Unitarization effects in the EChL predictions of *WZ* scattering at the LHC

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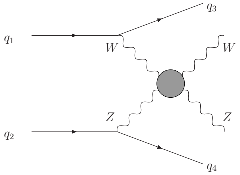
WG1 periodic meeting

Based on Phys. Rev. **D100** (2019) 096003 [1907.06668]

$pp \to WZj_1j_2$

by $WZ \to WZ$ scattering



Outline

- Introduction: linear vs. non-linear EFT's
- The Electroweak Chiral Lagrangian (EChL)
- Aspects on unitarity violation
- Restoring unitarity in WZ scattering at the LHC
- Parameter determination uncertainties
- Conclusions

The (EW) Chiral symmetry in the SM and BSM

The Spontaneously Breaking Sector of the SM can be written as

$$\mathcal{L}_{SBS} = \frac{1}{4}\langle \partial_\mu M^\dagger \partial^\mu M \rangle - \frac{\lambda}{4}\left(\frac{1}{2}\langle M^\dagger M\rangle + \frac{\mu^2}{\lambda}\right)^2$$

where $M = \sqrt{2}\begin{pmatrix} \phi_0^* & \phi^+ \\ -\phi^- & \phi_0 \end{pmatrix}$ and the Φ doublet is $\begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$.

$\Rightarrow$ the $\mathcal{L}_{SBS}$ is manifestly invariant under the global transformation:

$$M \to M' = g_L M g_R^\dagger \quad \text{with } g_L \subset SU(2)_L \text{ and } g_R \subset SU(2)_R$$

This global $SU(2)_L \times SU(2)_R$ is called the Chiral symmetry and it is spontaneously broken down to the diagonal subgroup

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R} \equiv SU(2)_{Custodial}$$

Gauge interactions ($g' \neq 0$) and different fermion masses (in the same doublet) explicitly break the Chiral and Custodial symmetries.

Main implication: the ρ parameter value is close to 1!

Linear approach to BSM: SMEFT

- The Higgs and the Goldstone bosons (GB) form a left $SU(2)$ doublet. In particular, the Higgs always appears in the combination $H + v$.
- The GB **transform linearly** under the Chiral symmetry.
- Typical situation when H is a fundamental field.
- Based on a **cutoff Λ expansion** (canonical dimension):

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{f_i^{(6)}}{\Lambda^2} \hat{\mathcal{O}}_i^{d=6} + \sum_i \frac{f_i^{(8)}}{\Lambda^4} \hat{\mathcal{O}}_i^{d=8} + \ldots$$

- SMEFT typically emerging from **weakly interacting** UV theory.

Our approach to BSM: the non-linear EChL or HEFT

- The Goldstone bosons π^a are independent from the Higgs boson. In particular, the Higgs is a $SU(2)$ singlet.
- The π^a **transform non-linearly** under the Chiral symmetry.
- Appropriate for composite models of the EWSB (H as a GB).
- Based on a **derivative expansion** $\leftrightarrow$ Chiral expansion (powers of p). Derivates and masses are soft scales of the EFT with power counting $\mathcal{O}(p) \Rightarrow$ the $\mathcal{L}$ is organized in terms of operators $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$, ...
- Associated to **strongly interacting** UV theory. Natural scenario to generate dynamically resonances.
- Non-trivial relation between linear and non-linear representations! Some higher order operators, that were dim-8 in the linear representation, can contribute to a lower order in the non-linear one (dim-4 in the Chiral expansion).

The Electroweak Chiral Lagrangian (EChL)

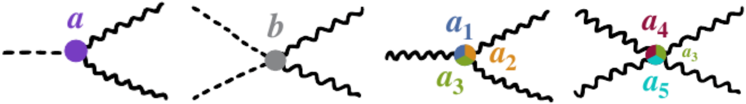
- Symmetries are Lorentz, CP, EW gauge $SU(2)_L \times U(1)_Y$ and Chiral $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$. Based on ChPT of QCD.
- Light degrees of freedom and building blocks are:
  Higgs boson as a singlet $\Rightarrow \mathcal{F}(H) = 1 + 2a\frac{H}{v} + b\left(\frac{H}{v}\right)^2 + \ldots$
  EW gauge bosons $\Rightarrow \hat{W}_\mu = gW_\mu^a\tau^a/2,\ \hat{B}_\mu = g'\,B_\mu\tau^3/2,\ \hat{W}_{\mu\nu},\ \hat{B}_{\mu\nu}$.
  EW GB in $U = \exp\left(\frac{i\pi^a\tau^a}{v}\right)$ that transforms linearly $U \to g_L U g_R^\dagger$
  $\Rightarrow D_\mu U = \partial_\mu U + i\hat{W}_\mu U - iU\hat{B}_\mu$ and $\mathcal{V}_\mu = (D_\mu U)U^\dagger$.
  Our assumptions: fermion ints as in SM. Custodial sym preserved.

