Two-loop resummation for QCD hard scattering

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- **Resummation**
- **Soft anomalous dimensions**
- **Two-loop eikonal calculations**
- **LHC and Tevatron phenomenology**

Resummation

Soft-gluon corrections important in many processes, particularly near threshold

Needed at higher-orders for increased accuracy in theoretical predictions

Terms $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+ \,, \quad k \le 2n-1 \,, \quad s_4 \to 0$ **at threshold arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons**

Soft corrections exponentiate

Resummation follows from factorization

At NLL (NNLL) accuracy requires one-loop (two-loop) calculations in the eikonal approximation

Many phenomenological applications:

Drell-Yan processes;

top pair and single top production;

jet, direct photon, or W production at high p_T**;**

(charged) Higgs, squark and gluino production; etc.

Resummed cross section

Resummation follows from factorization properties of the cross section

- performed in moment space

Use RGE to evolve function associated with soft-gluon emission

H**: hard-scattering function** S**: soft-gluon function**

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N)\right] \, H(\alpha_s) \times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \, \Gamma_S^\dagger(\alpha_s(\mu))\right] \, S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \, \Gamma_S(\alpha_s(\mu))\right]$$

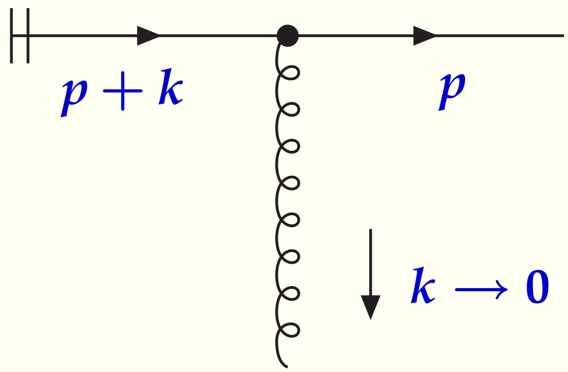
where

Γ_S **is the soft anomalous dimension - a matrix in color space**

and a function of kinematical invariants s, t, u

Calculate Γ_S **in eikonal approximation**

Eikonal approximation



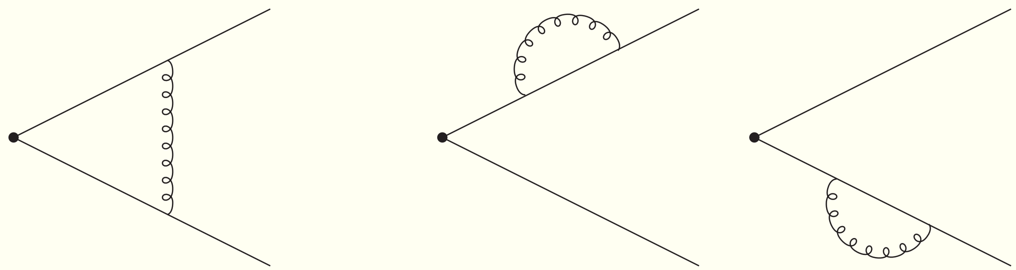
$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon} \longrightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not{p}+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c \frac{v^\mu}{v\cdot k+i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

Perform calculation for massive quarks in momentum space and Feynman gauge

Complete two-loop results for soft (cusp) anomalous dimension for $e^+e^- \longrightarrow t\bar{t}$

One-loop eikonal diagrams



(a) (b)

$$\Gamma_S = \frac{\alpha_s}{\pi}\Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2}\Gamma_S^{(2)} + \cdots$$

