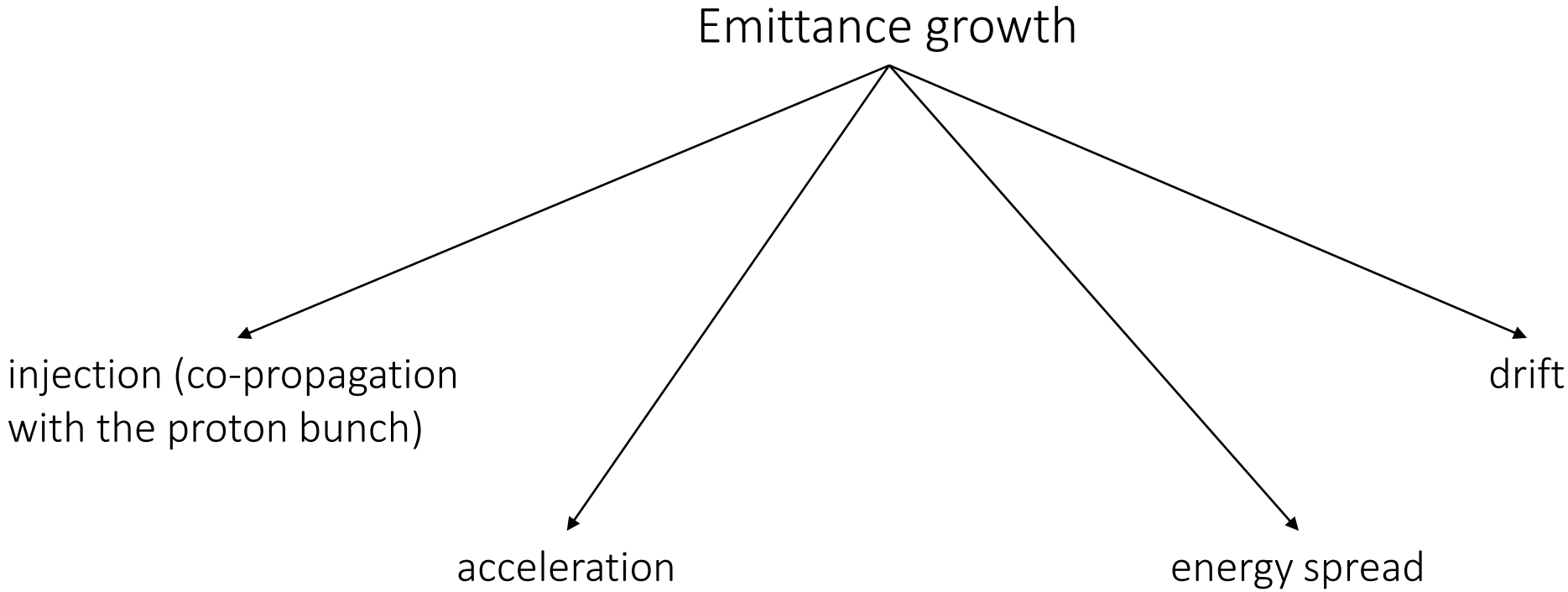
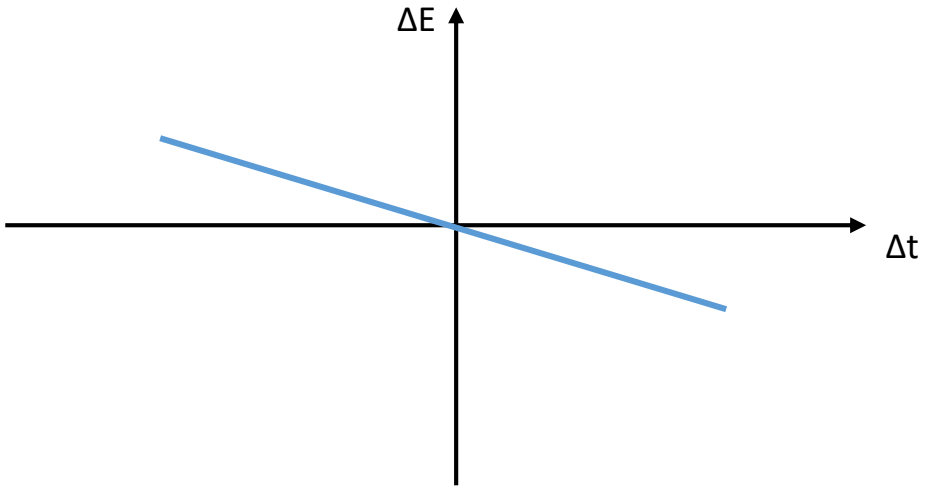