$\mathcal{L}_{EChL} = \mathcal{L}_2 + \mathcal{L}_4$ (relevant for VBS)

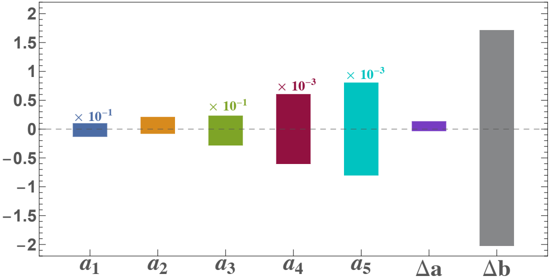
$$\mathcal{L}_2 = -\frac{1}{2g'}\langle\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle - \frac{1}{2g}\langle\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle + \frac{1}{2}\partial_\mu H\partial^\mu H - V(H) + \frac{v^2}{4}\mathcal{F}(H)\langle D_\mu U^\dagger D^\mu U\rangle$$

$$\mathcal{L}_4 = a_1\langle U\hat{B}_{\mu\nu}U^\dagger\hat{W}^{\mu\nu}\rangle + ia_2\langle U\hat{B}_{\mu\nu}U^\dagger[\mathcal{V}^\mu,\mathcal{V}^\nu]\rangle - ia_3\langle\hat{W}_{\mu\nu}[\mathcal{V}^\mu,\mathcal{V}^\nu]\rangle + a_4\langle\mathcal{V}_\mu\mathcal{V}_\nu\rangle\langle\mathcal{V}^\mu\mathcal{V}^\nu\rangle + a_5\langle\mathcal{V}_\mu\mathcal{V}^\mu\rangle\langle\mathcal{V}_\nu\mathcal{V}^\nu\rangle$$

EChL parameters and interactions



- SM predictions recovered for $\Delta a = a - 1 = 0$, $\Delta b = b - 1 = 0$ and $a_i = 0$.
- Only a, b, a_4 and a_5 survive *switching off* gauge interactions (limit $g, g' \to 0$). Relevant parameters applying Equivalence Theorem (ET): $A(V_L V_L \to V_L V_L) \simeq A(\pi\pi \to \pi\pi)$
- Exp. bounds derived from [Pyhs. Rev. **D98** (2018) 030001 (PDG) Pyhs. Rev. **D99** (2019) 033001 (ATLAS) Phys. Lett. B 798 (2019)134985 (CMS) ATLAS-CONF-2019-005 (1909.02845) ATLAS-CONF-2019-030]



Unitarity violation in VBS

- VBS is a powerful observable to look for New Physics: extremely sensitive to SM deviations introduced by EChL operators. Quasi-direct access to Goldstone dynamics through the longitudinal components (Equivalence Theorem).
- In the EChL context, interactions among gauge bosons scale with the external momenta $\Rightarrow$ pathological predictions when energy increases $\Rightarrow$ violation of unitarity of the S matrix!
- Unitarity requires on each J^{th} partial wave of $A(V_{\lambda_1} V_{\lambda_2} \to V_{\lambda_3} V_{\lambda_4})$

