# Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

Reinventing the Parton Shower Monte Carlo

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**in collaboration with**

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More in http://jadach.web.cern.ch/

# The aim of KRKMC project

**Can we construct NLO Parton Shower Monte Carlo for QCD Initial State Radiation:**

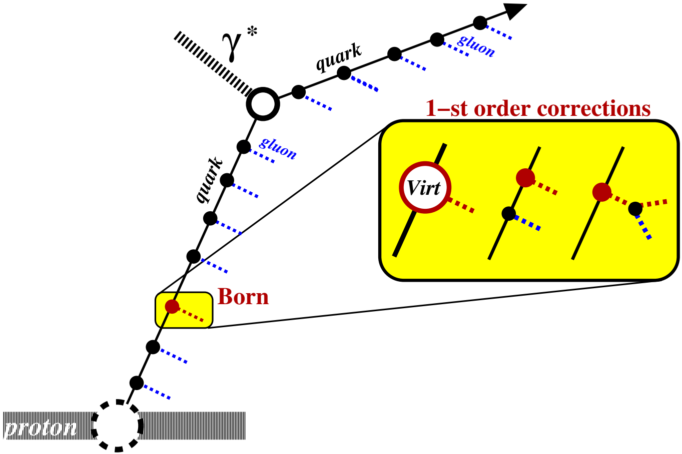
- based firmly on Feynman Diagrams (ME) and LIPS,
- based rigorously on the collinear factorisation (EGMPR, CSS, Bodwin...),
- implementing *exactly* NLO DGLAP evolution,
- for fully unintegrated exclusive PDFs (ePDFs);
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels**???**

We are going to show that **YES!** We can do it!
And report on the first Monte Carlo implementation
– the proof of the concept for non-singlet NLO DGLAP.

# Leading Order (LO) ladder vertex is our "Born"

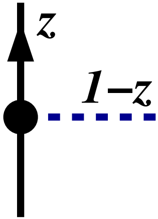
## Emission of gluons out of quark

The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).

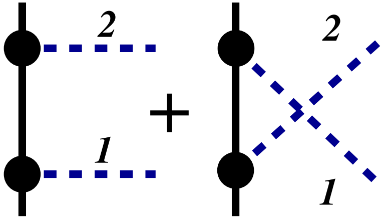


# 1-st order virtual and real correction diagrams

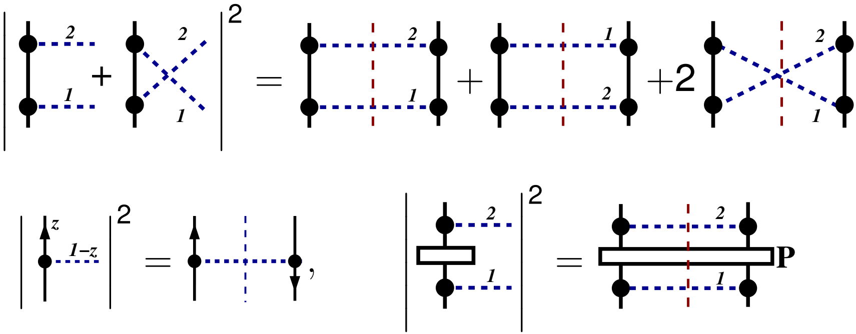
Virtual : $\left(1 + \Delta^{(1)}_{ISR}(z)\right)$



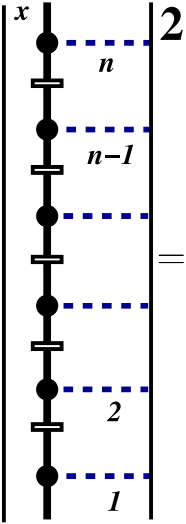

Real :



