Generalized and transverse-momentum dependent parton distributions

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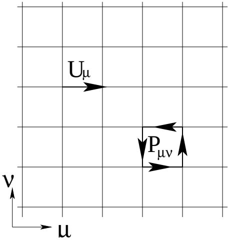
Outline

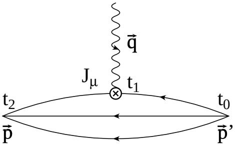
- “control” quantities: nucleon mass, axial charge and isovector momentum fraction
- spin structure of the nucleon
- electromagnetic form factors F_1 and F_2
- generalized form factors and their radii
- conclusion.

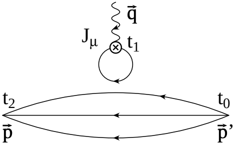
Results by:

- “QCD Structure Functions” Collaboration (QCDSF)
- European Twisted Mass Collaboration (ETMC)
- Riken-BNL-Columbia / UKQCD Collaboration (RBC-UKQCD)
- Lattice Hadron Physics Collaboration (LHPC)

Lattice QCD







Dynamical variables: $U_\mu(x) = e^{iag_0 A_\mu(x)}$

Wilson action (1974):

$$S_g = \frac{1}{g_0^2} \sum_x \sum_{\mu \neq \nu} \mathrm{Re}\,\mathrm{Tr}\,\{1 - P_{\mu\nu}(x)\}$$

Euclidean time: $\tau = -it$

Continuum limit: $g_0 \sim 1/\log(1/a)$

Quarks:

- Grassmann variables: $\psi_1\psi_2 = -\psi_2\psi_1$
- quadratic action $\Rightarrow$

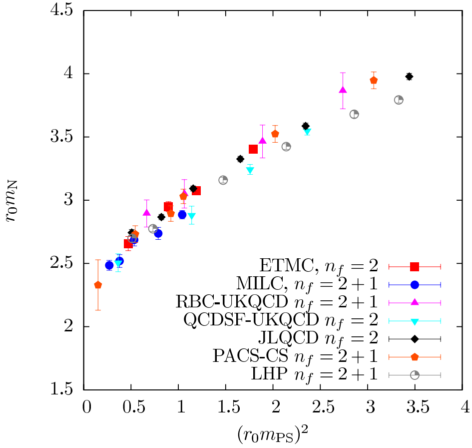
$$\langle \psi(y)\bar{\psi}(x) \rangle = M^{-1}([U]; x, y)$$

“Control” quantities

$$M_N,\ g_A,\ \langle x \rangle_{u-d}$$

Quark-mass dependence of the nucleon mass

$$M_N(M_\pi) = M_N^0 - 4c_1 M_\pi^2 + c_{NA} M_\pi^3 + c_4 M_\pi^4 + c'_{NA} M_\pi^4 \log M_\pi + c_6 M_\pi^6$$



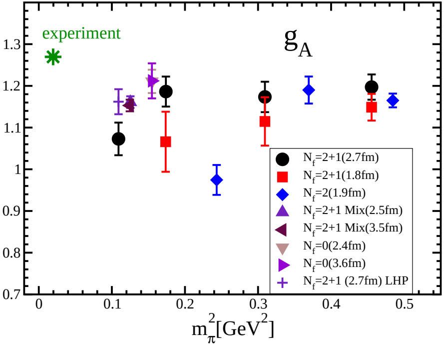
c_1: the sigma-term

M_π^3: $\quad N \to N\pi \to N$

$M_\pi^4 \log M_\pi$: second branch cut from $N \to \Delta\pi \to N$

compiled by K. Jansen, LATTICE 08

The axial charge g_A



Bjorken sum rule:

$$\int_0^1 [g_1^{ep}(x) - g_1^{en}(x)]dx = \frac{1}{6}(1 - \frac{\alpha_s(Q)}{\pi})g_A$$

compilation taken from 0904.2039

(RBC-UKQCD collab.)