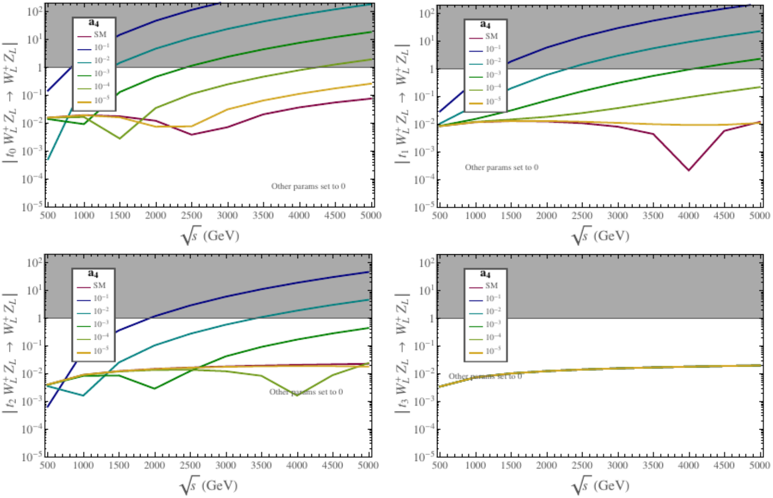
$$\mathrm{Im}[a^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s)] = \sum_{\lambda_a,\lambda_b} [a^J_{\lambda_1\lambda_2\lambda_a\lambda_b}(s)][a^J_{\lambda_a\lambda_b\lambda_3\lambda_4}(s)]^*$$

  It is a **coupled system** among all helicity states!
- Unitarity condition can be rewritten as $|a^J(s)| \leq 1$ and defines the unitary violation energy scale. This scale depends on EChL parameters.
- As in the ChPT, unitarity condition is fulfilled perturbatively

$$\mathrm{Im}[a^J_{\mathcal{O}(p^4)}(s)] = |a^J_{\mathcal{O}(p^2)}(s)|^2$$

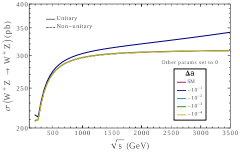
Unitarity violation in WZ scattering at partial wave level

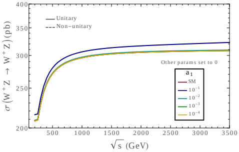
As an example, consider the helicity state $LL \to LL$ and study the effect of a_4 in the partial wave amplitudes t_J corresponding to $J = 0, 1, 2, 3$.

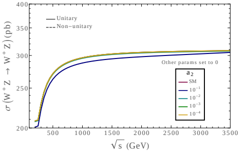


Next, consider the total cross section $\sigma(W^+ Z \to W^+ Z)$

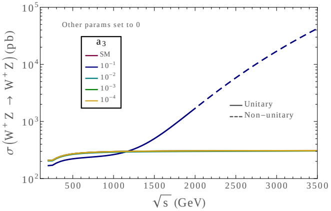
Unitarity violation in *WZ* scattering at subprocess level



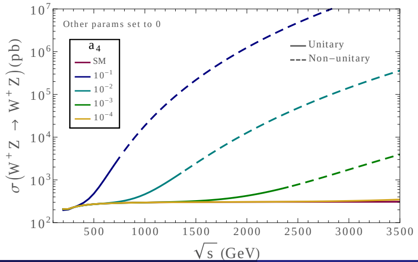


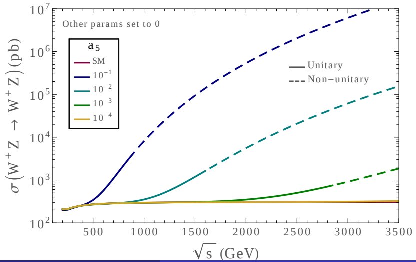


a_4 and a_5 are the **most relevant** parameters to unitarity violation!



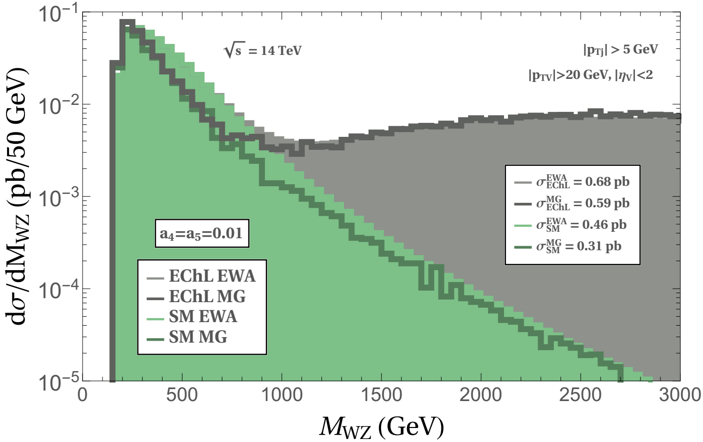
$a^{J=0}_{LLLL}(s)$ is the dominant contribution to unitarity violation!