Emittance growth
injection (co-propagation with the proton bunch)
acceleration
energy spread
drift

Energy spread

- matching is defined for only one energy: $K_\beta = \frac{\omega_{pe}}{c\sqrt{2\,\gamma}}, \beta_m = \frac{1}{K_\beta}$
- energy spread leads to mismatching



Slice emittance

- For every longitudinal “slice”, we can calculate the emittance and betatron functions.
- If mismatched (because of energy spread), the emittance of the slice grows, following:

$$\frac{\epsilon_{N\_sat}(\xi)}{\epsilon_{N\_in}} = \frac{1}{2} \cdot \left(\frac{1+\alpha^2}{\beta_0} \cdot \beta_m(\xi) + \frac{\beta_0}{\beta_m(\xi)}\right)$$

matched beta for the single slice

beta of the whole beam, defined by the matching condition

Projected emittance

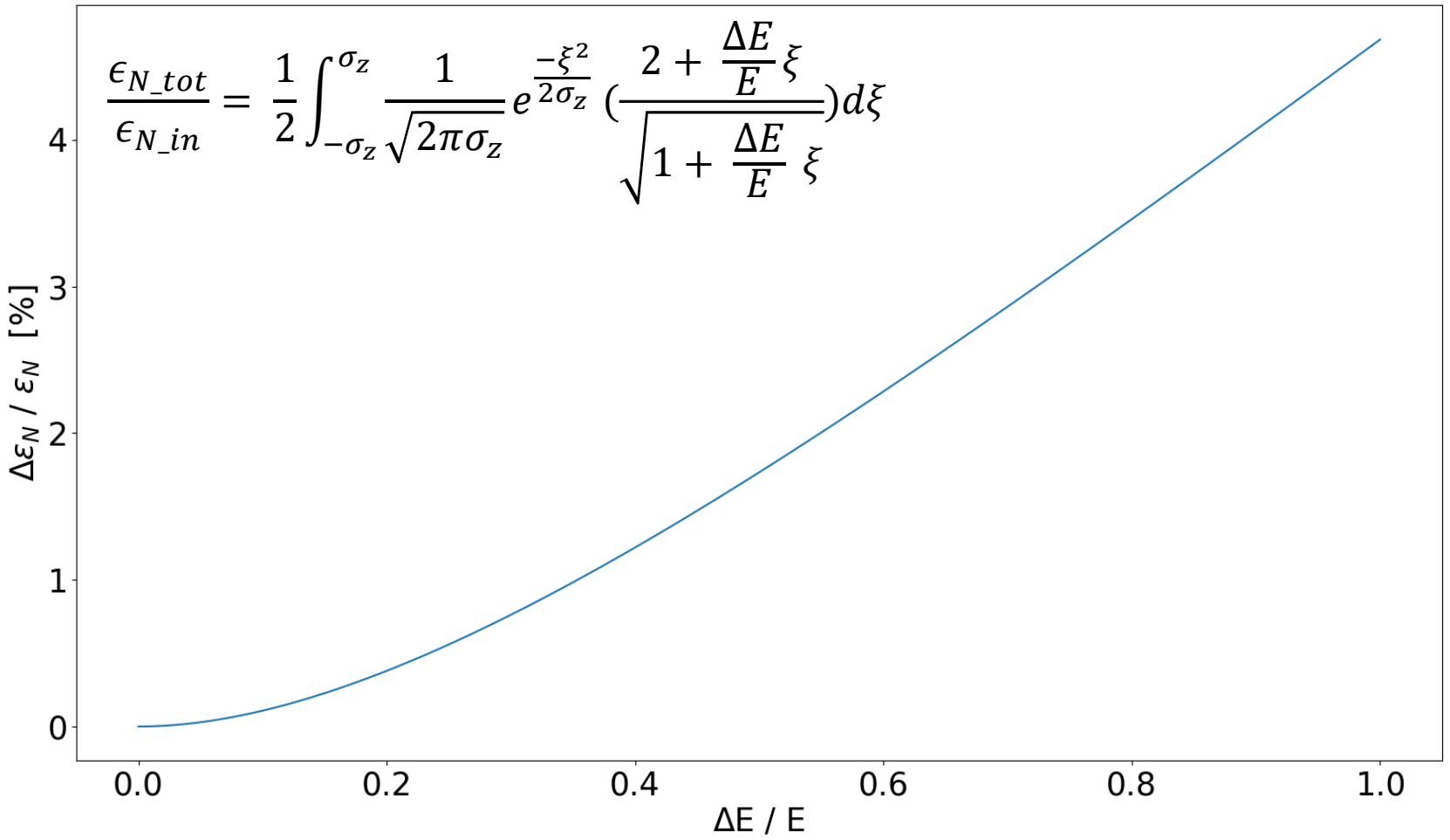
- Integrating over the bunch, one obtains the total emittance

