QCD and the strong force

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ABSTRACT: This is a lecture given in the third Tirana school of particle physics and astrophysics. After a general introduction to the physics of strong interactions we give the basic ideas of lattice quantum chromodynamics.

Contents

1 Basic facts

The physics of strong force is the physics of nuclear and subnuclear phenomena. It deals with the structure of the basic constituents of matter such as proton and neutron which are part of atomic nucleus. The first striking observation is the stability of nucleus against Coulomb repulsion of protons. This means that nuclear forces are stronger in magnitude than electromagnetic ones. Furthermore, they do not fill the space like the latter, they are confined within the nucleus.

1.1 Yukawa theory

Yukawa suggested that the nuclear potential decays exponentially with the distance

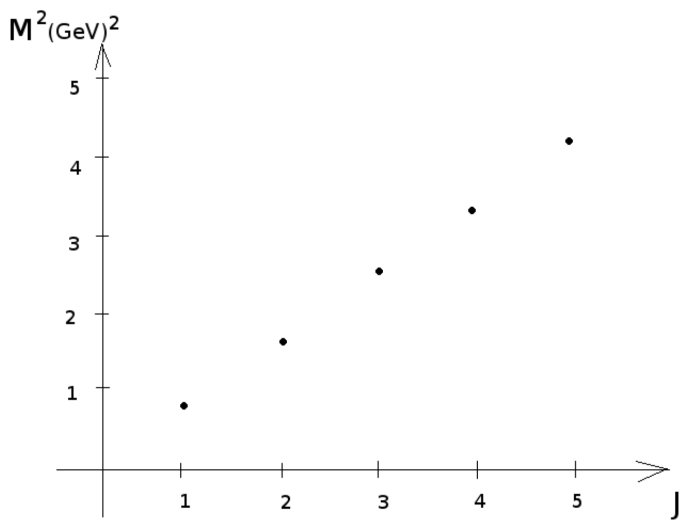
$$V(r) \propto \frac{e^{-r/\xi}}{r} \, ,$$

where ξ is the characteristic correlation length of strong interactions. Since the radius of proton is of the order of one Fermi, it turns out that the particle mediating the nuclear force has the mass of the order $200\ MeV$, which is indeed close to the pi-mesons with mass $140\ MeV$.

Another important fact is the smallness in the mass difference among pi-mesons as well as among nucleons, the latter having a mass of $940\ MeV$. If one neglects the difference one may describe the degeneracy as a manifestation of the SU(2) symmetry, which rotates unitarily the wave functions of protons and neutrons. The proliferation of hadron resonances may be explained by a larger symmetry, namely the SU(3) symmetry and larger. This leads to the hypothesis that hadrons have structure composed of fermion particles called partons or quarks. These have not been isolated in Nature, which brings us to the confinement hypothesis. But how do quarks interact with each-other? In the following we will describe two basic facts that lead to a theory of strong interactions, the linear potential and scaling.

1.2 String picture of hadrons

When confronted with rho-meson resonance spectroscopy one encounters a linear relationship between their mass squared as a function of their spin, which is the Regge plot.



We may describe the energy of a quark and one antiquark inside the rho-meson as the kinetic energy of rotation around the center of mass of the system. This is proportional to the radius squared, i.e. r^2. Since the angular momentum of the system is proportional to r^2, we may not be able to explain the Regge plot within this model. However, we may explain the plot if we model the interaction according to the string potential

$$V(r) = Kr\ ,$$

where K is the string tension. One can even infer from the plot that the string tension is $(440\ MeV)^2$, which corresponds to an energy of one GeV within one Fermi or a mass of 14 tons. Now let us describe the scaling phenomenon.

1.3 Scaling

In deep inelastic scattering of electrons off the nucleons one observes that the structure functions of hadrons change only a little with the energy transfer of the reaction. This phenomenon, called scaling, is explained by the parton model of Feynman, whereby the target is composed of freely interacting point particles. The partons behave like fermions which means they are quarks interacting weakly at high energies. On the other hand we have strongly interacting quarks at low energies and even confinement.

These are two essential qualities of the strong force based on quarks. How to make an interacting theory out of these key ingredients? In the following we will briefly discuss the Yang-Mills theory and quantum chromodynamics (QCD).