Unitarity violation in WZ scattering at the LHC

The pathological behaviour at subprocess level translates in the prediction at the LHC process $pp \to WZ + jj$.
For example in the differential cross section:

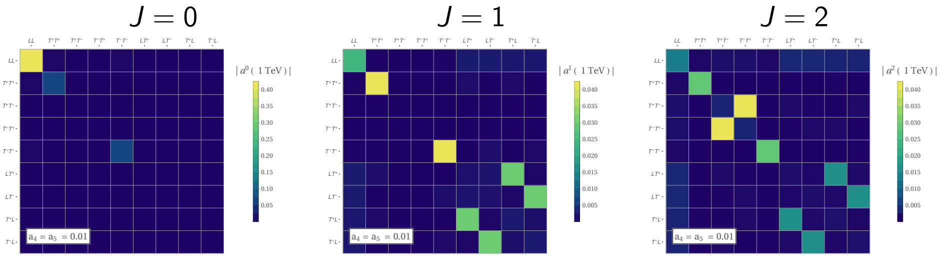


⇒ unitarization for realistic predictions is mandatory!

Coupled helicities system

Looking at the partial wave amplitudes:

- All the helicities channels $(9 \times 9 = 81)$ have to be considered consistently:



- The i^{th} helicity amplitude grows with the CM energy like $A_i \sim s^{\xi_i}$
  $\Rightarrow$ eventually reach the unitarity limit $|a_i^J| = 1$ at some scale $s = \Lambda_i$.
- Longitudinal modes only dominant for some J's or (a_4, a_5) values. In particular, $\xi_{LLLL} = 2$ can be understood through the ET.

Unitarization methods applied to the total amplitude

In order to provide unitary amplitude $\hat{A}$, several methods are implemented:

- **Cut-Off**: limit the validity range of the EFT up to the minimal unitarity violation scale Λ

$$\hat{A}(WZ \to WZ) = A(WZ \to WZ) \quad \text{for } s \leq \Lambda^2$$

- **Form Factor (FF)**: suppress the pathological behaviour via multiplying the amplitude by a smooth, continuous function

$$\hat{A}(WZ \to WZ) = A(WZ \to WZ) f^{\mathrm{FF}} \quad \text{with } f^{\mathrm{FF}} = (1 + s/\Lambda^2)^{-\xi}$$

- **Kink**: now the suppression is not smooth, but through a step function

$$f^{\mathrm{Kink}} = \begin{cases} 1 & \text{if } s \leq \Lambda^2 \\ (s/\Lambda^2)^{-\xi} & \text{if } s > \Lambda^2 \end{cases}$$

Unitarization methods applied to the partial waves

In the other two methods, unitarity is recovered from partial waves directly. Our proposal:

$$\hat{A}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\cos\theta) = A_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\cos\theta) + 16\pi \sum_{J=0}^{2} (2J+1)\, d^J_{\lambda,\lambda'}(\cos\theta) \left(\hat{\mathbf{a}}^J_{[\lambda_1\lambda_2\lambda_3\lambda_4]}(s) - a^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s)\right)$$

- **K-matrix**: an imaginary part is added such that the unitarity limit is saturated. The 9×9 matrix **a** containing the whole helicity system is reconstructed as

$$\hat{\mathbf{a}}^J = \mathbf{a}^J \cdot [\mathbf{1} - i\, \mathbf{a}^J]^{-1}$$

- **Inverse Amplitude Method (IAM)**: from the contributions of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ in the chiral expansion, the partial wave matrix amplitude is reconstructed as