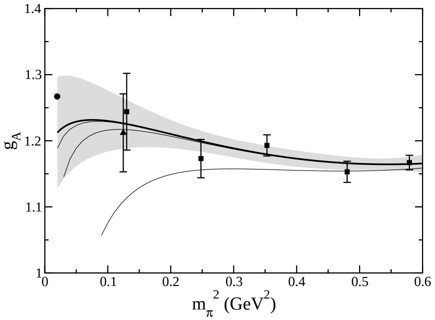
lattice results systematically lower than the phenomenological value 1.2695(29).

$$g_A(m_\pi) = g_A - \frac{g_A^3 m_\pi^2}{16\pi^2 f_\pi^2} + 4m_\pi^2 \left\{ C(\lambda) + \frac{c_A^2}{4\pi^2 f_\pi^2}\left[\frac{155}{972}g_1 - \frac{17}{36}g_A\right] + \gamma \log\frac{m_\pi}{\lambda} \right\}$$

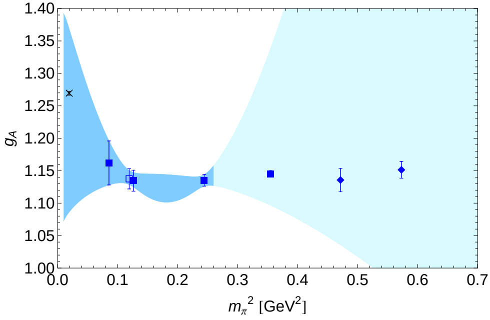
$$+ \frac{4c_A^2 g_A}{27\pi f_\pi^2 \Delta} m_\pi^3 + \frac{8c_A^2 g_A m_\pi^2}{27\pi^2 f_\pi^2}\left[1 - \frac{m_\pi^2}{\Delta^2}\right]^{\frac{1}{2}} \log R + \frac{c_A^2 \Delta^2}{81\pi^2 f_\pi^2}(25g_1 - 57g_A)\left\{ \log\frac{2\Delta}{m_\pi} - \left[1 - \frac{m_\pi^2}{\Delta^2}\right]^{\frac{1}{2}} \log R \right\}$$

$\gamma = \frac{1}{16\pi^2 f_\pi^2}\left[\frac{50}{81}c_A^2 g_1 - \frac{1}{2}g_A - \frac{2}{9}c_A^2 g_A - g_A^3\right]; \quad R = \frac{\Delta}{m_\pi} + \left[\frac{\Delta^2}{m_\pi^2} - 1\right]^{\frac{1}{2}}$ Hemmert, Procura, Weise PRD**68** 075009 ('03)

g_A: LHPC mixed-action calculation



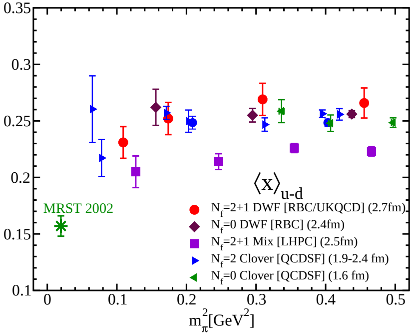
LHPC, Phys.Rev.Lett.96:052001,2006
3-param. SSE fit (g_A^0, $g_{\Delta\Delta}$, C)



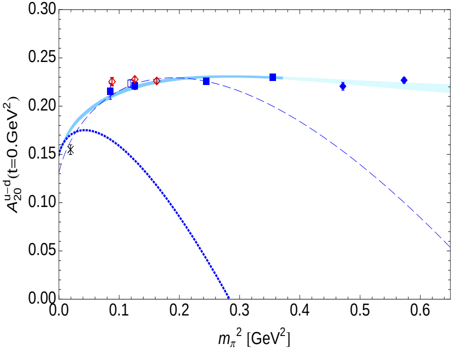
LHPC 1001.3620
3-parameter SSE fit (g_A^0, $g_{\Delta\Delta}$, C)

- new, more accurate results tend to be lower than previously
- chiral logs could make g_A bend up sharply as $m_\pi \stackrel{>}{\rightarrow} m_\pi^{\text{phys}}$

Isovector Momentum Fraction $\langle x \rangle_{u-d}$



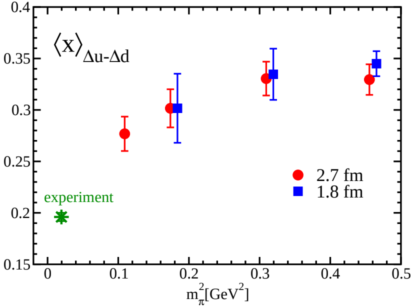
World's published lattice data



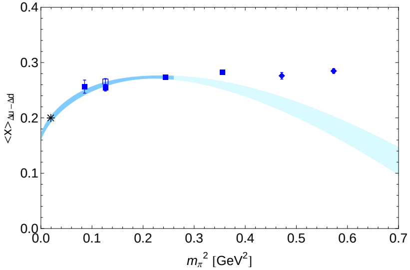
LHPC 1001.3620 and in preparation (CBChPT fit)

- pion mass dependence similar in all calculations (very flat)
- quite a spread in the absolute normalization
- unclear whether/how data points will converge to the MRST ’02 value.

