Multiple parton interactions in QCD

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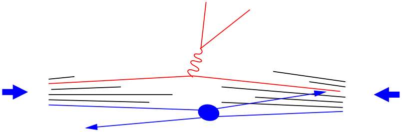
DIS 2010, Firenze, 20 April 2010

Multi-parton interactions

- generically take place in hadron-hadron collisions
- effects average out in sufficiently inclusive quantities
  but do affect final state properties

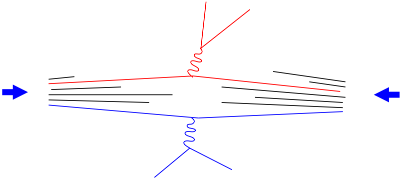


- estimated to be important for many LHC processes
  see e.g. Procs. of the Workshop on HERA and the LHC, 2005 and 2008
- many studies in the literature (theory + experiment)
  but so far no systematic derivation in QCD
- aim of this talk: discuss some steps in this direction

work in progress with A. Schäfer

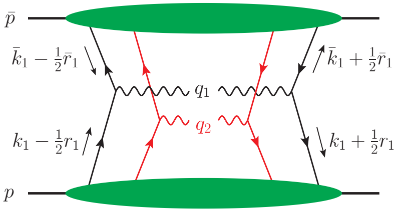
Theoretical framework

- require all interactions to be hard
  - predictive power from factorization and pert. theory
  - collision energy at LHC is huge
- consider gauge boson pair production (pairs of γ^*, W, Z)



  - jet production highly relevant in practice
    but gluon exchange with spectator partons $\gg$ complicated
- keep transverse gauge boson momentum differential
  - since are interested in final-state details
  - need k_T dependent parton distributions

Basic structure: momentum



- transverse momentum balance:

  $$\boldsymbol{q}_1 = \boldsymbol{k}_1 \pm \tfrac{1}{2}\boldsymbol{r}_1 + \bar{\boldsymbol{k}}_1 \pm \tfrac{1}{2}\bar{\boldsymbol{r}}_1 \quad \Rightarrow \quad \boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$$

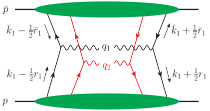
  $\rightsquigarrow$ transverse parton momenta not the same on left and right of final-state cut

- Fourier transform to impact parameter: $\boldsymbol{r}_1 \to \boldsymbol{y}_1$ and $\bar{\boldsymbol{r}}_1 \to \bar{\boldsymbol{y}}_1$

  $\boldsymbol{r}_1 + \bar{\boldsymbol{r}}_1 = \boldsymbol{0}$ implies $\boldsymbol{y}_1 = \bar{\boldsymbol{y}}_1$

fully analogous for partons with index 2

Basic structure: cross section



- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \Bigg[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big) \Bigg]$$

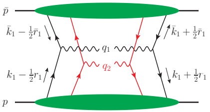
$$\times \Bigg[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \Bigg] \int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)\, \bar{F}(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y}_1)$$

$\hat{\sigma}_i =$ parton-level cross section

$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) =$ k_T dependent two-parton distribution

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation involved
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\rightarrow$ collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$$



Operator definitions

- k_T dependent distribution

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{i x_2 z_2^- p^+ - i \boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{i x_1 z_1^- p^+ - i \boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times \, 2p^+ \int dy_1^- \, \langle p| \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2 \, q\big(\tfrac{1}{2} z_2\big) \, \bar{q}\big(y_1 - \tfrac{1}{2} z_1\big) \Gamma_1 \, q\big(y_1 + \tfrac{1}{2} z_1\big) |p\rangle \Big|_{z_i^+ = y_1^+ = 0}$$

- collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int \frac{dz_2^-}{2\pi} \, e^{i x_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi} \, e^{i x_1 z_1^- p^+}$$