1.4 Yang-Mills theory

Many years before the experimental evidence Yang and Mills introduced a generalization of electrodynamics with a non-Abelian gauge degree of freedom [1]. At that time they did not know that the theory confines the color and the field is massive as it is suggested by many years of computer simulations. When Gross and Wilczek as well as by Politzer [2, 3] showed that the theory is asymptotically free even in the presence of quarks no one new that the theory confines the quarks.

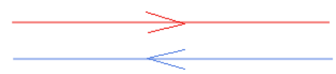
1.5 Lattice QCD

Wilson showed that a quantum field has to start with a cutoff in place such as a space-time lattice [4]. He showed that the Yang-Mills field confines the charges at strong coupling. Therefore, QCD was born. It was immediately clear that a direct evaluation of QCD path integral was only possible using Monte Carlo simulations. Creutz was the first to show numerically that the weak and strong regimes are in the same phase in four dimensions as well as to confirm asymptotic freedom on the lattice [5]. Since then, lattice QCD has grown into a separate numerical discipline and has delivered results of growing accuracy. Nonetheless, we have no direct access to the mechanism of confinement unless the theory is solved analytically. In a recent development, we have shown that a small extension of the Wilson theory at vanishing weak coupling renders the theory solvable, in which case the Yang-Mills theory turns out to be a theory of free massive bosons where the color confinement is trivial [6]. Although further calculations are needed to understand better the theory, we have an analytical tool to investigate the strong force.

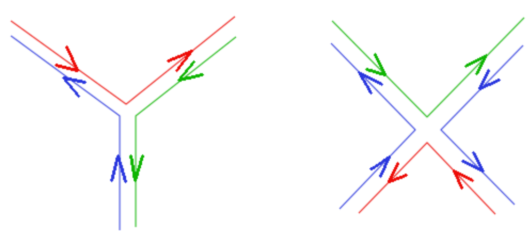
2 A colorful theory

What is quark in the theory of strong interactions? They are primitive fermions i.e. elementary particles with spin one half. They carry a color charge, one of the three conventionally described as blue, red and green. Following Feynman graph description of elementary processes we will describe a quark as a source of color as in the diagram below.

The Yang-Mills field describes the spin one gauge bosons, called gluons, which are described by two color lines of one quark and one antiquark.

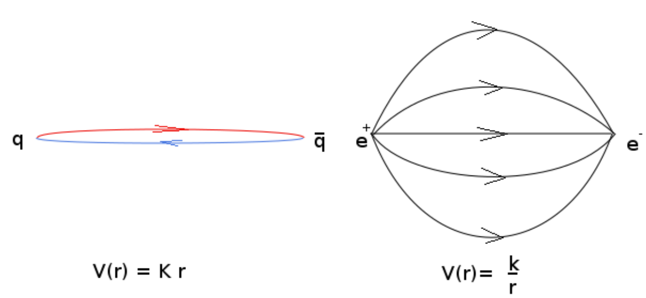


They join and split in many ways using three and four gluon vortices of the type:

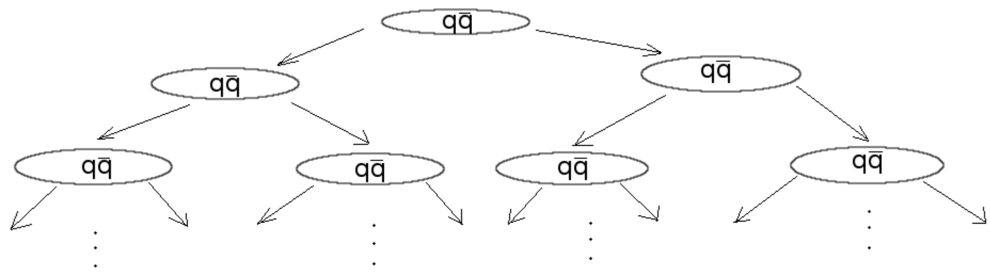


2.1 Mesons as strings

The strong interaction of color does not allow the field lines to spread as in the case of electromagnetism. Therefore, inevitably, the strong force follows the string law as opposed to the Coulomb law.



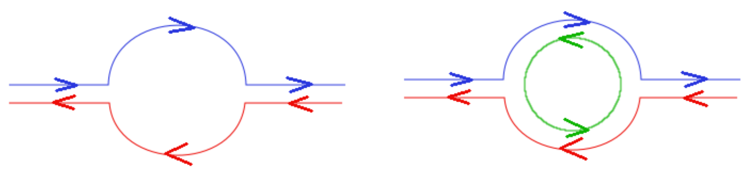
If we try to break the string we won't be able to isolate quarks but generate instead a cascade of smaller meson strings.