Isovector Helicity Fraction $\langle x\rangle_{\Delta u-\Delta d}$



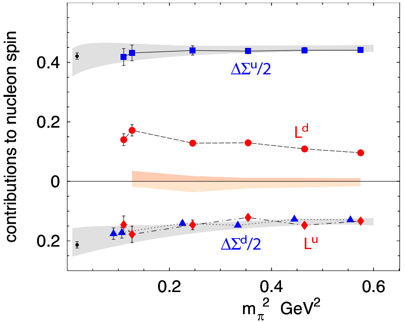
RBC-UKQCD 1003.3387

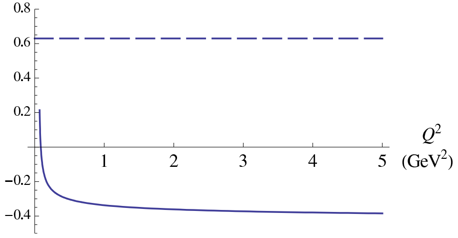


LHPC 1001.3620 (CBChPT fit)

- pion mass dependence quite flat
- reasonable agreement with the HERMES result [Phys. Rev. D75, 012007].

Spin Structure of the Nucleon [LHPC, Phys.Rev.D77:094502,2008; 1001.3620]





one-loop evolution of $\frac{1}{2}\Delta\Sigma_{u-d}$ and L_{u-d}

(Fig. by J.W. Negele)

- non-singlet $J = L + S$ renormalizes multiplicatively
- spin conserved $\Rightarrow$ large change in L [A.W. Thomas PRL 101:102003,2008]

$$L^{u-d}(t) + \frac{\Delta\Sigma^{u-d}}{2} = \left(\frac{t}{t_0}\right)^{-\frac{32}{81}} \left(L^{u-d}(t_0) + \frac{\Delta\Sigma^{u-d}}{2}\right), \quad t = \log(Q^2/\Lambda_{\rm QCD}^2)\,.$$

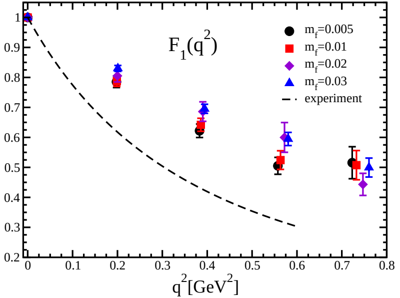
disconnected diagrams remain to be calculated for non-isovector quantities

Electromagnetic Form Factors

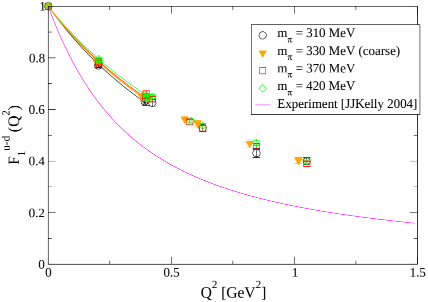
$$\langle p', s' | J^{\mu} | p, s \rangle_{\theta} = \bar{u}_{s'}(p') \Gamma^{\mu}(q^2) u_s(p) \,,$$

$$\Gamma^{\mu}(q^2) = \gamma^{\mu} F_1(q^2) + i\sigma^{\mu\nu} \frac{q_{\nu}}{2m} F_2(q^2)$$

Nucleon isovector Dirac form factor F_1



$N_f = 2 + 1$ Domain-wall-fermion calculation (RBC-UKQCD, 0904.2039)
$a = 0.114$fm $\quad m_\pi \geq 350$MeV



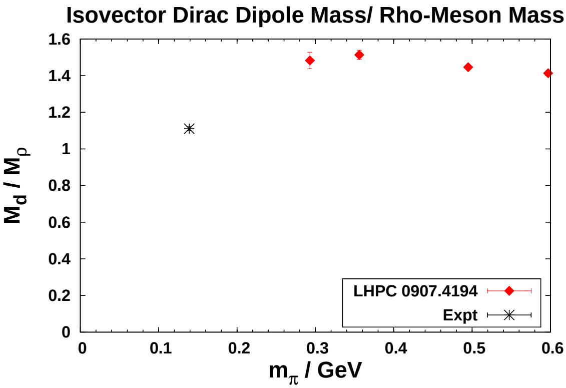
$N_f = 2 + 1$ DWF
LHPC, 0907.4194
$a = 0.084$fm $\quad m_\pi \geq 310$MeV

- new level of accuracy achieved
- pion mass dependence is (very) weak between 500 and 300 MeV.
- dipole form $1/(1 + Q^2/M_D^2)^2$ provides a good fit up to $Q^2 \approx 1$GeV^2.