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still $\boldsymbol{y}_1 \neq \boldsymbol{0}$ $\Rightarrow$ finite transverse distance between two partons
$\Rightarrow$ not a twist-four operator
but product of two twist-two operators

Dirac matrices Γ_i → spin structure → talk Thu 9:00 in spin/diffraction session

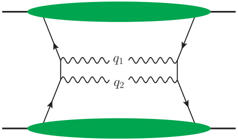
Power behavior: single versus double hard scattering

- from scattering formulae readily find

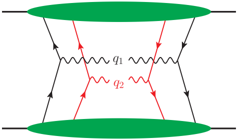
$$s \, \frac{d\sigma}{dx_1 \, d\bar{x}_1 \, d^2\boldsymbol{q}_1 \;\, dx_2 \, d\bar{x}_2 \, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2 \Lambda^2}$$

$Q^2 \sim q_i^2, \; \Lambda^2 \sim \text{GeV}^2$

for both



and



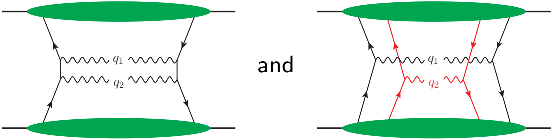
$\Rightarrow$ double scattering not power suppressed

Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\,d^2\boldsymbol{q}_1\;dx_2\,d\bar{x}_2\,d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2} \qquad Q^2 \sim q_i^2,\ \Lambda^2 \sim \mathrm{GeV}^2$$

for both and



$\Rightarrow$ double scattering not power suppressed

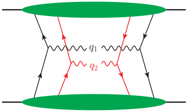
- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

single: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim 1$$ since $\int d^2(\boldsymbol{q}_1 + \boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) \sim Q^2$

double: $$s\,\frac{d\sigma}{dx_1\,d\bar{x}_1\;dx_2\,d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space in transv. momenta

Color structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$

$$\times \, 2p^+ \int dy_1^- \; \langle p| \bar{q}(-\tfrac{1}{2}z_2) \Gamma_2 \, q(\tfrac{1}{2}z_2) \; \bar{q}(y_1 - \tfrac{1}{2}z_1) \Gamma_1 \, q(y_1 + \tfrac{1}{2}z_1) |p\rangle$$

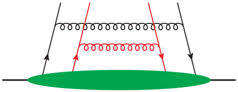
- operators $\bar{q}_2 \, q_2$ and $\bar{q}_1 \, q_1$ can couple to color singlet or octet:

$$F_1 \longrightarrow (\bar{q}_2 \mathbb{1} q_2) \; (\bar{q}_1 \mathbb{1} q_1)$$
$$F_8 \longrightarrow (\bar{q}_2 t^a q_2) \; (\bar{q}_1 t^a q_1) = \tfrac{1}{2} (\bar{q}_2 \mathbb{1} q_1) \; (\bar{q}_1 \mathbb{1} q_2) - \tfrac{1}{2N_c} \; (\bar{q}_2 \mathbb{1} q_2) \; (\bar{q}_1 \mathbb{1} q_1)$$

- relative weight in gauge boson pair production:
  $\frac{1}{N_c^2} \, F_1 \bar{F}_1 + \frac{4}{N_c^2 - 1} \, F_8 \bar{F}_8$
- color octet distributions essentially unknown
  (no probability interpretation as a guide)

High q_T: more predictive power

- consider region $\Lambda \ll q_T \ll Q$ with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n = hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- case 1: $|\boldsymbol{r}_1| \sim \Lambda$, i.e. $|\boldsymbol{y}| \sim 1/\Lambda$ of hadronic size
  $\rightsquigarrow$ independent hard scatters for pair 1 and 2



- color factors: C_F^2 for singlet F_1
  $\left(\frac{1}{2N_c}\right)^2$ for octet F_8
- singlet also favored in gluon sector

power behavior: $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \sim \frac{\Lambda^2}{q_T^4}$ and $s \, \frac{d\sigma}{d^2\boldsymbol{q}_1 \, d^2\boldsymbol{q}_2} \sim \frac{\Lambda^2}{Q^2 q_T^4}$