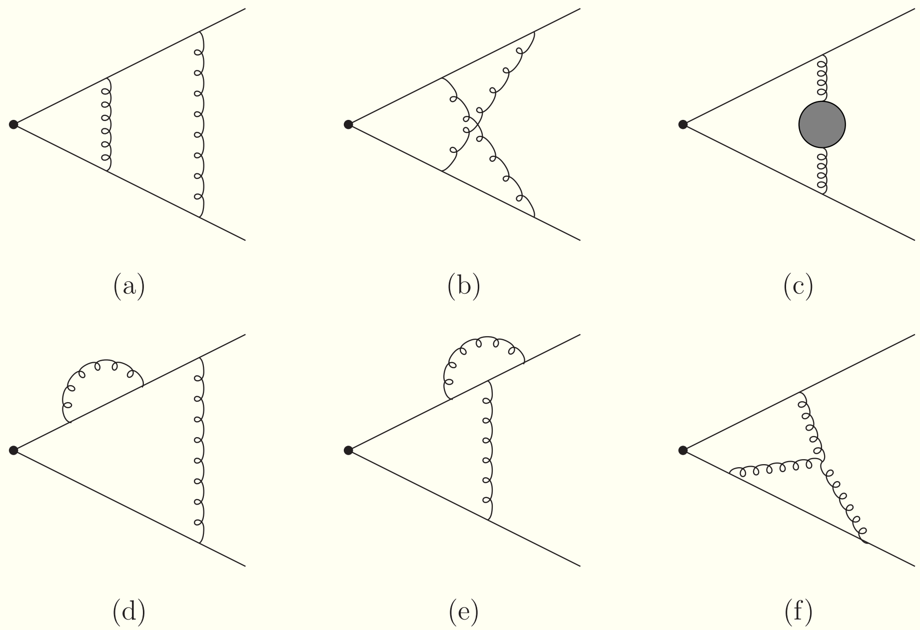
The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

$$\Gamma_S^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln\left(\frac{1-\beta}{1+\beta}\right) - 1 \right]$$

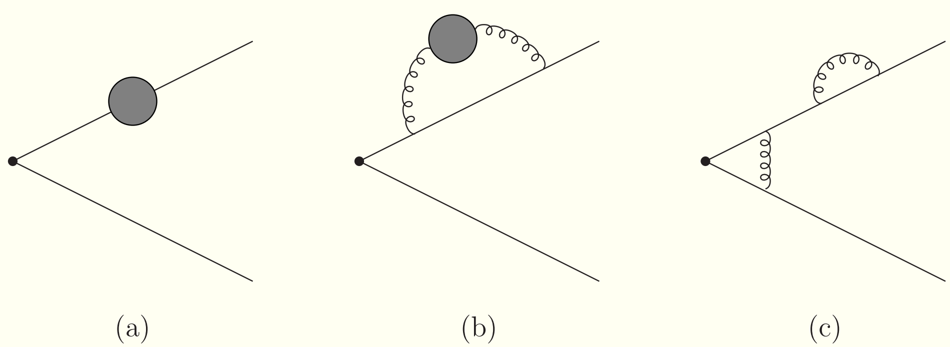
with $\beta = \sqrt{1 - \frac{4m^2}{s}}$

Two-loop eikonal diagrams

Vertex correction graphs



Heavy-quark self-energy graphs



Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)} = \left\{\frac{K}{2} + \frac{C_A}{2}\left[-\frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right) - \zeta_2\right]\right.$$
$$\left. + \frac{(1+\beta^2)}{4\beta} C_A \left[\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) + \zeta_2\right]\right\} \Gamma_S^{(1)}$$
$$+ C_F C_A \left\{\frac{1}{2} + \frac{1}{2}\ln\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{3}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \frac{(1+\beta^2)^2}{8\beta^2}\left[-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) + \zeta_3\right]\right.$$
$$\left. - \frac{(1+\beta^2)}{2\beta}\left[\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right) - \frac{1}{6}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

where $K = C_A(67/18 - \zeta_2) - 5n_f/9$

In terms of the cusp angle $\gamma = \ln[(1+\beta)/(1-\beta)]$ **we get**

$\Gamma_S^{(1)} = C_F(\gamma\coth\gamma - 1)$ **and**

$$\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \left\{\frac{1}{2} + \frac{\zeta_2}{2} + \frac{\gamma^2}{2} - \frac{1}{2}\coth^2\gamma\left[\zeta_3 - \zeta_2\gamma - \frac{\gamma^3}{3} - \gamma\,\mathrm{Li}_2\left(e^{-2\gamma}\right) - \mathrm{Li}_3\left(e^{-2\gamma}\right)\right]\right.$$
$$\left. - \frac{1}{2}\coth\gamma\left[\zeta_2 + \zeta_2\gamma + \gamma^2 + \frac{\gamma^3}{3} + 2\gamma\ln\left(1 - e^{-2\gamma}\right) - \mathrm{Li}_2\left(e^{-2\gamma}\right)\right]\right\}$$

N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

$\Gamma_S^{(2)}$ **vanishes at $\beta = 0$, the threshold limit, and diverges at $\beta = 1$, the massless limit**

Large β behavior: as $\beta \rightarrow 1$, $\Gamma_S^{(2)} \longrightarrow \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \frac{(1-\zeta_3)}{2}$

