The On-Shell Viewpoint of Effective Field Theory

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May 5, 2020
Describe an EFT: Lorentz invariant, gauge invariant operators

\[
\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i = \sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}.
\]

Equivalent to a complete set of physical observables.

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On-Shell Amplitudes

On-shell amplitudes are expressed in terms of spinor-helicity variables, as $SL(2,\mathbb{C})$ irreps

$$p_{\alpha\dot{\alpha}} \equiv p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{\dot{\alpha}I}, \quad I = 1, \ldots, \text{rank}(p).$$

General form of helicity amplitudes for $N$ massless particles

$$\mathcal{A}(\ldots, p_i, h_i, \ldots) = f(s_{\mathcal{I}}) \times \prod_{i=1}^{N} \lambda_i^{r_i} \tilde{\lambda}_i^{2h_i+r_i}, \quad s_{\mathcal{I}} = \left(\sum_{i \in \mathcal{I}} p_i\right)^2.$$

The kinematic function $f(s_{\mathcal{I}})$ has analytic structures on the complex $s_{\mathcal{I}}$ planes, indicating the existence of intermediate on-shell phase spaces $\{\langle \gamma \rangle\}$ in the channel $\mathcal{I} \rightarrow \bar{\mathcal{I}}$:

Factorization: $$2 \ Im\mathcal{A}_{\alpha\beta} = \int d\gamma \mathcal{A}_{\alpha\gamma} \mathcal{A}^{*}_{\beta\gamma}.$$
Generically, full amplitudes are determined by the analytic structures, and recursively by the basic unfactorizable building blocks (the \textit{amplitude basis}), up to analytic functions (new amplitude basis).

- Constructible examples: Yang-Mills, NL$\sigma$M, \ldots 
  Counter-examples: Scalar QED, EFT, \ldots

- \textit{Hidden symmetries} (like SUSY, soft theorem) constrain the magnitudes of \textit{new amplitude basis} so that the theory is still constructible from the original building blocks.

- Constructibility is similar to renormalizability, which is not necessary for all theories (like for EFT).

- In this talk, I will focus on applications \textbf{NOT} demanding the full on-shell constructibility.
Examples of amplitude basis

Denote amplitude basis as $\mathcal{M}(\Phi_1 \ldots \Phi_N)$, expressed in terms of spinor brackets $\langle ij \rangle \equiv \lambda_i^\alpha \lambda_i \dot{\alpha}$ and $[ij] \equiv \tilde{\lambda}_i \tilde{\lambda}_i$. 

- $N = 3$ special kinematics: either $\langle ij \rangle = 0$ or $[ij] = 0$

$$
\mathcal{M}(\psi \psi \phi) \sim \langle 12 \rangle, \quad \mathcal{M}(FF\phi) \sim \langle 12 \rangle^2,
\mathcal{M}(F\psi \psi) \sim \langle 12 \rangle \langle 13 \rangle, \quad \mathcal{M}(FFF) \sim \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle,
$$

**spurious pole** is allowed (the minimal gauge couplings):

$$
\mathcal{M}(F\phi \phi) \sim \frac{\langle 12 \rangle \langle 13 \rangle}{\langle 23 \rangle}, \quad \mathcal{M}(F\psi \tilde{\psi}) \sim \frac{\langle 12 \rangle^2}{\langle 23 \rangle}, \quad \mathcal{M}(FFF) \sim \frac{\langle 12 \rangle^3}{\langle 13 \rangle \langle 23 \rangle}.
$$

- $N \geq 4$ polynomials of spinor brackets, e.g.

$$
\mathcal{M}(\psi \tilde{\psi} \phi \phi) \sim \langle 13 \rangle [23], \quad \mathcal{M}(FF\phi \phi) \sim \langle 12 \rangle^2.
$$
Gauge Invariance

Gauge bosons in non-minimal gauge couplings only come from field strength $F$. How about the gauge field in $D_\mu = \partial_\mu - i A_\mu$?