Then, how do we understand the vanishing strength at high energies?

2.2 Paramagnetic vacuum

It is the same strong force of color that allows the following diagrams to occur in the QCD vacuum. The first one is the quark-antiquark creation whereas the second is the creation of a color loop.



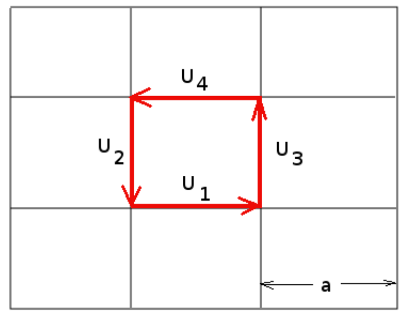
The second diagram has a larger probability making color loops ubiquitous. These are currents equivalent to elementary magnetic pins of color. Therefore the QCD vacuum has a paramagnetic structure which turns the color dielectric constant smaller than one. This makes for the antiscreening of the color charge which is enhanced at high energies since more color loops are created. At low energies loop diagrams are suppressed favoring the more stable string diagrams. The inescapable consequence of this picture is that at high energy collisions the quark-antiquark potential is of the Coulomb type, i.e.

$$V(r)_{\text{high energy}} \propto \frac{\alpha_s(r)}{r} ,$$

where α_s is a dimensionless number, the strong coupling "constant", which changes with the distance as we will see below.

3 Continuum limit

Wilson was the first to suggest a lattice cutoff in the formulation of the theory in order to measure a physical quantity as a function of the lattice spacing a and then take the continuum limit.



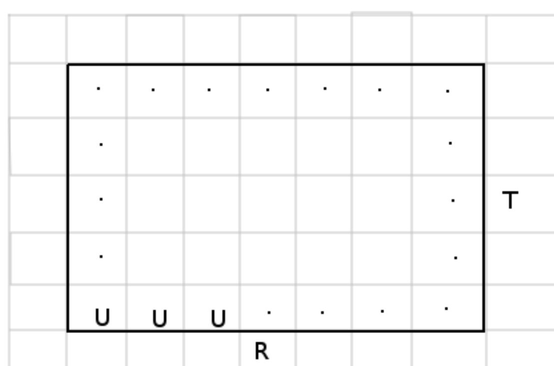
He defined the Yang-Mills action as the sum over tile or plaquette contributions:

$$S = -\frac{1}{g^2} \sum_{\text{plaquettes}} \text{Tr}\, U_1 U_2 U_3 U_4 + h.c.\ ,$$

where g is the coupling constant of the theory and U_1, U_2, U_3, U_4 are gluon fields of the theory, 3×3 unitary matrices. The idea is to find a value g_c where the physical quantities scale. This is the desired continuum limit.

3.1 Wilson loop

In order to measure the quark-antiquark potential we should measure the a large loop, eg. a rectangular Wilson loop with sider $R \times T$.



Its expectation value is given by

$$W(R, T, g) = \int_{\text{over U fields}} \text{Tr}\, \underbrace{UU \ldots U}_{R \times T}\, e^{-S}$$

At large T the loop falls off exponentially

$$W(R, T, g) = A e^{-V(R,g)T}$$

and the potential data are fitted according to the expression

$$V(R,g) = -\frac{g^2}{3\pi R} + KR\,,$$

which gives the Coulomb law for small R and the string law for large R. Repeating the calculation for different values of g we find a lattice scale $\tilde{\Lambda}$ in the range of small values of g by fitting the string tension according to the expression

$$\tilde{\Lambda}\sqrt{K} = e^{-\frac{1}{2b_o g^2}}\,,$$

where b_o is a positive constant. This shows that g^2 runs with the dimensionless string tension measured on the lattice. If we replace $\tilde{\Lambda}\sqrt{K}$ by $R\Lambda$ we get the law

