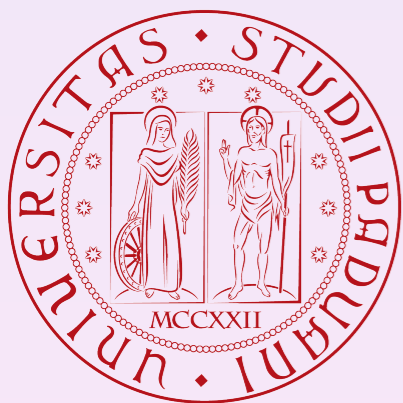


# Top quark mass effects in $gg \rightarrow ZZ$ at 2-loops and off-shell Higgs interference

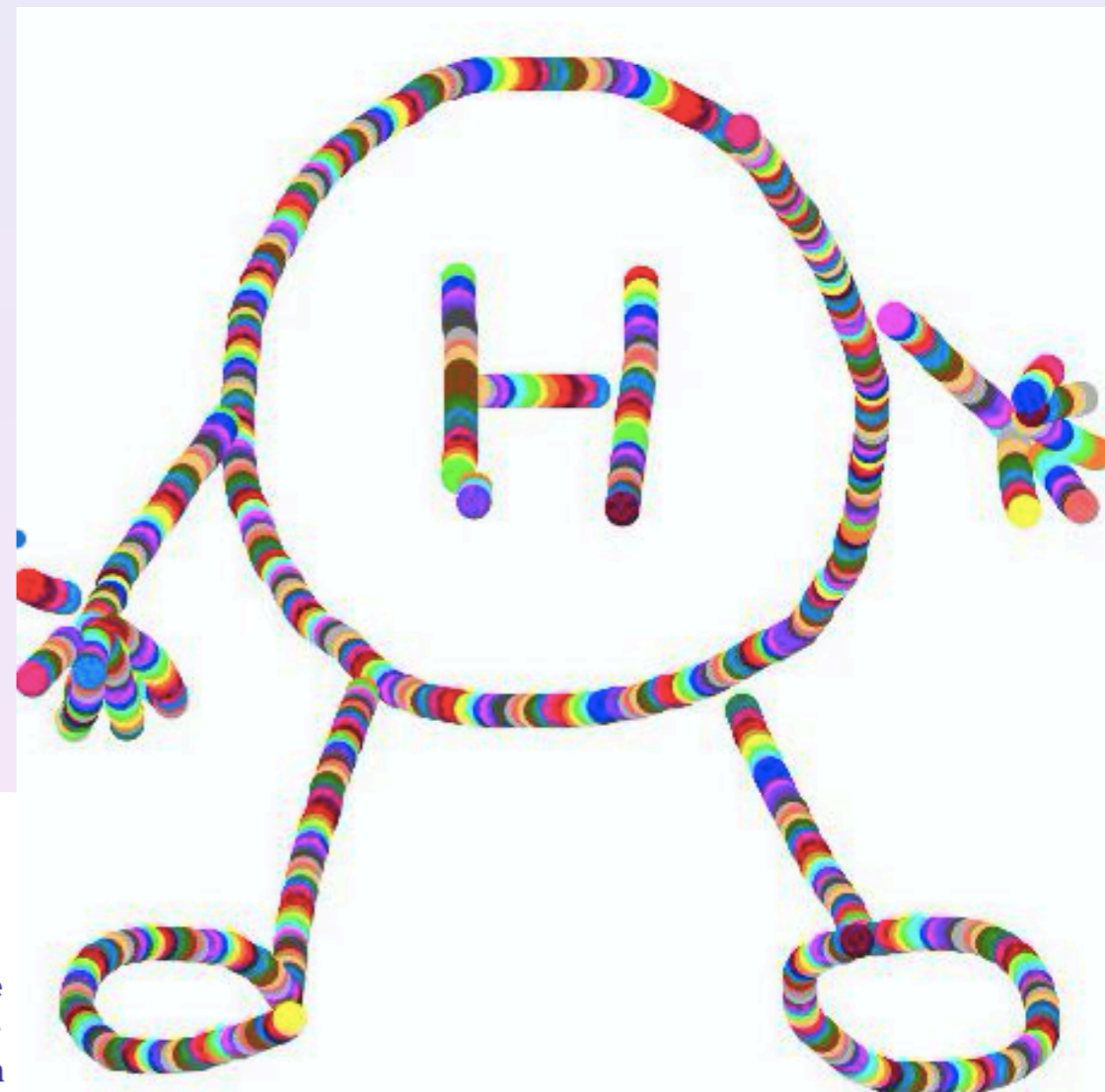
Ramona Gröber

work in collaboration with A. Maier  
and T. Rauh, arXiv: 1908.04061 (to  
appear in PRD)

partially based also on work with A. Maier, T.  
Rauh, JHEP 1803 (2018) 020  
and work with J. Davies, A. Maier, T. Rauh and  
M. Steinhauser, PRD100 (2019) 034017



25/11/2019

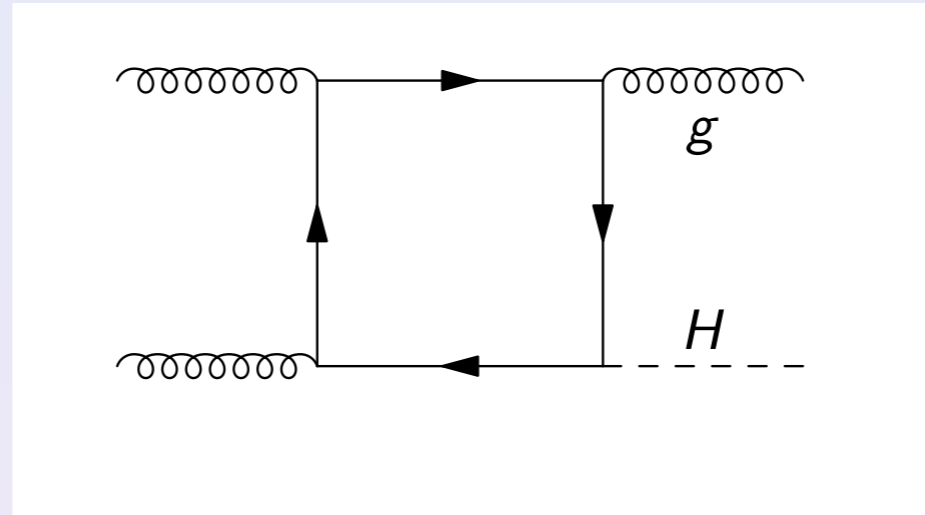
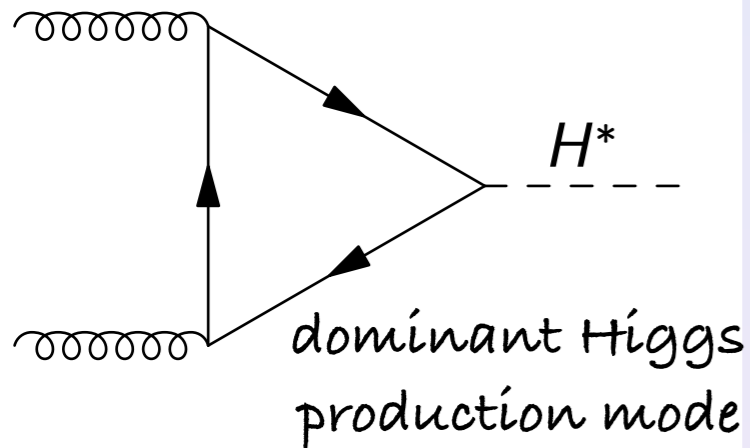


# Outline

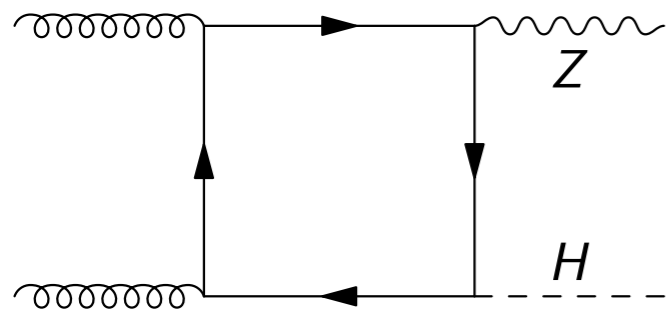
- Gluon fusion processes and top quark mass dependence
- Our method
- Proof of Method: Di-Higgs production
- ZZ production
- Conclusion/Outlook

# Gluon fusion processes at LO

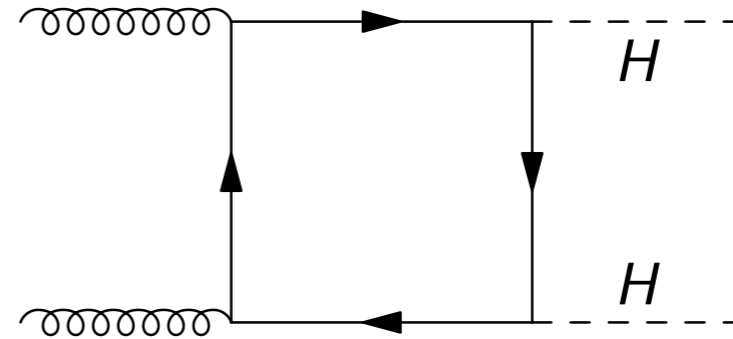
Gluon fusion processes of high phenomenological relevance for Higgs physics at LHC



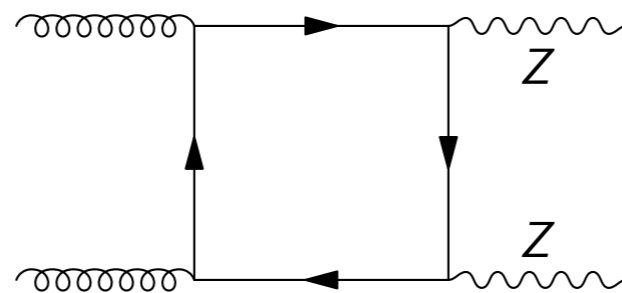
measurement light  
Yukawa couplings  
[Bishara et al '16, Soreq et  
al. '16], resolves  
degeneracy top  
Yukawa effective  $ggH$   
[Azatov, Paul '13, Grojean et  
al. '13]



NNLO to  $ZH$   
production, in  
particular  
important at large  
 $M_{HZ}$



measurement  
trilinear  
Higgs self-  
coupling



Interference  
off-shell  
Higgs,  
measurement  
Higgs width

[Kauer, Passarino '12,  
Caola, Melnikov '13,  
Campbell, Ellis, Williams  
'13]

# Gluon fusion processes at higher orders

Computation of  $2 \rightarrow 2$  multi-scale processes at two-loop order difficult

Bottleneck: virtual corrections, dependence on several scales

Well-established method:

Asymptotic expansion in large top mass (LME)

$$\frac{1}{(p+q)^2 - m^2} \approx \frac{1}{p^2 - m^2} \left( 1 - \frac{2p \cdot q + q^2}{p^2 - m^2} + \dots \right) \quad \text{simplifies integrals dramatically}$$

