

The GS09 Double Parton Distribution Functions

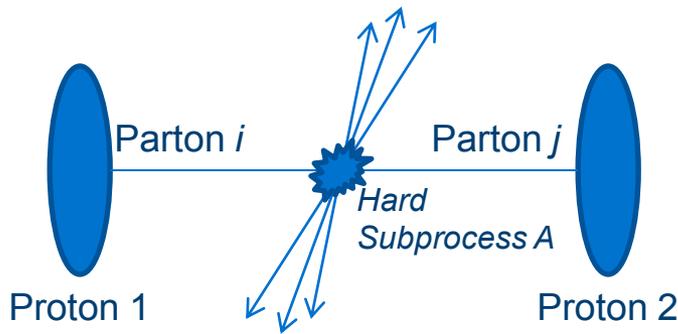
Jo Gaunt

DIS 2010, 21 April 2010, Florence, Italy

Work performed in collaboration with W.J. Stirling (arXiv:0910.4347), and C.H. Kom, A. Kulesza and W.J. Stirling (arXiv:1003.3953).

Single vs. Double Parton Scattering

Single Parton Scattering (SPS):

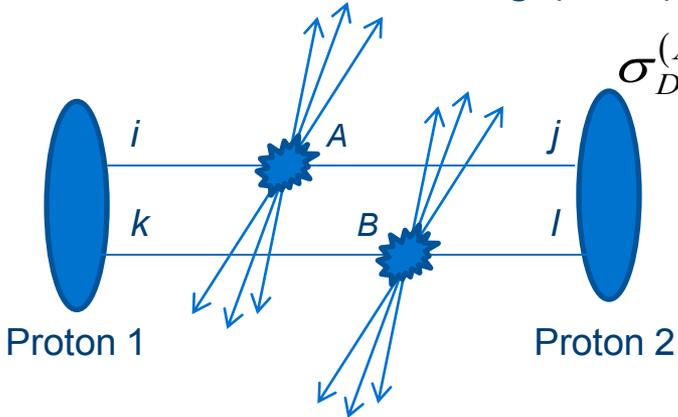


$$\sigma_S^{(A)} = \sum_{i,j} \int D_h^i(x_1; Q_A) D_h^j(x'_1; Q_A) \hat{\sigma}_{ij}^A(x_1, x'_1) dx_1 dx'_1$$

(Single) Parton
Distribution Functions

Hard subprocess
cross section

Double Parton Scattering (DPS):



$$\sigma_D^{(A,B)} = \frac{m}{2\sigma_{eff}} \sum_{i,j,k,l} \int D_h^{ik}(x_1, x_2; Q_A, Q_B) D_h^{jl}(x'_1, x'_2; Q_A, Q_B) \times \hat{\sigma}_{ij}^A(x_1, x'_1) \hat{\sigma}_{kl}^B(x_2, x'_2) dx_1 dx'_1 dx_2 dx'_2$$

Symmetry factor

**Double Parton Distribution
Functions (dPDFs)**

Factor related to correlations of partons in transverse space (nonperturbative, expected to vary little with \sqrt{s}).

Why should we care about double parton scattering at the LHC?

Crudest approximation for dPDFs:

$$D_h^{ij}(x_1, x_2; Q_A, Q_B) \approx D_h^i(x_1; Q_A) D_h^j(x_2; Q_B)$$

$$\Rightarrow \sigma_D^{(A,B)} \approx \frac{m}{2} \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