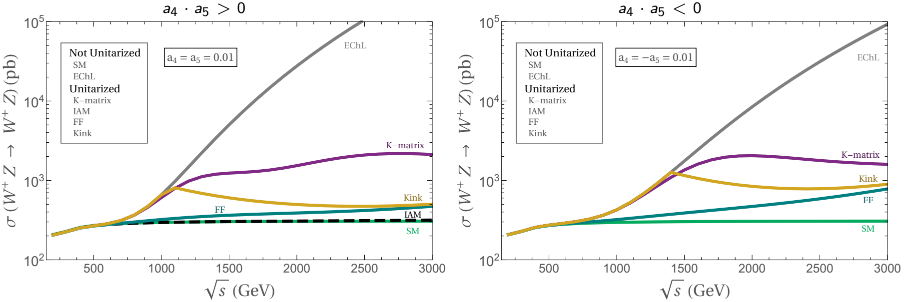
$$\hat{\mathbf{a}}^J = \mathbf{a}^{(2)\,J} \cdot [\mathbf{a}^{(2)\,J} - \mathbf{a}^{(4)\,J}]^{-1} \cdot \mathbf{a}^{(2)\,J}$$

  Not only unitary predictions arise, but also the appropriate analytical structure $\Rightarrow$ dynamically generated resonances can be accommodated with this procedure (as in ChPT for pion-pion scattering).

  We work here with non-resonant scenarios below $4\pi v \sim 3$ TeV.

Implications of unitarity at subprocess level

Very different predictions for the $WZ \to WZ$ total cross section using different methods! $\Rightarrow$ the experimental constraints interpreted using one method or another will be different.

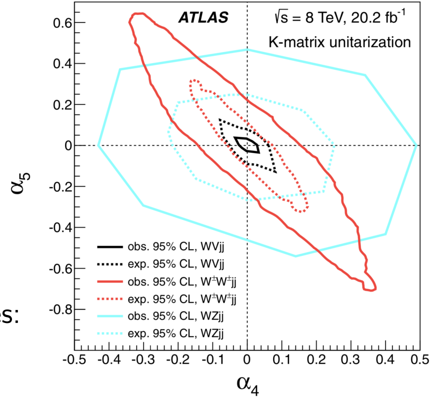


Then our aim is to give an estimate of the theoretical uncertainty in the experimental determination of a_4 and a_5 due to the unitarization scheme choice.

Present experimental constraints: no consensus yet

Current bounds are given using one method at a time or no method at all.

- Some ATLAS analyses:
  [Phys. Rev. **D95** (2017) 032001]
  use K-matrix
  for $a_{4(5)} = \alpha_{4(5)}$
  contours at 95% C.L.
  Our work focused in this Run 1



- Other ATLAS and CMS analyses:
  no unitarization method applied
  $a_{4(5)} = \frac{v^4}{16}\frac{f_{S0(S1)}}{\Lambda^4}$

	Expected (WV) (TeV^{-4})	Observed (ZV) (TeV^{-4})	Expected (ZV) (TeV^{-4})
f_{S0}/Λ^4	$[-4.2, 4.2]$	$[-40, 40]$	$[-31, 31]$
f_{S1}/Λ^4	$[-5.2, 5.2]$	$[-32, 32]$	$[-24, 24]$

- Other searches: Cut-Off used.

95% C.L. limits for $\sqrt{s} = 13$ TeV, 35.9 fb^{-1}

[Phys. Lett. B 798 (2019)134985 (CMS)]

Our computation of unitarity effects at *pp* collisions

1.- Unitarization applied to VBS subprocess amplitude $\Rightarrow \sigma(pp \to WZjj)$ computed with a Python code using the Effective W Approximation
That is by means of a factorization connecting the subprocess with the process.

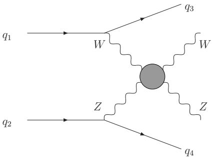
2.- Then we check the goodness of the EWA by comparing with full MG5 $pp \to WZjj$ events (VBS+others). Both in SM and EChL.

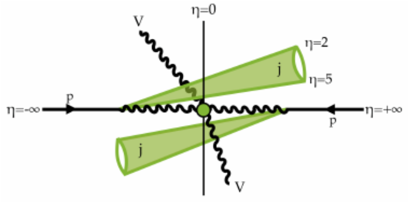
3.- We compare our predicted $\sigma^{\mathrm{EWA}}(pp \to WZjj)$ for a given unitarization method with LHC data in the (a_4, a_5) plane.

4.- VBS events usually selected by specific VBS-cuts: large pseudorapidity gap and large invariant mass (like $\Delta\eta_{jj} > 4$ and $M_{jj} > 500$ GeV).

