

Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

Reinventing the Parton Shower Monte Carlo

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More in <http://jadach.web.cern.ch/>



Can we construct **NLO Parton Shower Monte Carlo for QCD Initial State Radiation:**

- based firmly on Feynman Diagrams (ME) and LIPS,
- based rigorously on the collinear factorisation (EGMPR, CSS, Bodwin...),
- implementing *exactly* NLO DGLAP evolution,
- for fully unintegrated exclusive PDFs (ePDFs);
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels???

We are going to show that YES! We can do it!

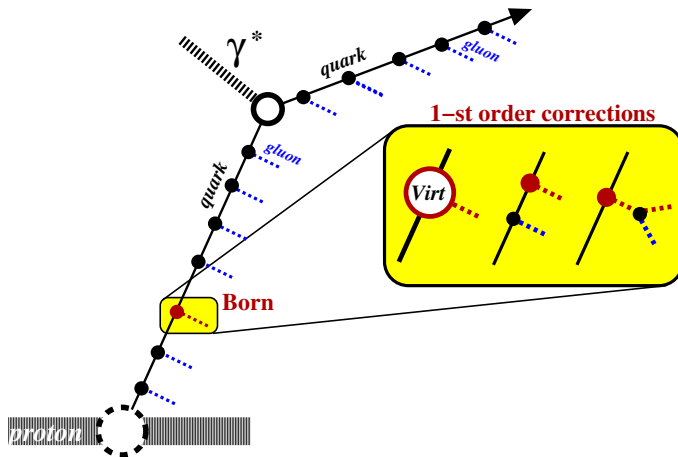
And report on the first Monte Carlo implementation
– the proof of the concept for non-singlet NLO DGLAP.



Leading Order (LO) ladder vertex is our “Born”

Emission of gluons out of quark

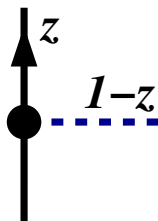
The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).



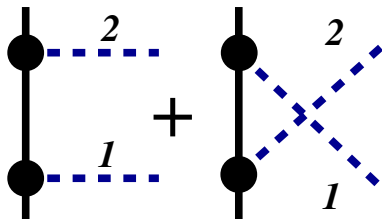
1-st order virtual and real correction diagrams

Virtual :

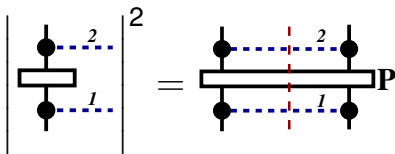
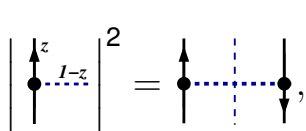
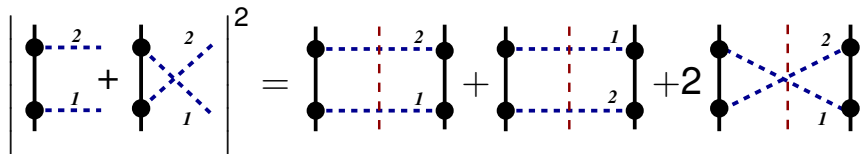
$$\left(1 + \Delta_{ISR}^{(1)}(z)\right)$$



Real :



NOTATION: squared MEs = cut-diagrams, C_F^2 only

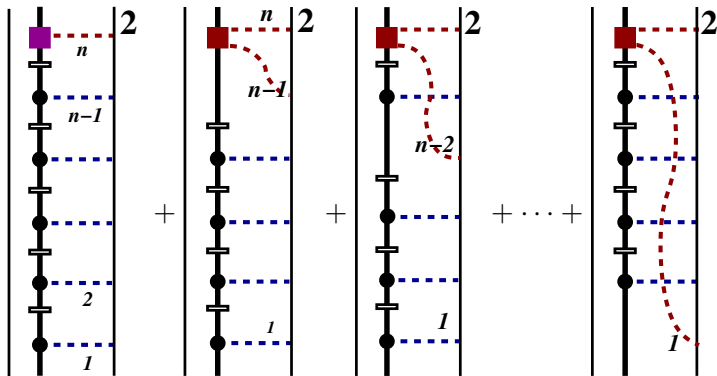


LO ladder = parton shower MC

$$\sum_{n=0}^{\infty} = e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^n \frac{d^3 k_i}{k_i^0} \theta_{Q > a_i > a_{i-1}} \rho_{1B}^{(0)}(k_i) \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}$$

LO with NLO-corrected kernel at the end of the ladder



Virt. multiplicative

Undoing LO simplificat.

