The On-Shell Viewpoint of Effective Field Theory

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arXiv:1902.06752 with T. Ma and J. Shu
arXiv:2001.04481 with M. Jiang, J. Shu and Y.-H. Zheng
arXiv:2005.00008 with H.-L. Li, Z. Ren, J. Shu J.-H. Yu and Y.-H. Zheng

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An Alternative Viewpoint

Describe an EFT: Lorentz invariant, gauge invariant operators

$$\mathcal{L}_{\mathrm{EFT}} = \sum_i C_i \mathcal{O}_i = \sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}.$$

Equivalent to a complete set of physical observables.

Off-Shell Operators	**On-Shell Amplitude**
Operator Basis (EOM redundancy)	Amplitude Basis ($p^2 = m^2$)
Feynman Rules	Unitarity and Recursion Relations
Gauge dependent sub-diagrams	Always gauge invariant

On-Shell Amplitudes

On-shell amplitudes are expressed in terms of spinor-helicity variables, as $SL(2,\mathbb{C})$ irreps

$$p_{\alpha\dot{\alpha}} \equiv p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{\dot{\alpha}I}, \quad I = 1, \ldots, \mathrm{rank}(p) \ .$$

General form of helicity amplitudes for N massless particles

$$\mathcal{A}(\ldots, p_i, h_i, \ldots) = f(s_\mathcal{I}) \times \prod_{i=1}^N \lambda_i^{r_i} \tilde{\lambda}_i^{2h_i + r_i}, \quad s_\mathcal{I} = \left(\sum_{i \in \mathcal{I}} p_i \right)^2 \ .$$

The kinematic function $f(s_\mathcal{I})$ has analytic structures on the complex $s_\mathcal{I}$ planes, indicating the existence of intermediate on-shell phase spaces $\{|\gamma\rangle\}$ in the channel $\mathcal{I} \to \bar{\mathcal{I}}$:

Factorization: $$2\,\mathrm{Im}\mathcal{A}_{\alpha\beta} = \int \mathrm{d}\gamma\, \mathcal{A}_{\alpha\gamma}\mathcal{A}^*_{\beta\gamma} \ .$$

Amplitude Basis

Generically, full amplitudes are determined by the analytic structures, and recursively by the basic unfactorizable building blocks (the **amplitude basis**), up to analytic functions (new amplitude basis).

- Constructible examples: Yang-Mills, NLσM, . . .
  Counter-examples: Scalar QED, EFT, . . .
- *Hidden symmetries* (like SUSY, soft theorem) constrain the magnitudes of ***new amplitude basis*** so that the theory is still constructible from the original building blocks.
- Constructibility is similar to renormalizability, which is not necessary for all theories (like for EFT).
- In this talk, I will focus on applications **NOT** demanding the full on-shell constructibility.

Examples of amplitude basis

Denote amplitude basis as $\mathcal{M}(\Phi_1 \dots \Phi_N)$, expressed in terms of spinor brackets $\langle ij\rangle \equiv \lambda_i^{\alpha}\lambda_{i\alpha}$ and $[ij] \equiv \tilde{\lambda}_{i\dot{\alpha}}\tilde{\lambda}_i^{\dot{\alpha}}$.

- $N = 3$ special kinematics: either $\langle ij\rangle = 0$ or $[ij] = 0$

$$\mathcal{M}(\psi\psi\phi) \sim \langle 12\rangle, \quad \mathcal{M}(FF\phi) \sim \langle 12\rangle^2,$$
$$\mathcal{M}(F\psi\psi) \sim \langle 12\rangle\langle 13\rangle, \quad \mathcal{M}(FFF) \sim \langle 12\rangle\langle 23\rangle\langle 31\rangle,$$

  spurious pole is allowed (the minimal gauge couplings):

$$\mathcal{M}(F\phi\phi) \sim \frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}, \; \mathcal{M}(F\psi\bar{\psi}) \sim \frac{\langle 12\rangle^2}{\langle 23\rangle}, \; \mathcal{M}(FF\bar{F}) \sim \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\,.$$

- $N \geq 4$ polynomials of spinor brackets, *e.g.*

$$\mathcal{M}(\psi\bar{\psi}\phi\phi) \sim \langle 13\rangle[23], \quad \mathcal{M}(FF\phi\phi) \sim \langle 12\rangle^2\,.$$

Gauge Invariance

Gauge bosons in non-minimal gauge couplings only come from field strength F. How about the gauge field in $D_\mu = \partial_\mu - iA_\mu$?

