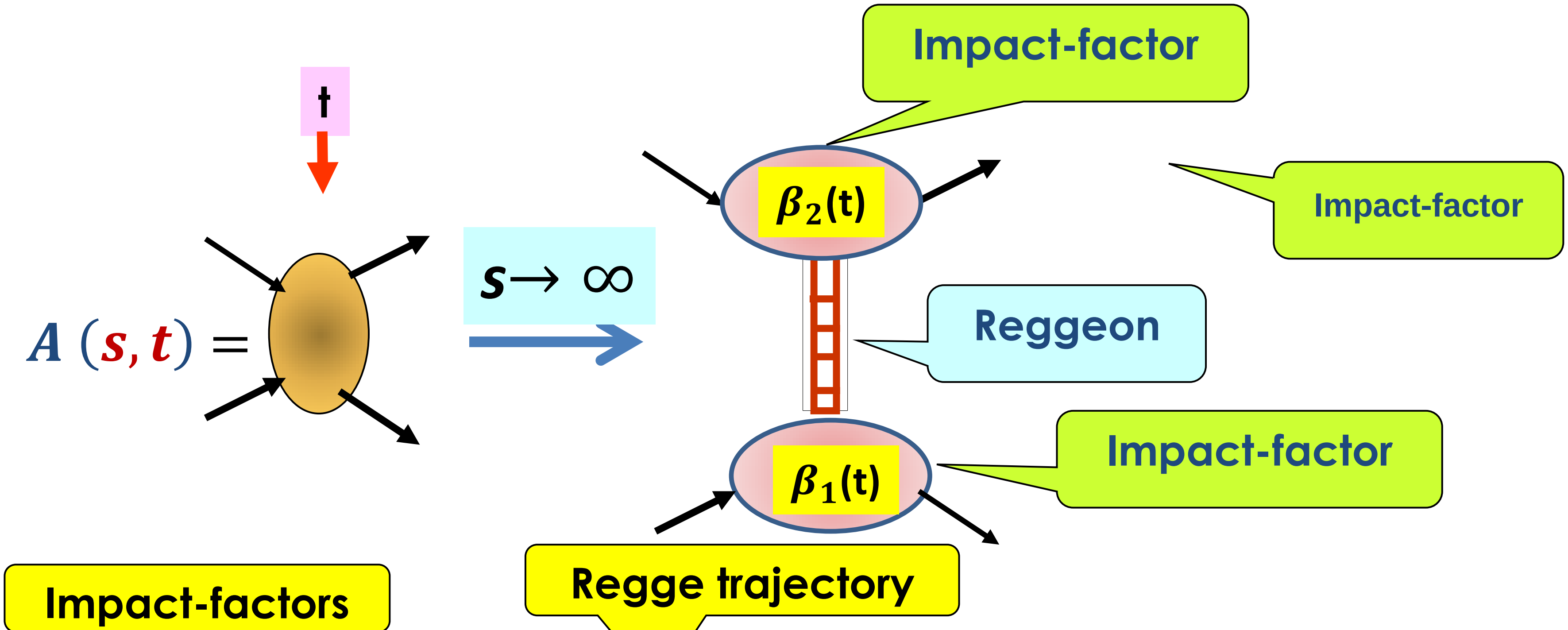
Forward Physics and Diffraction CERN December 16 2019

B. I. Ermolaev

Double-logarithmic contribution to Pomeron and its applications

talk based on results obtained in collaboration with S.I. Troyan

Classic/phenomenological Regge theory is based on very general Concepts such as **ANALITICITY, CAUSALTY, UNITARITY**.
It predicts high-energy asymptotics of any scattering amplitude in the forward kinematics:



$$A(s,t) \sim \beta_1(t)\,\beta_2(t)\,s^{\alpha(t)}$$

Impact-factors depend on properties of external hadrons while Reggeon is a more general object: it depends on quantum numbers of the two-hadron states in the t-channel

The trajectories are expanded in the series in *t*. However *t* is small, so the linear approximation can be used:

$$\alpha(t) = \alpha(0) + \alpha'(0)\,t$$

intercept

slope

Optical theorem:

$$\sigma_{tot} \sim Im\ A_{el}(s,0)/s \sim (s/s_0)^{\alpha(0)-1}$$

Total cross section

UNITARITY → Froissart-Martin bound

$$\boxed{\sigma_{tot}(s) \leq c\,ln^2 s} \rightarrow \boxed{\alpha(0) \leq 1}$$

I.Y. Pomeranchuk (1958) suggested existence of Reggeon with intercept = 1 when the Reggeon has the vacuum quantum numbers in t-channel

Pomeron $\boxed{\alpha_P(0) \equiv \alpha(0)_{max} = 1}$

$$\sigma_{tot} \sim (s/s_0)^{\alpha_P(0)-1}$$

no power dependency on **s**

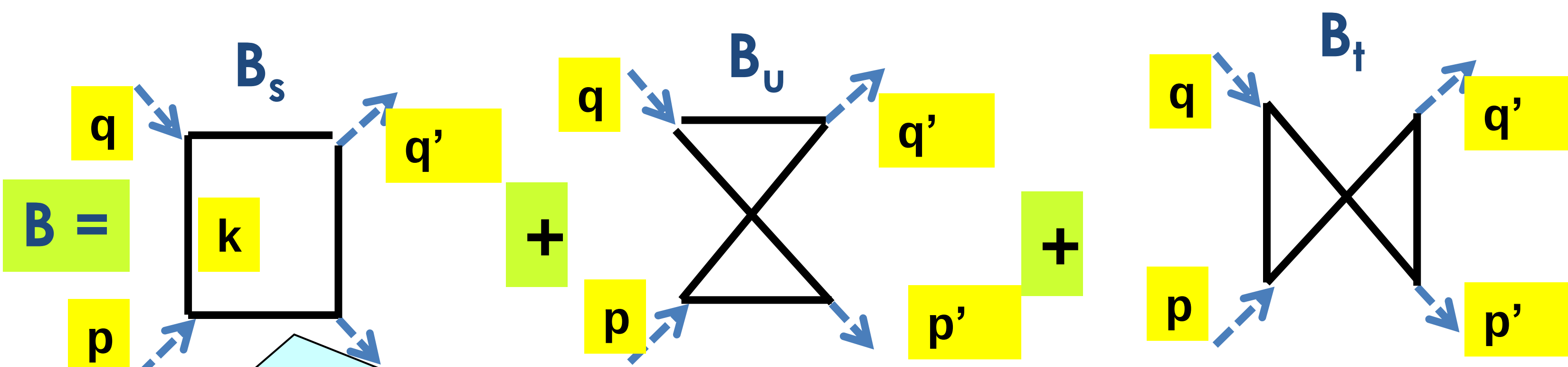
Further development of the Regge theory was done with QCD methods

I present a double-logarithmic (DL) contribution to Pomeron and start with considering amplitudes of the elastic scattering of virtual photons $\gamma^*(p)\,\gamma^*(q)\to\gamma^*(p')\,\gamma^*(q')$ in the forward kinematics $s = (p+q)^2 \gg -t = (p-p')^2$

All the photons are non-polarized

This process, apart of its experimental importance, is interesting from the theoretical point of view because, in contrast to hadronic reactions, it is free of non-perturbative contributions, so it can be regarded as a test-field for various theoretical approaches

