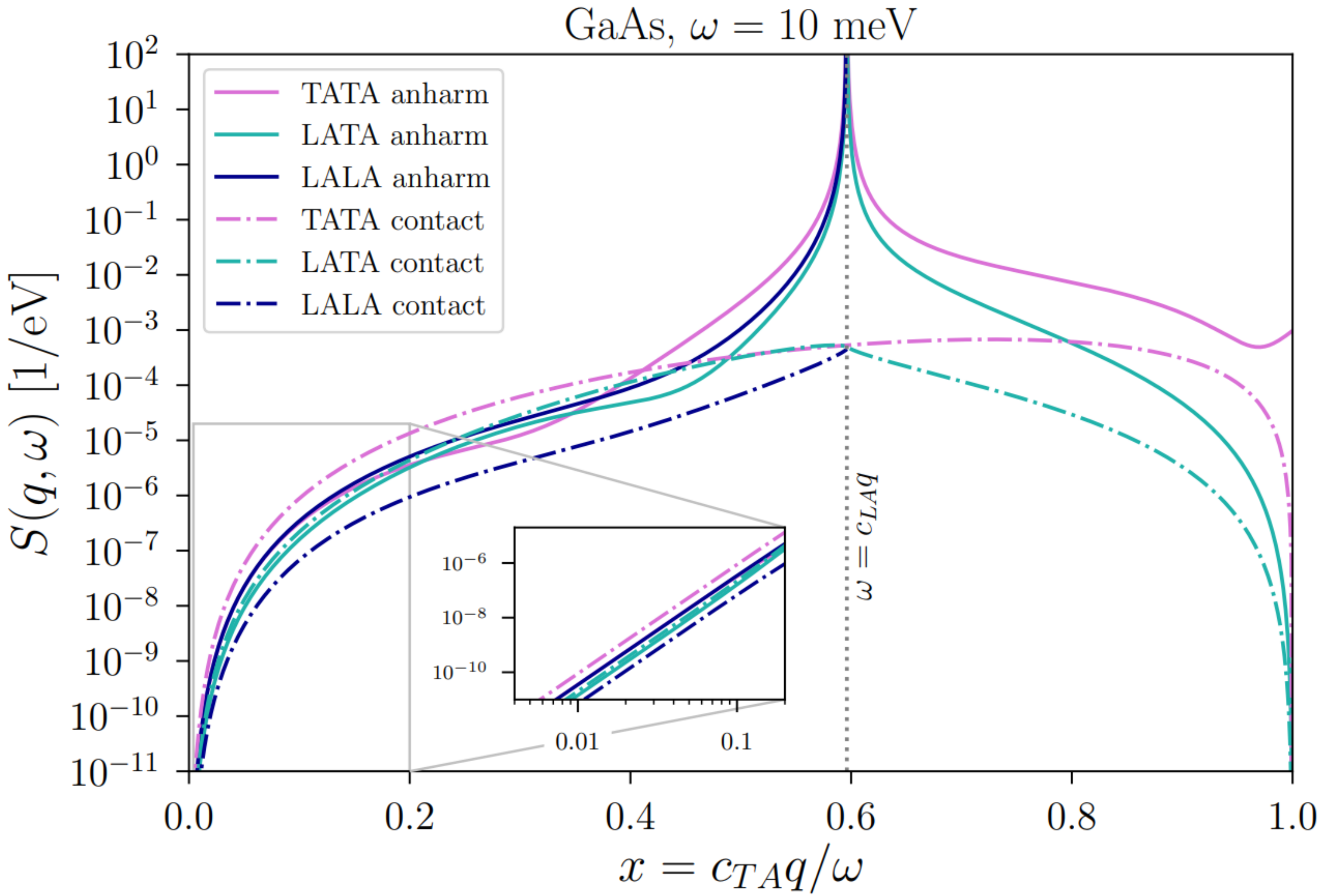
$$\delta H = \int d^3\mathbf{r}\ \frac{1}{2}(\beta + \lambda) u_{ii} u_{jk} u_{jk} + (\gamma + \mu) u_{ij} u_{ki} u_{kj} + \frac{\alpha}{3!} u_{ii} u_{jj} u_{kk} + \frac{\beta}{2} u_{ii} u_{jk} u_{kj} + \frac{\gamma}{3} u_{ij} u_{jk} u_{ki}$$

- Complementarity between crystals, etc.