$p$  loop momentum,  $q$  external momentum

At LO: Taylor expansion in

$$\frac{1}{m_t^2}$$

At NLO: Taylor expansion in

$$\frac{1}{m_t^2}$$

+ log terms from IR divergent diagrams

# Gluon fusion processes at higher orders

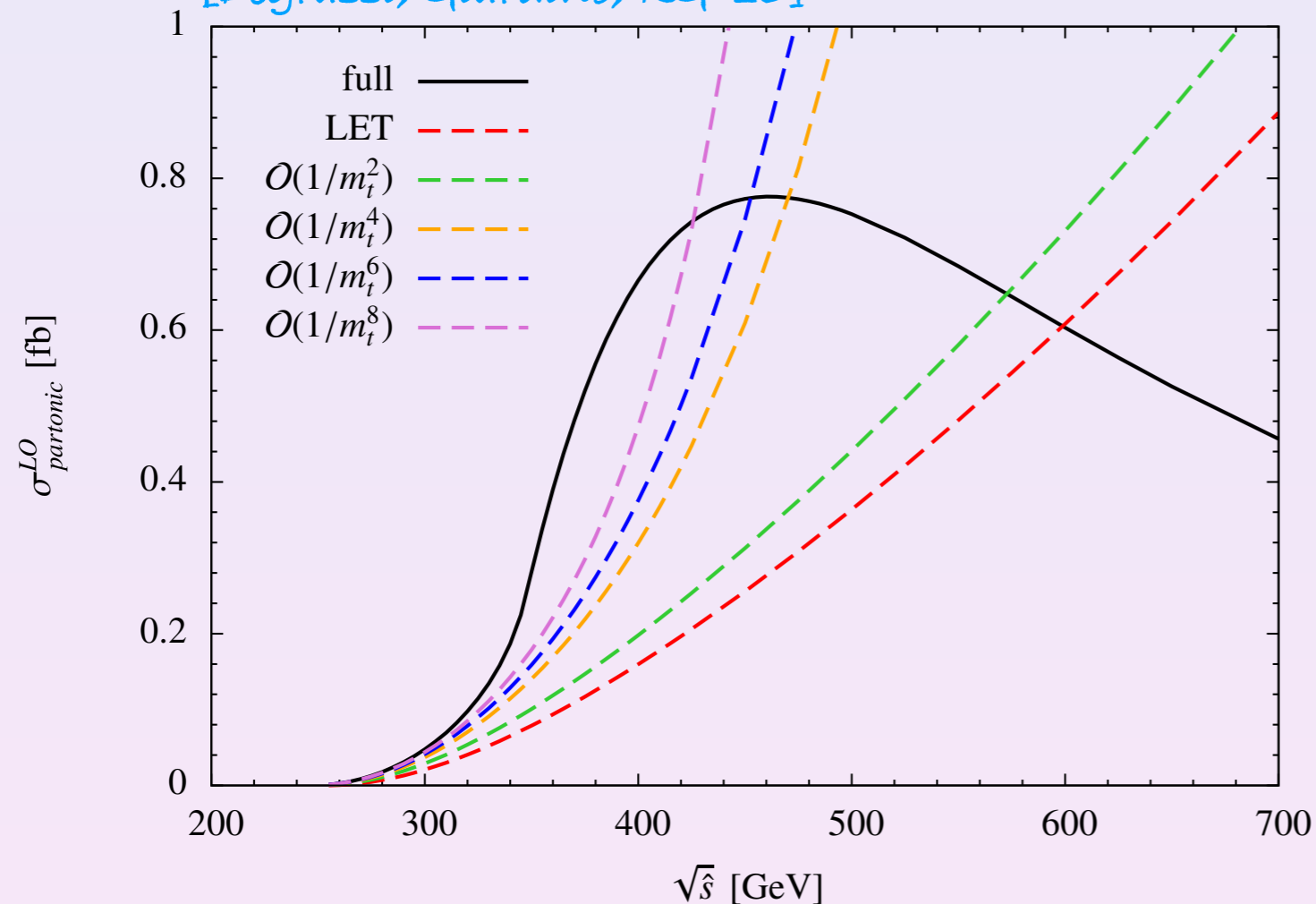
Computation of  $2 \rightarrow 2$  multi-scale processes at two-loop order difficult

Difficult part: virtual corrections, dependence on several scales

Well-established method:

Asymptotic expansion in large top mass (LME)

[Degrassi, Giardino, RG '16]



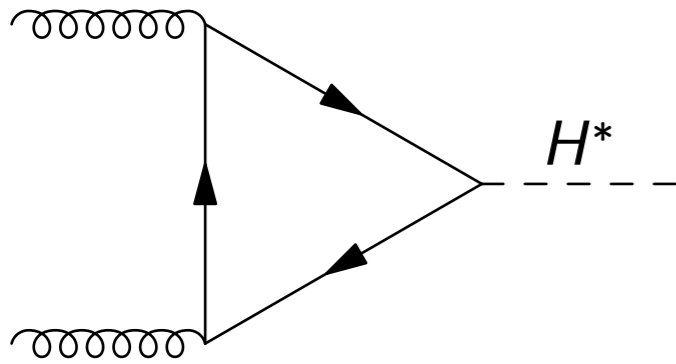
HH at LO

valid for  $\hat{s}, \hat{t}, \hat{u}, m_H^2 \ll 4m_t^2$

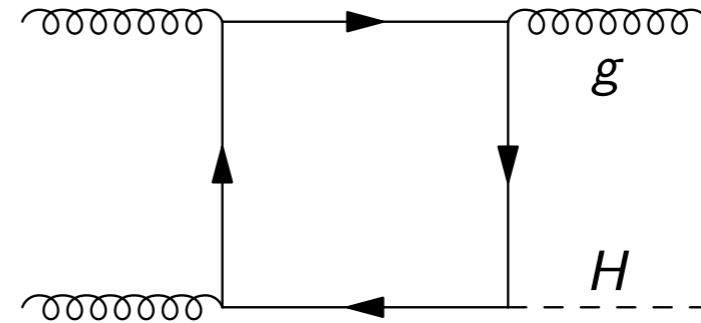
at NLO: improvement by reweighting with full LO cross section

# Gluon fusion processes at NLO

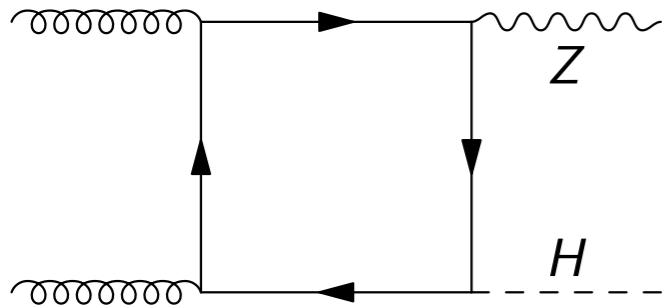
## Results in LME



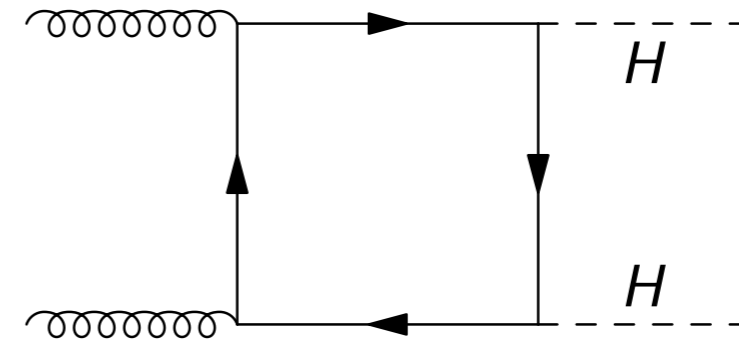
[Dawson '91,  
Djouadi, Spira,  
Zerwas '91]



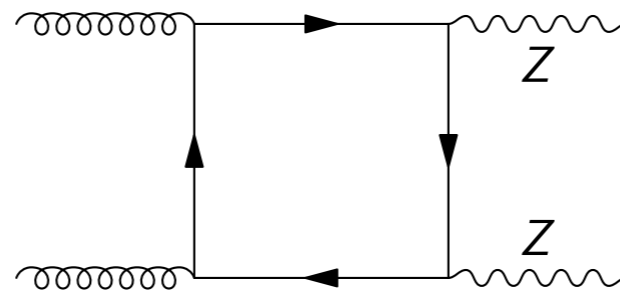
[Harlander,  
Neumann,  
Ozeren,  
Wiesemann '12,  
...]



[Hasselhuhn,  
Luthe,  
Steinhauser  
'16]



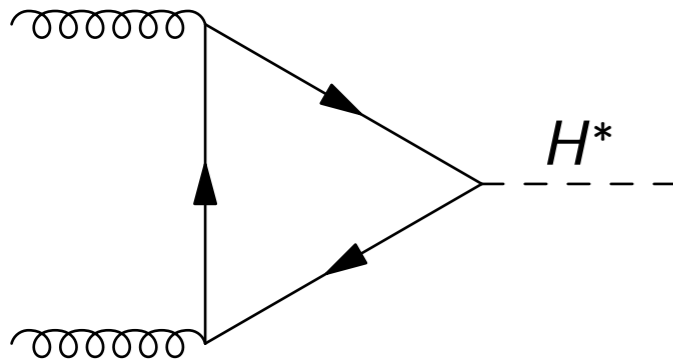
[Grigo, Hoff,  
Steinhauser '15,  
Degrassi, Giardino,  
RG '16]



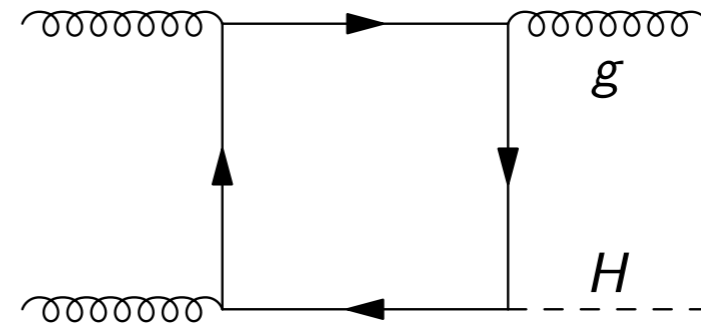
[Melnikov,  
Dowling '15,  
Campbell et al.  
'16, Caola et al.  
'16]

# Gluon fusion processes at NLO

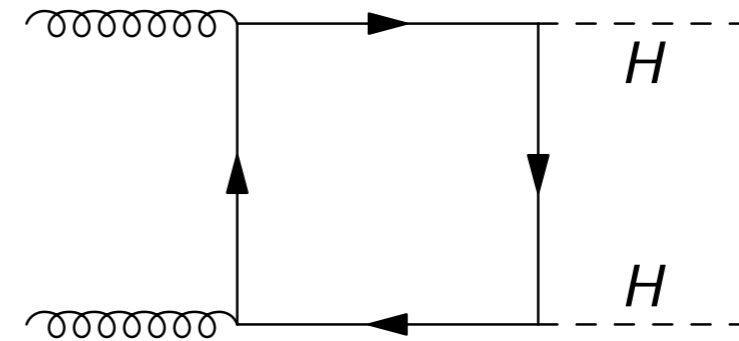
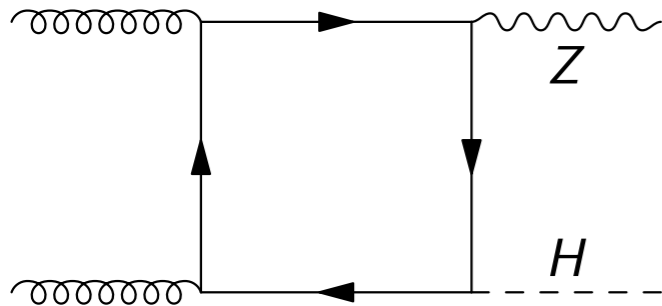
Results in full mass dependence



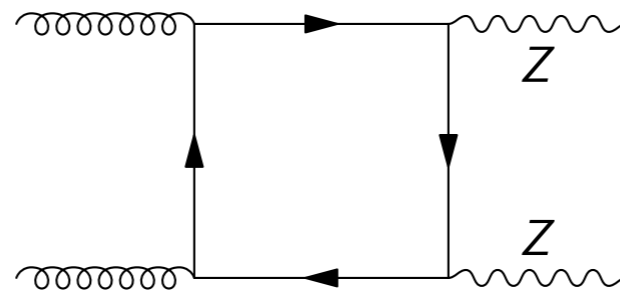
[Spira, Djouadi,  
Graudenz, Zerwas  
'95, Harlander,  
Kant '05,  
Anastasiou et al.  
'06, Aglietti et al.  
'06]



[Jones, Kerner  
Luisoni '18]



[Borowka et al.  
'16,  
Baglio et al. '18]



[massless loops  
in Caola et al.  
'15, '15, '16, von  
Manteuffel,  
Tancredi '15]



# Our idea

- Construct an approximation that works in (nearly) whole phase space based on simpler expansion

Based on LME and expansion around non-relativistic top threshold (THR) combined by Padé approximants

- Demonstrate method on a process that is known in full mass dependence

HH as it carries full complexity of  $2 \rightarrow 2$

[RG, Maier, Rauh '17]

- Apply to other cases

ZZ

[RG, Maier, Rauh '19]

- Apply to higher loop orders

off-shell single Higgs production

[Davies, RG, Maier, Rauh, Steinhauser '19]



# Padé approximants

- Combine different expansions using Padé approximants

$$[n/m](\omega) = \frac{\sum_{i=0}^n a_i \omega^i}{1 + \sum_{j=1}^m b_j \omega^j}$$

Construct them from large mass expansion and threshold expansion

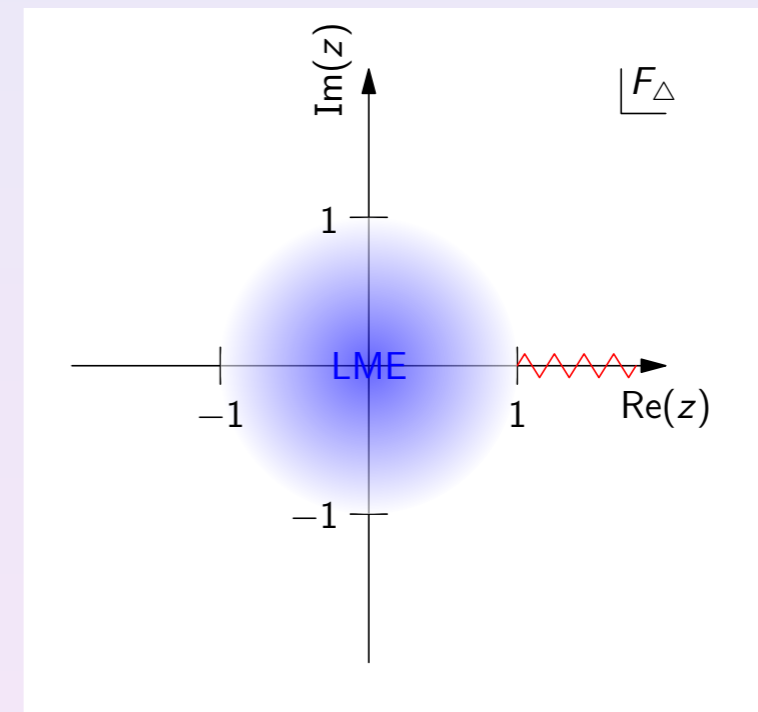
→ LME from literature, THR computed by us