Sum over trailing LO spectators, essential (BE, YFS61)

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \square \\ \text{---} \\ \uparrow \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR}^{(1)})) \left| \begin{array}{c} \uparrow \\ \text{---} \\ \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2,$$

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \square \\ \text{---} \\ \uparrow \end{array} \right|^2 = \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2 + \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2 - \left| \begin{array}{c} \bullet \\ \text{---} \\ \square \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2$$



LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

With more details:

$$\bar{D}_B^{[1]}(x, Q) = e^{-S_{ISR}} \left[\text{Diagram 1} + e^{-S_{ISR}} \left[\text{Diagram 2} + e^{-S_{ISR}} \sum_{j=1}^{n-1} \text{Diagram 3} \right] \right] = e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},$$

where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\beta_0^{(1)} = \frac{\text{Diagram 4}}{\text{Diagram 5}}$, $W(k_2, k_1) = \frac{\text{Diagram 6}}{\text{Diagram 7}} = \frac{\text{Diagram 8} + \text{Diagram 9}}{\text{Diagram 7}} - 1$.

Mapping $k_i \rightarrow \tilde{k}_i$, essential and instrumental.

S_{ISR} = double-log Sudakov.



Algebraic crosscheck

For NLO part the analytical integration gives us:

$$\bar{D}_B^{[1]}(x, Q) = \left(\prod_{i=1}^{n-1} \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \delta_{x=u \prod_{j=1}^{n-1} z_j} \Bigg\} = e^{-S_{ISR}} \left\{ \delta_{x=1} + \right. \\ \left. + \sum_{n=1}^{\infty} \int du \int_{Q > a_n > a_{n-1}} \frac{da_n}{a_n} \mathcal{P}_{qq}^{(1)}(u) \left(\prod_{i=1}^{n-1} \int_{a_{i+1} > a_i > a_{i-1}} \frac{da_i}{a_i} \mathcal{P}_{qq}^{(0)}(z_i) \right) \delta_{x=u \prod_{j=1}^{n-1} z_j} \right.$$

where we recover precisely NLO part of standard DGLAP kernel $\mathcal{P}_{qq}^{(1)}(u)$ defined according to:

$$\mathcal{P}_{qq}^{(1)}(u) \ln \frac{Q}{q_0} = \int_{Q > a_n > a_0} d^3 \eta_n \rho_{1B}^{(1)}(k_n) \beta_0^{(1)}(z_n) \delta_{u=z_n} + \int_{a_n > a_{nn'} > 0} d^3 \eta_n \int d^3 \eta_{n'} \beta_1^{(1)}(\tilde{k}_n, \tilde{k}_{n'}) \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.



NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p :

$$\bar{D}_B^{[1]}(x, Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{array}{l} \text{Diagram 1: Ladder with } n \text{ rungs, top vertex } x, \text{ bottom vertex } I, \text{ and } n-1 \text{ internal vertices.} \\ \text{Diagram 2: Ladder with } n \text{ rungs, top vertex } x, \text{ bottom vertex } I, \text{ and } n-1 \text{ internal vertices. A purple square is inserted at rung } p. \\ \text{Diagram 3: Ladder with } n \text{ rungs, top vertex } x, \text{ bottom vertex } I, \text{ and } n-1 \text{ internal vertices. A red dashed loop is inserted at rung } p \text{ between vertices } j \text{ and } j+1. \end{array} \right\} = e^{-S_{ISR}} \left\{ \delta_{x=1} + \right. \\ \left. + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^n \beta_0^{(1)}(z_p) + \sum_{p=1}^n \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},$$

Next step is to add more insertions, for instance 2 at positions p_1 and p_2 and sum up over positions... then 3 insertions and so non – in this way we build up LO+NLO kernels all over along the ladder! See next slide...



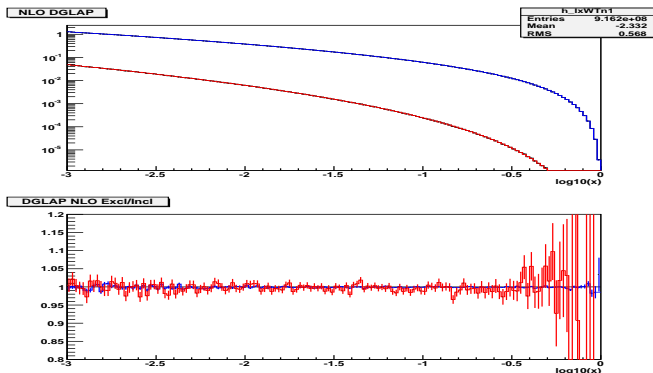
NLO-corrected kernels all over the ladder, $\sim C_F^2$

$$\begin{aligned}
 \bar{D}_B^{[1]}(x, Q) &= e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{array}{l} \text{Diagram 1: Ladder with } n \text{ rungs, rungs } p, 2, \dots, 1. \\ \text{Diagram 2: Ladder with } n \text{ rungs, rungs } p_1, j_1, \dots, 1. \\ \text{Diagram 3: Ladder with } n \text{ rungs, rungs } p_1, p_2, j_1, j_2, \dots, 1. \end{array} \right\} \\
 &= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \right) \left[1 + \sum_{p=1}^n \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) + \right. \right. \\
 &\quad \left. \left. + \sum_{p_1=1}^n \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1 \neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2 \neq p_1, j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \right] \delta_{x=\prod_{j=1}^n x_j} \right\},
 \end{aligned}$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.