Intermediate phonon can go on shell

- regulated with single phonon lifetime

Good for acoustic phonons, but no good one for optical



Numerical input for GaAs shows the two are comparable

Approximations

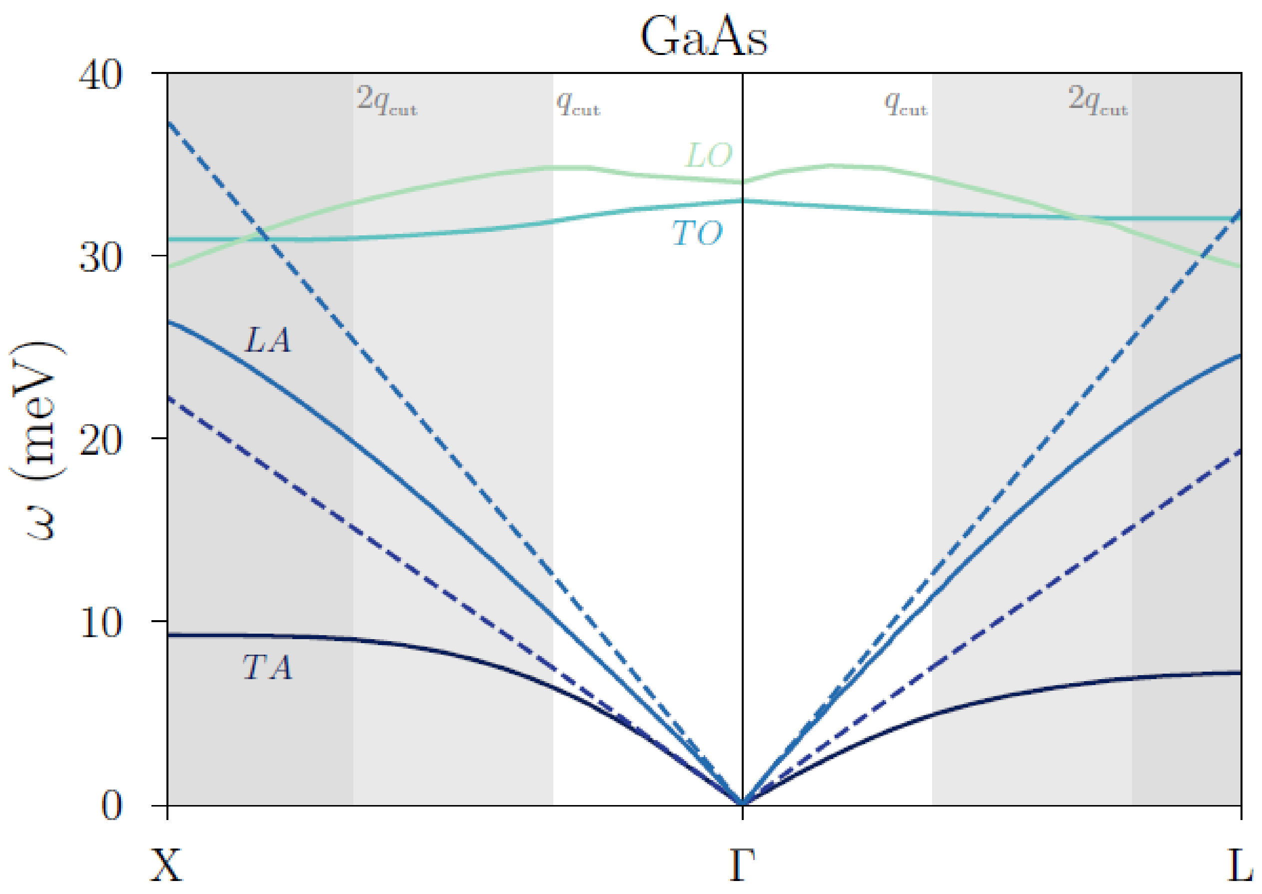
Anharmonic Hamiltonian describes long-wavelength phonons, and breaks down as you approach the edge of the first BZ