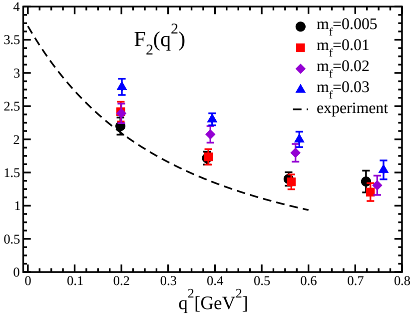
A test of vector-meson dominance

The approximate dipole behavior of the FF $F_1(Q^2) = \frac{1}{(1+Q^2/0.71\mathrm{GeV}^2)^2}$ can be understood as being due to the contribution of two nearby vector meson poles with opposite residua, [Perdrisat, Punjabi, Vanderhaeghen, Prog Part Nucl Phys 59 (2007) 694]

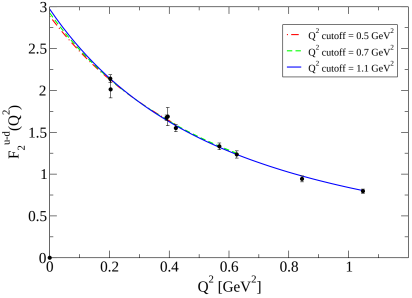
$$F_1(q^2) \approx \frac{a}{q^2 - m_{V1}^2} + \frac{-a}{q^2 - m_{V2}^2} = \frac{a(m_{V1}^2 - m_{V2}^2)}{(q^2 - m_{V1}^2)\,(q^2 - m_{V2}^2)}\,.$$



Nucleon isovector Pauli form factor F_2



$N_f = 2+1$ Domain-wall-fermion calculation (RBC-UKQCD, 0904.2039)

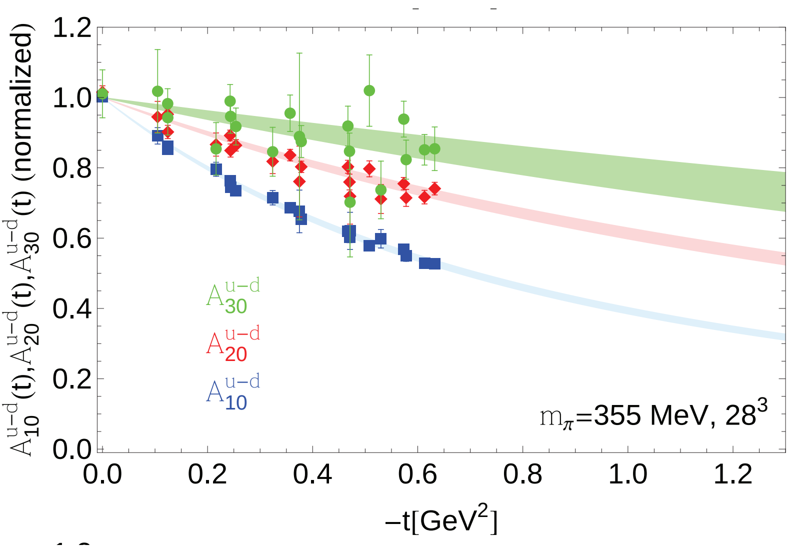


$N_f = 2+1$ DWF calculation $m_\pi = 300$MeV (LHPC 0907.4194)

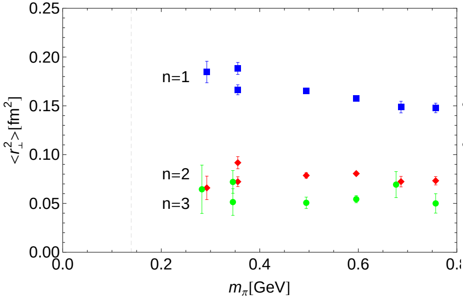
- extrapolation to $Q^2 = 0$ using dipole form $\Rightarrow$ magnetic moment $\kappa_v = F_2(0)$.