$$\epsilon_{N\_tot} = \frac{1}{Q_{tot}} \int_{-\sigma_z}^{\sigma_z} \epsilon_{N\_slice}(\xi) \cdot q(\xi)\, d\xi$$

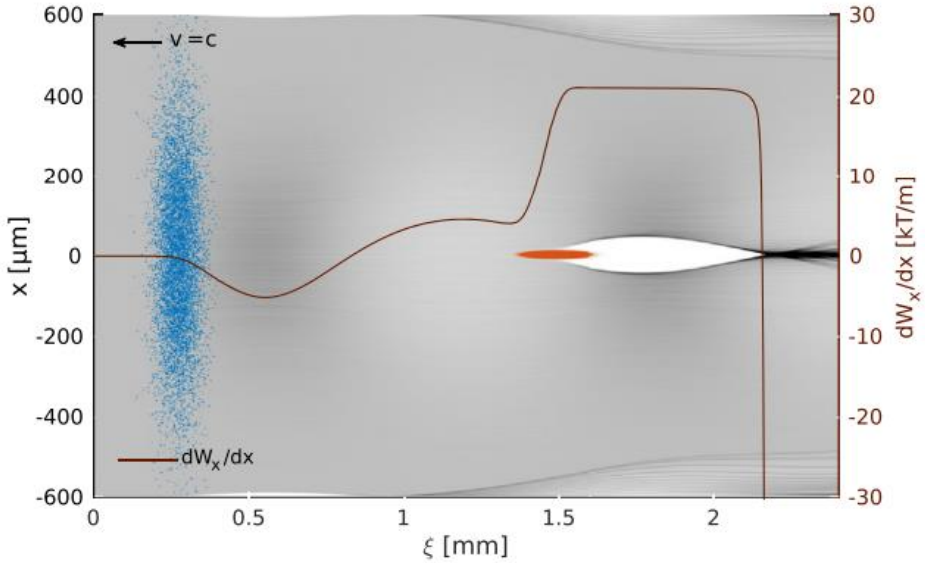
Gaussian distribution

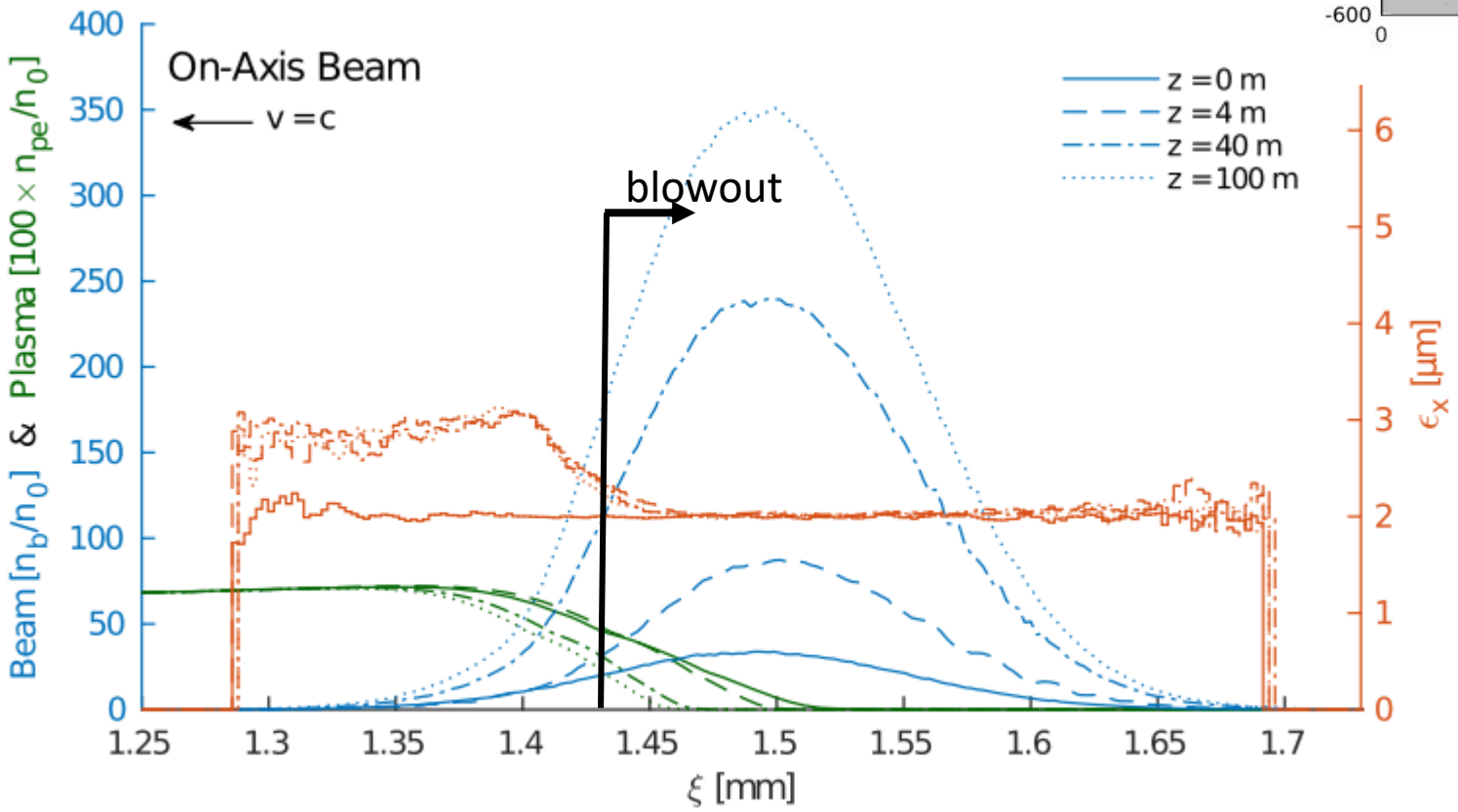
Matched beam (blowout regime)

$$\frac{\epsilon_{N\_tot}}{\epsilon_{N\_in}} = \frac{1}{2}\int_{-\sigma_z}^{\sigma_z} \frac{1}{\sqrt{2\pi\sigma_z}} e^{\frac{-\xi^2}{2\sigma_z}} \left(\frac{2+\frac{\Delta E}{E}\xi}{\sqrt{1+\frac{\Delta E}{E}\xi}}\right) d\xi$$