In massive-massless case

$$\Gamma_S^{(1)} = C_F \left[\ln\left(\frac{\sqrt{2}\, v_i \cdot v_j}{\sqrt{v_i^2}} \right) - \frac{1}{2} \right]$$

$$\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

QCD processes

Color structure gets more complicated with more than two colored partons in the process

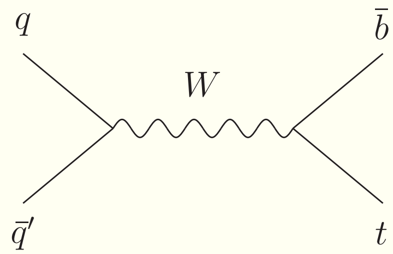
Cusp anomalous dimension an essential component of other calculations

Next, we compute two-loop soft anomalous dimensions for:

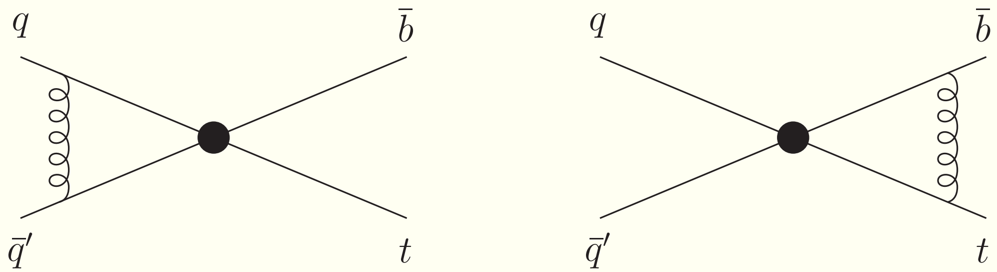
Single top production in s-channel (also direct photon production)