Numerical test of ISR pure C_F^2 NLO MC



Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos. **Blue curve** is single NLO insertion, **red curve** is double insertion component. LO+NLO is off scale. Evolution $10\text{GeV} \rightarrow 1\text{TeV}$ starting from $\delta(1-x)$. The ratio demonstrates 3-digit agreement, in units of LO.



THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:

$$\left| \text{diagram with red square} \right|^2 = \left| \text{diagram 1} + \text{diagram 2} + \text{diagram 3} \right|^2 - \left| \text{diagram with white box} \right|^2$$

and $-S_{FSR}$ in the virtual correction:

$$\left| \text{diagram with purple square} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR})) \left| \text{diagram with black dot} \right|^2.$$

SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.



NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

$$e^{-S_{ISR} - S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^m \left| \begin{array}{c} \text{Diagram with } n-2, n-1, 1, 2, r, m \text{ labels} \end{array} \right|^2$$

where Sudakov S_{FSR} is subtracted in the virtual part:

$$\left| \begin{array}{c} \text{Diagram with } z \text{ label} \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})) \left| \begin{array}{c} \text{Diagram with } 1-z \text{ label} \end{array} \right|^2$$

and FSR counterterm is subtracted in the 2-real-gluon part:

$$\left| \begin{array}{c} \text{Diagram with } 2 \text{ label} \end{array} \right|^2 = \left| \begin{array}{c} \text{Diagram 1} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram 4} \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram 5} \end{array} \right|^2$$

The miracle: both are free of any collinear or soft divergency!!!



ISR+FSR NLO scheme, NLO corr. at end of the ladder

$$\bar{D}_{NS}^{[1]}(x, Q) =$$

$$e^{-S} \sum_{n,m=0}^{\infty} \left\{ \left| \begin{array}{c} \text{Diagram 1: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is purple square.} \\ \text{Diagram 2: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square.} \\ \text{Diagram 3: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square, with } r \text{ rungs in red.} \end{array} \right. \right\}$$

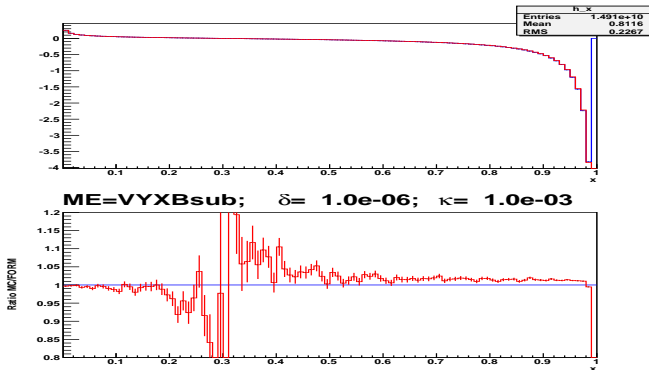
$$= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) e^{-S_{FSR}} \sum_{m=0}^{\infty} \left(\prod_{j=1}^m \int_{Q > a_{nj} > a_{n(l-1)}} d^3 \eta'_j \rho_{1V}^{(1)}(k'_j) \right) \right. \\ \left. \times \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^m W(\tilde{k}_n, \tilde{k}'_r) \right] \delta_{x=\prod_{j=1}^n x_j} \right\}$$

$$\beta_0^{(1)} \equiv \frac{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is purple square.} \end{array} \right|^2}{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines.} \end{array} \right|^2}, \quad W(k_2, k_1) \equiv \frac{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square.} \end{array} \right|^2}{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines.} \end{array} \right|^2} = \dots - 1.$$



3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.



Summary and Prospects

- First serious **feasibility study** of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game (similar to MC@NLO).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.



DGLAP Collinear QCD ISR Evolution and Monte Carlo. The state of art.

1970

1980

1990

2000

2010

Moments OPE

(74) QCD: Georgi+Politzer

Diagramatic

(72) QED: Gribov+Lipatov

(77) Altarelli+Parisi

Monte Carlo

10 years

(85) Sjostrand

(88) Marchesini, Webber

NLO

Moments OPE

(78) Floratos+Ross+Sachrajda

WE ARE HERE!!!

Diagramatic

(81) Curci+Furmanski+Petronzio

Monte Carlo

27 years later

(08) Jadach Skrzypek

NNLO

Moments

(03) Moch+Verm.+Vogt

Diagramatic

(03) Moch+Verm.+Vogt

Monte Carlo

(15) ???

NNLO