$pp \to WZj_1j_2$

by $WZ \to WZ$ scattering





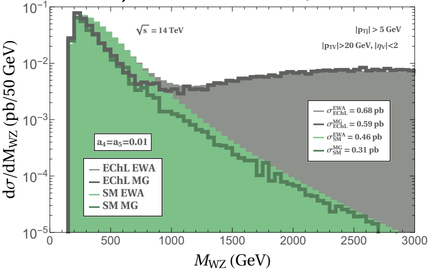
Effective W Approximation (EWA)

- W's and Z's considered as partons inside the proton.
  Generalization of the Weiszäcker-Williams approximation for photons.
- They are emitted collinearly from the fermions (quarks) with probability functions $f_V(\hat{x})$ and then scatter on-shell.
- Factorization using a sort of PDFs

$$\sigma(pp \to (V_1 V_2 \to V_3 V_4) + X) =$$
$$\sum_{i,j} \int \int dx_1 dx_2 f_{q_i}(x_1) f_{q_j}(x_2) \int \int d\hat{x}_1 d\hat{x}_2 f_{V_1}(\hat{x}_1) f_{V_2}(\hat{x}_2) \hat{\sigma}(V_1 V_2 \to V_3 V_4)$$

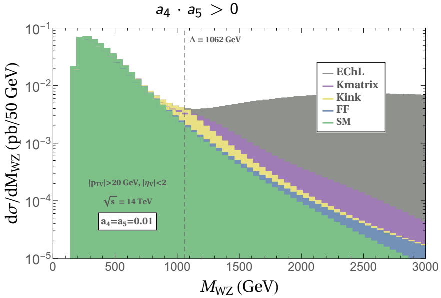
- We have tested with MG5 the accuracy of various probability functions (SM and EChL): Dawson's Improved formulas work best!

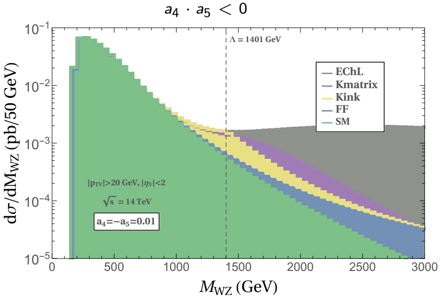
[Nucl. Phys. **B249** (1985) 42]



Predictions with different unitarization methods at LHC

- Different results depending on unitarization method also at the LHC.
- Both distributions and cross sections result to be different.



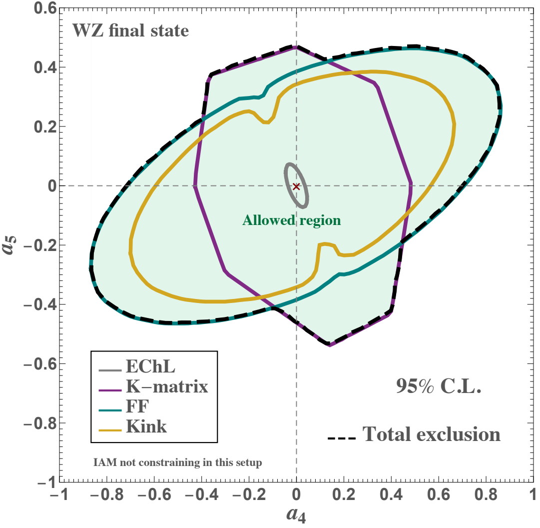


SS a_4 and a_5 from non-unitarized EChL is more constrained than OS.

Our results: parameter uncertainty in (a_4, a_5) plane

- We focus on *WZ* ATLAS analysis at Run 1, $\sqrt{s} = 8$ TeV, 20.2 fb^{-1}.
- From the ATLAS 'ellipse' (contour at 95% C.L.) for K-matrix we extract our equivalent cross section.

- For the other unitarization methods, we construct the contours at 95% C.L.
  Main assumption: selection cuts affect equaly all predictions.
- No unitarization gives strong constraints (small ellipse).
  $a_4.a_5 > 0$ more constrained.
- Overlap corresponds to the uncertainty in (a_4, a_5)
- Shape and orientation change from one unitarization method to another.
  Size enhances by a factor 10 approx the uncertainty respect non-unitarized constraints.