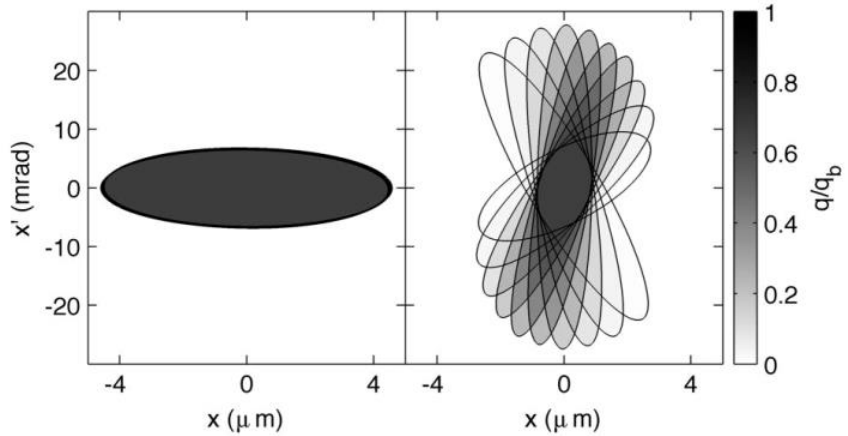


Matched beam (blowout regime)





Emittance growth during the drift (extraction)