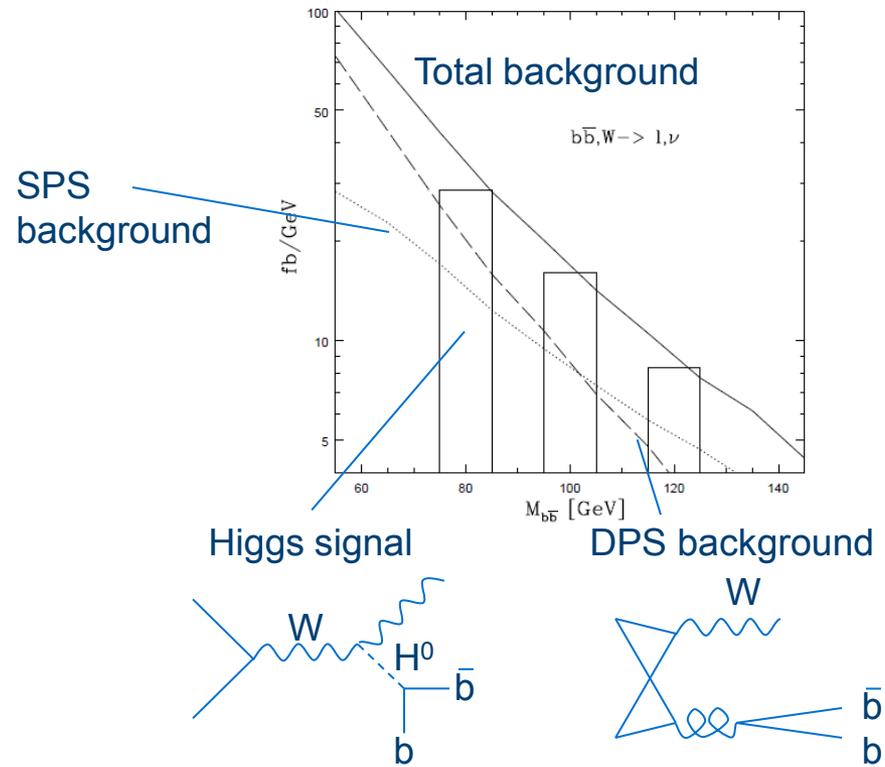
\Rightarrow DPS cross sections go like the product of SPS ones!

\Rightarrow DPS cross sections **grow faster with energy than SPS σ** .

DPS processes...

- provide significant backgrounds to Higgs and new physics signals.
- reveal information about the structure of the proton.

DPS background to Higgs + W production (Del Fabbro and Treleani, hep-ph/9911358, 1999):

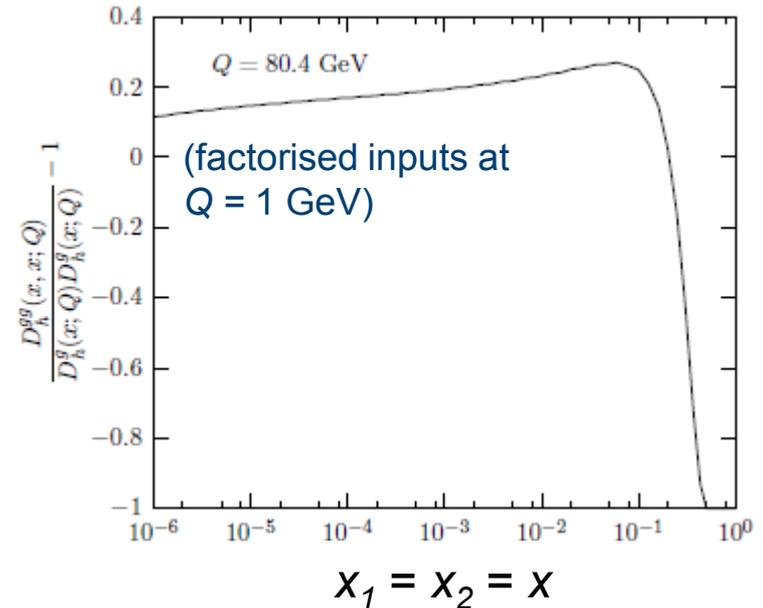


What information do we have on the dPDFs?

Experimental: CDF/D0 investigations of DPS in $\gamma + 3$ jet production (DPS contribution corresponds to $A = 2j$, $B = \gamma j$). Their findings are consistent with the factorised approximation for dPDFs (but low statistics & looking at sea quarks at low x only).