Associated top production with a W boson or a charged Higgs

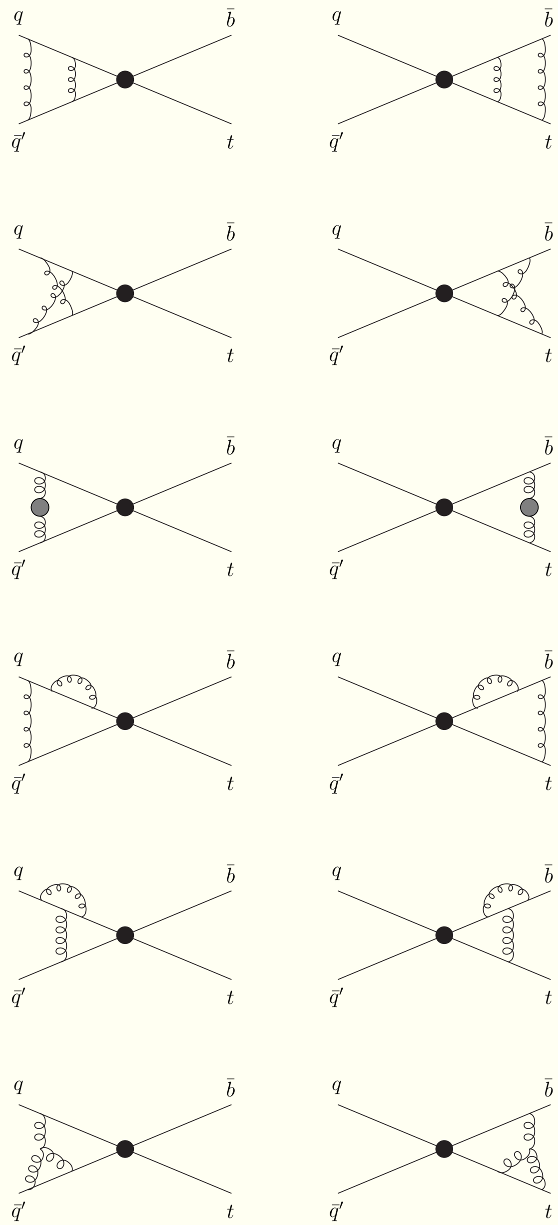
s-channel single top production

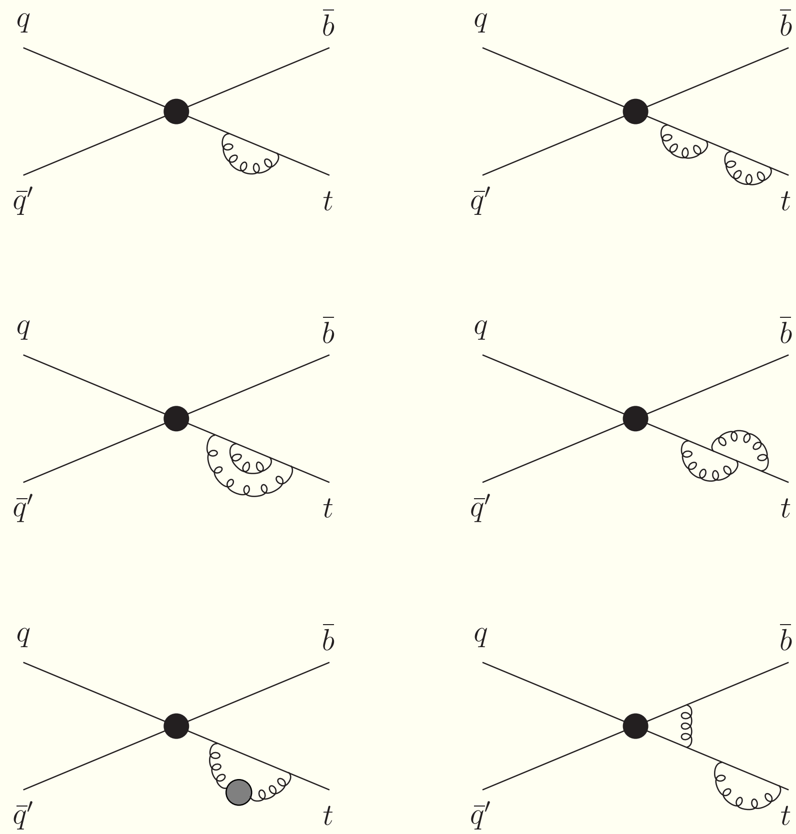


One-loop eikonal diagrams



Two-loop eikonal diagrams

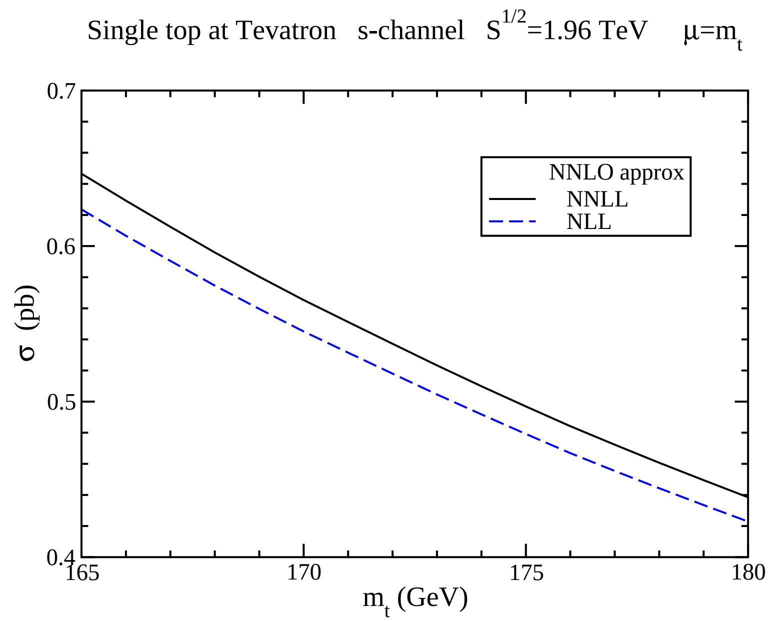


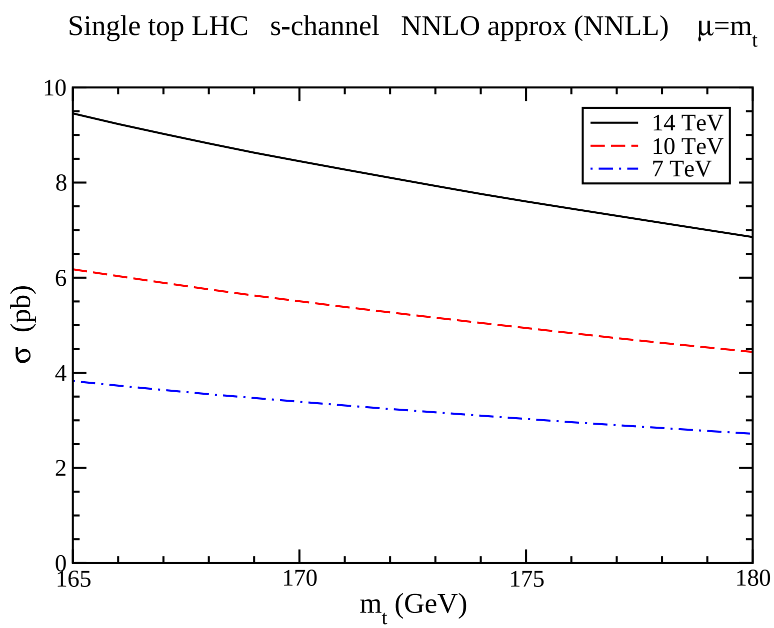