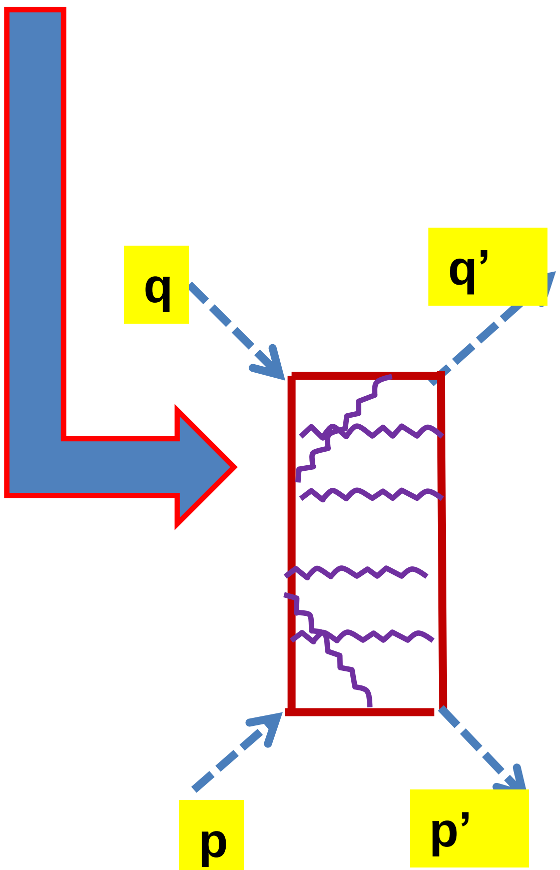
Born (lowest order) approximation is well-known:



Yields the largest contribution when s-cut is done

Beyond the Born approximation

Step 1 Amplitudes of photon- photon scattering via
overall quark loop in DLA



Bartels-Lublinsky 2003
Explicit DLA expressions for such amplitudes at $t = 0$ (collinear scattering) obtained with several different approaches

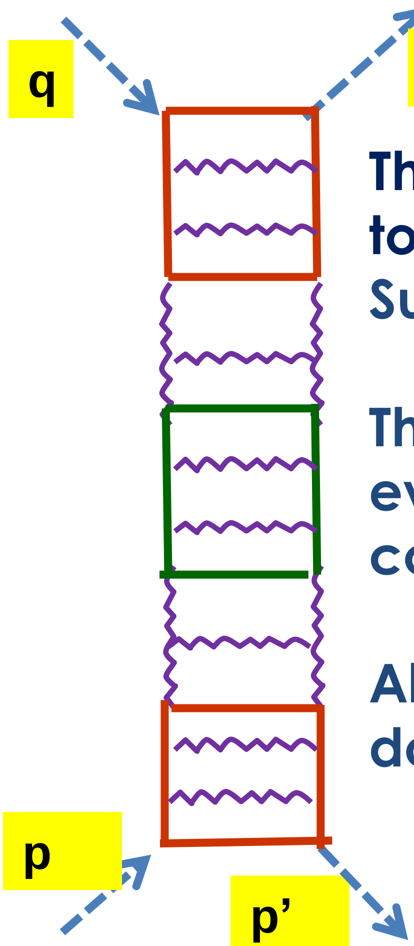
Ermolaev-Ivanov-Troyan 2017
Generalization to $s \gg -t \neq 0$ and accounting for the running coupling

We used InfraRed Evolution Equations (IREE)
This method was invented by L.N. Lipatov and applied to both Gravity (Lipatov, 1982) and quark-quark elastic scattering (Kirschner-Lipatov, 1982)

After that IREE approach was developed to calculate many problems of QED, QCD and EW interactions, and proved to be simple and reliable

STEP 2: photon scattering via mix of quark and gluon loops

First, there are ladder graphs with quark and gluon rungs
Then there are non-ladder graphs which are not depicted though also important



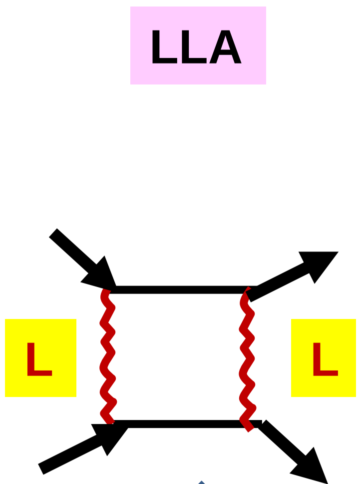
The aim is to calculate all involved Feynman graphs to all orders in α_s

Such resummations can be done approximately only.

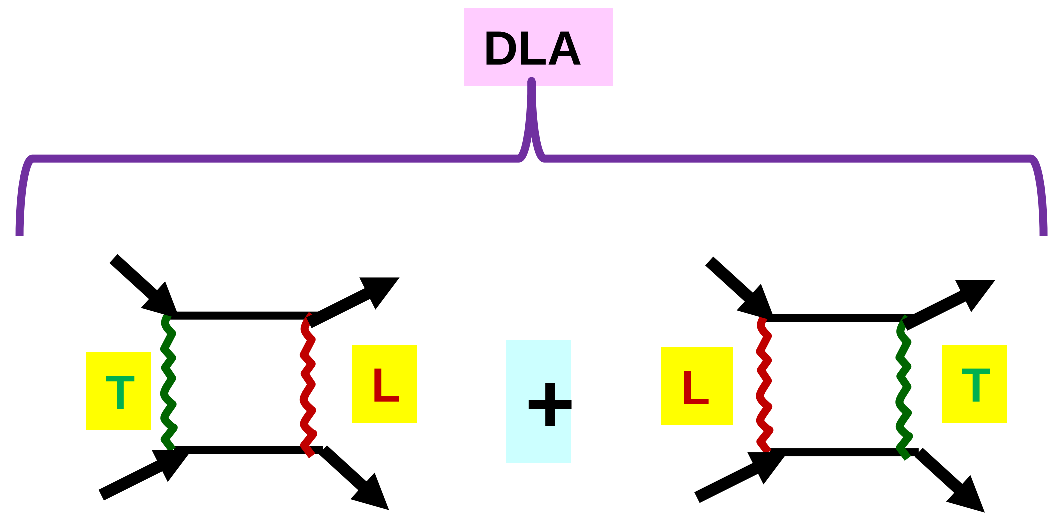
The best-known is BFKL (Balitski-Fadin-Kuraev-Lipatov) evolution equation. It sums leading logarithmic (LL) contributions

Alternatively, one can calculate such graphs in double-logarithmic approximation (Ermolaev-Troyan)

LLA and DLA deal with the same graphs but account for different polarizations of the ladder (vertical) gluons. For instance, for the first-loop to quark-quark scattering the difference is:



Both gluons have longitudinal polarizations (L)



One of the gluons has longitudinal (L) polarization while the other has transverse (T)