Generalized Form Factors

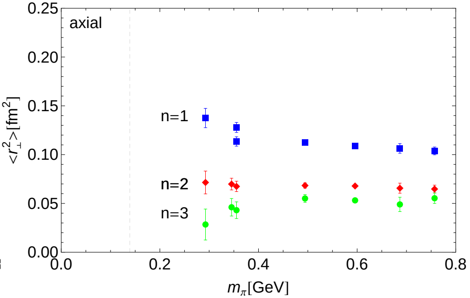
Generalized Form Factors (unpolarized, isovector case)



Transverse radii in the infinite-momentum frame



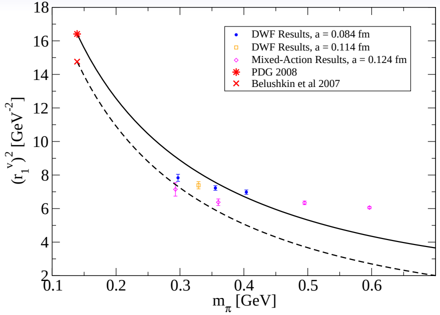
Unpolarized case

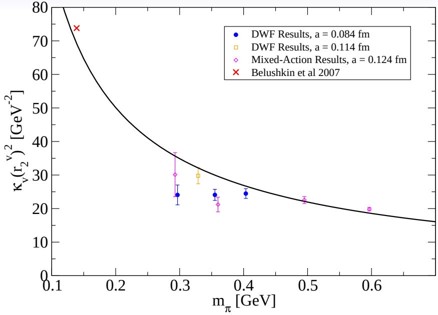


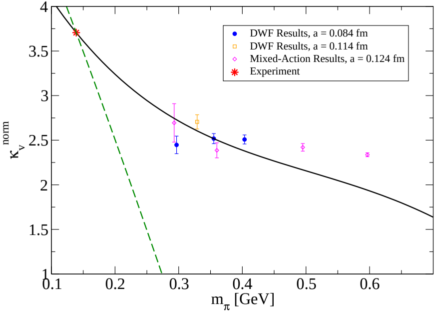
Polarized case

- transverse radii decrease with n, as expected – most of the contribution to higher moments comes from the large-x region; a parton carrying by itself most of the nucleon's momentum must be located near the center of mass in the transverse plane
- for $n \geq 2$, the radii are weakly dependent on the pion mass and similar in the polarized and unpolarized case [LHPC 1001.3620].

Chiral extrapolation? Dirac radius r_1, Pauli radius r_2, magnetic moment κ_v







LHPC 0907.4194
curves = chiral formula with Δ dof,
phenom. param. $c_A = 1.5$, $\Delta = 293$MeV:

$$\kappa_v(m_\pi)(r_2^v)^2 = \frac{g_A^2 M_N}{8\pi f_\pi^2 m_\pi} + \frac{c_A^2 M_N}{9\pi^2 f_\pi^2 \sqrt{\Delta^2 - m_\pi^2}} \log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right].$$

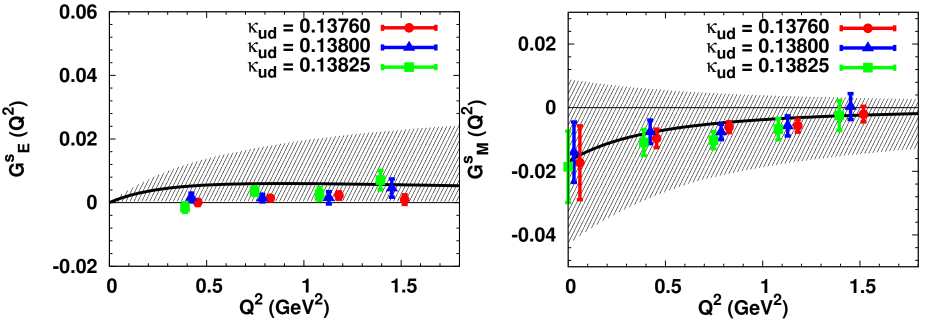
$(r_1^v)^2 \sim \log m_\pi \qquad (r_2^v)^2 \sim m_\pi^{-1} \qquad \kappa_v \sim \text{cst}$

Conclusion

- many (isovector) nucleon observables now known quite accurately for $300\mathrm{MeV} \leq m_\pi < 800\mathrm{MeV}$
- general observation: very mild pion mass dependence; much milder than suggested by chiral perturbation theory
- contact with the non-analytic behavior at small m_π remains to be established
- transverse structure: transverse mass distribution of nucleon is more compact than the transverse charge distribution, $R_d^{n=2}/R_d^{n=1} \approx 1.4$.