**Example:** $O^\mu D_\mu \Psi$ where $\Psi$ is charged for gauge boson $\gamma$:

$$A(O\Psi\gamma) = \langle O|O^\mu(-iA_\mu)\Psi|\Psi\gamma\rangle + \langle O|(O^\mu\partial_\mu\Psi)(J^\mu_\Psi A_\mu)|\Psi\gamma\rangle$$

The **gauge invariant** full amplitude involves a pole of the charged particle, whose residue factorizes due to unitarity

$$A(O\Psi\gamma) \rightarrow \left\langle O|O^\mu\partial_\mu\Psi|\Psi\right\rangle \times \left\langle \Psi|J^\mu_\Psi A_\mu|\Psi\gamma\right\rangle .$$

The operator-amplitude correspondence $[1902.06752]$:

$$D_\mu \leftrightarrow p_\mu, \quad i[D_\mu, D_\nu] = F_{\mu\nu} \leftrightarrow F \oplus \bar{F}.$$
The anomalous dimension matrix for effective operators

\[ \dot{C}_m = (4\pi)^{-2} \gamma_{mn} C_n \] at one loop is determined by

\[ 16\pi^2 A_{\text{UV}}^{1\text{-loop}} = -\left( \sum_{m,n} \gamma_{mn} C_n M_m + A' \right) \frac{1}{\epsilon} \]

By finding the partial wave amplitude basis $M^j$ with definite angular momentum $j$ at particular channel $|\Psi\rangle \rightarrow \cdots$, $\gamma_{mn}$ can be non-vanishing only for $j = j'$ in $M^j_m$ and $M^{j'}_n$. e.g. [2001.04481]

\[ \dot{C}^{(1)}_{Hl} \propto C_{HD} + C_{H\Box}, \quad \dot{C}^{(3)}_{Hl} \propto C_{H\Box}. \]
Generically, loop amplitudes do not put constraints on the contributing operators. Two exceptions [2001.04481]:

1. Orbital angular momentum $l \perp \hat{z}$ for two particles, hence

$$ \hat{O} |\Psi\rangle \geq j_{\Psi} \geq j_{z} = s_{z} = \Delta h = |h_{1} - h_{2}|. $$

Example: $|\Psi\rangle = |\psi\phi\phi\rangle$, $h_{i} = (-1, 1/2)$, take $O$ from $[\psi^{2}\phi^{3}D^{2}]$, which includes 6 independent operators c.f. [2005.00008].

$$ j_{\Psi} \geq 3/2 \Rightarrow \begin{cases} O_{1}^{j=3/2} = -2O_{1} + O_{2} + 3O_{4} + O_{5}, \\ O_{2}^{j=3/2} = -O_{2} + 2O_{3} + O_{6}, \end{cases} $$

Only two combinations contribute to $A^{1-\text{loop}}(\psi\phi\phi \rightarrow F\bar{\psi})$. 
2. When the two particles on the RHS are identical [1709.04891]:

\[ h \langle \Psi | h \Psi \rangle = 1. \]

**Example:** All of \( \mathcal{M}(\bar{\psi}\psi\phi^2D) \) have \( j = 1 \) in the channel \( \bar{\psi}\psi \leftrightarrow \phi\phi \).

None of \( \mathcal{O} \) in the type \( [\bar{\psi}\psi\phi^2D] \) contribute to \( \mathcal{A}^{1-\text{loop}}(\bar{\psi}\psi\gamma^\pm\gamma^\pm) \) or \( \mathcal{A}^{1-\text{loop}}(\phi\phi\gamma^\pm\gamma^\pm) \).

Could be generalized when we consider full permutation symmetry involving group factors, e.g.

\[ \epsilon^{IJK} |\Psi\rangle^K \]

\[ \langle O | W^I \]

\[ j_\Psi = 1. \]
Summary

- The on-shell viewpoint is an alternative way of thinking about QFT, which has distinct building blocks (amplitude basis) and calculations (recursion relations).
- We established a correspondence between effective operators and amplitude basis.
- We are able to define angular momentum of an amplitude basis in a certain channel, which induces an eigenbasis of amplitudes – partial wave amplitude basis.
- With partial wave amplitude basis, we obtained non-trivial selection rules for anomalous dimension matrix and full loop calculations in certain cases.
- We obtain a complete set of operator basis, with both Lorentz and gauge group structures specified, for dimension 8 standard model EFT.
Thank You