It is convenient to use the Mellin transform in DLA and LLA calculations

$$A^{(\pm)} = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega)\, F^{(\pm)}(\omega, Q_1{}^2, Q_2{}^2)$$

$$\xi^{(\pm)} = -\left(1 \pm e^{-i\pi\omega}\right)/2$$

IR cut-off

signature factor

virtualities of external photons

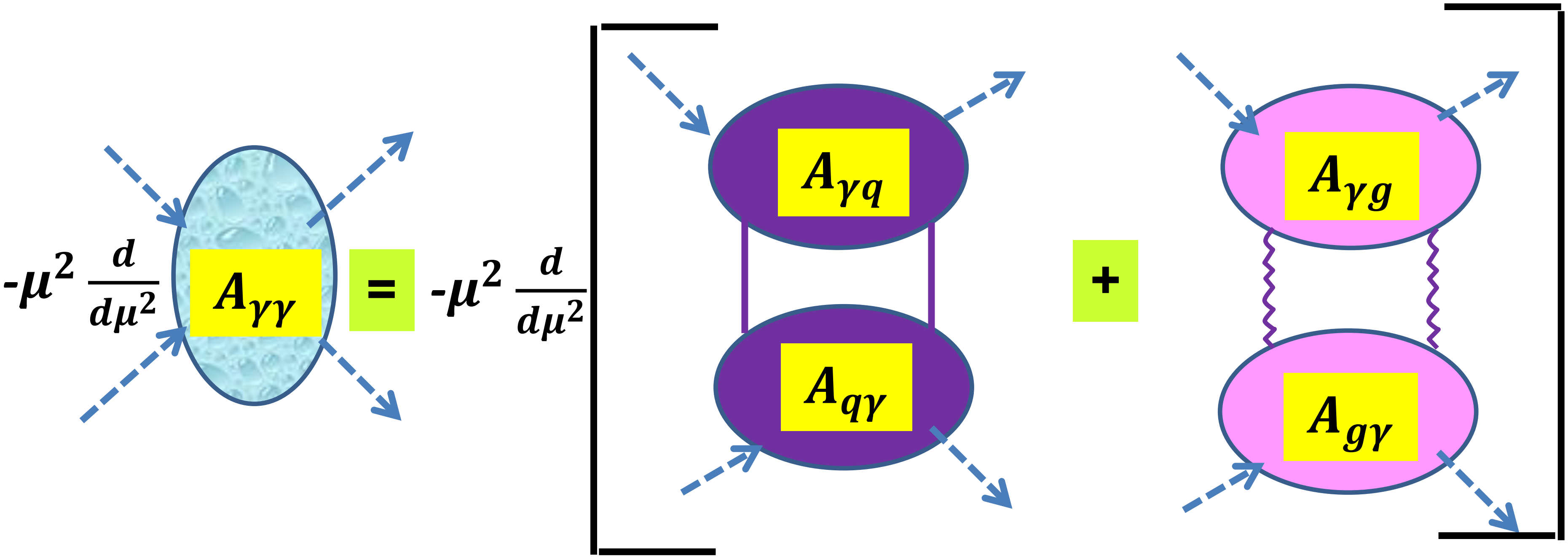
It is convenient to use logarithmic variables:

$$\rho = \ln\left(s/\mu^2\right) \quad y_1 = \ln\left(Q_1{}^2/\mu^2\right) \quad y_2 = \ln\left(Q_2{}^2/\mu^2\right)$$

There are two regions where scattering amplitudes in DLA are given by different expressions, depending on virtualities of the external photons:

1. Moderately virtual photons: $s\,\mu^2 \gg Q_1{}^2 Q_2{}^2$
2. Deeply virtual photons: $s\,\mu^2 \ll Q_1{}^2 Q_2{}^2$

R.h.s. of IREEs for $A_{\gamma\gamma}(s, Q_1^{\,2}, Q_2^{\,2})$ involve auxiliary amplitudes $A_{\gamma q}(s, Q_1^{\,2})$, $A_{\gamma g}(s, Q_2^{\,2})$



The partons with minimal transverse momenta (softest partons) factorize amplitude $A_{\gamma\gamma}$ in auxiliary amplitudes.

Only two-parton intermediate states yield DL contributions

Auxiliary amplitudes become on-shell after differentiation over μ^2

Applying the standard Feynman rules , we arrive at IREEs in analytic form

They look simpler in the Melin space

l.h.s of the IREE is

$$-\mu^2 \frac{d}{d\mu^2} A(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F(\omega, y_1, y_2)$$

The l.h.s. is the same for both Moderately and Deeply Virtual photons but r.h.s. are differerent:

Moderately virtual photons $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(M)}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y_1) F_{q\gamma}(\omega, y_2)$$
$$+ \frac{1}{8\pi^2} F_{\gamma g}(\omega, y_1) F_{g\gamma}(\omega, y_2)$$

Deeply virtual photons $s\,\mu^2 \ll {Q_1}^2 {Q_2}^2$ is IR-stable

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(D)}{}_{\gamma\gamma}(\omega, y_1, y_2) = 0$$

Solving these equations allows us to express amplitudes $A_{\gamma\gamma}(s, {Q_1}^2, {Q_2}^2)$
In terms of the auxiliary amplitudes

For instance, solution for deeply virtual photons $s\,\mu^2 \gg Q_1{}^2 Q_2{}^2$

$$\mathbf{A}^{(\mathrm{M})}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\,\pi^2} \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\sqrt{Q_1{}^2 Q_1{}^2}} \right)^{\omega} \sum_{r=q,g} \left[{}^{1}\!/_{\omega}\, f_{\gamma r}(\omega) f_{r\gamma}(\omega) + f_{r\gamma}(\omega) \int_o^{\eta} dz F_{\gamma r}(\omega, z) + {}^{1}\!/_{2} \int_{\eta}^{2\rho-\xi} dz\ e^{\omega z/2} F_{\gamma r}(\omega, z) F_{r\gamma}(\omega, z) \right]$$