Conclusions

- EFT is a powerful tool to study New Physics in a model-independent way.
- EChL is the most general EFT suitable for strongly interacting scenarios of EWSB. This EFT approach might lead to event predictions that violate unitarity.
- VBS is the key observable of this kind of physics.
- Unitarization methods must be applied in order to provide unitary predictions:
  $\Rightarrow$ different unitarization procedures lead to different predictions for VBS.
  $\Rightarrow$ a theoretical uncertainty is associated with this ambiguity.
- We provide a first approximation to quantify this uncertainty in the experimental determination of (a_4, a_5) due to the unitarization scheme choice through the elastic WZ scattering at the LHC.

Backup slides

Transformations under $SU(2)_L \times SU(2)_R$

The rotations under $SU(2)_L$ and $SU(2)_R$ correspond to

$$g_L = e^{i\vec{\tau}\cdot\vec{\alpha}_L/2} \quad \text{and} \quad g_R = e^{i\vec{\tau}\cdot\vec{\alpha}_R/2}$$

Then building blocks transform under the global $SU(2)_L \times SU(2)_R$ as

$$\begin{aligned}
U \mapsto U' &= g_L\, U\, g_R^\dagger && \text{with chiral dim.} = 0\\
\hat{B}_\mu \mapsto \hat{B}'_\mu &= \hat{B}_\mu && \text{with chiral dim.} = 1\\
\hat{W}_\mu \mapsto \hat{W}'_\mu &= g_L\, \hat{W}_\mu\, g_L^\dagger && \text{with chiral dim.} = 1\\
D_\mu U \mapsto (D_\mu U)' &= g_L\, D_\mu U\, g_R^\dagger && \text{with chiral dim.} = 1\\
\hat{B}_{\mu\nu} \mapsto \hat{B}'_{\mu\nu} &= \hat{B}_{\mu\nu} && \text{with chiral dim.} = 2\\
\hat{W}_{\mu\nu} \mapsto \hat{W}'_{\mu\nu} &= g_L\, \hat{W}_{\mu\nu}\, g_L^\dagger && \text{with chiral dim.} = 2
\end{aligned}$$

For the EW gauge symmetry $SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R$, the association of the generator of $U(1)_Y$ as the third one of the $SU(2)_R$ and the generator of $U(1)_{EM}$ as the third one of the $SU(2)_{L+R}$:

$$Y \leftrightarrow X_R^3 \;\text{ and }\; Q \leftrightarrow X_{L+R}^3 = T^3 + Y$$

Unitarized amplitudes

The partial wave decomposition is

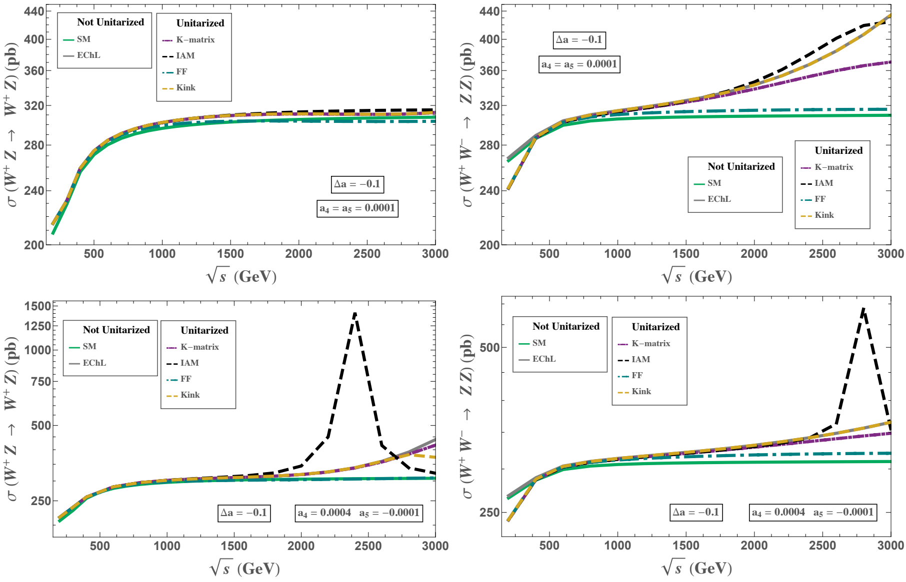
$$a^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\ A(V_{\lambda_1}V_{\lambda_2} \to V_{\lambda_3}V_{\lambda_4})(s,\cos\theta)\, d^J_{\lambda,\lambda'}(\cos\theta)$$

where J is the total angular momentum of the system, $\lambda = \lambda_1 - \lambda_2$, $\lambda' = \lambda_3 - \lambda_4$, being λ_i the helicity states of the external gauge bosons, and where $d^J_{\lambda,\lambda'}(\cos\theta)$ are the Wigner functions.