- Conformal mapping

[Fleischer, Tarasov '98]

$$z = \frac{4\omega}{(1+\omega)^2}$$

$$z = (\hat{s} + i0)/(4m_t^2)$$



# Padé approximants

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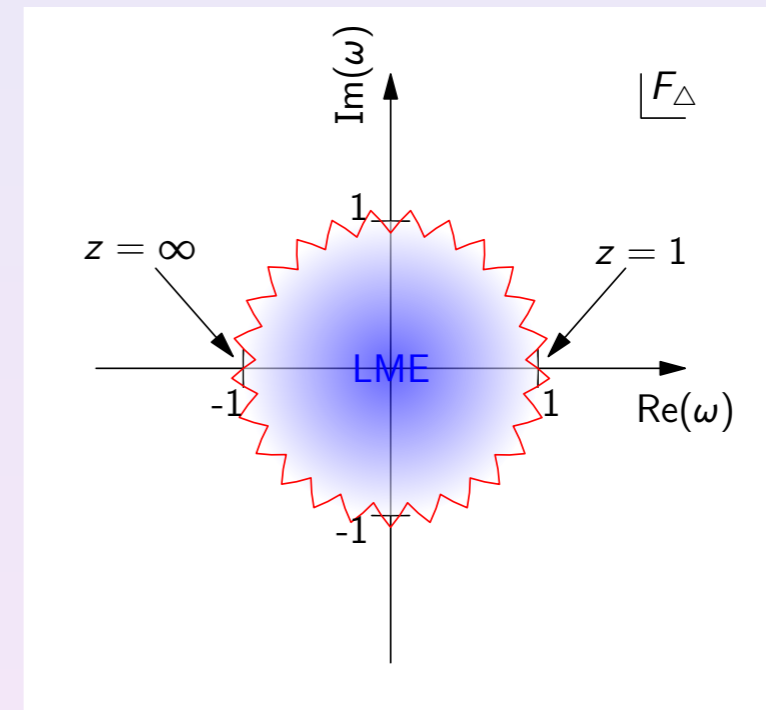
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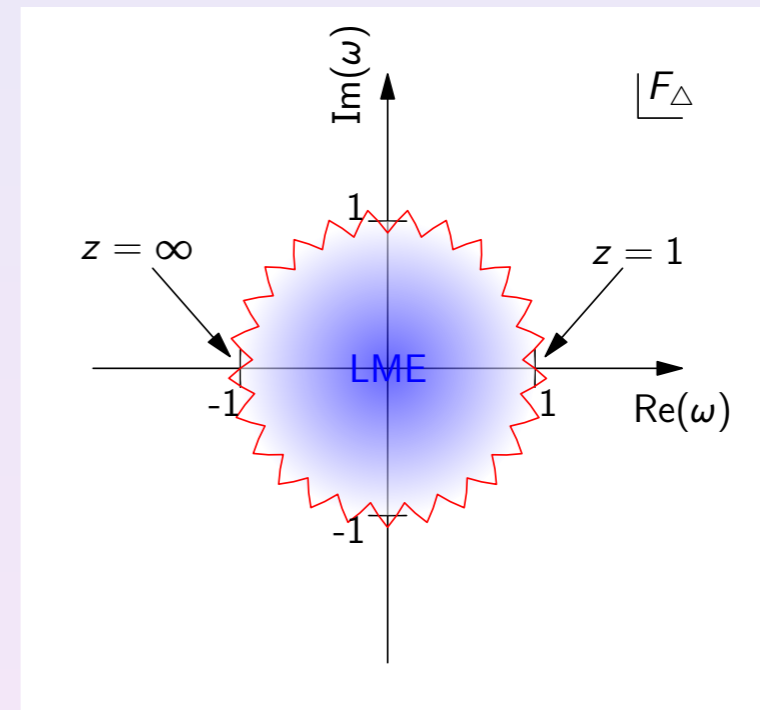
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[Fleischer, Tarasov '98]

$$z = \frac{4\omega}{(1 + \omega)^2}$$

$$z = (\hat{s} + i0)/(4m_t^2)$$



- Form factor for  $z \rightarrow \infty$

implemented by a rescaling

$$[n/m](\omega) = \frac{(1 + a_R z(\omega))}{z(\omega)^i} F$$

$i=0$  for HH,  $i=0,1$  for ZZ

varying  $a_R$  allows for an error estimate

# Threshold expansion (THR)

- Near threshold: tops can only be on-shell when non-relativistic

Hierarchy

$$E_t \sim m_t(1-z) \ll p_t \sim m_t\sqrt{1-z} \ll m_t$$

THR by using based on PNRQCD and SCET [Caswell, Lepage '85; Pineda, Soto '97; Beneke '98, Luke et al '00, ... Bauer et al. '00, '01, ...]  
"Unstable-Particle Effective Theory" [Beneke, Chapovsky, Signer, Zanderighi '03, '04]

for higher powers in THR also expansion by regions [Beneke, Smirnov '98, Jantzen '11]

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 "Unstable-Particle Effective Theory" [Beneke, Chapovsky, Signer, Zanderighi '03, '04]

for higher powers in THR also expansion by regions [Beneke, Smirnov '98, Jantzen '11]

- Result is power series in  $\sqrt{1-z}$ ,  $\ln(1-z)$

$$\begin{aligned}
 F_{\Delta}^{(0)} &\stackrel{z \rightarrow 1}{\sim} 2\pi(1-z)^{3/2} + \frac{13\pi}{3}(1-z)^{5/2} + \mathcal{O}((1-z)^{7/2}), \\
 F_{\Delta}^{(1),\text{fin}} &\stackrel{z \rightarrow 1}{\sim} \frac{4\pi^2}{3}(1-z)\ln(1-z) + \frac{\pi}{12} \left[ -5\pi^2 - \frac{124}{3} \right] (1-z)^{3/2} \\
 &\quad + \frac{8\pi^2}{9}(1-z)^2\ln(1-z) + \mathcal{O}((1-z)^{5/2}), \\
 F_{\Delta}^{(2),\text{fin}} &\stackrel{z \rightarrow 1}{\sim} -\frac{8\pi^3}{27}(3+\pi^2)\sqrt{1-z} + \frac{\pi^2}{54} \left[ (458 - 15\pi^2 - 44n_f)\ln(1-z) \right. \\
 &\quad \left. - (99 - 6n_f)\ln^2(1-z) \right] (1-z) + \mathcal{O}((1-z)^{3/2}).
 \end{aligned}$$

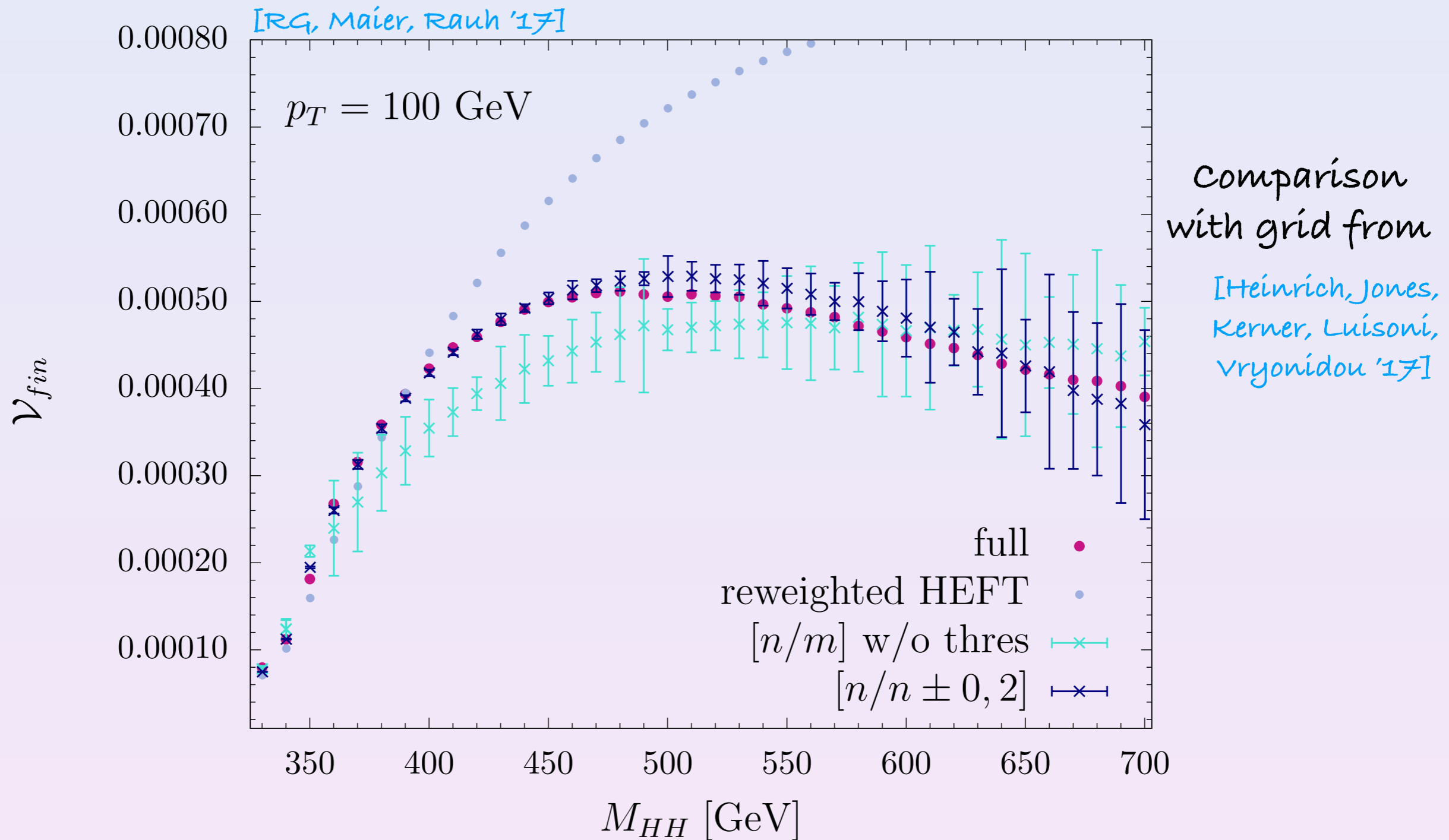
Log-terms can't be reproduced by Padé ansatz

Building Padés for subtracted functions

(Instead log terms from IR divergent diagrams in LME are accounted for by building separate Padés)

Dí-Higgs production

# Results for di-Higgs at NLO



THR expansion important to describe full top mass dependence correctly



ZZ production

# ZZ production

- We computed the top mass dependence of the virtual corrections of the on-shell form factors that interfere with Higgs-amplitude by means of Padé approximants

- Two form factors interfering with Higgs amplitude

$$|\mathcal{B}\rangle = \frac{\delta^{AB}}{N_A} (p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu) P_Z^{\alpha\rho}(p_3) P_{Z,\rho}^\beta(p_4) |B_{\mu\nu\alpha\beta}^{AB}\rangle$$

$$|\mathcal{B}\rangle = \frac{ig_W^2}{4\cos^2\theta_W} \left( v_t^2 |\tilde{\mathcal{B}}_{VV}\rangle + a_t^2 |\tilde{\mathcal{B}}_{AA}\rangle \right),$$

Instead we build Padés for

vanishes for  $z \rightarrow \infty$   $\longrightarrow$   $|\tilde{\mathcal{B}}_{AA-VV}\rangle \equiv |\tilde{\mathcal{B}}_{AA}\rangle - |\tilde{\mathcal{B}}_{VV}\rangle$