Example: $O^\mu D_\mu \Psi$ where Ψ is charged for gauge boson γ:

$$\mathcal{A}(\mathcal{O}\Psi\gamma) = \langle \mathcal{O}|\mathcal{O}^\mu(-iA_\mu)\Psi|\Psi\gamma\rangle + \langle \mathcal{O}|(\mathcal{O}^\mu\partial_\mu \Psi)(J^\mu_\Psi A_\mu)|\Psi\gamma\rangle$$

The **gauge invariant** full amplitude involves a pole of the charged particle, whose residue factorizes due to unitarity

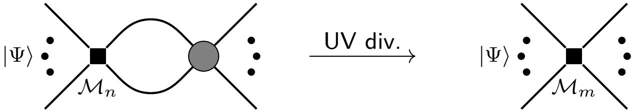
$$\mathcal{A}(\mathcal{O}\Psi\gamma) \to \underbrace{\langle \mathcal{O}|\mathcal{O}^\mu\partial_\mu\Psi|\Psi\rangle}_{\mathcal{M}(O\Psi)} \times \underbrace{\langle \Psi|J^\mu_\Psi A_\mu|\Psi\gamma\rangle}_{\mathcal{M}(F_\gamma\Psi\bar{\Psi})}.$$

The operator-amplitude correspondence [1902.06752]:

$$D_\mu \leftrightarrow p_\mu, \quad i[D_\mu, D_\nu] = F_{\mu\nu} \leftrightarrow F \oplus \bar{F}.$$

Application I: Renormalization Selection Rule

The anomalous dimension matrix for effective operators $\dot{C}_m = (4\pi)^{-2}\gamma_{mn}C_n$ at one loop is determined by



$$16\pi^2 \mathcal{A}_{\text{UV}}^{\text{1-loop}} = -\left(\sum_{m,n} \gamma_{mn} C_n \mathcal{M}_m + \mathcal{A}'\right)\frac{1}{\epsilon}$$

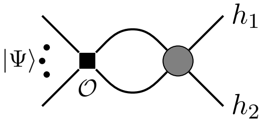
By finding the partial wave amplitude basis $\mathcal{M}^j$ with definite angular momentum j at particular channel $|\Psi\rangle \to \cdots$, γ_{mn} can be non-vanishing only for $j = j'$ in $\mathcal{M}_m^j$ and $\mathcal{M}_n^{j'}$. *e.g.* [2001.04481]

$$\dot{C}_{Hl}^{(1)} \propto C_{HD} + C_{H\Box}, \quad \dot{C}_{Hl}^{(3)} \propto C_{H\Box}.$$

Application II: Loop Selection Rule

Generically, loop amplitudes do not put constraints on the contributing operators. Two exceptions [2001.04481]:

1. Orbital angular momentum $l \perp \hat{z}$ for two particles, hence



$$j_\Psi \geq j_z = s_z = \Delta h = |h_1 - h_2|.$$

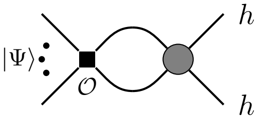
Example: $|\Psi\rangle = |\psi\phi\phi\rangle$, $h_i = (-1, 1/2)$, take $\mathcal{O}$ from $[\psi^2\phi^3 D^2]$, which includes 6 independent operators c.f. [2005.00008].

$$j_\Psi \geq 3/2 \Rightarrow \begin{cases} \mathcal{O}_1^{j=3/2} = -2\mathcal{O}_1 + \mathcal{O}_2 + 3\mathcal{O}_4 + \mathcal{O}_5, \\ \mathcal{O}_2^{j=3/2} = -\mathcal{O}_2 + 2\mathcal{O}_3 + \mathcal{O}_6, \end{cases}$$

Only two combinations contribute to $\mathcal{A}^{1-\text{loop}}(\psi\phi\phi \to F\bar{\psi})$.

Application II: Loop Selection Rule

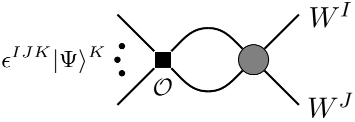
2. When the two particles on the RHS are identical [1709.04891]:



$$j_\Psi \neq 1.$$

Example: All of $\mathcal{M}(\bar{\psi}\psi\phi^2 D)$ have $j = 1$ in the channel $\bar{\psi}\psi \leftrightarrow \phi\phi$. None of $\mathcal{O}$ in the type $[\bar{\psi}\psi\phi^2 D]$ contribute to $\mathcal{A}^{1-\text{loop}}(\bar{\psi}\psi\gamma^\pm\gamma^\pm)$ or $\mathcal{A}^{1-\text{loop}}(\phi\phi\gamma^\pm\gamma^\pm)$.

Could be generalized when we consider full permutation symmetry involving group factors, *e.g.*



$$j_\Psi = 1.$$

Summary

- The on-shell viewpoint is an alternative way of thinking about QFT, which has distinct building blocks (amplitude basis) and calculations (recursion relations).
- We established a correspondence between effective operators and amplitude basis.
- We are able to define angular momentum of an amplitude basis in a certain channel, which induces an eigenbasis of amplitudes – partial wave amplitude basis.
- With partial wave amplitude basis, we obtained non-trivial selection rules for anomalous dimension matrix and full loop calculations in certain cases.
- We obtain a complete set of operator basis, with both Lorentz and gauge group structures specified, for dimension 8 standard model EFT.

Thank You

Fine