For the K-matrix and IAM methods, the unitarized amplitude is reconstructed from the corresponding unitarized partial wave and the non-unitary amplitudes following:

$$\begin{aligned}\hat{A}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\cos\theta) = {} & A_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\cos\theta)\\ & - 16\pi\sum_{J=0}^{2}(2J+1)\, d^J_{\lambda,\lambda'}(\cos\theta)\, a^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s)\\ & + 16\pi\sum_{J=0}^{2}(2J+1)\, d^J_{\lambda,\lambda'}(\cos\theta)\, \hat{\mathbf{a}}^J_{[\lambda_1\lambda_2\lambda_3\lambda_4]}(s)\end{aligned}$$

Dynamical resonances in the IAM



More about the EWA

The most accurate EWA expression in our setup is the Dawson's Improved (yellow line)

$$f_{V_T}^{Improved}(x) = \frac{C_V^2 + C_A^2}{8\pi^2 x} \left[\frac{-x^2}{1 + M_V^2/(4E^2(1-x))} + \frac{2x^2(1-x)}{M_V^2/E^2 - x^2} + \left\{ x^2 + \frac{x^4(1-x)}{(M_V^2/E^2 - x^2)^2} \left(2 + \frac{M_V^2}{E^2(1-x)} \right) - \frac{x^2}{(M_V^2/E^2 - x^2)^2} \frac{M_V^4}{2E^4} \right\} \log\left(1 + \frac{4E^2(1-x)}{M_V^2} \right) + x^4 \left(\frac{2-x}{M_V^2/E^2 - x^2} \right)^2 \log \frac{x}{2-x} \right] \eta$$

$$f_{V_L}^{Improved}(x) = \frac{C_V^2 + C_A^2}{\pi^2} \frac{1-x}{x} \frac{\eta}{(1+\eta)^2} \left\{ \frac{1 - x - M_V^2/(8E^2)}{1 - x + M_V^2/(4E^2)} - \frac{M_V^2}{4E^2} \frac{1 + 2(1-x)^2}{1 - x + M_V^2/(4E^2)} \frac{1}{M_V^2/E^2 - x^2} - \frac{M_V^2}{4E^2} \frac{x^2}{2(1-x)(x^2 - M_V^2/E^2)^2} \left[(2-x)^2 \log \frac{x}{2-x} - \left(\left(x - \frac{M_V^2}{E^2 x} \right)^2 - (2(1-x) + x^2) \right) \log\left(1 + \frac{4E^2(1-x)}{M_V^2} \right) \right] - \frac{M_V^2}{8E^2} \frac{x}{\sqrt{x^2 - M_V^2/E^2}} \left[\frac{2}{x^2 - M_V^2/E^2} + \frac{1}{1-x} \right] \left[\log \frac{2 - x - \sqrt{x^2 - M_V^2/E^2}}{2 - x + \sqrt{x^2 - M_V^2/E^2}} - \log \frac{x - \sqrt{x^2 - M_V^2/E^2}}{x + \sqrt{x^2 - M_V^2/E^2}} \right] \right\}$$

with $C_{V(A)}$ the vector(axial) couplings Vqq, x the fraction of quark energy E carried by V and $\eta \equiv \left(1 - \frac{M_V^2}{x^2 E^2}\right)^{1/2}$

In the limit $M_V \ll E \Rightarrow$

$$f_{V_T}^{LLA}(x) = \frac{C_V^2 + C_A^2}{8\pi^2 x} \left[x^2 + 2(1-x) \right] \log\left(\frac{4E^2}{M_V^2} \right)$$

$$f_{V_L}^{LLA}(x) = \frac{C_V^2 + C_A^2}{\pi^2} \frac{1-x}{x}$$