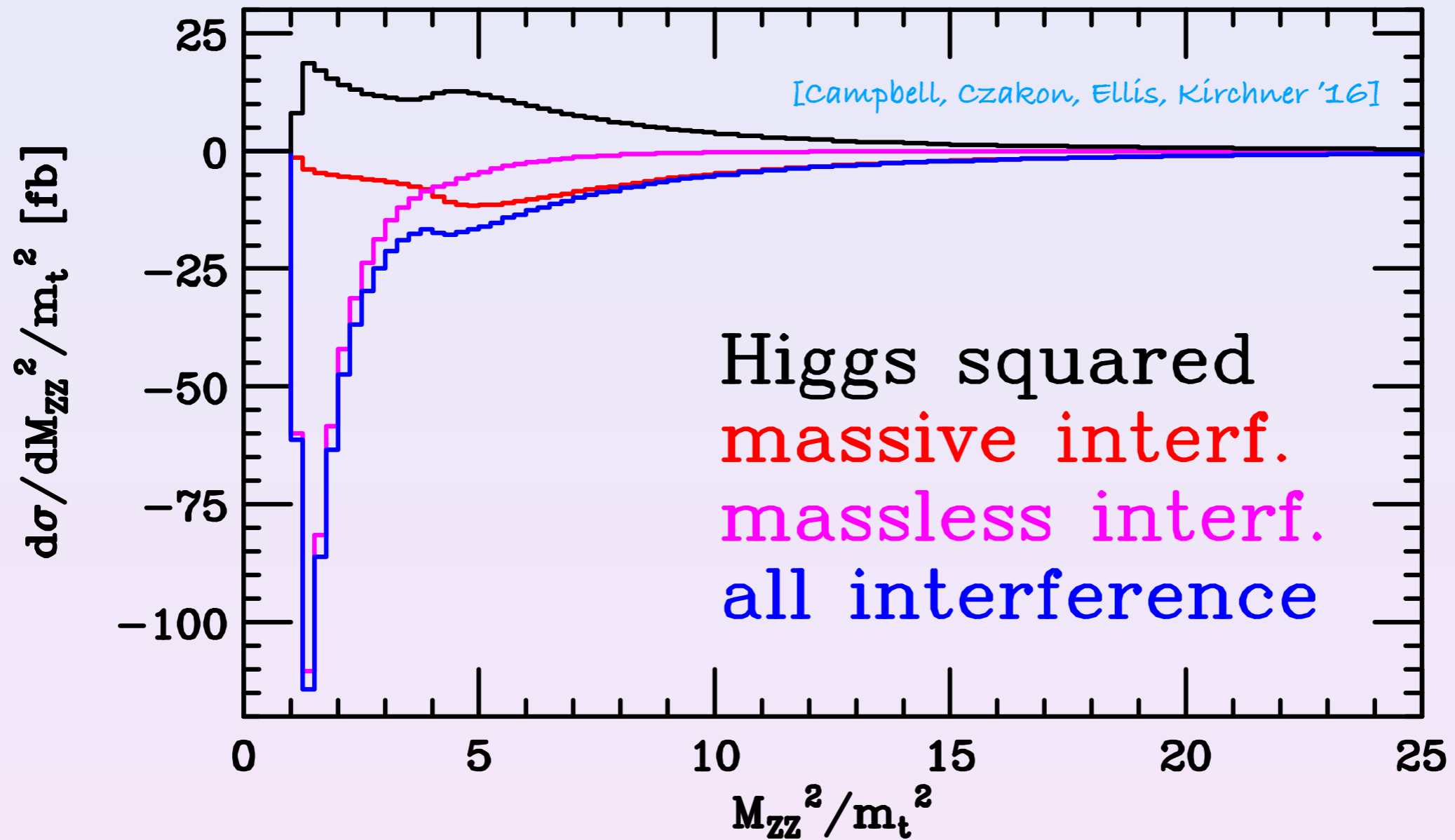
- Exclude Padé approximants with poles in

$$\text{Re}(z(\omega)) > 0 \quad \text{and} \quad |\omega| < 1.2.$$

- Construct 100 Padés for varying  $[m/m]$  (diagonal and next-to-diagonal) for varying

$$a_R \in [0.1, 10]$$

# Importance of top loops



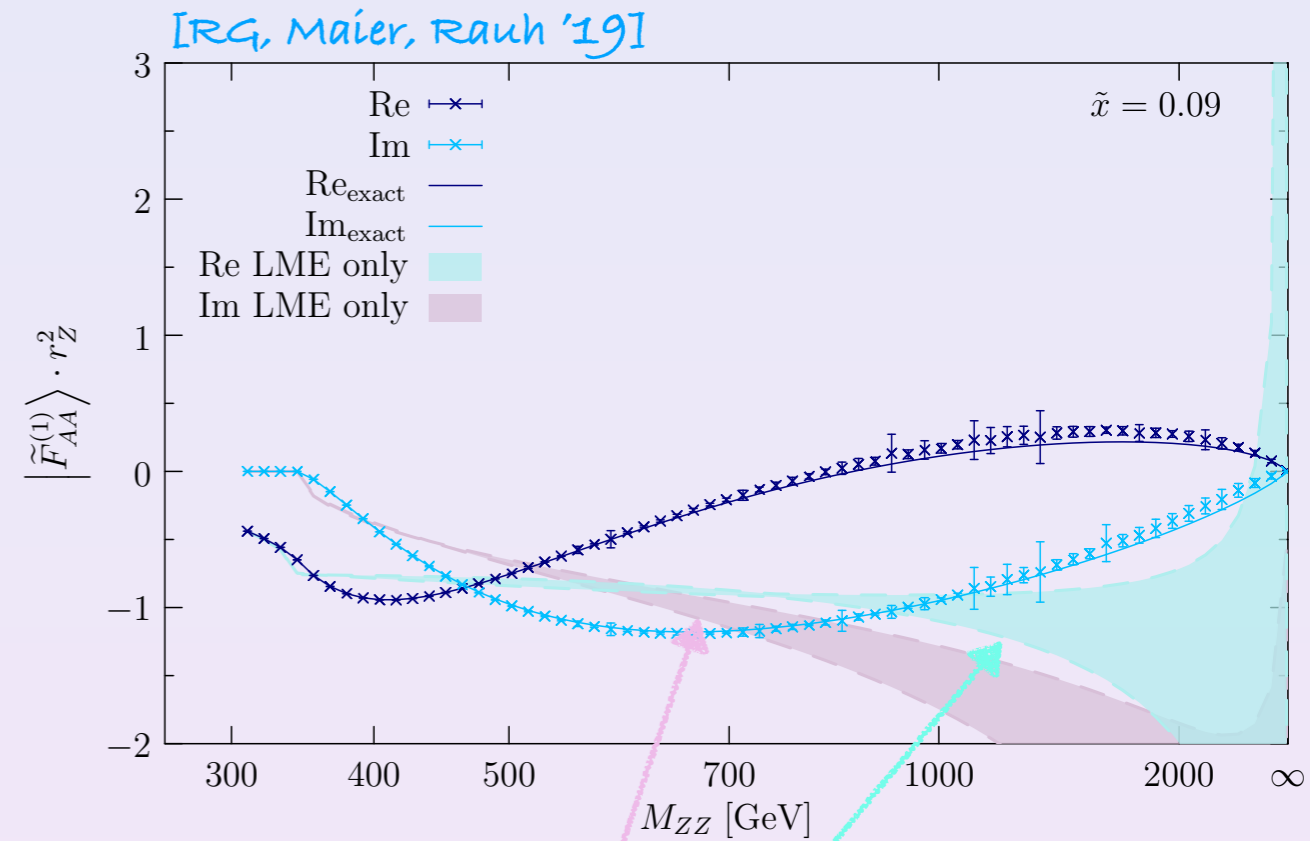
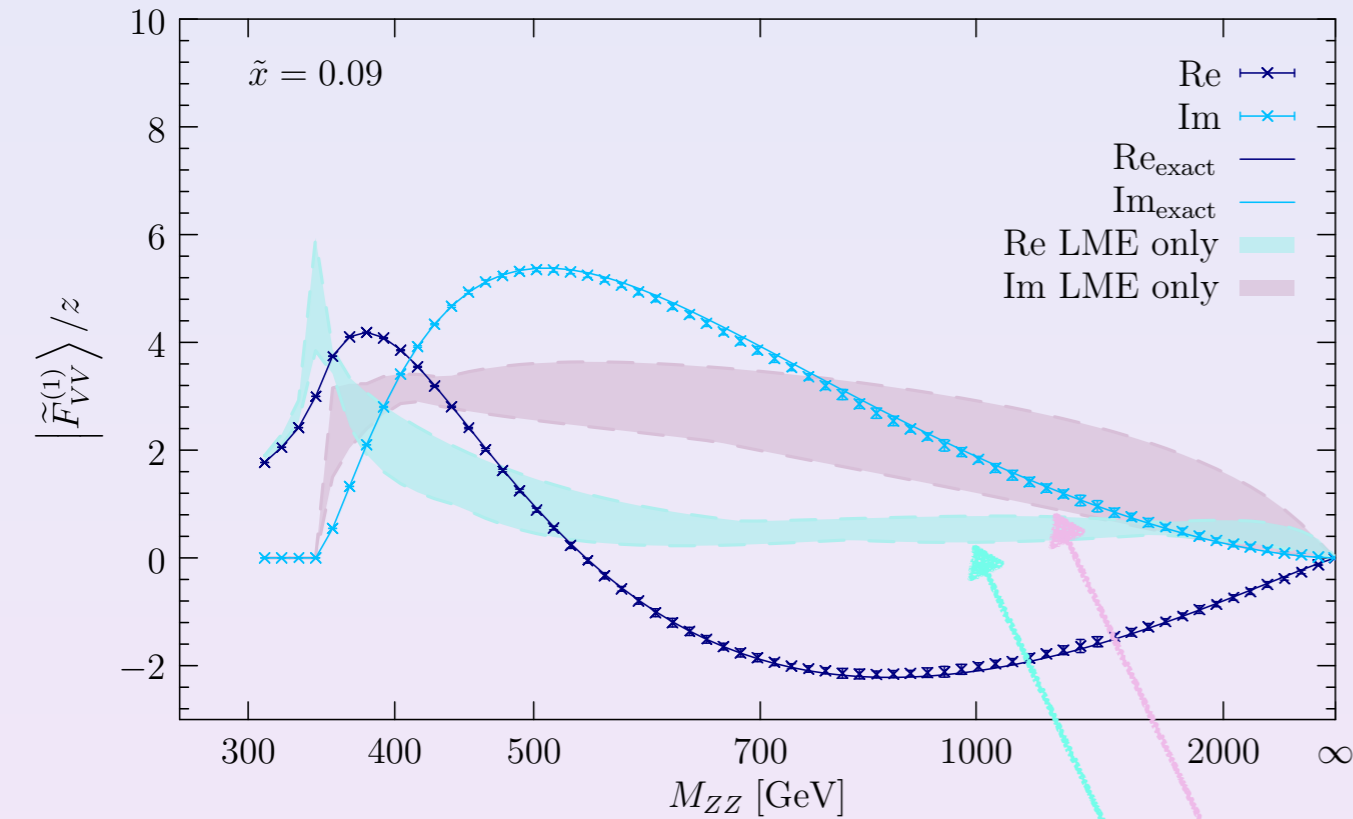
Top loops especially important in part of phase space where LME can't be applied.