Theoretical: 'double DGLAP equation' describing the change in the dPDFs with factorisation scale, for the dPDFs with $Q_A = Q_B = Q$. (Kirschner, Phys.Lett.B84:266, 1979 and Shelest, Snigirev, and Zinovjev, Phys.Lett.B113:325,1982).

Crucial prediction of this equation: **pQCD evolution causes dPDFs to deviate from factorised forms!**



JG and Stirling,
0910.4347, 2009

Input dPDFs

The most accurate approach to modelling the (equal scale) dPDFs is to use the double DGLAP equation along with some suitably chosen inputs at a low scale Q_0 .

But what should the inputs look like? **Can we get any theoretical insight?**

First reaction - **NO!** A dPDF at any particular scale receives contributions from **non-perturbative physics**.

The dPDF Sum Rules

Actually – **YES**, we can! We have shown that the following equalities (**sum rule equalities**) are preserved by double DGLAP:

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1 j_2}(x_1, x_2; t) = (1-x_2) D_h^{j_2}(x_2; t)$$
$$\int_0^{1-x_2} dx_1 D_h^{j_1 j_2}(x_1, x_2; t) = \begin{cases} N_{j_1 v} D_h^{j_2}(x_2; t) & \text{when } j_2 \neq j_1 \text{ or } \bar{j}_1 \\ (N_{j_1 v} - 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = j_1 \\ (N_{j_1 v} + 1) D_h^{j_2}(x_2; t) & \text{when } j_2 = \bar{j}_1 \end{cases}$$

These equalities are no more than the statements of conservation of momentum and quark number for the dPDFs, and have an interpretation in terms of conditional probabilities.

In general, we expect there to be a hierarchy of such relations, relating the integrals of n parton distributions to $(n-1)$ parton distributions.

The sum rules impose important constraints on the type of input dPDFs that are allowable...although non-trivial to implement them!