where the symmetrical variables are used

$\xi = y_1 + y_2$,

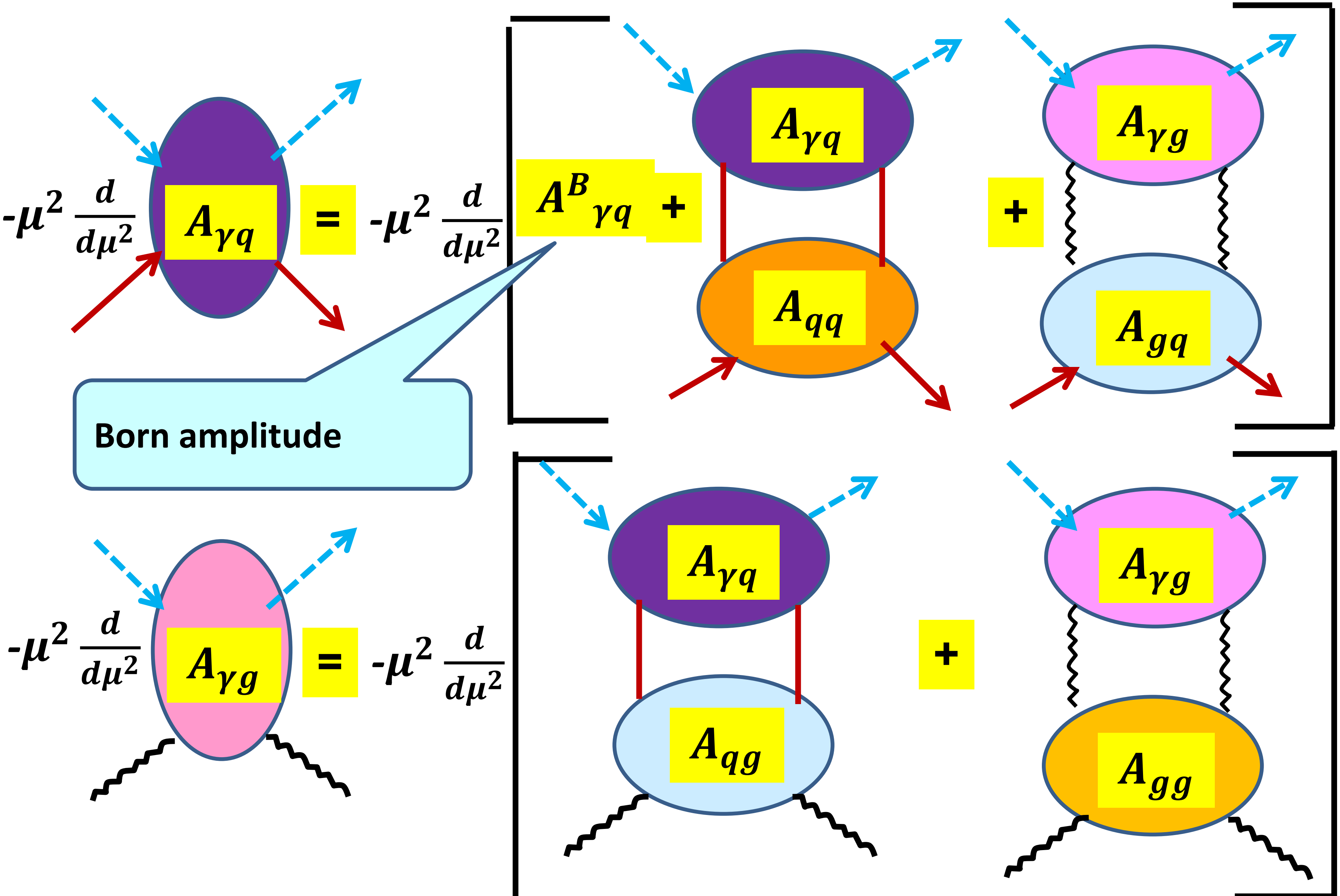
$\eta == |y_1 - y_2|$

Auxiliary amplitudes $F_{\gamma r}(\omega, z)$, $f_{\gamma r}(\omega)$ describe photon-parton scattering

$Q^2 \gg \mu^2$

$Q^2 \sim \mu^2$

IREE for auxiliary amplitudes



Applying the standard Feynman rules , arrive at IREEs in analytic form

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma q}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qq}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gq}(\omega)$$

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma g}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qg}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gg}(\omega)$$

IREEs involve new objects:
parton-parton amplitudes

$f_{ik}(\omega)$

Eqs are linear, so it is easy to find general solutions in terms of the parton-parton amplitudes.
It is more convenient to use

$$h_{ik}(\omega) = \left(1/8\pi^2\right) f_{ik}(\omega)$$

General solutions are specified with the matching

$F_{\gamma q}(\omega, y = 0) = f_{\gamma q}(\omega)$ $\quad$ $F_{\gamma g}(\omega, y = 0) = f_{\gamma g}(\omega)$

amplitudes of photon-parton scattering at y =0. They are unknown and must be calculated independently

IREEs for them are algebraic because *y=0*

$$\omega f_{\gamma q}(\omega) = a_{\gamma q} + f_{\gamma q}(\omega) h_{qq}(\omega) + f_{\gamma g}(\omega) h_{gq}(\omega)$$

$$\omega f_{\gamma g}(\omega) = f_{\gamma q}(\omega) h_{qg}(\omega) + f_{\gamma g}(\omega) h_{gg}(\omega)$$

with $a_{\gamma q} = \alpha/2\,\pi$,

Solving these equations allows us to express the auxiliary amplitudes in terms of the parton-parton amplitudes

IREE for the parton-parton amplitudes are algebraic but non-linear

$$\omega h_{qq} = b_{qq} + h_{qq}h_{qq} + h_{qg}h_{gq}$$
$$\omega h_{qg} = b_{qg} + h_{qq}h_{qg} + h_{qg}h_{gg}$$
$$\omega h_{gq} = b_{gq} + h_{gq}h_{qq} + h_{gg}h_{gq}$$
$$\omega h_{gg} = b_{gg} + h_{gq}h_{qg} + h_{gg}h_{gg}$$

$$\mathrm{b}_{\mathrm{ik}}(\omega) = a_{ik}(\omega) + V_{ik}(\omega)$$

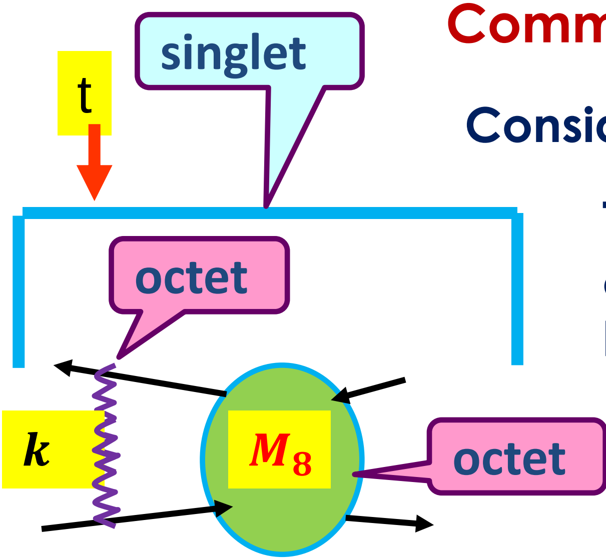
Born contributions. They are independent of ω when QCD coupling is fixed but depend on it when the coupling is running

Contributions of the color octet (non-ladder graphs)

$a_{ik}(\omega)$ coincide with analogous factors for g_1 singlet, see e.g. Bartels-Ermolaev-Ryskin (1996)

However such coincidence does not take place for $V_{ik}(\omega)$

Comment on the color octets $V_{ik}(\omega)$



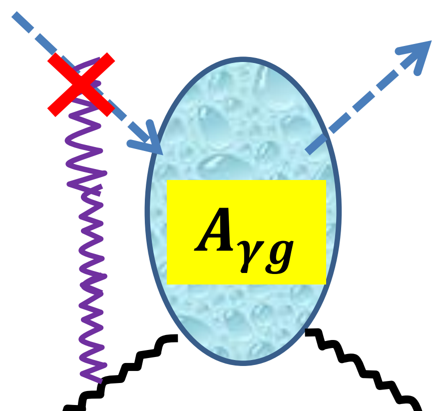
Consider a term of IREE with the factorized gluon.

The gluon belongs to the vector representation of the color group SU(3). Hence, the amplitude M_8 belongs to the octet representation too

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_A \oplus \ \ldots$$

We have to compose IREEs for the parton-parton octet amplitudes.
It is done absolutely in the same way

Fortunately, we need not octet components for photon-parton amplitudes:
Photon-gluon vertices are absent in the QCD Lagrangian



Born contributions at fixed QCD coupling:

$$a_{qq} = (\alpha_s/2\pi)\ C_F\,, \qquad a_{qg} = (\alpha_s/\pi) C_F,$$
$$a_{gq} = -(\alpha_s/2\pi)\ n_f\,, \qquad a_{gg} = (\alpha_s/\pi)\ 2N$$

Generalization to running QCD coupling

One cannot use the DGLAP parametrization $\alpha_s = \alpha_s\,(Q^2)$
based on the parametrization
in every ladder rung $\alpha_s = \alpha_s\left({k_T}^2\right)$
because it fails at small x (in the high-energy limit)

In this case the more adequate parametrization should be used.

$$A(\omega) = (1/b)\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty dz\,\frac{e^{-z\omega}}{(z+\eta)^2+\pi^2}\right]$$