# ZZ form factors at LO

$$\tilde{x} = \frac{p_T^2 + m_Z^2}{m_{ZZ}^2}$$

$$z = \frac{M_{ZZ}^2}{4m_t^2}$$

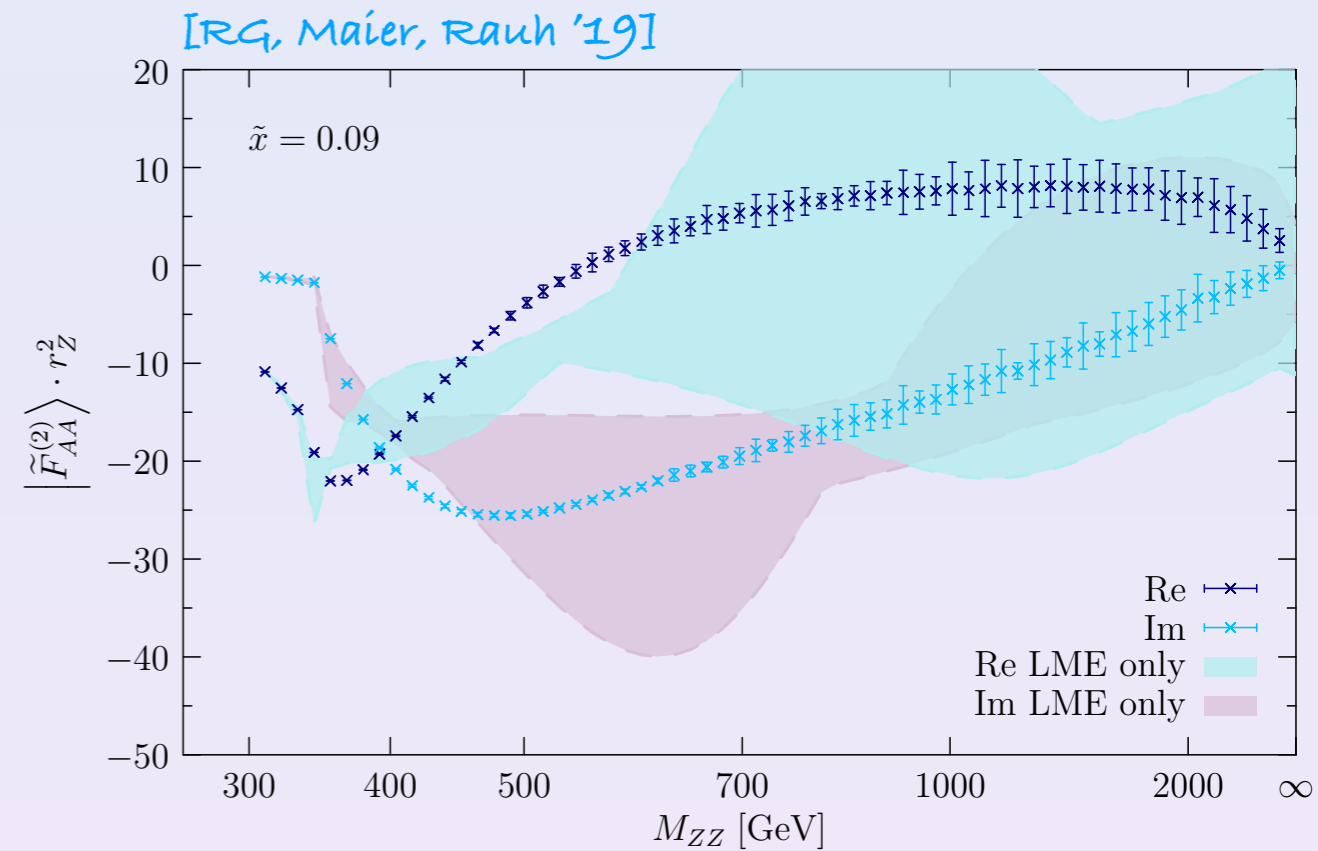
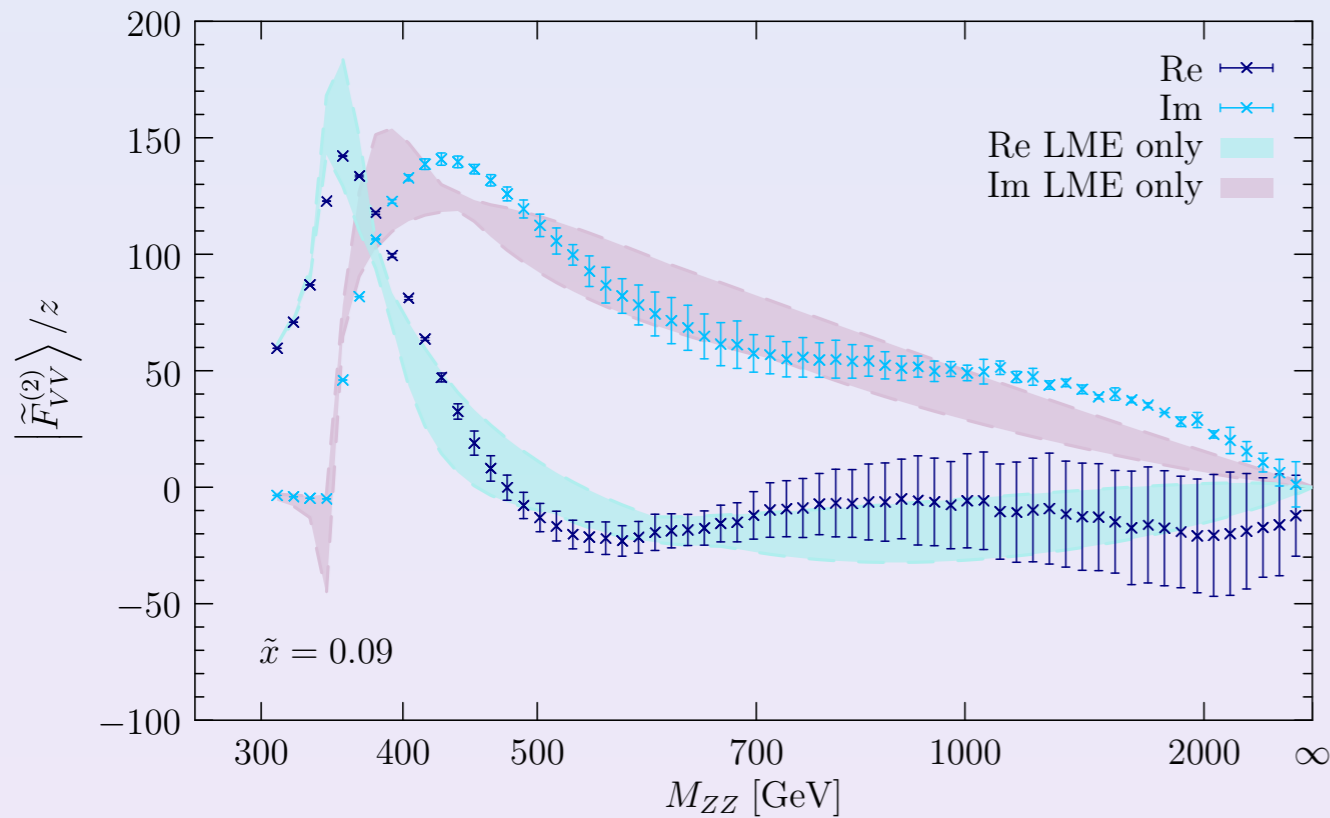
$$r_Z = \frac{M_Z^2}{M_{ZZ}^2}$$



Padé approximants as constructed from LME by

[Campbell, Czakon, Ellis, Kirchner '16]

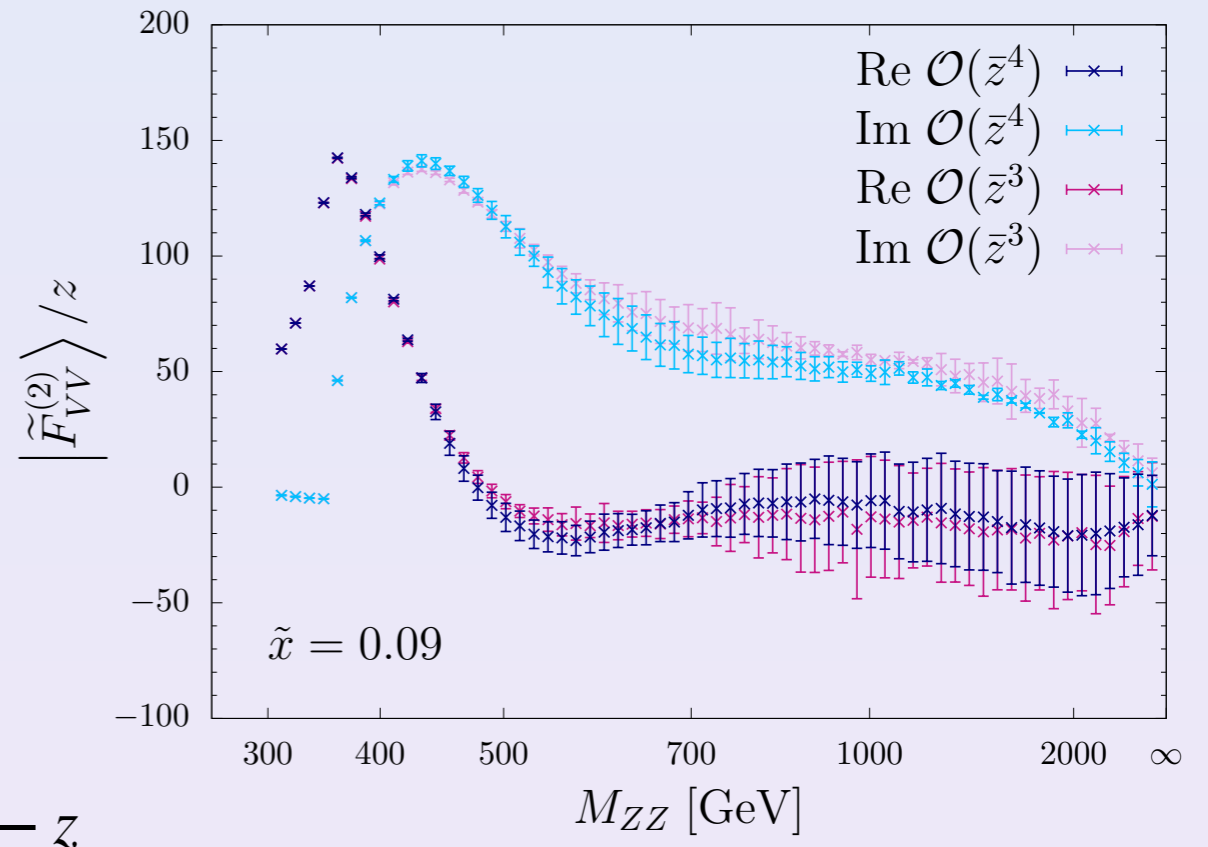
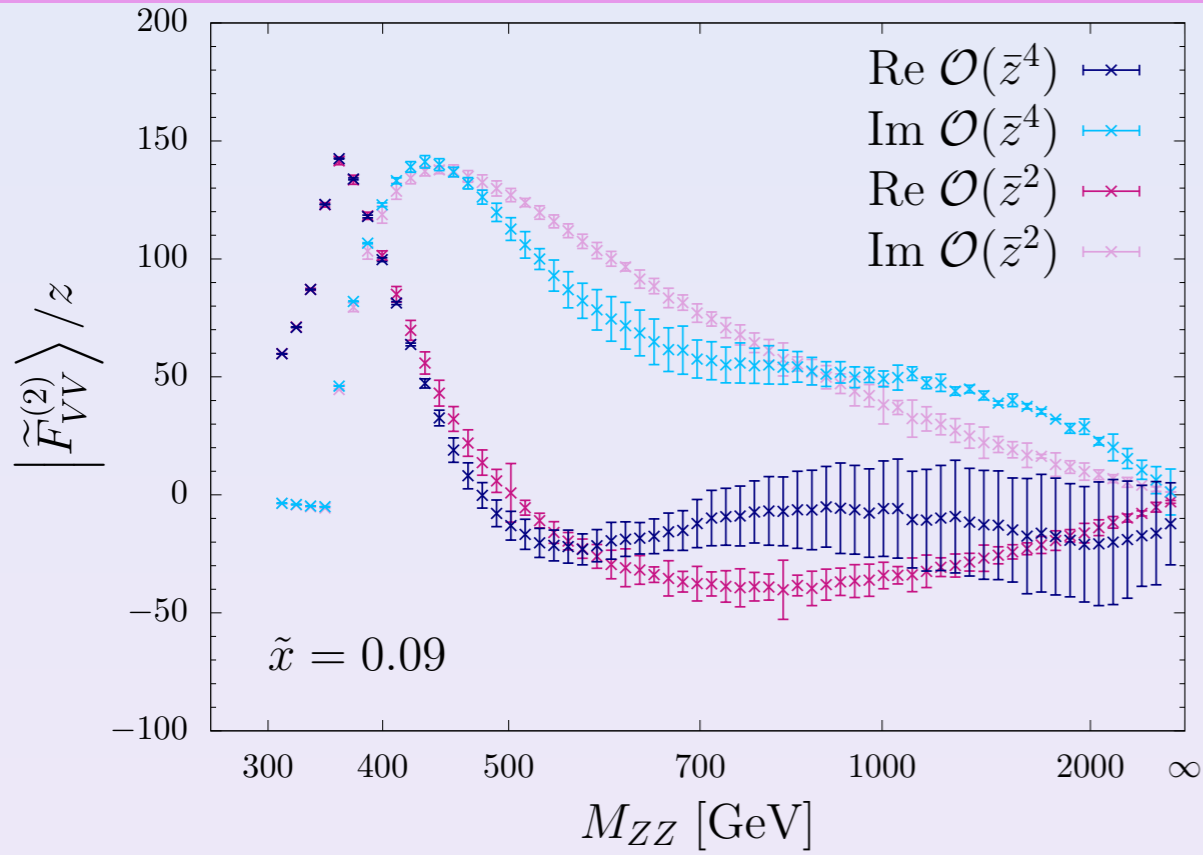
# ZZ form factors at NLO