$$\Gamma^{(1)}_{S,\,\mathrm{top\,s-ch}} = C_F \left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}} \right) - \frac{1}{2} \right]$$

$$\Gamma^{(2)}_{S,\,\mathrm{top\,s-ch}} = \frac{K}{2}\Gamma^{(1)}_{S,\,\mathrm{top\,s-ch}} + C_F C_A \frac{(1-\zeta_3)}{4}$$

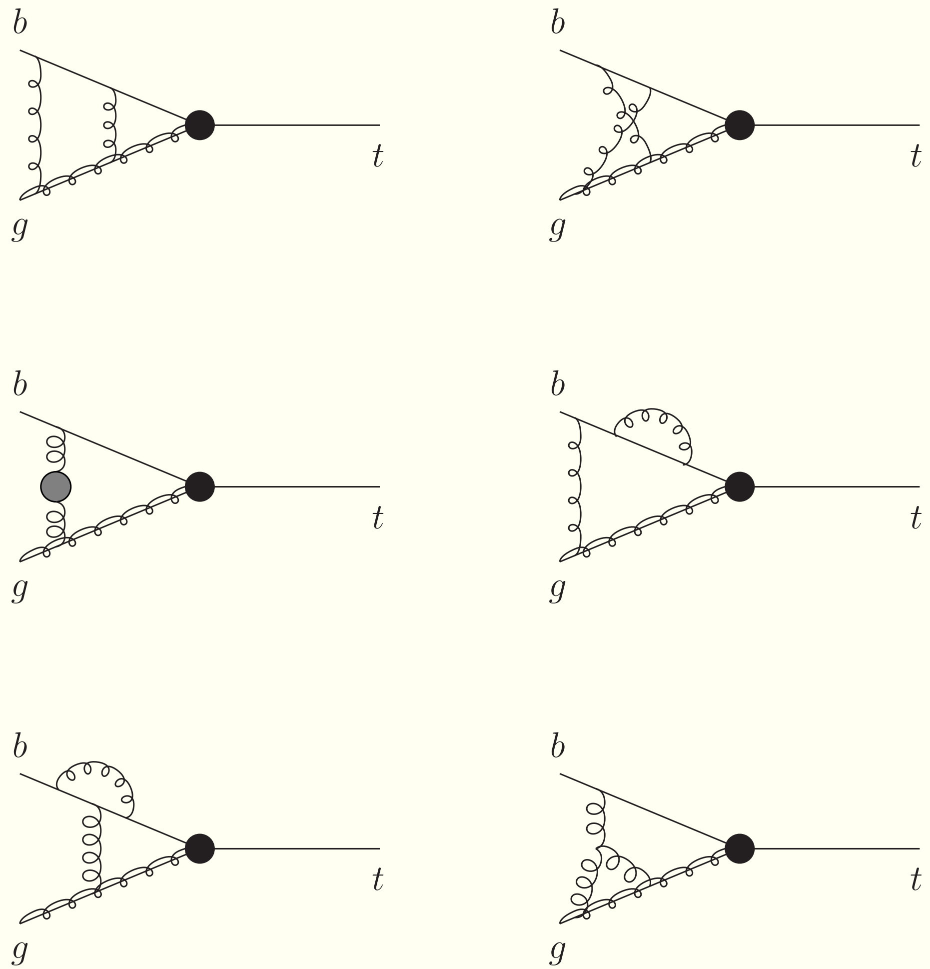
N. Kidonakis, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]