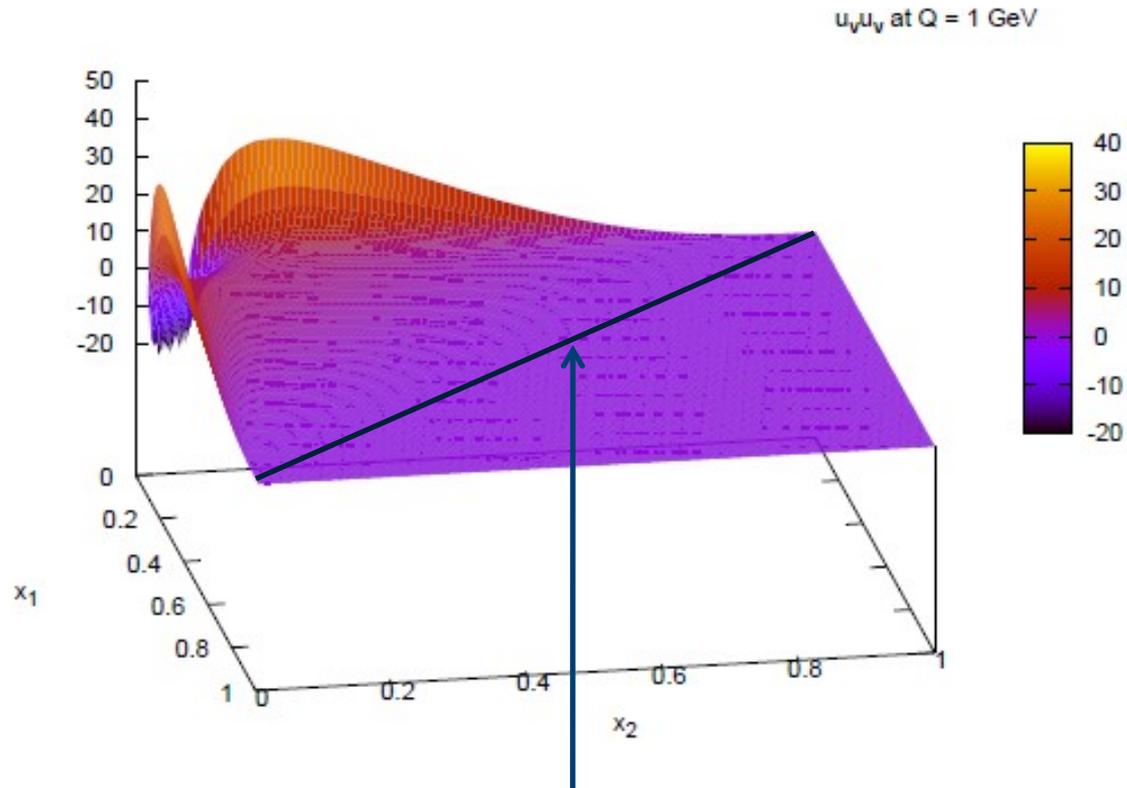
The GS09 dPDFs

First set of publicly available LO equal-scale dPDFs (available from **HepForge***). Grid of dPDF values obtained by applying numerical double DGLAP evolution to certain inputs.

Inputs used at $Q_0 = 1$ GeV are based on products of MSTW2008LO single PDFs, but contain a number of **key features** to ensure that they approximately satisfy the sum rules.

*<http://projects.hepforge.org/gsdpdf/>

The GS09 dPDFs

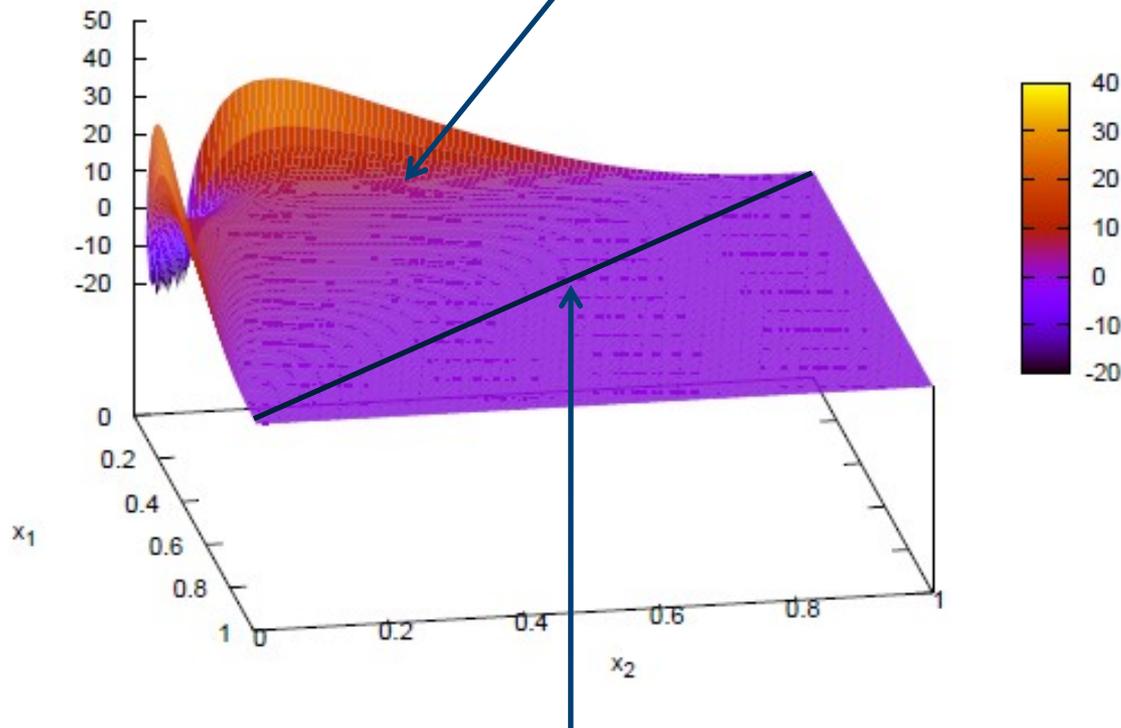


All dPDFs are suppressed near the kinematic boundary $x_1 + x_2 = 1$ to take account of **phase space considerations**.