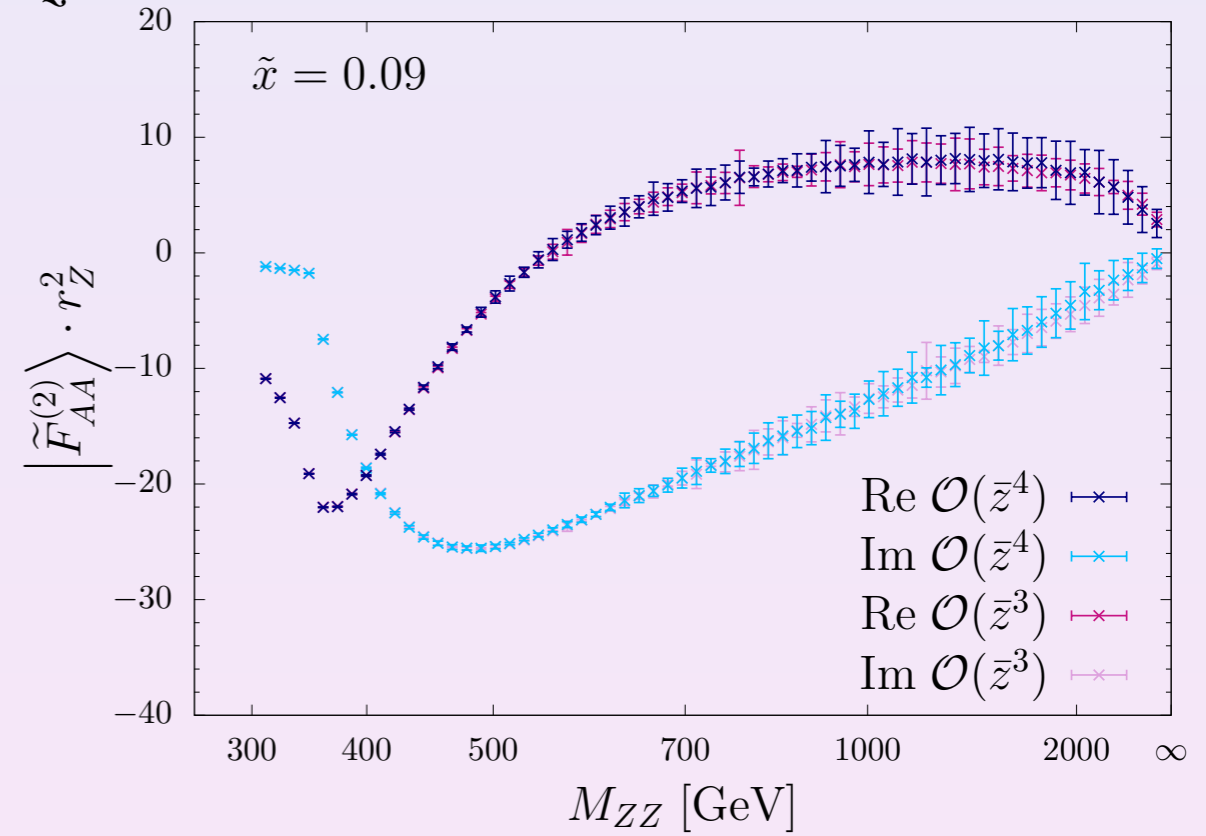
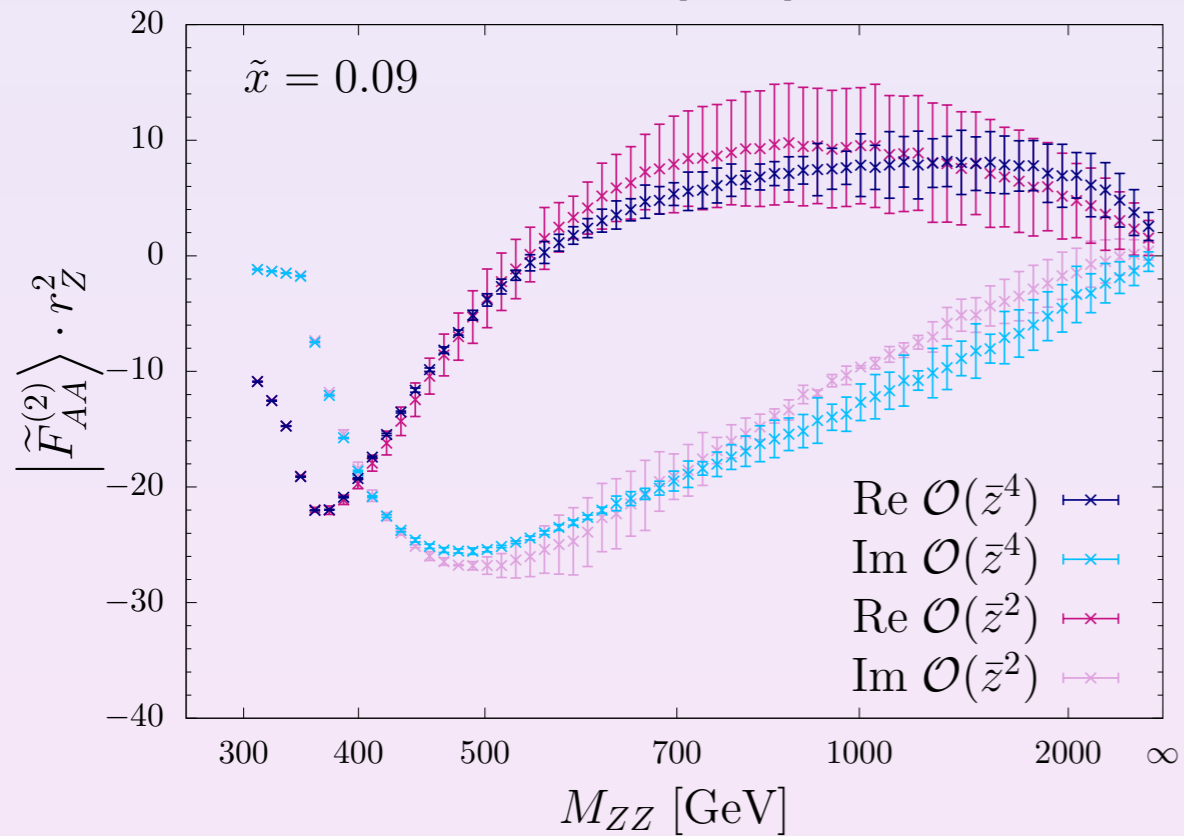
uncertainty from Padé construction also under control at NLO

check though convergence adding different amount of terms in expansion

# Convergence at NLO



$$\bar{z} = 1 - z$$



[RG, Maier, Rauh '19]

# Conclusion and Outlook

- Top quark mass effects important for many gluon fusion processes
- Can be approximated well by Padé approximants constructed from LME and THH expansions
- Last Higgs off-shell meeting: Full top mass effects currently computed by [\[Agrawal, von Manteuffel\]](#), looking forward for comparison

## Outlook:

- more form factors (non-interfering with the Higgs amplitude)
- combination with real corrections and massless loops
- application for off-shell Z bosons
- our method can also be applied at higher loop orders, see [\[Davies, RG, Maier, Rauh, Steinhauser '19\]](#) (and appendix) for single Higgs production at NNLO



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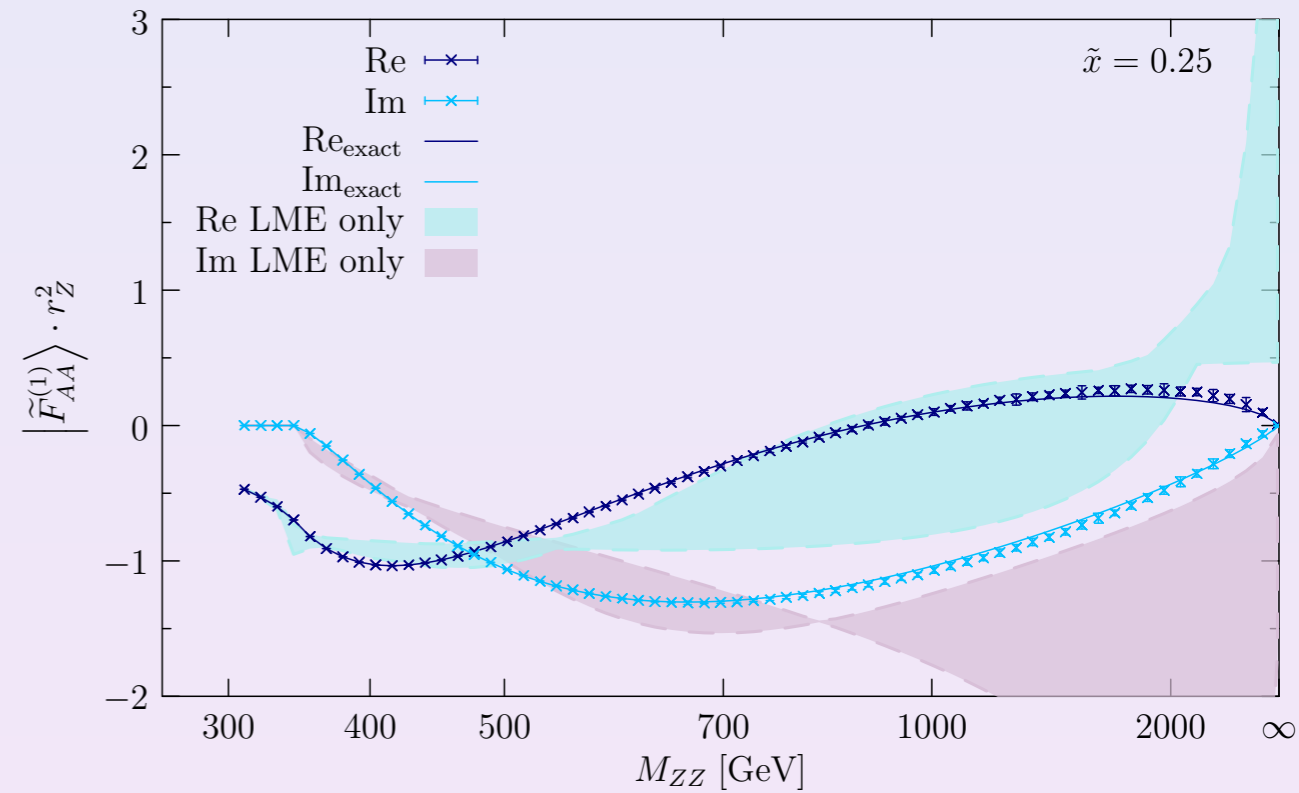
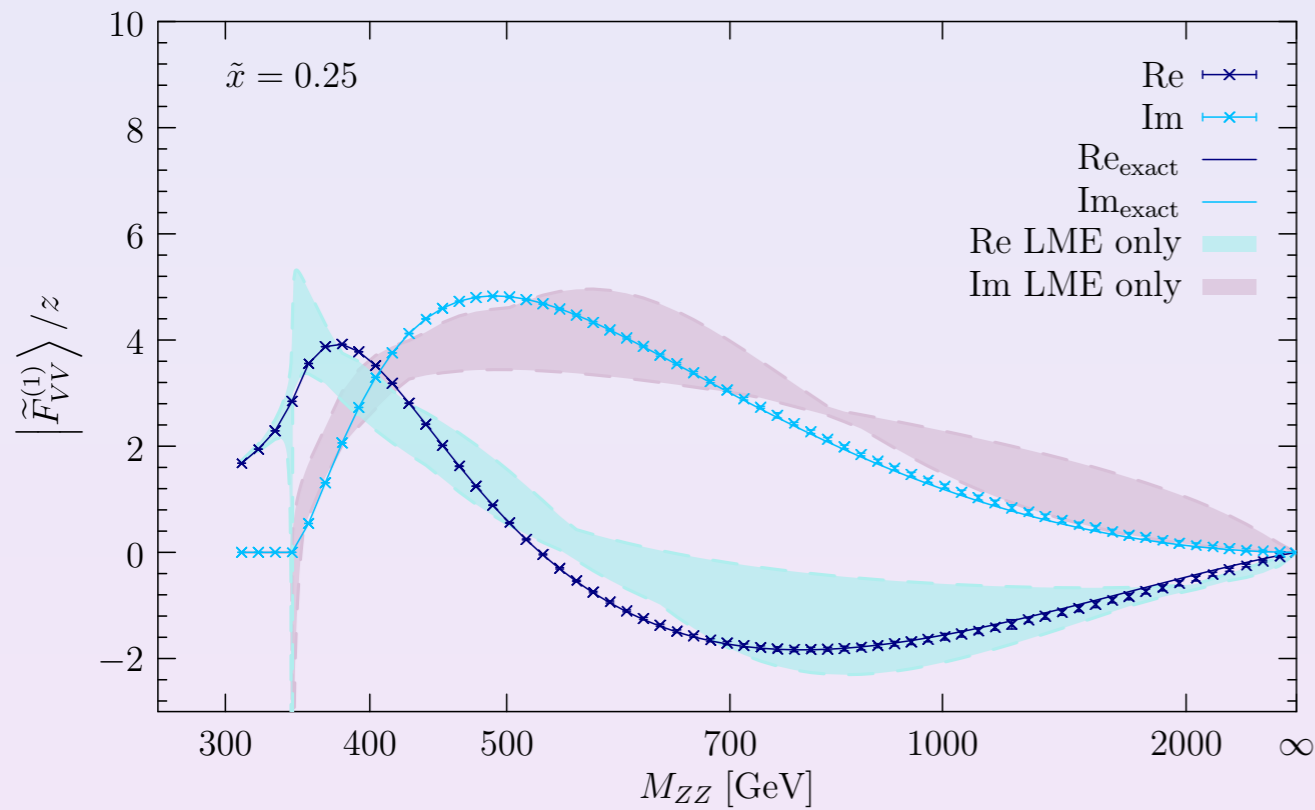
Thanks for your attention!