Linear dispersion approximation assumed everywhere

- Implement cut method to get idea of “error”

Isotropic approximation

- Assume same BZ size, sound speeds, etc. in all directions



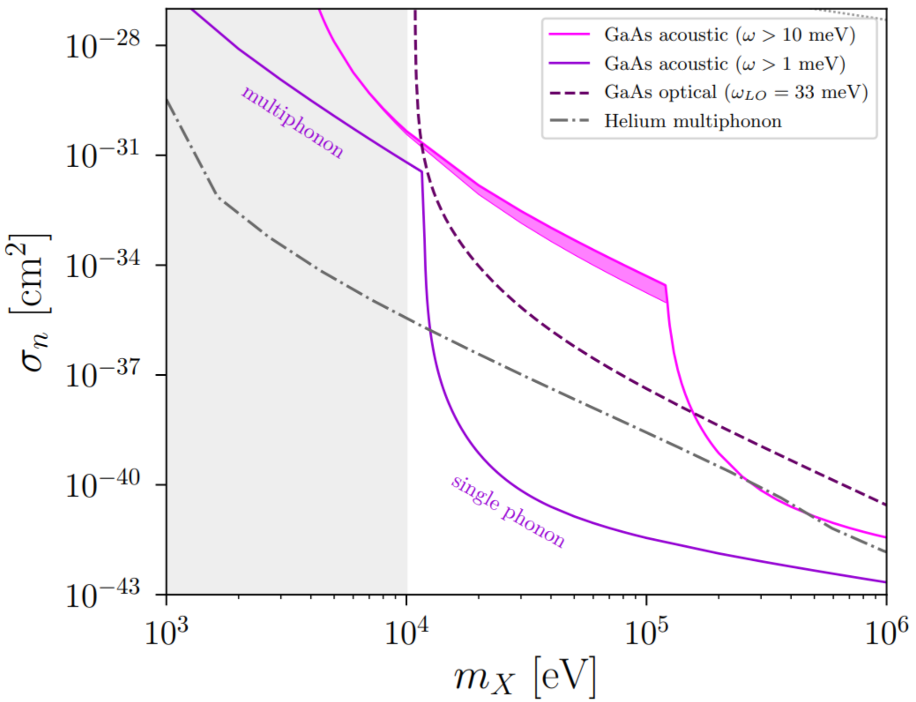
Rate Calculation and Results

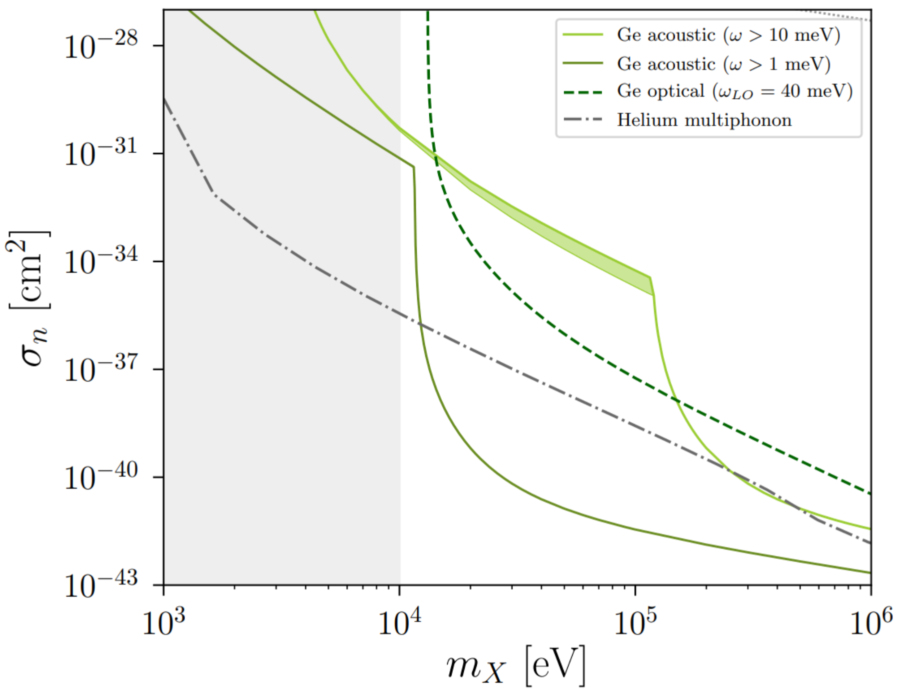
Ultimately, we desire $S(q,\omega)$ to calculate the rate; integrate over momentum and energy transfers, and initial velocity