Ermolaev-Greco-Troyan

k is time-like momentum

$$A'(\omega) = (1/b)\left[\frac{1}{\eta} - \int_0^\infty dz\,\frac{e^{-z\omega}}{(z+\eta)^2}\right]$$

k is space-like momentum

where $\eta = ln(\mu^2/\Lambda_{QCD}{}^2)$, $b = (11N - 2\,n_f)/(12\,\pi)$

Principle of Minimal Sensitivity (P.M.Stevenson) was used to specify the scale of μ

It leads to the following expressions

$$a_{qq} = (A(\omega)/2\pi)\ C_F\,, \qquad a_{qg} = (A'(\omega)/\pi) C_F,$$
$$a_{gq} = -(A'(\omega)/2\pi)\ n_f\,, \qquad a_{gg} = (A(\omega)/\pi)\ 2N$$

The scale of α_s for NLO BFKL Pomeron intercept was set With using both Principle of Minimal sensitivity as well as Principle of Maximum Conformality (Brodsky-Di Giustino, Brodsky-Fadin-Kim-Lipatov-Pivovarov, Ermolaev-Troyan)

The system of non-linear algebraic equations for parton-parton amplitudes h_{ik} can be solved exactly for both the case of fixed and running QCD coupling. Explicit expressions for h_{ik} are

$$h_{qq} = \frac{1}{2}\left[\omega - Z - \frac{b_{gg} - b_{qq}}{Z}\right] \qquad h_{qg} = \frac{b_{qg}}{Z}$$

$$h_{gq} = \frac{b_{gq}}{Z} \qquad h_{gg} = \frac{1}{2}\left[\omega - Z + \frac{b_{gg} - b_{qq}}{Z}\right]$$

where

$$Z = \frac{1}{\sqrt{2}}\left[U + W\right]$$

$$U = \omega^2 - 2\left(b_{qq} + b_{gg}\right)$$

$$W = \left[\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq}\right]^{1/2}$$

Substituting them in expressions for auxiliary amplitudes $F_{\gamma q}$ $F_{\gamma q}$ and then in expressions for photon-photon amplitudes, we arrive at explicit expressions for $A^{(M)}{}_{\gamma\gamma}$, $A^{(D)}{}_{\gamma\gamma}$

Substituting the explicit expressions for parton-parton amplitudes into the expressions for the auxiliary photon-parton amplitudes and then substituting the latter into the expressions for the photon-photon amplitudes we arrive at the explicit expressions to them

These expressions are complicated, so we do not write them

Instead, let us focus on
the high-energy asymptotics of amplitudes $A_{\gamma\gamma}(s, Q_1^{\,2}, Q_2^{\,2})$

The standard mathematical tool to calculate asymptotics at $s \to \infty$ is Saddle-Point method:

Saddle-Point method:

$$A(s) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} F(\omega) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho + \Psi(\omega)}$$

We remind that $\rho = ln(s/\mu^2)$

Asymptotics

Let $s \to \infty, \Longrightarrow \rho \to \infty$

$$A \sim \frac{e^{\Psi(\omega_0)}}{\sqrt{2\pi\Psi''(\omega_0)}} \left(\frac{s}{\mu^2}\right)^{\omega_0}$$

$$\rho + \Psi'(\omega_0) = 0$$

$\rho \to \infty$

$\Psi'(\omega) \to -\infty$ when $\omega \to \omega_0$

Singularity point

There can be many singularities but the most important is the rightmost singularity. It can be a pole or a branching point of $\Psi'(\omega)$

Using explicit expression for $A_{\gamma\gamma}$ we conclude that ω_0 is the rightmost singularity (branching point) of the parton-parton amplitudes . It corresponds to the largest root of the equation

$$\boxed{\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq} = 0}$$

Stationary point equation

This equation can be solved analytically, when QCD coupling is fixed because b_{ik} are fixed

When the coupling runs only numerical solution can be obtained because b_{ik} depend on ω

Asymptotics of light-by-light amplitudes is of the Regge form:

impact-factors

Reggeon

$$A_{\gamma\gamma} \sim \bar{A}_{\gamma\gamma} = \frac{N}{ln^{3/2}(s/\mu^2)} \left(\frac{s}{\sqrt{Q_1{}^2 Q_1{}^2}} \right)^{\omega_0}$$

This Reggeon has vacuum quantum numbers, so potentially it can be a new, DL contribution to Pomeron. It has nothing in common with BFKL Pomeron where Leading Logs are accounted for

We introduce $x = \left(\sqrt{Q_1{}^2 Q_1{}^2} \right) / s$

and will write the asymptotics $\bar{A}_{\gamma\gamma}$ in the BFKL manner:

$$\bar{A}_{\gamma\gamma} = x^{-(1+\Delta)}$$

Before doing it, let us compare LLA and DLA series

BFKL Pomeron is asymptotics of the LL series:

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

Asymptotics $x^{-\omega_0}$

$\omega_0 =$ 1 $+ \Delta$

comes from resummation

comes from the overall factor 1/x

DL Pomeron is asymptotics of the DL series:

$$1 + c'_1\left(\alpha_s ln^2(1/x)\right) + c'_2\left(\alpha_s ln^2(1/x)\right)^2 + c'_3\left(\alpha_s ln^2(1/x)\right)^3 + \cdots$$

The factor 1/x is absent, so the whole ω_0 comes altogether from calculations

For comparison with BFKL, it is convenient to introduce

$$\Delta \equiv \omega_0 - 1$$

We calculate the intercept for several particular cases:

1. QCD coupling is fixed

Then the equation for the stationary point is algebraic and can be solved analytically:

$$\omega_{0_{fix}} = (\alpha_s/\pi)^{1/2} \left[4N + C_F + \sqrt{(4N - C_F)^2 - 8 n_f C_F} \right]^{1/2}$$

α_s=0.24

Ermolaev-Greco-Troyan on basis of Principle of Minimal Sensitivity

A. quark contributions neglected, i.e. purely gluonic Pomeron

Δ_{fix}= 0.35

close to LO BFKL intercept

$\Delta_{LO\ BFKL}$=0.34

B. both gluon and quark contributions accounted for

Δ_{fix}= 0.29

2. Accounting for the running α_s effects

C. Purely gluonic Pomeron $\Delta = 0.25$

D. Both gluon and quark contributions are taken into account

$\Delta = 0.066$ ← Close to NLO BFKL intercept

$\Delta_{NLO\ BFKL}$ =0.08

We think that there is no a physical reason whatsoever for DL intercepts be close to BFKL ones and consider it as coincidence