Backup slides

Example of disconnected diagrams: strangeness form factor



χQCD collaboration 0903.3232 $\quad N_f = 2+1 \quad a = 0.12$fm $\quad m_\pi \geq 600$MeV

Electromagnetic form factors of the proton: (NB. $G_E^u(0) = 2$)

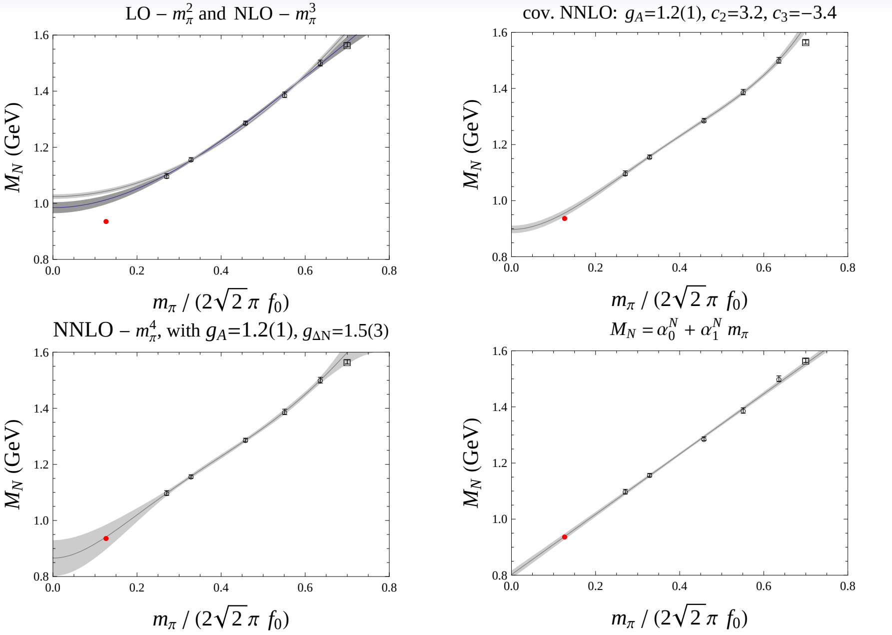
$$G_{E,M} = \frac{2}{3}G^u - \frac{1}{3}G^d - \frac{1}{3}G^s = \frac{1}{2}G^{u-d} + \frac{1}{6}G^{u+d-2s} + 0 \cdot G^{u+d+s}$$

$$= \frac{1}{2}G^{u-d}_{\rm conn} + \frac{1}{6}G^{u+d}_{\rm conn} + \frac{1}{6}(G^{u+d}_{\rm disc} - 2G^s_{\rm disc})$$

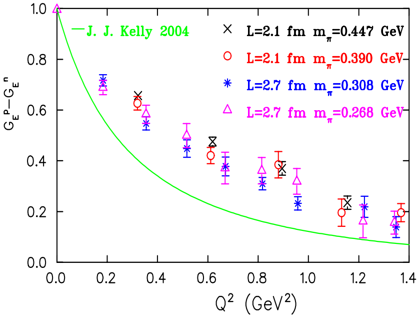
These Figs suggest disconnected diagrams contribution $< 0.01 \Rightarrow$ negligible at low Q^2.

Nucleon mass: LHP mixed-action calculation

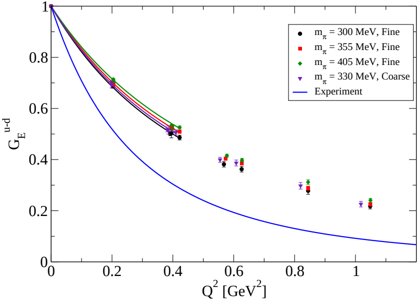
PhysRevD.79.054502 (2009)



The isovector electric form factor $G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_N^2} F_2(q^2)$



$N_f = 2$ twisted-mass-fermion calculation (ETMC, 0906.4137)



$N_f = 2+1$ Domain-wall-fermion calculation (LHPC, in preparation)

- good agreement between $N_f = 2$ and $N_f = 2+1$ calculations.