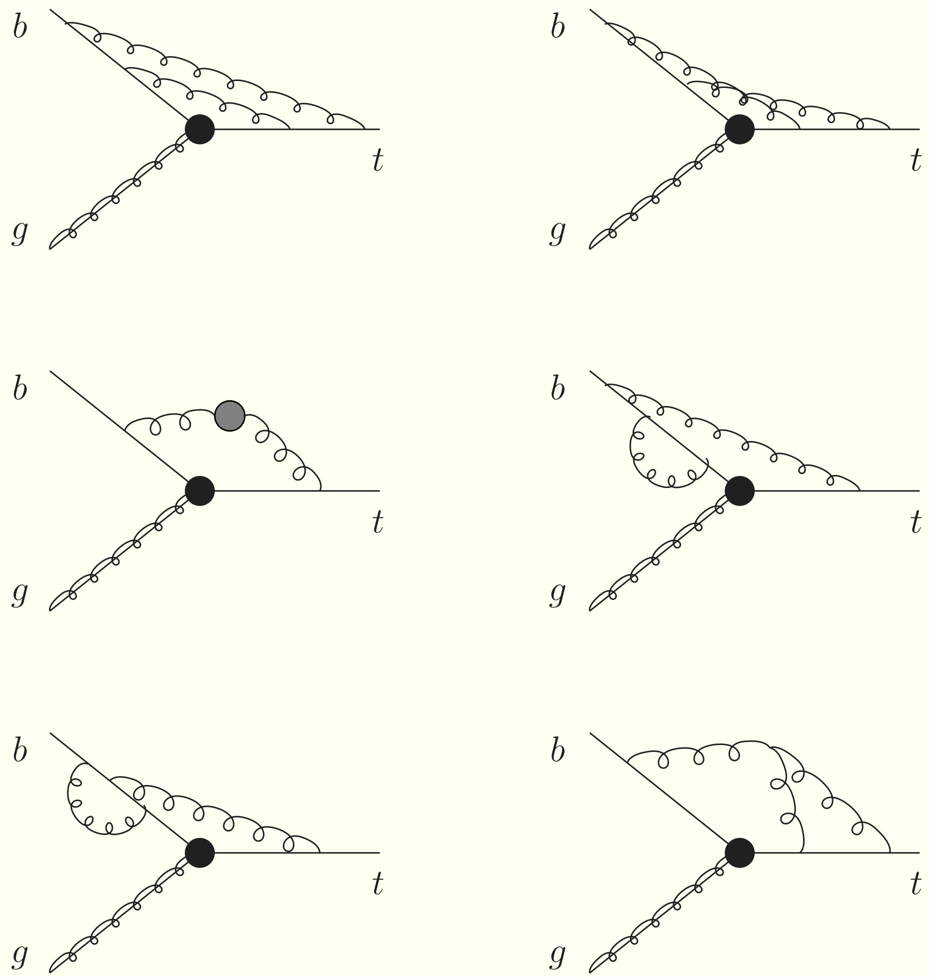

Single top at Tevatron s-channel S^1/2=1.96 TeV μ=m_t
NNLO approx
NNLL
NLL
σ (pb)
m_t (GeV)


Single top LHC s-channel NNLO approx (NNLL) μ=m_t
14 TeV
10 TeV
7 TeV
σ (pb)
m_t (GeV)