The GS09 dPDFs

Terms have been added/subtracted from dPDFs to take account of **number effects**.

$u_\nu u_\nu$ at $Q = 1$ GeV

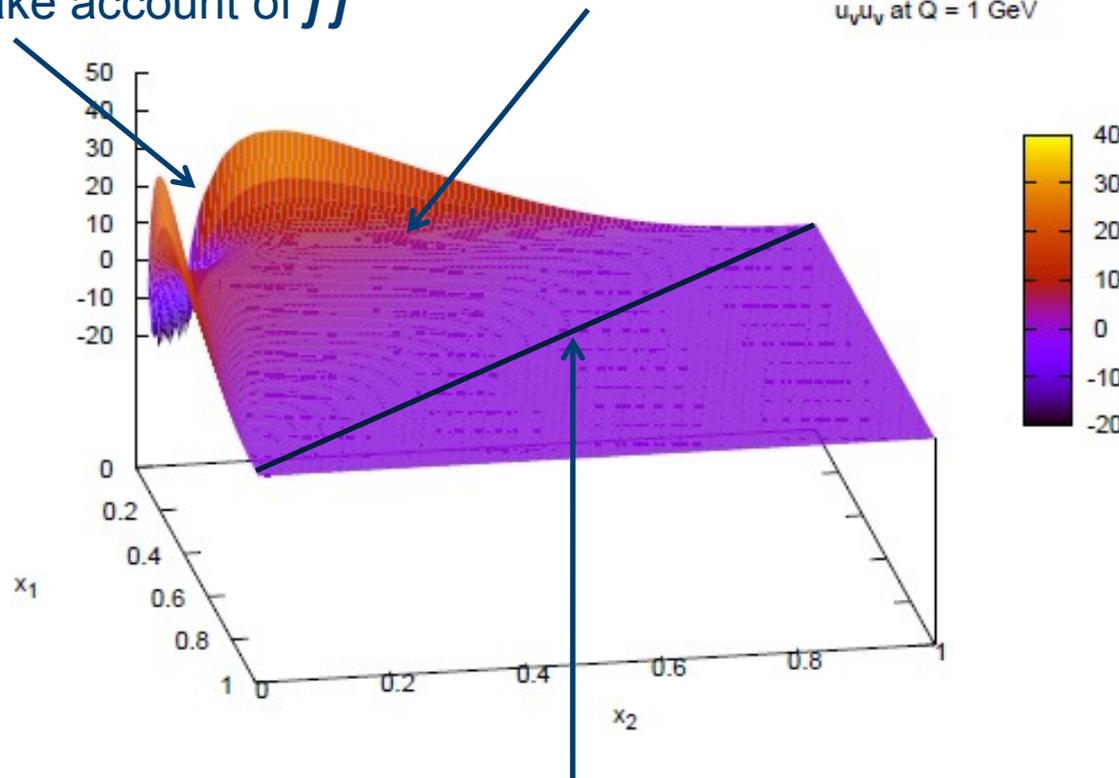


All dPDFs are suppressed near the kinematic boundary $x_1 + x_2 = 1$ to take account of **phase space considerations**.

The GS09 dPDFs

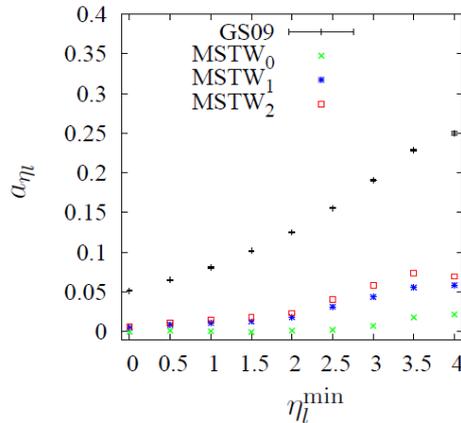
Terms have been added to the $j\bar{j}$ distributions to take account of $j\bar{j}$ correlations.