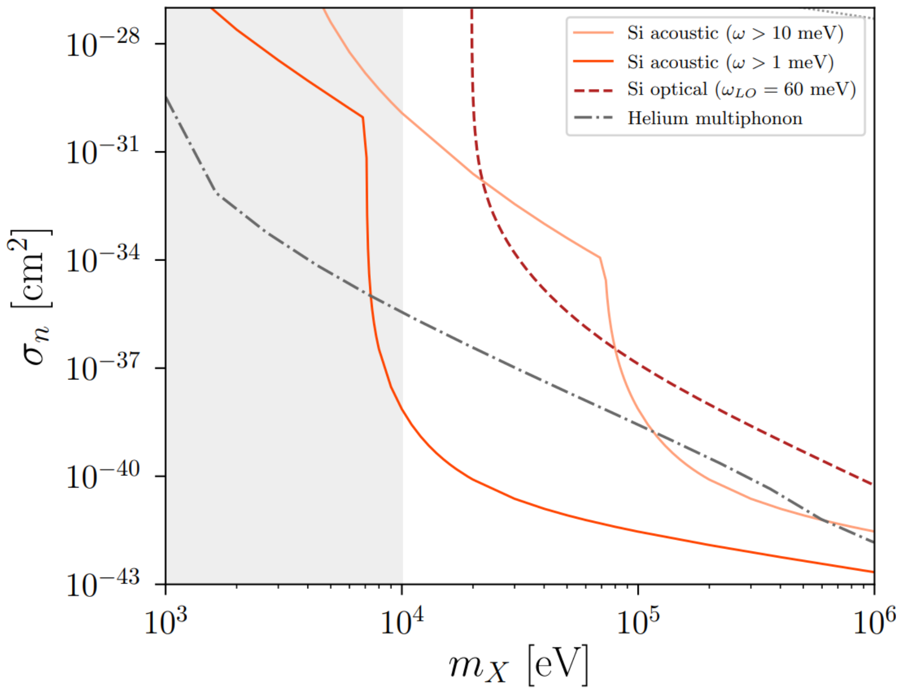
$$R = \frac{\sigma_n}{\sum_d A_d m_p} \frac{\rho_\chi}{m_{DM}} \int d^3\mathbf{v}_i\, f(\mathbf{v}_i) \int_{\omega_-}^{\omega_+} d\omega \int_{q_-}^{q_+} dq\, \frac{q}{2p_i m_{DM}} |\tilde{F}(q)|^2 S(q,\omega)$$