Associated production of a top quark with a W^- or H^-

Two-loop eikonal diagrams

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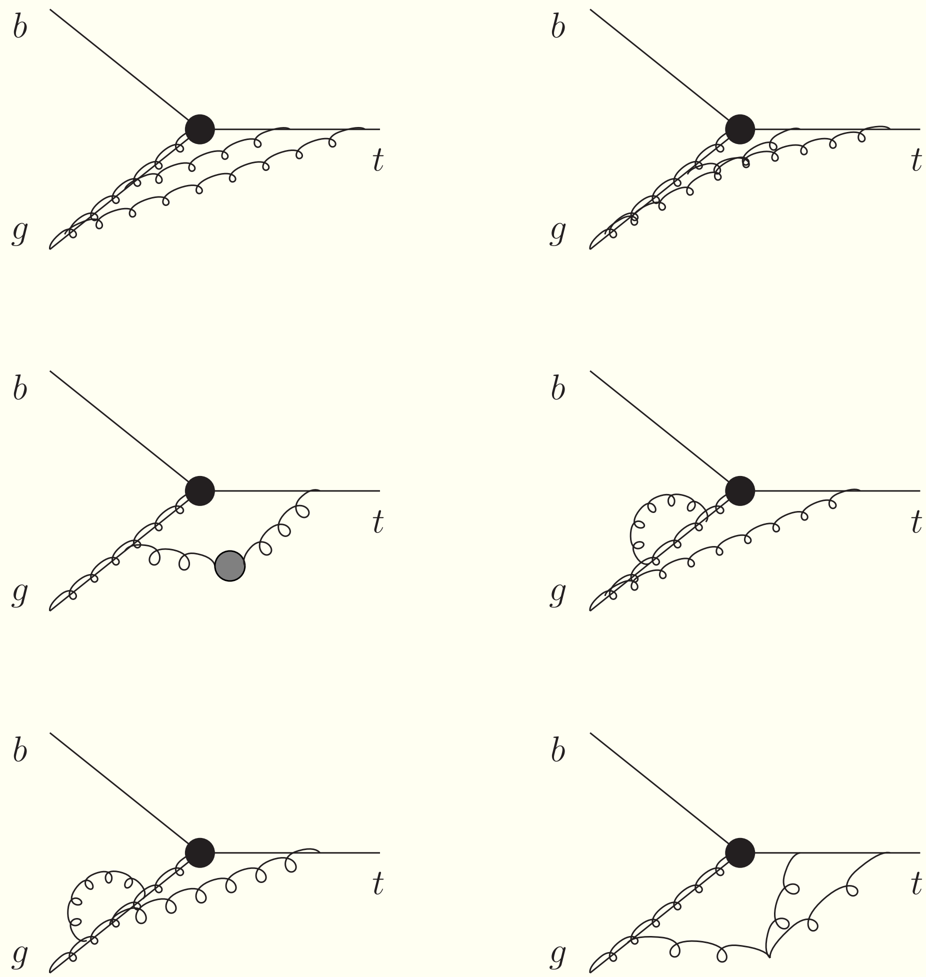
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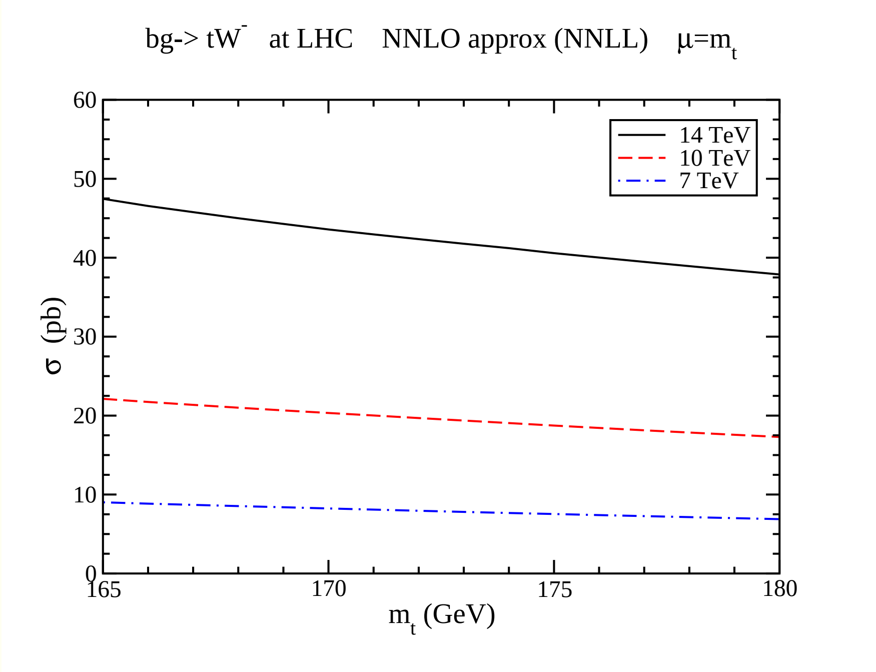
Soft anomalous dimension for $bg \to tW^-$

$$\Gamma_{S,tW^-}^{(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{m_t^2 - u}{m_t^2 - t}\right)$$

$$\Gamma_{S,tW^-}^{(2)} = \frac{K}{2}\Gamma_{S,tW^-}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$



Same analytical result for Γ_S for $bg \to tH^-$

Similar results are derived for direct photon production

Summary

- **Soft-gluon corrections and resummation**
- **Two-loop calculations in eikonal approximation**
- **Massive quarks involve further complications**
- **Two-loop soft anomalous dimensions**
- **Application to single top production and other processes at LHC and Tevatron energies**