# NOTATION: squared MEs = cut-diagrams, $C_F^2$ only



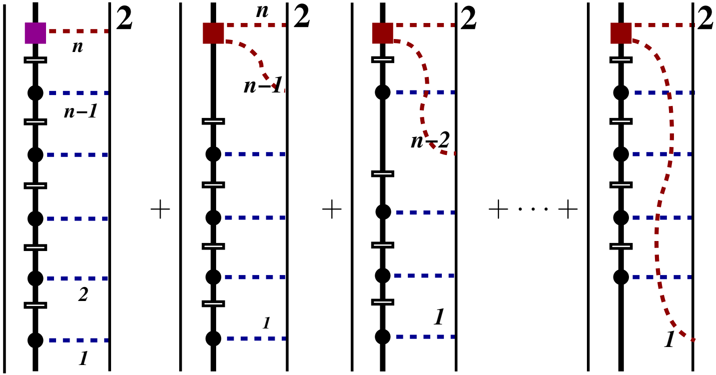
# LO ladder = parton shower MC



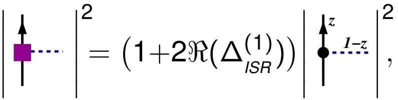

$$\sum_{n=0}^{\infty} \left| \cdots \right|^2 = e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \; \theta_{Q>a_i>a_{i-1}} \, \rho_{1B}^{(0)}(k_i) \; \delta_{x=\prod z_i}$$

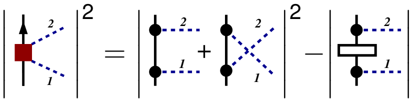
$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

# LO with NLO-corrected kernel at the end of the ladder




Virt. multiplicative  Undoung LO simplificat.  Sum over trailing LO spectators, essential (BE, YFS61)

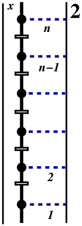


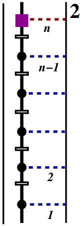
# LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

With more details:

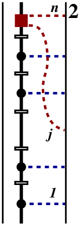
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$$




$$+\, e^{-S_{ISR}}$$

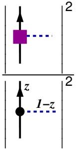



$$+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$$

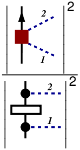



$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left( \prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[ \beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\quad \beta_0^{(1)} =$




, $\quad W(k_2, k_1) =$

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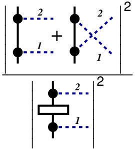


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Mapping $k_i \rightarrow \tilde{k}_i$, essential and instrumental. $\quad S_{ISR}$ = double-log Sudakov.

# Algebraic crosscheck

For NLO part the analytical integration gives us:

$$\bar{D}_B^{[1]}(x,Q) = \left( \prod_{i=1}^{n-1} \int\limits_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j} \Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \, \mathcal{P}_{qq}^{(1)}(u) \left( \prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_n}{a_i} \mathcal{P}_{qq}^{(0)}(z_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part of standard DGLAP kernel $\mathcal{P}_{qq}^{(1)}(u)$ defined according to:

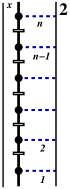
$$\mathcal{P}_{qq}^{(1)}(u) \ln\frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n \; \rho_{1B}^{(1)}(k_n) \; \beta_0^{(1)}(z_n) \delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{nn'}>0} d^3\eta_{n'} \; \beta_1^{(1)}(\tilde{k}_n, \tilde{k}_{n'}) \; \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.

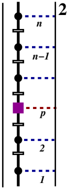
# NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion $p$ can be anywhere in the ladder and we sum up over $p$:

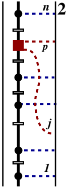
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$




$$+ \sum_{p=1}^{n}$$




$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



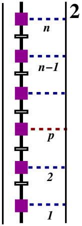

$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left( \prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[ \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

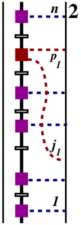
Next step is to add more insertions, for instance 2 at positions $p_1$ and $p_2$ and sum up over positions... then 3 insertions and so non – in this way we build up LO+NLO kernels all over along the ladder! See next slide...