$$g(R)^2 = -\frac{1}{2b_o \ln(R\Lambda)}\,.$$

Substituting this expression into the potential one has

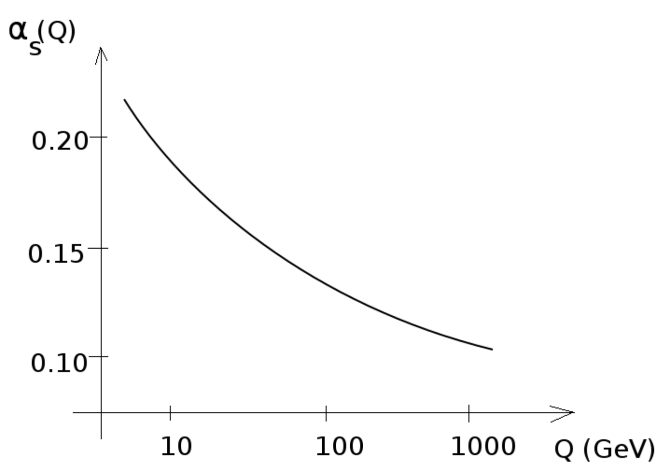
$$V(R) = -\frac{4\alpha_s(R)}{3R}\,, \qquad \alpha_s(R) = -\frac{1}{2b_o \ln(R\Lambda)}\,,$$

where we have defined the strong coupling constant

$$\alpha_s(R) = \frac{g(R)^2}{4\pi}\,.$$

If R corresponds to the energy Q then the strong coupling constant runs according to

$$\alpha_s(Q) = \frac{1}{2b_o \ln\left(\frac{Q}{\Lambda}\right)}\,.$$



Fitting the experimentally measured running coupling with the above law one finds $\Lambda \sim 200\ MeV$, which is the scale of strong interactions. We learn that in order to take the continuum limit one must send g to zero. This quality makes QCD a parameter free theory and conceptually the forefront theory of particle physics. However, the numerical solution does not tell us about the confinement mechanism. In fact, at strong coupling we cen get an idea of chiral symmetry breaking as we show in the following.

4 Strong coupling

In the strong coupling QCD and in the case of large number of colors N the action of the theory is written in terms of the scalar field $\phi(x)$

$$S(\phi) = N \sum_x \ln[m_f + \phi(x)] - 2N \sum_{x,y} \phi(x)(A^{-1})(x,y)\phi(y) \ ,$$

where $A(x,y) = \sum_\mu(\delta_{x+\hat{\mu},y} + \delta_{y+\hat{\mu},x})$.[1] The theory confines the color and $N \to \infty$ is its classical limit. In this limit we can compute the effective theory at the saddle point

$$\frac{\partial S(\phi)}{\partial \phi(x)} = 0 \ .$$

Resulting equations

$$\frac{1}{G(x)} = m_f + \frac{1}{4}\sum_\mu [G(x+\hat{\mu}) + G(x-\hat{\mu})]$$

with $G(x) = 1/[m_f + \phi(x)]$ are solved using the Ansatz

$$G(x) = G_o + \sigma(x) \ ,$$

where the field $\sigma(x)$ is a perturbation around the uniform solution G_o. Substituting the Ansatz in the system and staying with the first order in $G_o^{-1}\sigma(x)$ one has a quadratic equation for G_o

$$\frac{1}{G_o} - m_f - \frac{d}{2}G_o = 0 \qquad \Rightarrow \qquad G_o = \frac{-m_f + \sqrt{m_f^2 + 2d}}{d} \ ,$$

as well as a constraint for the perturbation $\sigma(x)$

$$\sigma(x) + \frac{1}{4}G_o^2 \sum_\mu [\sigma(x+\hat{\mu}) + \sigma(x-\hat{\mu})] = 0 \ .$$

[1] See the appendix of reference [6] and references there in.

Expanding the action up to the second order in $G_o^{-1}G(x)$ and using the constraint one has

$$S(\phi)/N = -V\left(\ln G_o + \frac{d}{4}G_o^2\right) + \frac{1}{8}\left(\frac{4}{G_o^2} - 2d\right)\sum_x \sigma(x)^2 + \frac{1}{8}\sum_{x,\mu}\left[\sigma(x+\hat{\mu}) - \sigma(x)\right]^2 \ .$$

We can read off the free energy

$$\mathcal{F} = NV\left(\ln G_o + \frac{d}{4}G_o^2\right) \ ,$$

and the effective theory

$$S_{\text{eff}}(\sigma)/N = \frac{M^2}{4}\sum_x e^{i\sigma(x)} + \frac{1}{4}\sum_{x,\mu} e^{i\sigma(x+\hat{\mu})}e^{-i\sigma(x)} + h.c. \ ,$$

with

$$M^2 = \frac{4}{G_o^2} - 2d = 2\left(m_f^2 + m_f\sqrt{m_f^2 + 2d}\,\right) \ .$$