Backup

# ZZ form factors at LO

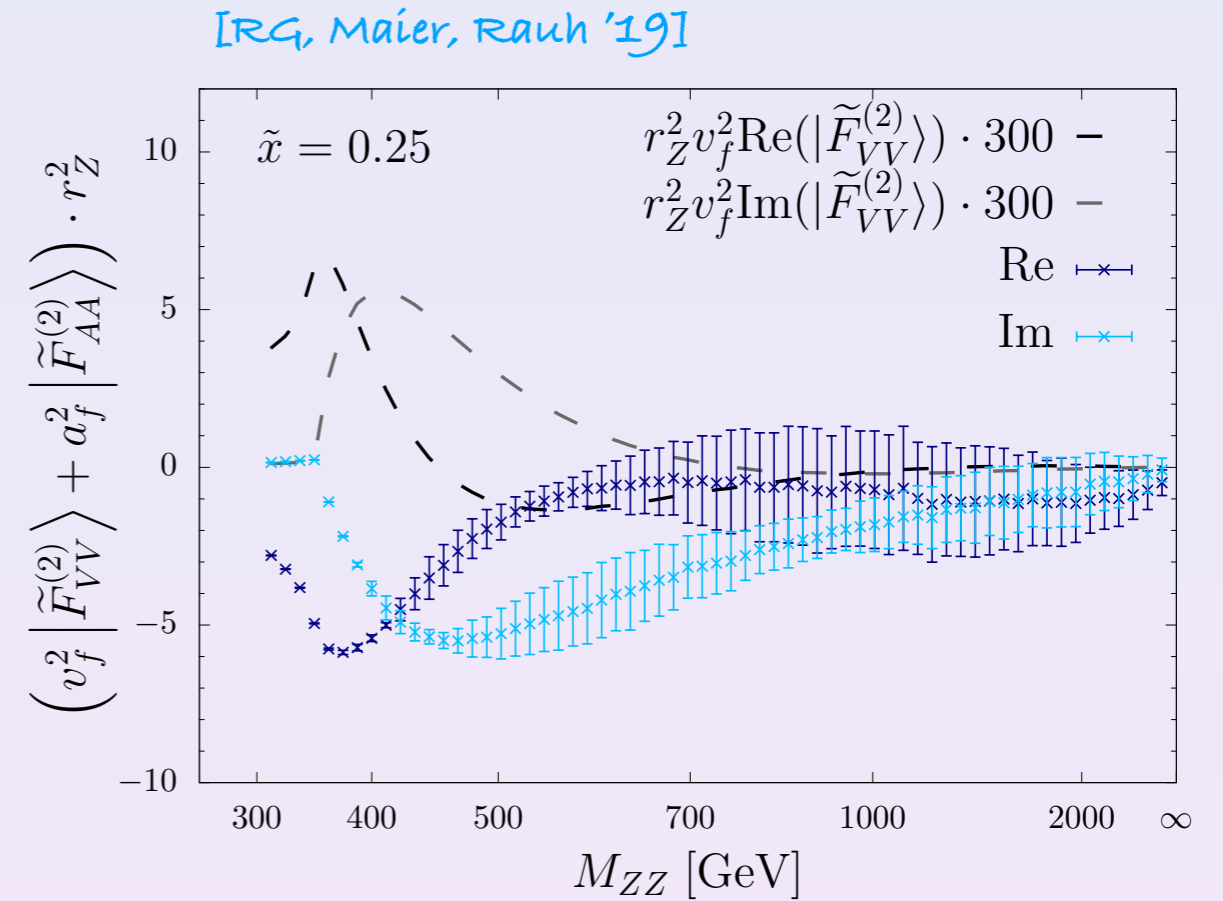
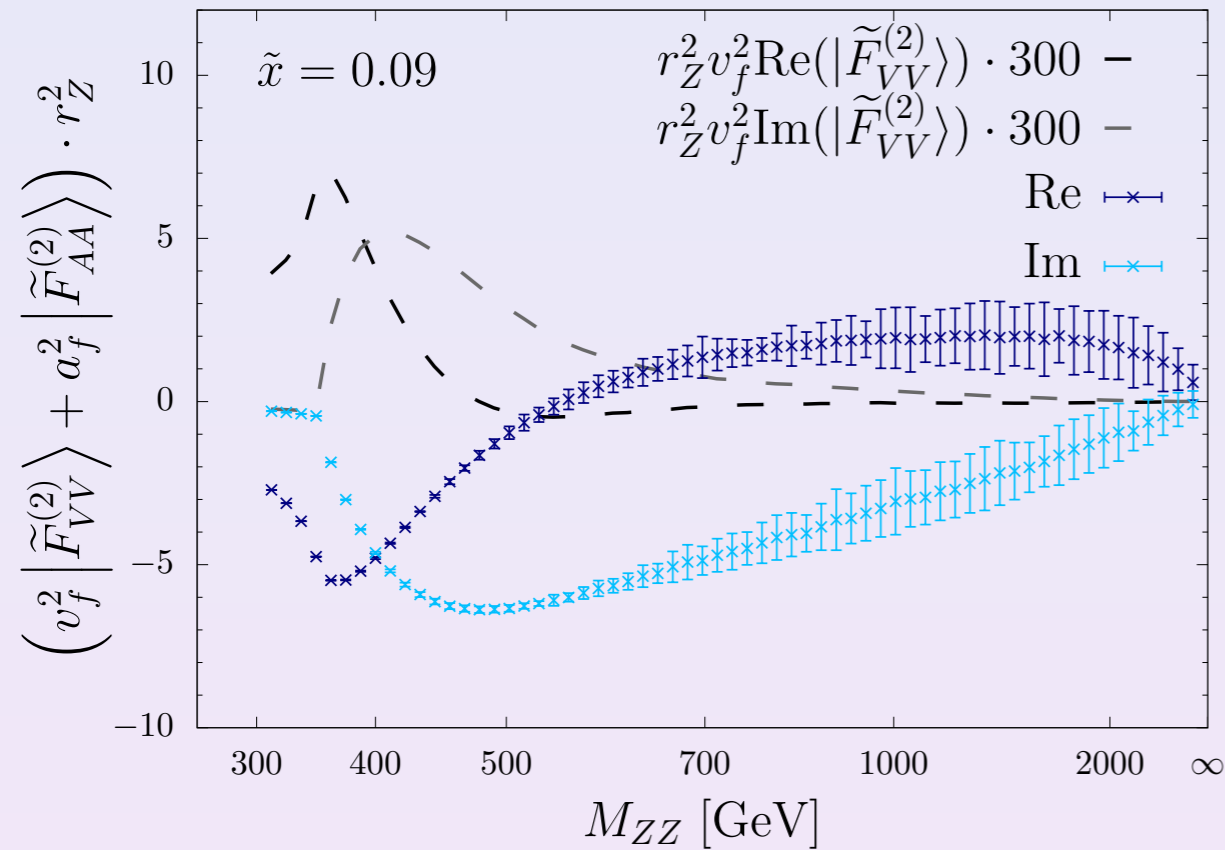
Plots for  $\tilde{x} = 0.25$

[RG, Maier, Rauh '19]



# Oscillatory behaviour for VV form factor

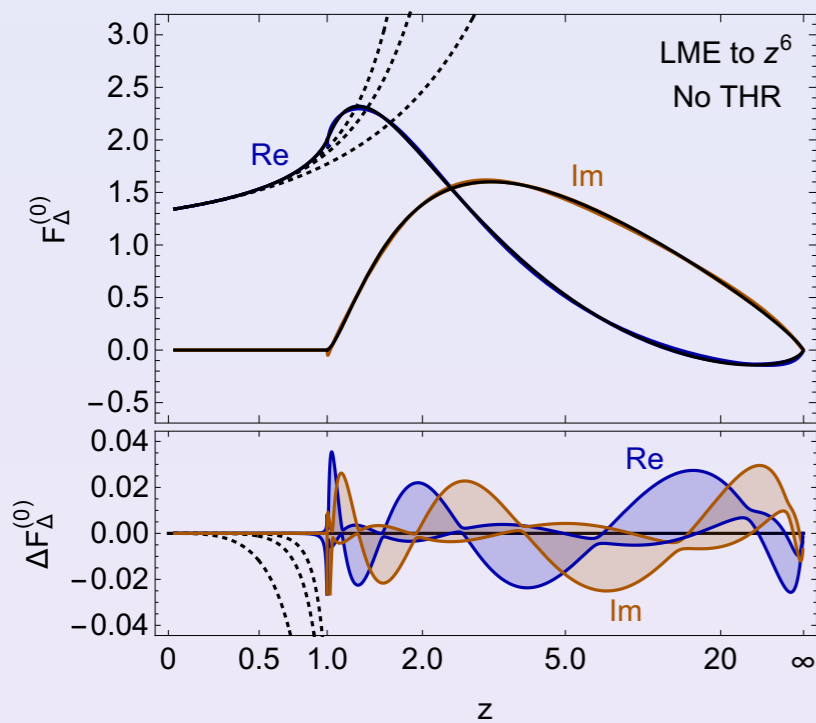
Traced back to the Padé approximate for the IR divergent diagrams  $\sim \ln(-4z)$



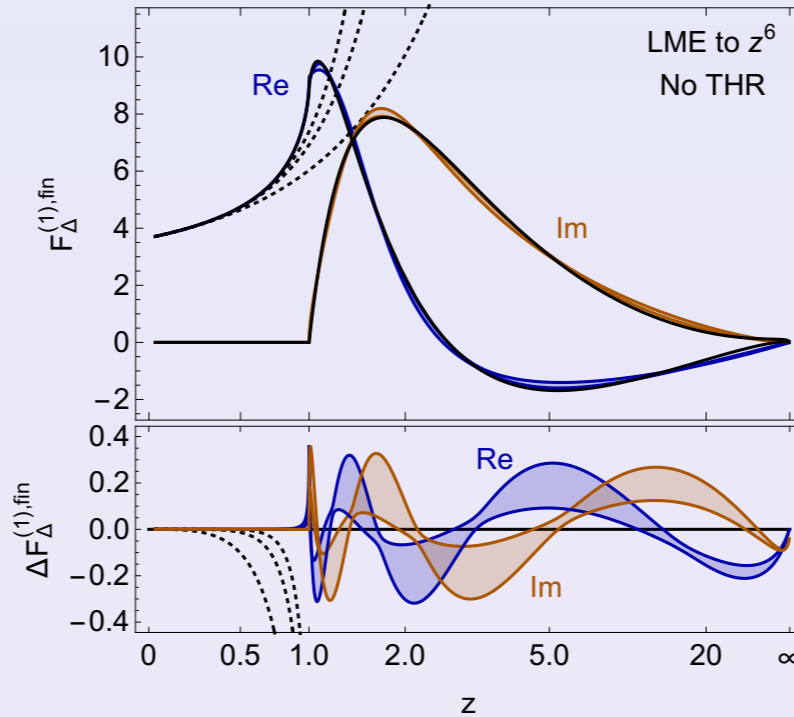
$|\tilde{F}_{VV}^{(2)}\rangle$  though negligible in finite part of virtual corrections