# NLO-corrected kernels all over the ladder, $\sim C_F^2$

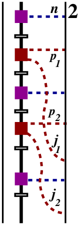
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{ \Bigg|$$




$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$




$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1}$$



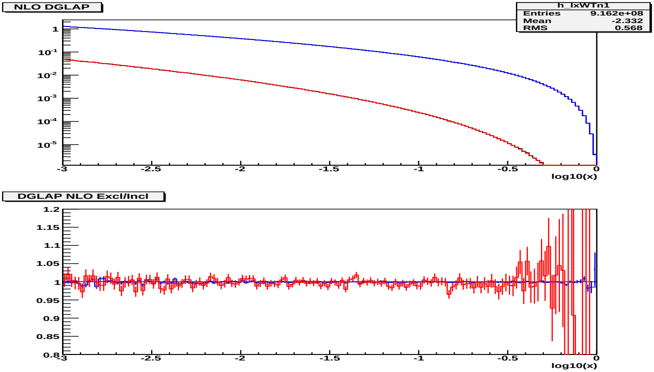

$$\Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg( \prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[ 1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

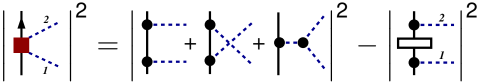
# Numerical test of ISR pure $C_F^2$ NLO MC



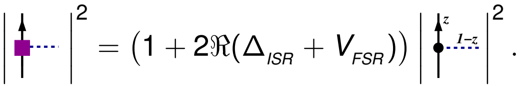

Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV→1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

# THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:




and $-S_{FSR}$ in the virtual correction:

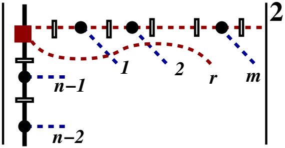



SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

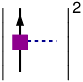
# NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

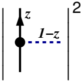
$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



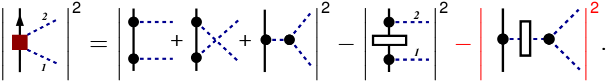

where Sudakov $S_{FSR}$ is subtracted in the virtual part:




$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$



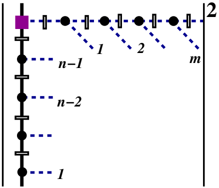

and FSR counterterm is subtracted in the 2-real-gluon part:



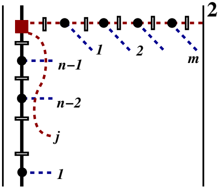

The miracle: both are free of any collinear or soft divergency!!!

$$\bar{D}^{[1]}_{NS}(x,Q) =$$

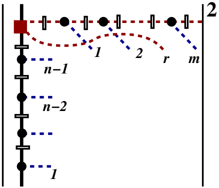
$$e^{-S} \sum_{n,m=0}^{\infty} \Bigg\{$$




$$+ \sum_{j=1}^{n-1}$$



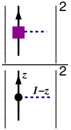

$$+ \sum_{r=1}^{m}$$

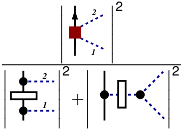


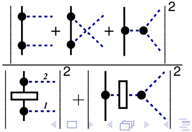

$$\Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg( \prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho^{(1)}_{1B}(k_i) \Bigg) e^{-S_{FSR}} \sum_{m=0}^{\infty} \Bigg( \prod_{j=1}^{m} \int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j \; \rho^{(1)}_{1V}(k'_j) \Bigg)$$

$$\times \Bigg[ \beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^{m} W(\tilde{k}_n, \tilde{k}'_r) \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

$\beta_0^{(1)} \equiv$ 

, $\quad W(k_2, k_1) \equiv$ 

$=$ 

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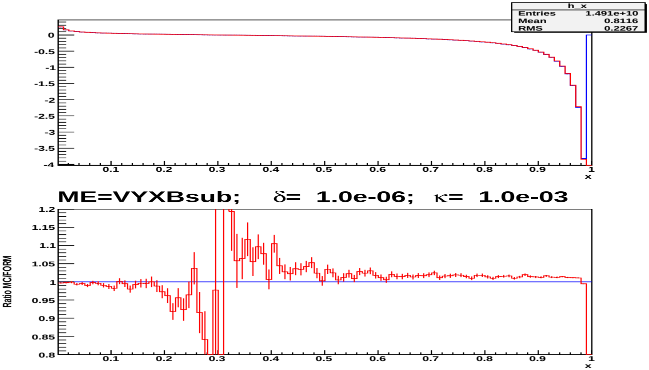
# 3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:




because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

# Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game (similar to MC@NLO).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.

# DGLAP Collinear QCD ISR Evolution and Monte Carlo. The state of art.