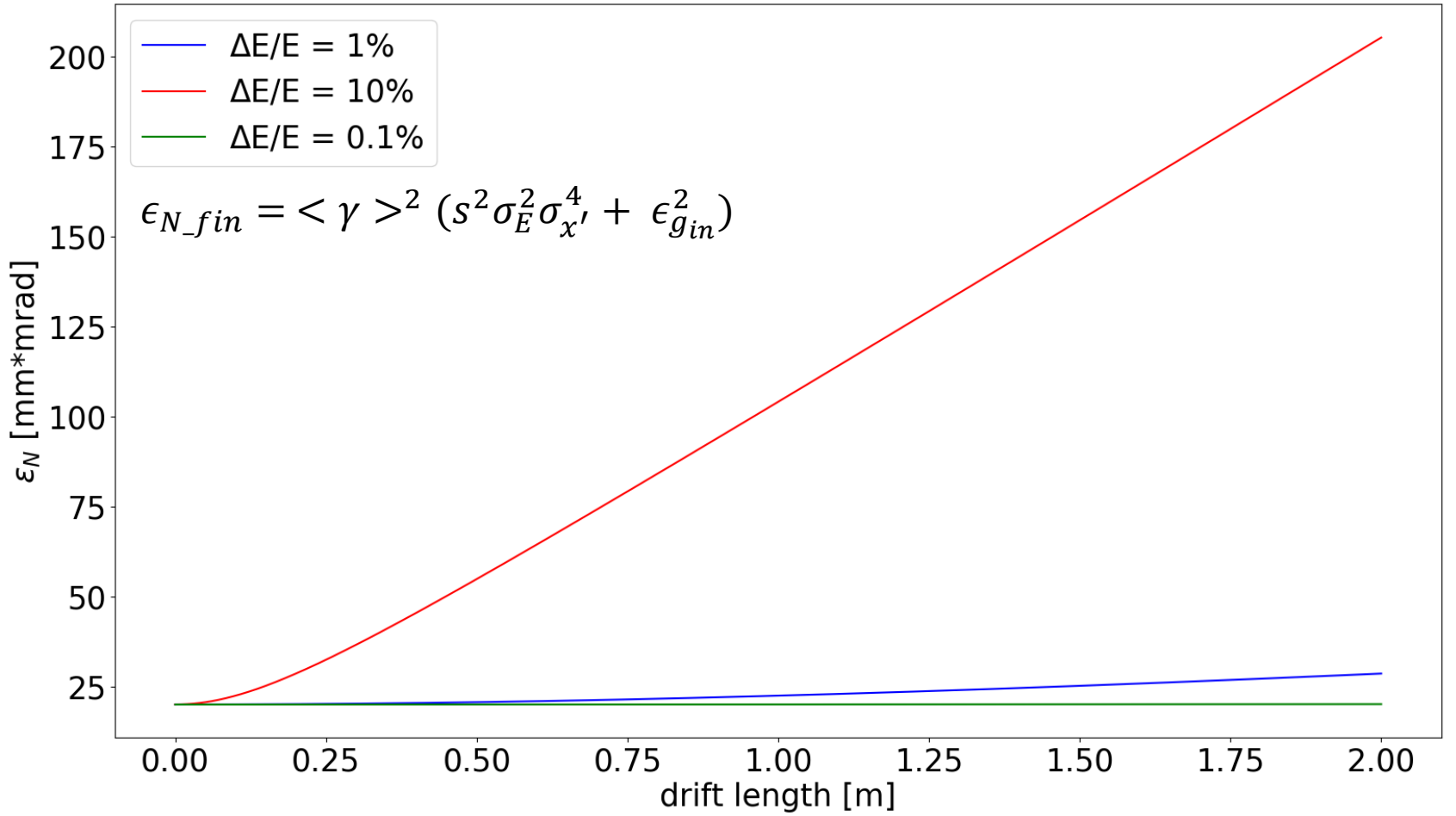


- Betatron frequency of a particle depends on its energy

→particles with different energies rotate with different velocities