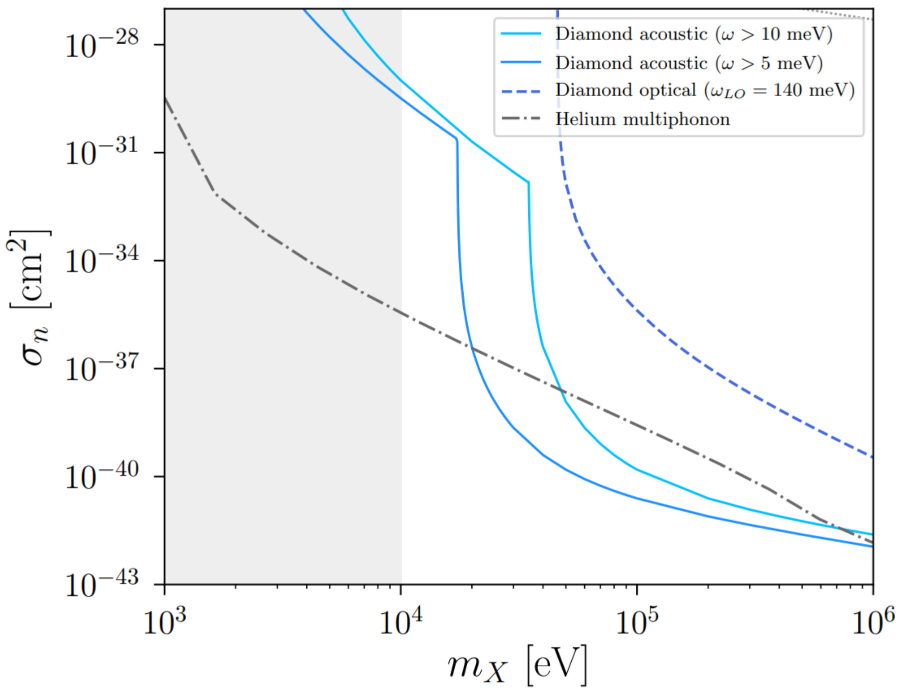
Impose a specific rate and exposure (3 events per kg•year), invert to solve for cross section.

- Thus, can rule out parameter space dependent on lack of observation


GaAs acoustic ($\omega > 10$ meV)
GaAs acoustic ($\omega > 1$ meV)
GaAs optical ($\omega_{LO} = 33$ meV)
Helium multiphonon
multiphonon
single phonon
σ_n [cm^2]
m_X [eV]


Ge acoustic ($\omega > 10$ meV)
Ge acoustic ($\omega > 1$ meV)
Ge optical ($\omega_{LO} = 40$ meV)
Helium multiphonon
σ_n [cm^2]
m_X [eV]


Si acoustic ($\omega > 10$ meV)
Si acoustic ($\omega > 1$ meV)
Si optical ($\omega_{LO} = 60$ meV)
Helium multiphonon
σ_n [cm^2]
m_X [eV]


Diamond acoustic ($\omega > 10$ meV)
Diamond acoustic ($\omega > 5$ meV)
Diamond optical ($\omega_{LO} = 140$ meV)
Helium multiphonon
σ_n [cm^2]
m_X [eV]