OBSERVATION: The higher accuracy, the smaller the Pomeron intercept

$\Delta_{fix} = 0.35$ — Fixed coupling, gluons only

$\Delta_{fix} = 0.29$ — Fixed coupling, gluons and quarks

$\Delta = 0.25$ — Coupling runs, gluons only

$\Delta = 0.07$ — Coupling runs, gluons and quarks

Hard Pomeron

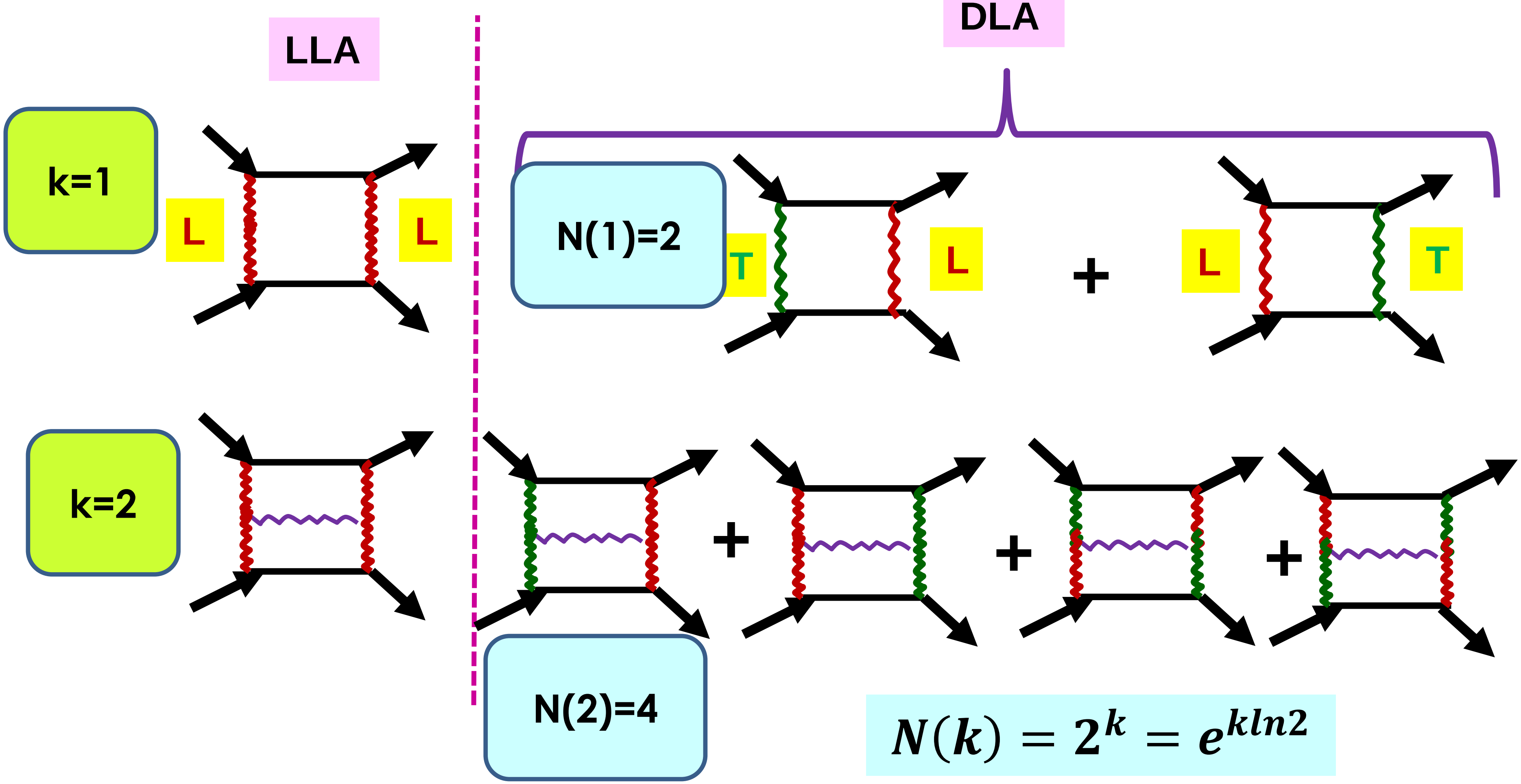
Soft Pomeron

SUGGESTION: Increase of accuracy converts hard Pomeron into Soft one, so eventually the intercept could go down to zero, which would restore Unitarity

This tendency is clearly seen for both BFKL and DL Pomerons

The fact that DL intercepts are comparable with BFKL ones looks surprising. A possible explanation is:

Number of graphs *N* contributing in DLA increases in higher loops:



$$N(k) = 2^k = e^{k\ln 2}$$

Exponential rise of *N* explains why DL contribution, being much smaller in the first loops than LL one, grows in higher loops so fast that its asymptotics, DL Pomeron is comparable with BFKL Pomeron

Applicability region of Regge asymptotics

Regge asymptotics are given by simple and elegant expressions. Let us fix their applicability region

We introduce $R_{as} = \bar{A}_{\gamma\gamma}/A_{\gamma\gamma}$

and study its x-dependence at fixed Q^2 e.g. at $Q^2 = 10$ GeV^2

NB The asymptotics reliably represents the parent amplitude $A_{\gamma\gamma}$ when R_{as} is close to 1.

$R_{as} = 0.9$	$x = 10^{-6}$
$R_{as} = 0.67$	$x = 10^{-4}$
$R_{as} = 0.5$	$x = 10^{-3}$

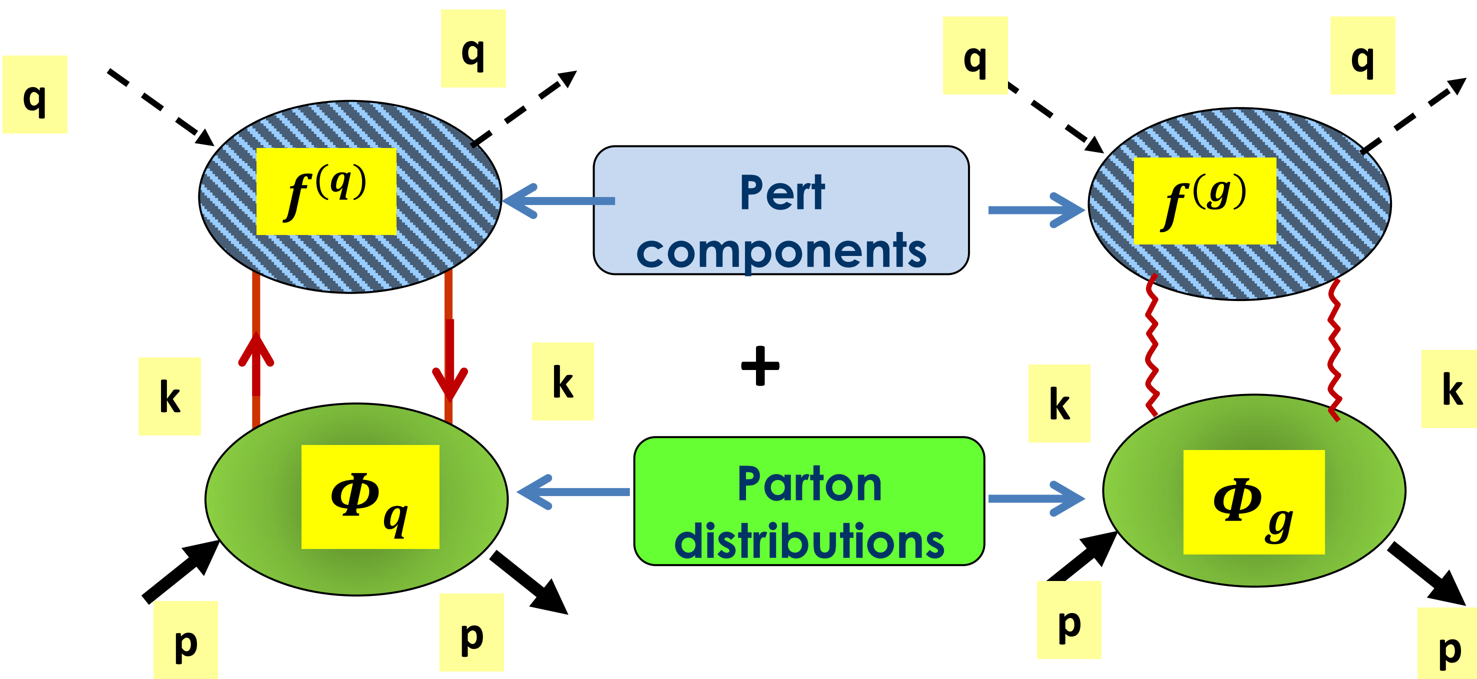
Appicability region for DL Pomeron

$x < x_{max} = 10^{-6}$

$$s > s_{min} = 10^6 \sqrt{{Q_1}^2 {Q_2}^2}$$

Does not depend on the specific process but is the same for all reactions involving DL Pomeron