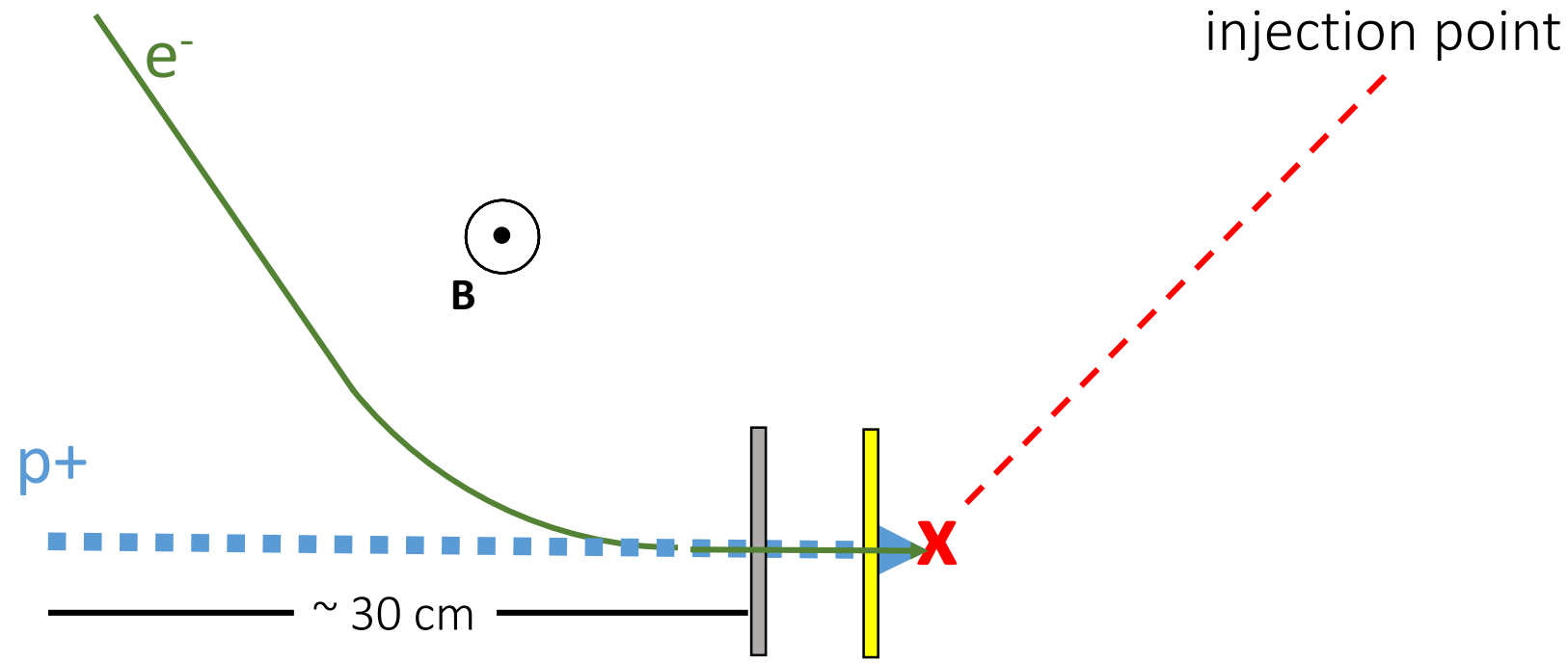
→resulting projected emittance becomes a function both of drift length and energy spread