Terms have been added/subtracted from dPDFs to take account of **number effects**.

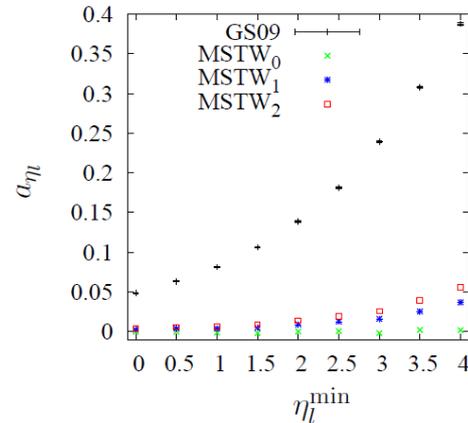


All dPDFs are suppressed near the kinematic boundary $x_1 + x_2 = 1$ to take account of **phase space considerations**.

Comparison of GS09 with factorised dPDFs



(a) Positively charged leptons



(b) Negatively charged leptons

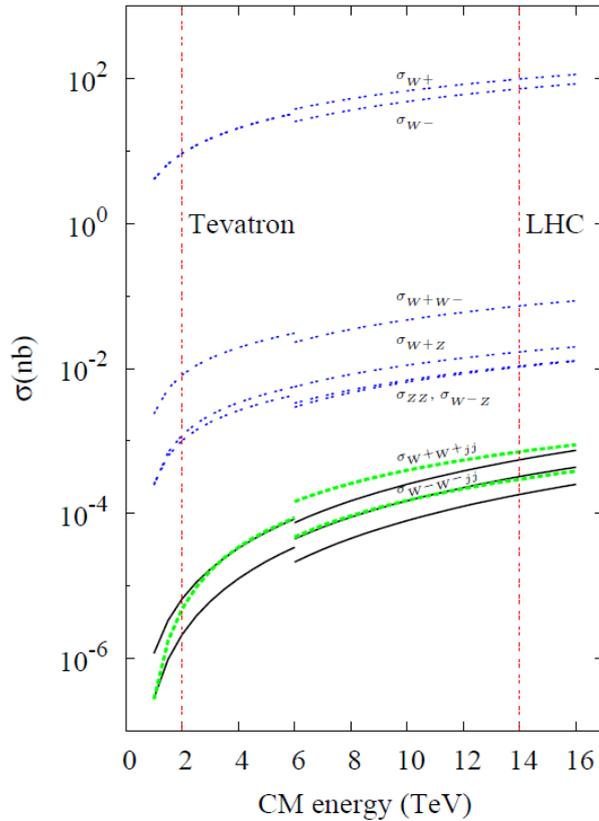
Comparison in the context of a particular process – **equal sign W pair production.**

$$a_{\eta l} = \frac{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) - \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}{\sigma(\eta_{l_1} \times \eta_{l_2} < 0) + \sigma(\eta_{l_1} \times \eta_{l_2} > 0)}$$

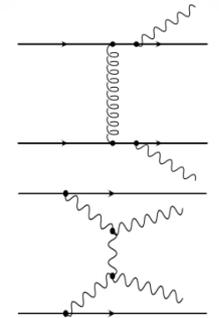
$a_{\eta l}$ larger for GS09 due to number effect subtractions, especially for large η_l^{\min} (i.e. large x, where number effect subtractions have the largest impact).

JG, Kom, Kulesza, Stirling, 1003.3953, 2010

Possibility of observing same-sign WW DPS at LHC



SPS same-sign WW production is forbidden at order $\alpha_W^2 \Rightarrow \sigma$ for this process is comparable to DPS σ , and always involves 2j.



This SPS background can be efficiently removed via a jet veto – however, there are other SPS processes that can mimic the DPS same-sign lepton signal.