High q_T: more predictive power

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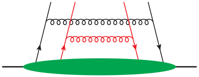
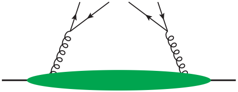
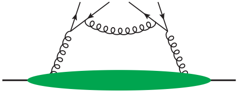


$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \sim \frac{\Lambda^2}{q_T^4}$ and $s \frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{\Lambda^2}{Q^2 q_T^4}$

- case 2: $|\boldsymbol{r}_1| \sim |\boldsymbol{q}_i|$, i.e. $|\boldsymbol{y}| \ll 1/\Lambda$ small





$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \sim \frac{1}{q_T^2}$ and $s \frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2 q_T^2}$

- case 1 is power suppressed by $\frac{\Lambda^2}{q_T^2}$, but has small-x enhancement

Evolution of collinear distributions

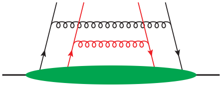
consider only color singlet combination F_1, situation for F_8 more complicated

$$F(x_i, \boldsymbol{y}_1) = \int \frac{dz_2^-}{2\pi}\, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi}\, e^{ix_1 z_1^- p^+}$$
$$\times\, 2p^+ \int dy_1^-\, \langle p| \bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\, \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1\, q(y_1 + \tfrac{1}{2}z_1)|p\rangle \Big|_{z_i^+ = y_1^+ = 0,\, \boldsymbol{z}_i = \boldsymbol{0}}$$

- $F(x_i, \boldsymbol{y}_1)$ for $\boldsymbol{y}_1 \neq \boldsymbol{0}$:

  separate DGLAP evolution for pair 1 and 2

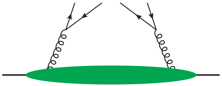
  $$\frac{d}{d\log\mu} F(x_i, \boldsymbol{y}_1) = P \otimes_{x_1} F + P \otimes_{x_2} F$$



- $\int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{y}_1)$:

  extra term from $2 \to 4$ parton transition

  recent study by Gaunt and Stirling



- $\rightsquigarrow$ consistency problem for frequently made ansatz

  $$F(x_i, \boldsymbol{y}_1; \mu) = G(\boldsymbol{y}_1)\left[\int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{y}_1)\right]_\mu$$

  if on r.h.s. include $2 \to 4$ term in evolution

Sudakov factors

- cross section differential in $\boldsymbol{q}_i$ contains Sudakov logarithms
- can adapt Collins-Soper-Sterman formalism

  (originally developed for single Drell-Yan and similar proc's)

  $\rightsquigarrow$ include and resum Sudakov logs in k_T dependent parton distr's

results of analysis (simplified):

- for $q_T \sim \Lambda$
  - Sudakov factors mix singlet and octet distr's F_1 and F_8
  - generically Sudakov factors of same size for singlet and octet
- for $q_T \gg \Lambda$ and $\boldsymbol{y}_1 \sim 1/\Lambda$
  - singlet F_1 decouples from octet F_8
  - octet distr'n has extra suppression by a power of $\frac{\Lambda}{q_T}$

Summary

- multiple hard interactions not power suppressed
  for cross section differential in $|\boldsymbol{q}_i| \ll Q$
- nontrivial color structure
  contributions without probability interpretation
- some simplification for transv. mom. $|\boldsymbol{q}_i| \gg \Lambda$
  - collinear distributions as input
  - enhancement of color singlet combinations

  further studies needed
- need multi-parton distr's depending on transverse distance $\boldsymbol{y}$ between partons
- for finite $\boldsymbol{y}$ (not $\int d^2\boldsymbol{y}$) multi-parton distr's evolve with simple sum of ordinary DGLAP kernels