Emittance growth during the drift (extraction)



Emittance growth during injection

- In the gap region, electron and proton bunch co-propagate for <30 cm
- Emittance growth occurs due to beam-beam space charge effect

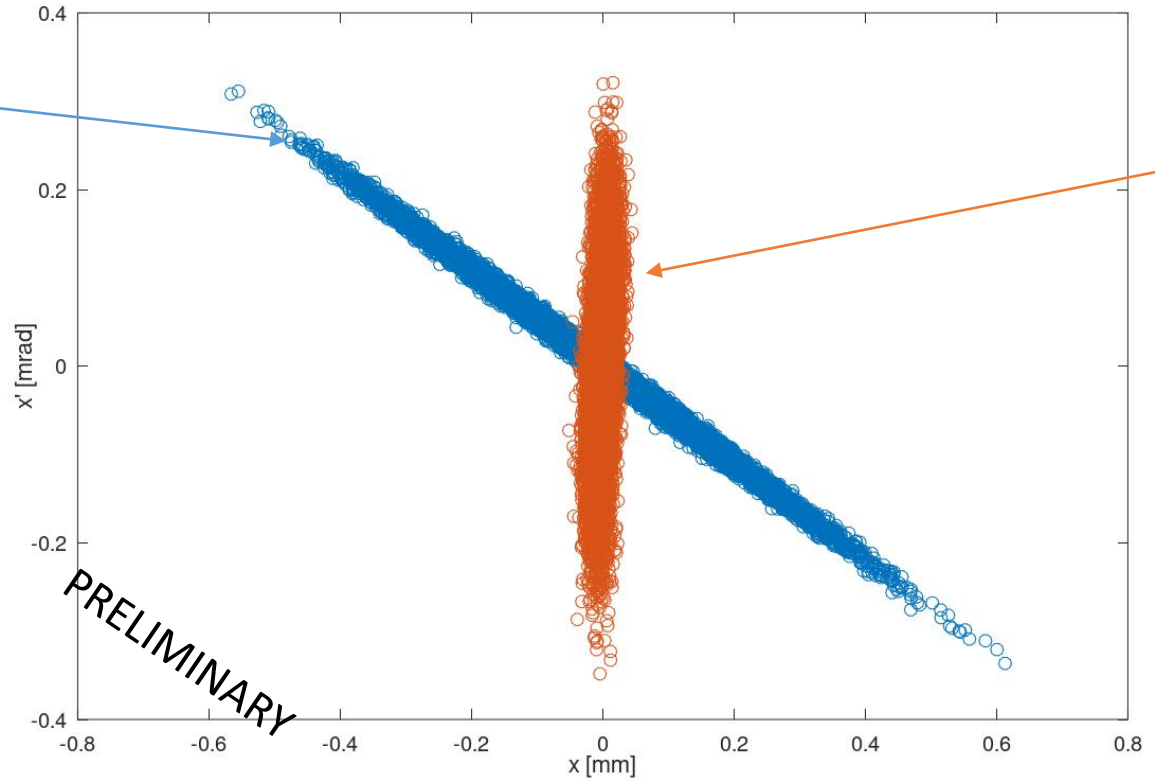


Emittance growth during injection

- Simulation of the converging electron beam (ε_N = 2 mm*mrad, β^* = 2 cm) co-propagating with unmodulated proton bunch

initial distribution

ε_N = 1.98 mm mrad



distribution at the focal point

ε_N = 2.02 mm mrad

(still need to check and compare without space charge effect)