# Single Higgs at NNLO

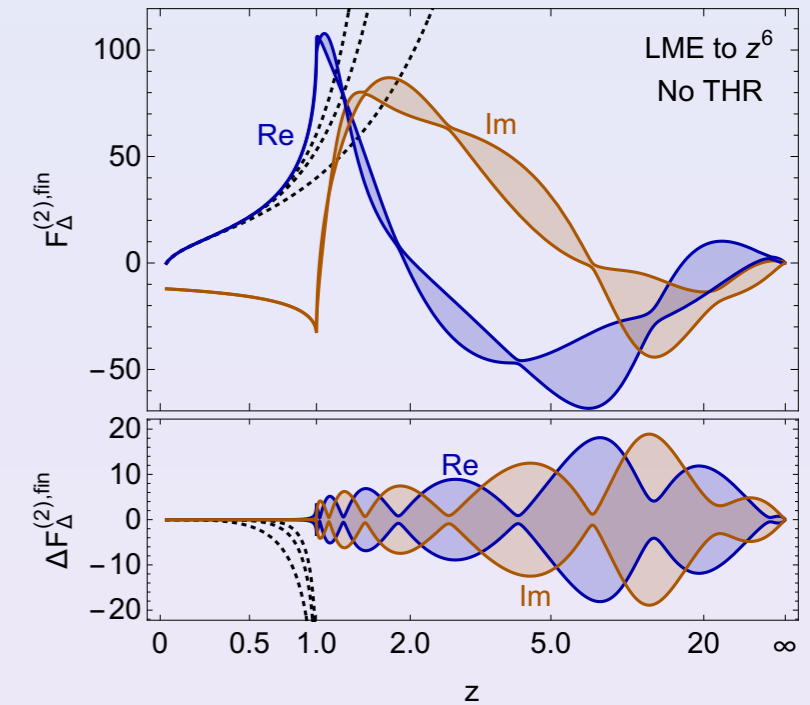
LO



NLO

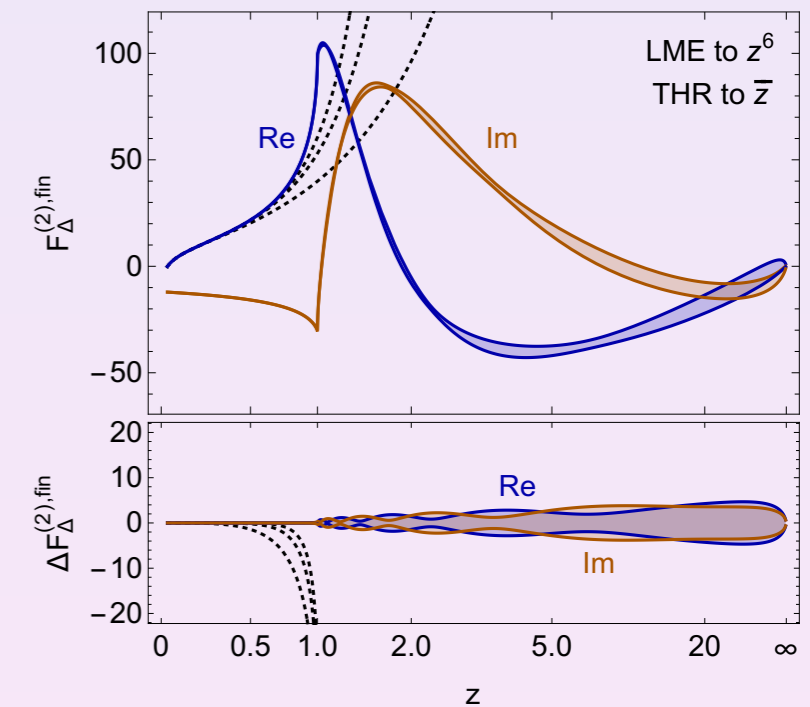
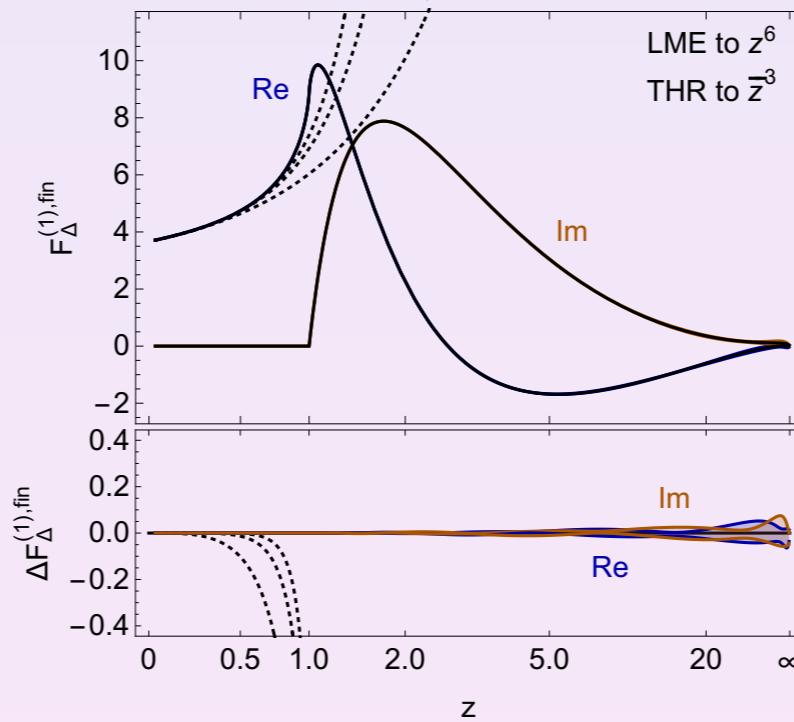
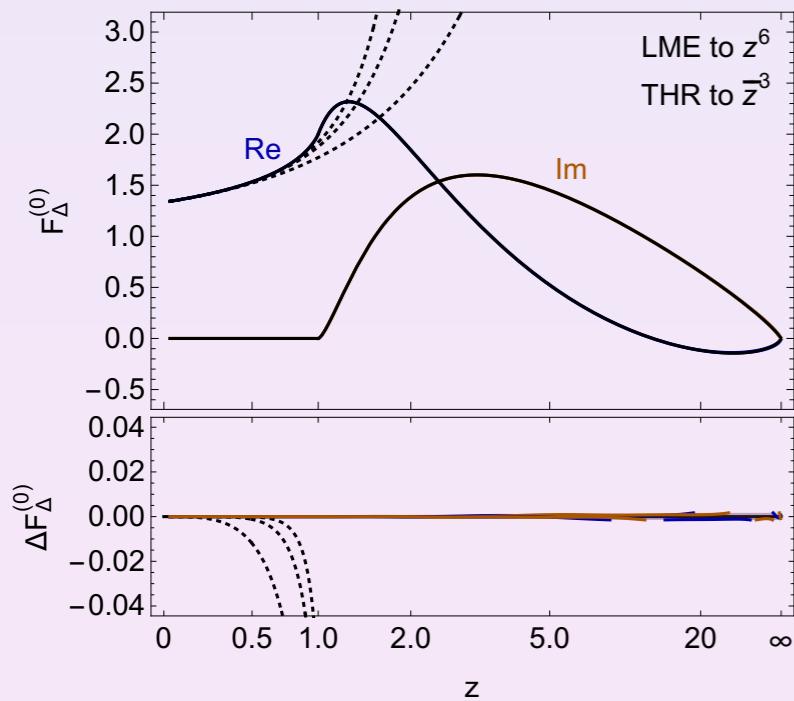


NNLO



adding THR expansion

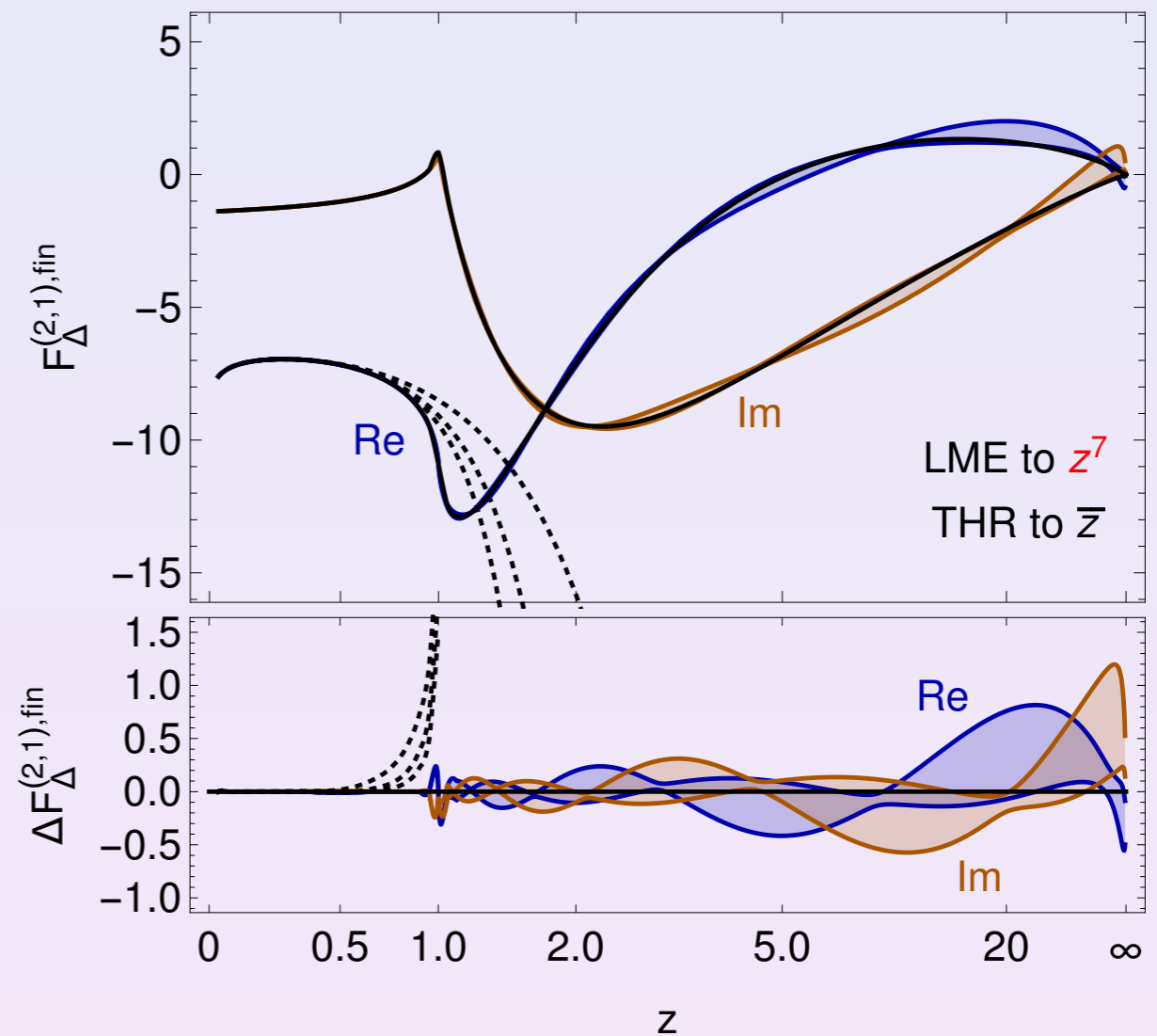
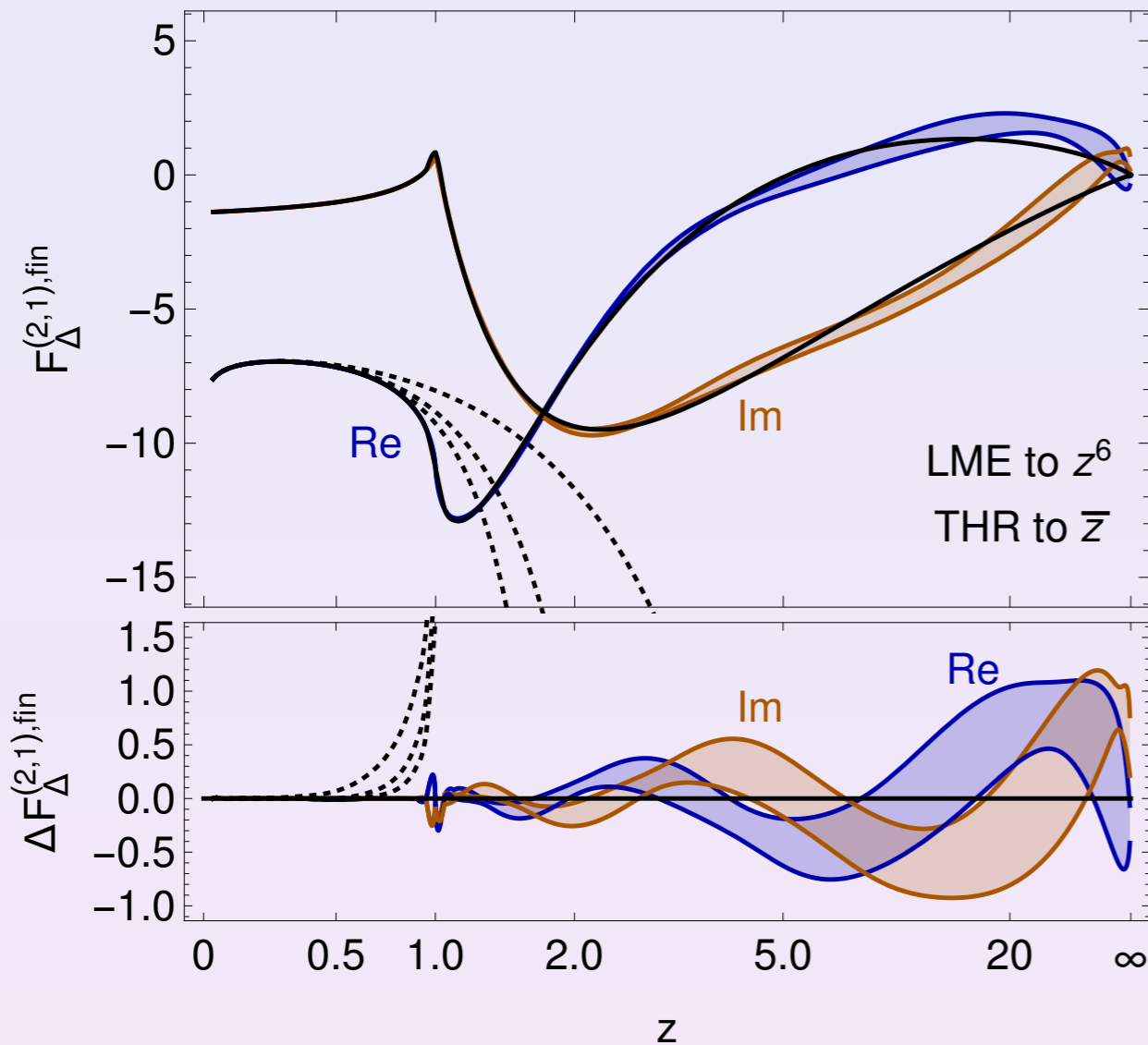
[Davies, RG, Maier, Rauh, Steinhauser '19]



# Single Higgs at NNLO

Analytical expressions for light-fermion contribution in the form factor computed recently in

[Harlander, Prausa, Usovitsch '19]



[slide copied from Thomas Rauh, RADCOR19]

Small deviations outside uncertainty estimate remedied with new  $z^7$  terms from

[Davies, Steinhauser '19]