- heavy flavour $\longrightarrow t \rightarrow W^+ b \rightarrow l^+ \nu b,$
 - electroweak gauge boson pair $\longrightarrow \bar{t} \rightarrow W^- \bar{b} \rightarrow q \bar{q}' l^+ \nu c.$
- (If these are not detected)
- $Z(\gamma^*) Z(\gamma^*) \rightarrow l^+ (l^-) l^+ (l^-)$

Thus this channel is not as 'clean' with regards to DPS as had been previously thought – carefully chosen cuts required to enhance S/B sufficiently.

JG, Kom, Kulesza, Stirling, 1003.3953, 2010

Future Work

Extend treatment to NLO!

- Need to compute $1 \rightarrow 2$ splitting functions at NLO (trivial at LO).
- Will need NLO coefficient functions for certain benchmark processes (e.g. equal sign WW production).

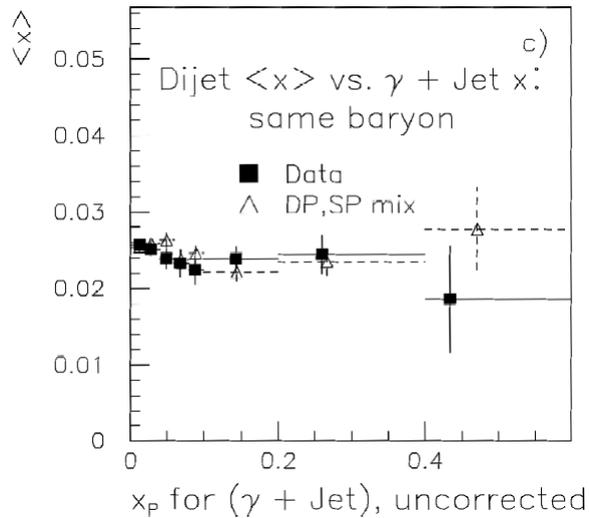
Summary

- Important to understand DPS – will produce significant backgrounds and interesting signals at the LHC.
- For DPS predictions, require dPDFs. A ‘double DGLAP’ equation exists dictating the evolution of the equal-scale dPDFs, and we have derived the number and momentum sum rules for these quantities.
- We have produced the first publicly available set of LO equal-scale dPDFs. Sum rules used to guide construction of inputs at $Q_0 = 1$ GeV, and double DGLAP equation used to obtain dPDF values at other scales.
- Number and momentum correlations in GS09 dPDFs affect the signatures of DPS processes – but may be difficult to see this at LHC due to SPS background.

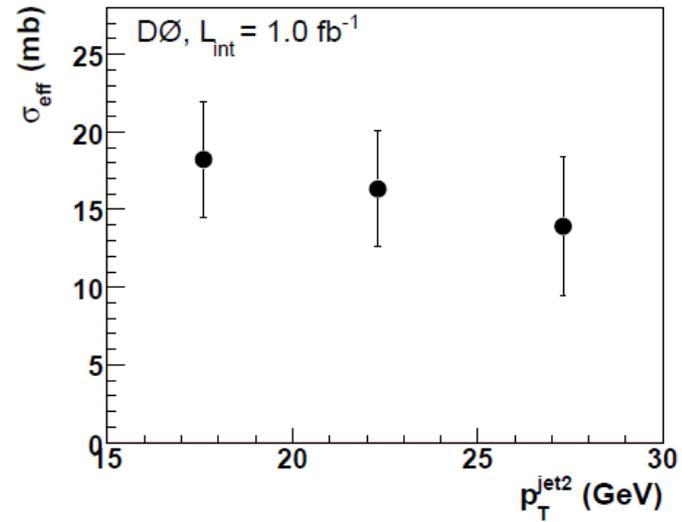
Backup Slides

CDF/D0 DPS Plots

Correlations in x



$\sigma_S^{(A)} \sigma_S^{(B)} / \sigma_D^{(A,B)}$ against second hardest jet p_T