Conclusion

Phonon excitation in crystals is a motivated method to detect light DM

- Want to analyze multiphonon excitations

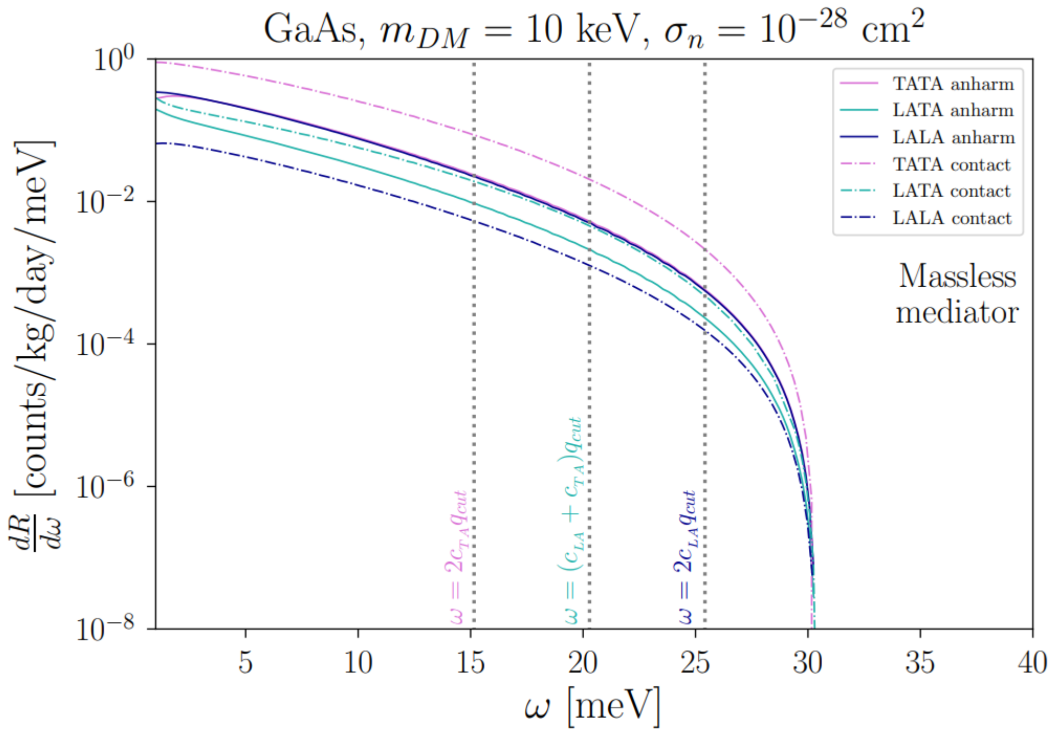
Expansion in terms of displacement operators and small q leads to two leading processes (anharmonic and contact)

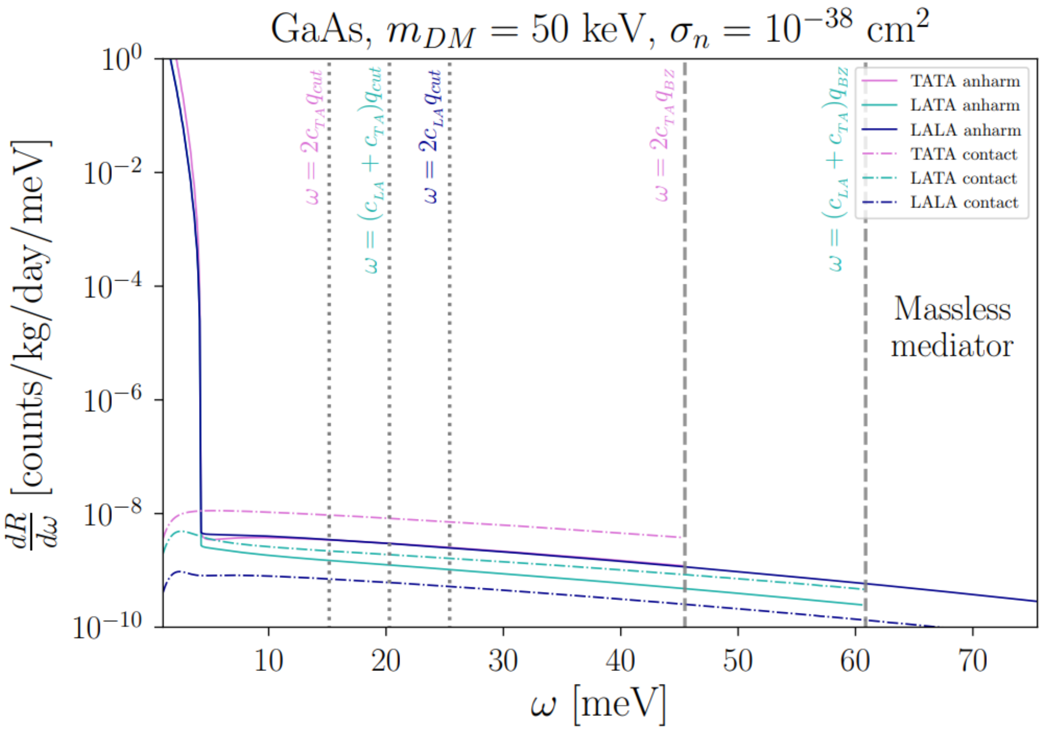
We find multiphonon processes are generally subdominant, but can contribute to ruling out additional parameter space (especially for higher energy detector thresholds)