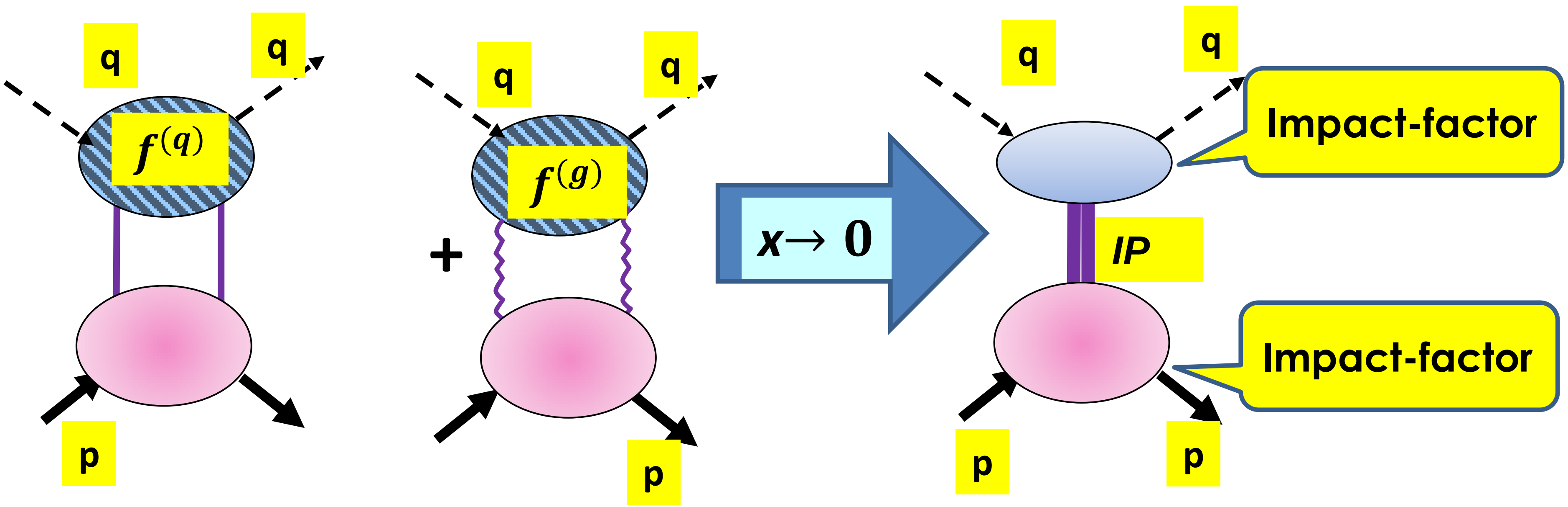
Pomeron applies to description of many high-energy processes.
For example, Deeply Inelastic Scattering (DIS)
and Diffractive DIS
QCD factorization represents DIS structure functions as convolutions:



$f^{(q,g)}$ is a generic notation for $F_{1,2}{}^{(q,g)}$,

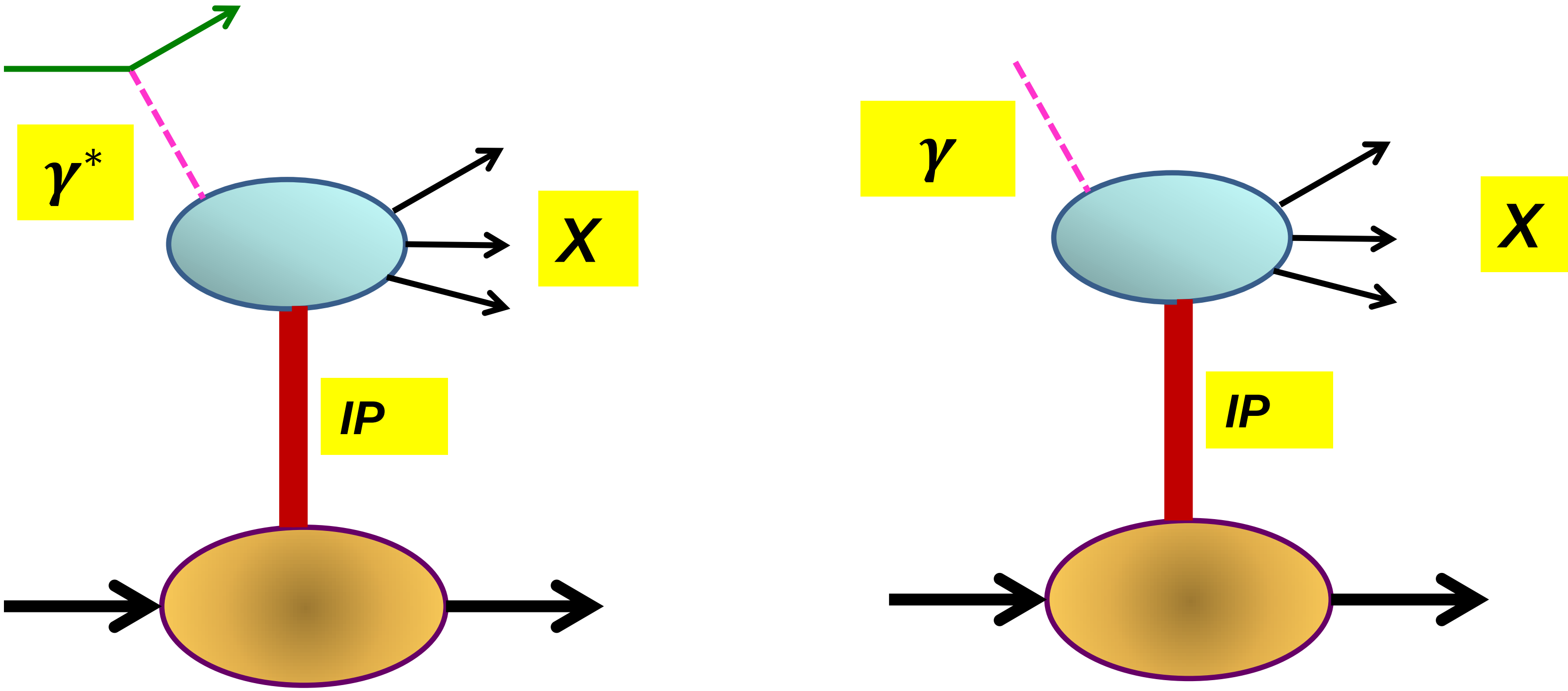
We will address $f^{(q,g)}$ as parent amplitude