Pictorial representation of double DGLAP equation

$$\Delta_+ \left[D_h^{j_1 j_2} (x_1, x_2; t) \delta x_1 \delta x_2 \right]$$

$$= \sum_{j'_1} \int_{x'_1=0}^{1-x_2} \frac{\alpha_s(t)\Delta t}{2\pi} P_{j'_1 \to j_1}^R \left(\frac{x_1}{x'_1} \right) \frac{\delta x_1}{x'_1} + \sum_{j'_2} \int_{x'_2=0}^{1-x_1} \frac{\alpha_s(t)\Delta t}{2\pi} P_{j'_2 \to j_2}^R \left(\frac{x_2}{x'_2} \right) \frac{\delta x_2}{x'_2}$$

$D_h^{j'_1 j_2} (x'_1, x_2; t) \delta x'_1 \delta x_2$

 $D_h^{j_1 j'_2} (x_1, x'_2; t) \delta x_1 \delta x'_2$

← Splitting processes acting to increase D^{ij} as the scale is increased from $t \rightarrow t + \Delta t$.

Splitting processes acting to decrease D^{ij} as the scale is increased from $t \rightarrow t + \Delta t$.

$$+ \sum_{j'} \frac{\alpha_s(t)\Delta t}{2\pi} P_{j' \to j_1 j_2} \left(\frac{x_1}{x_1+x_2} \right) \frac{\delta x_1}{x_1+x_2}$$

$D_h^{j'} (x_1 + x_2; t) \delta x_2$

“single PDF feed”

$$\Delta_- \left[D_h^{j_1 j_2} (x_1, x_2; t) \delta x_1 \delta x_2 \right]$$

$$= \frac{\alpha_s(t)\Delta t}{2\pi} P_{j_1 \to j_1}^V + \frac{\alpha_s(t)\Delta t}{2\pi} P_{j_2 \to j_2}^V$$

$D_h^{j_1 j_2} (x_1, x_2; t) \delta x_1 \delta x_2$

 $D_h^{j_1 j_2} (x_1, x_2; t) \delta x_1 \delta x_2$

Double DGLAP evolution as a branching process

$$\begin{aligned}
 & D_h^{j_1 j_2}(x_1, x_2; \tau) \delta x_1 \delta x_2 \\
 &= \sum_{j', j'_1, j'_2} \int_{\tau'=0}^{\tau} \int_{z_1=x_1}^{1-x_2} \int_{z_2=x_2}^{1-z_1} \Delta \tau P_{j' \rightarrow j'_1 j'_2} \left(\frac{z_1}{z_1+z_2} \right) \frac{\delta z_1}{z_1+z_2} \\
 & \quad \left(\text{Diagram: A node } j' \text{ at } z_1+z_2 \text{ branches into } j'_1 \text{ at } z_1 \text{ and } j'_2 \text{ at } z_2. \text{ A vertical dashed line separates } \tau' \text{ from } \tau. \text{ Labels include } D_h^{j'}(z_1+z_2; \tau') \delta z_2, D_{j'_1}^{j_1} \left(\frac{x_1}{z_1}; \tau - \tau' \right) \frac{\delta x_1}{z_1}, \text{ and } D_{j'_2}^{j_2} \left(\frac{x_2}{z_2}; \tau - \tau' \right) \frac{\delta x_2}{z_2}. \right) \\
 & + \sum_{j'_1, j'_2} \int_{z_1=x_1}^{1-x_2} \int_{z_2=x_2}^{1-z_1} \left(\text{Diagram: A node } j'_1 j'_2 \text{ at } (z_1, z_2; 0) \text{ evolves into } j'_1 \text{ at } z_1 \text{ and } j'_2 \text{ at } z_2. \text{ Labels include } D_h^{j'_1 j'_2}(z_1, z_2; 0) \delta z_1 \delta z_2, D_{j'_1}^{j_1} \left(\frac{x_1}{z_1}; \tau \right) \frac{\delta x_1}{z_1}, \text{ and } D_{j'_2}^{j_2} \left(\frac{x_2}{z_2}; \tau \right) \frac{\delta x_2}{z_2}. \right)
 \end{aligned}$$