Results, cont. (extra)

Differential rate analysis to further explore contact vs. anharmonic

- Dominance of single phonon when it's accessible





Extra

(Semi-)Explicit examples of the structure factor:

$$S^{(anh)}_{LALA}(q,\omega) = \frac{\sum_d A_d\, q^4\, \omega^4}{16\pi^2 c_{LA}^7 m_p \rho^3[(\omega^2-(c_{LA}q)^2)^2+(c_{LA}q)^2\Gamma^2_{LA,q}]} g^{(anh)}_{LALA}\left(\frac{qc_{LA}}{\omega}\right)\,\theta(\omega - c_{LA}q)\,,$$

$$S^{(anh)}_{TATAout}(q,\omega) = \frac{\sum_d A_d\, q^4\, \omega^4}{16\pi^2 c_{TA}^7 m_p \rho^3[(\omega^2-(c_{LA}q)^2)^2+(c_{LA}q)^2\Gamma^2_{LA,q}]} \delta^2 g^{(anh)}_{LALAout}\left(\frac{qc_{TA}}{\omega}\right)\,\theta(\omega - c_{TA}q)\,,$$

$$S^{(cont)}_{TATA}(q,\omega) = \frac{(\sum_d A_d)}{64\pi^2 c_{TA}^3 m_p \rho}\, q^4\, g^{(cont)}_{TATA}\left(\frac{c_{TA}q}{\omega}\right)\,\theta(\omega - c_{TA}q)\,,$$

$$S^{(cont)}_{LATA}(q,\omega) = \frac{(\sum_d A_d)}{64\pi^2 c_{LA} c_{TA}(c_{LA}+c_{TA}) m_p \rho} q^4\, g^{(cont)}_{LATA}\left(\frac{c_{TA}q}{\omega}\right)\theta(\omega - c_{TA}q)$$

Extra

Matrix element for anharmonic Hamiltonian:

$$\begin{aligned}
\widetilde{\mathcal{M}} = (\beta+\lambda)\Big[&(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{e}_1\cdot\mathbf{e}_2)+(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{q}\cdot\mathbf{k}_2)(\mathbf{e}\cdot\mathbf{e}_2)+(\mathbf{k}_2\cdot\mathbf{e}_2)(\mathbf{k}_1\cdot\mathbf{q})(\mathbf{e}_1\cdot\mathbf{e})\Big]\\
&+(\gamma+\mu)\Big[(\mathbf{q}\cdot\mathbf{k}_1)\big[(\mathbf{k}_2\cdot\mathbf{e}_1)(\mathbf{e}_2\cdot\mathbf{e})+(\mathbf{k}_2\cdot\mathbf{e})(\mathbf{e}_2\cdot\mathbf{e}_1)\big]\\
&\qquad+(\mathbf{k}_2\cdot\mathbf{k}_1)\big[(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{e}_2\cdot\mathbf{e})+(\mathbf{q}\cdot\mathbf{e}_2)(\mathbf{e}\cdot\mathbf{e}_1)\big]\\
&\qquad+(\mathbf{q}\cdot\mathbf{k}_2)\big[(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{e}_1\cdot\mathbf{e})+(\mathbf{k}_1\cdot\mathbf{e})(\mathbf{e}_1\cdot\mathbf{e}_2)\big]\Big]\\
&+\alpha(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{k}_2\cdot\mathbf{e}_2)\\
&+\beta\Big[(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{q}\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e})+(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e}_1)+(\mathbf{k}_2\cdot\mathbf{e}_2)(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{k}_1\cdot\mathbf{e})\Big]\\
&+\gamma\Big[(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e})+(\mathbf{q}\cdot\mathbf{e}_2)(\mathbf{k}_1\cdot\mathbf{e})(\mathbf{k}_2\cdot\mathbf{e}_1)\Big]\,,
\end{aligned}$$