Parent amplitudes can be replaced by their asymptotics at $x \to 0$



Such replacement can be done at $x < 10^{-6}$ only

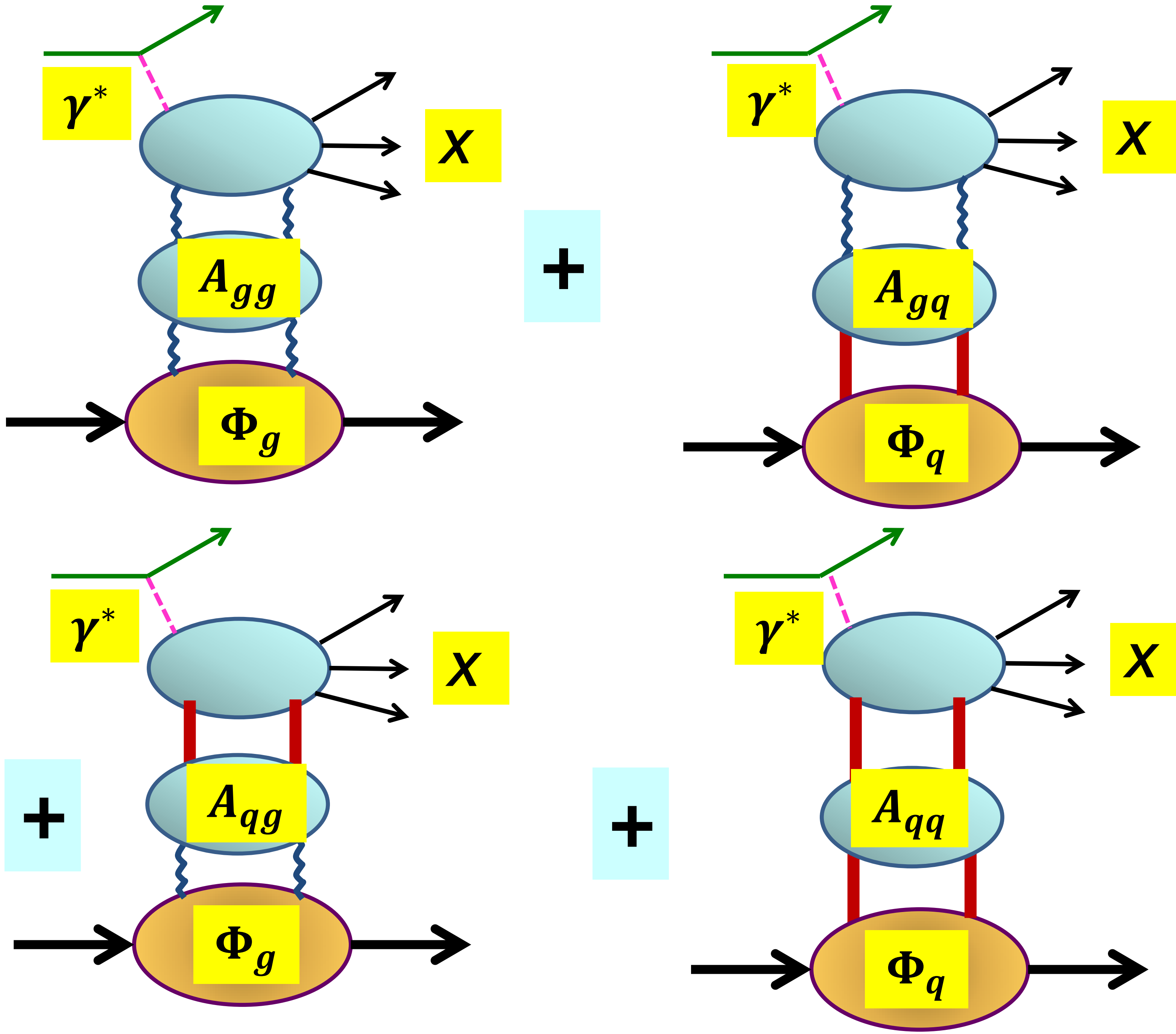
Pomeron participates in Diffractive DIS and inclusive photoproduction:



In all these processes Pomeron should also be used at $x < 10^{-6}$

At $x > 10^{-6}$ the parent amplitudes should be used

instead of Regge asymptotics :



γ^*
X
A_{gg}
Φ_g
+
γ^*
X
A_{gq}
Φ_q
+
γ^*
X
A_{qg}
Φ_g
+
γ^*
X
A_{qq}
Φ_q

.

However the asymptotics are often used outside their applicability region. It leads to various misconceptions. We consider most important ones:

1. Phenomenological/model hard Pomerons to explaine experimental data

Let us denote *M* a generic parent amplitude

By definition $x_{max}{}^{-\omega_0} \approx M$ where $x_{max} = 10^{-6}$

Model Pomeron $x^{-\sigma}$ is supposed to represent *M* at $x_1 > x_{max}$

Model intercept

Therefore $x_1{}^{-\sigma} \approx M$

Combining $x_{max}{}^{-\omega_0} \approx M$ and $x_1{}^{-\sigma} \approx M$

we obtain that $x_{max}{}^{-\omega_0} \approx x_1{}^{-\sigma}$

Choosing $x_1 = 10^{-4}$ we arrive at

$$\sigma = \frac{6}{4}\,\omega_0 = \frac{3}{2}\,1.07 = 1.6$$

soft

hard

So, we see that usage of soft Pomeron outside its applicability region generates an artificial hard Pomeron

2. Necessity of spin-dependent hard Pomerons

Asymptotics of spin-dependent structure function g_1 is also of the Regge type with the non-vacuum intercept $\omega_0^{(-)} = 0.87$ so it is not a Pomeron

Ermolaev-Greco-Troyan

Let a model Reggeon with intercept $\sigma^{(-)}$ mimic g_1 at 10^{-4}

$$\sigma^{(-)} = \frac{6}{4}\,\omega_0^{(-)} = \frac{3}{2}\,0.87 = 1.3$$

Non-vacuum Reggeon

Hard Pomeron

Mimicking parent amplitudes by Regge-like asymptotics inevitably leads to introducing model hard Pomerons, which is totally groundless

Such transformations of Pomerons can be wrongly interpreted as dependence of the intercepts on x or Q^2

CONCLUSIONS

We have obtained explicit expressions for light-by-light scattering amplitudes $A_{\gamma\gamma}$ in DLA, with fixed and running QCD coupling

Applying Saddle-Point method to these expressions, we arrive at the Regge asymptotics, with the Reggeon bearing the vacuum quantum numbers. So, it a new, DL contribution to Pomeron.

Altough intercepts of DL Pomeron are not far from the ones of BFKL and both of them are supercritical, DL Pomeron has nothing in common with BFKL Pomeron which is
asymptotics of total resummation of single-logarithmic contributions while we deal with double logarithms.

Value of the DL Pomeron intercept monotonically decreases with increase of accuracy of calculations. It tempts us to suggest that the further increase of accuracy should make the intercept be= 1, which would agree with the Froissart-Martin bound i.e. with Unitarity

Regge asymptotics are represent by simple exponential expressions, so they are often used instead of the parent amplitudes. Comparing $A_{\gamma\gamma}$ to their asymptotics , we fixed the applicability region of the high-energy asymptotics of $A_{\gamma\gamma}$

We found that usage of the asymptotics outside their applicability region inevitably leads to introducing fictitious hard Pomerons in both unpolarized and spin-dependent processes at high energies

We think that Interference of BFKL and DL Pomerons contributions to different reactions should be examined in detail