In the limit $m_f \to 0$, the action is invariant with respect to global chiral transformations $U(1)_L \times U(1)_R$, i.e.

$$e^{i\sigma(x+\hat{\mu})} \to U e^{i\sigma(x+\hat{\mu})} V^* \ , \qquad e^{i\sigma(x)} \to V e^{i\sigma(x)} U^* \ .$$

Note also that in the chiral limit the fermion condensate is non-zero

$$\Sigma = -\lim_{m_f \to 0}\lim_{V \to \infty}\frac{1}{V}\frac{\partial \mathcal{F}}{\partial m_f} = N\sqrt{\frac{2}{d}}$$

and one finds the Gell-Mann-Oakes-Renner relation

$$M^2 = 2m_f\sqrt{2d} + O(m_f^2) \ .$$

Therefore, chiral symmetry is spontaneously broken to the global $U(1)$ symmetry and the resulting degree of freedom σ describes the single pi-meson of the system. Using a different fermion formulation one recovers the $SU(2)$ flavor symmetry and three degenerate pi-mesons.

This picture is confirmed and enhanced in the subsequent years of research in lattice QCD. Today we are in the era of precision calculations of QCD spectrum, matrix elements, QED corrections and a plethora of other interesting physics phenomena within and beyond the Standard Model of particle physics. Nonetheless, more computer power is needed to reach a higher precision. In the following we give a brief introduction to a new analytical tool in lattice QCD, which allows to access a mechanism for the quark confinement.

5 A mechanism for quark confinement

There are many ways to define the Yang-Mills theory on the lattice. The only requirement is to have a unique continuum limit. From the Wilson theory we know how to approach the continuum limit, therefore this theory is a good basis for any other non-perturbative formulation. The new formulation proposed in reference [6] is an extension of the Wilson theory with larger Wilson loops

$$S_{\text{eff}}(U) = -\frac{1}{\kappa} \sum_{\text{plaquettes}} \text{Tr}\, U_1 U_2 U_3 U_4 + O(\kappa) + \text{h.c.}\ ,$$

where κ plays the role of g^2 in the Wilson theory. We note that the new formulation is local and coincides with the Wilson theory in the limit of vanishing κ, at least classically. The virtue of the formulation is that it may be solved analytically. In the large N limit one can write the theory in terms of a new boson matrix field $\Sigma(x,t,t')$, where t,t' label the lattice sites along an extra dimension of length

$$N_t \propto \frac{1}{\kappa^5}\ .$$

Since κ is related to the length scale in the Wilson theory, it is expected also here to play the same role, thereby giving the extra dimension the meaning of the scale dimension. The action has the form

$$S(\Sigma) = N \sum_{x,t} \left\{ \ln \left[1 + \hat{\gamma}_5(x)\hat{\partial}_t + \Sigma(x) \right] \right\}(t,t) - \frac{N}{2\kappa^2} \sum_{x,y,t,t'} \Sigma(x,t,t')(A^{-1})(x,y)\Sigma(y,t',t)\ ,$$

where A is the same matrix as in the previous section. We see that the color confinement is trivial in this theory. From here one can derive the effective action around the saddle point

$$S(\tilde{G})/N = -\sum_{x,\omega} M(\omega)^2 e^{i\tilde{G}(x,\omega)} - \sum_{x,\mu,\omega} e^{i\tilde{G}(x+\hat{\mu},\omega)} e^{-i\tilde{G}(x,\omega)} + h.c.\ ,$$

where ω are the momenta along the extra dimension. The mass spectrum of the theory is given by the expression

$$M(\omega)^2 = \frac{\mu(\omega)^2 + \mu(\omega)\sqrt{\mu(\omega)^2 + 8d\kappa^2}}{2\kappa^2}\ , \qquad \mu(\omega) = |1 + i\sin\omega|\ ,$$

bounded from below by the mass gap

$$M_o^2 = \frac{1 + \sqrt{1 + 8d\kappa^2}}{2\kappa^2}\ .$$

In the case of vanishing κ the mass gap diverges as $1/\kappa$. The theory shows that the Yang-Mills field is massive.

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