



Threshold resummation for Drell-Yan production: theory and phenomenology

Marco Bonvini

Dipartimento di Fisica, Università di Genova & INFN, sezione di Genova

DIS 2010
Firenze, April 19-23, 2010

In collaboration with:
Stefano Forte, Giovanni Ridolfi

For which x is resummation important?

Partonic $z \sim 1$ is logarithmically enhanced \rightarrow resummation.

$$\sigma(x) = \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}\right) \hat{\sigma}(z), \quad x = \frac{Q^2}{S}$$

For which x is resummation important?

Partonic $z \sim 1$ is logarithmically enhanced \rightarrow resummation.

$$\sigma(x) = \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}\right) \hat{\sigma}(z), \quad x = \frac{Q^2}{S}$$

- $z \sim 1$ always contained in the integration region

For which x is resummation important?

Partonic $z \sim 1$ is logarithmically enhanced \rightarrow resummation.

$$\sigma(x) = \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}\right) \hat{\sigma}(z), \quad x = \frac{Q^2}{S}$$

- $z \sim 1$ always contained in the integration region
- when does that region give the dominant contribution?

For which x is resummation important?

Partonic $z \sim 1$ is logarithmically enhanced \rightarrow resummation.

$$\sigma(x) = \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}\right) \hat{\sigma}(z), \quad x = \frac{Q^2}{S}$$

- $z \sim 1$ always contained in the integration region
- when does that region give the dominant contribution?

Traditional argument: x relevant for resummation when the region of partonic $z \sim 1$ is enhanced by pdfs.

For which x is resummation important?

Partonic $z \sim 1$ is logarithmically enhanced \rightarrow resummation.

$$\sigma(x) = \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}\right) \hat{\sigma}(z), \quad x = \frac{Q^2}{S}$$

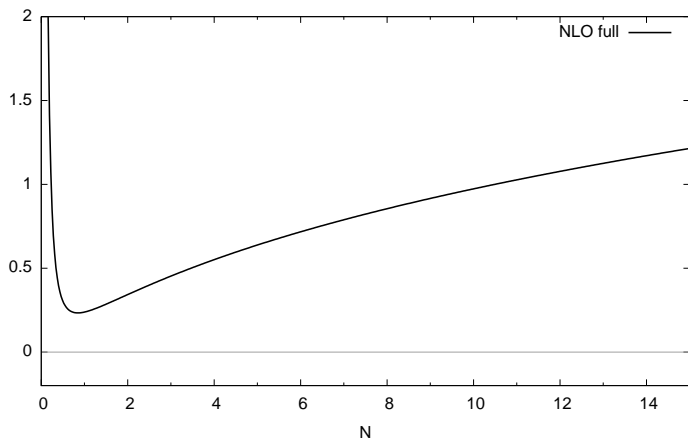
- $z \sim 1$ always contained in the integration region
- when does that region give the dominant contribution?

Traditional argument: x relevant for resummation when the region of partonic $z \sim 1$ is enhanced by pdfs.

N -space analysis
and saddle point approximation

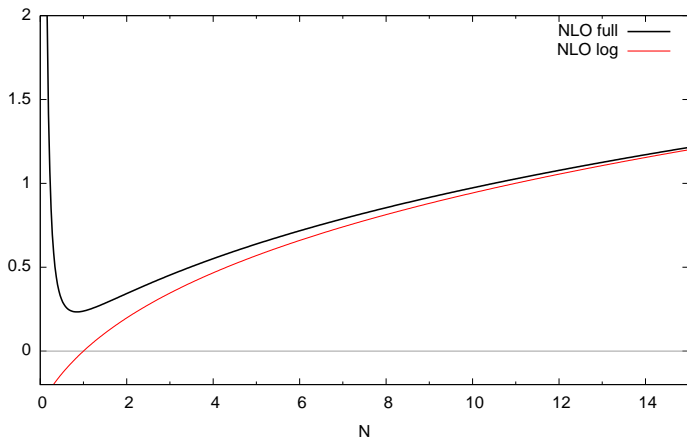
Drell-Yan $q\bar{q}$ at NLO in N -space

$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$



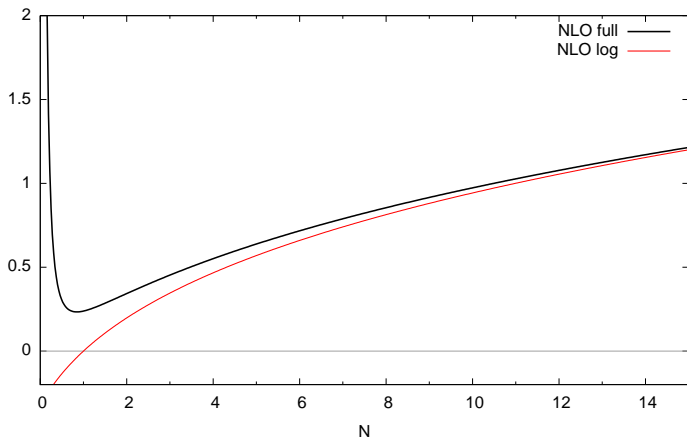
Drell-Yan $q\bar{q}$ at NLO in N -space

$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$



Drell-Yan $q\bar{q}$ at NLO in N -space

$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$



For $N \sim 2$ about 50% of the NLO is given by the log term

Saddle point approximation

$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN x^{-N} \mathcal{L}(N) \hat{\sigma}(N)$$

Saddle point approximation

$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{N \log \frac{1}{x} + \log \mathcal{L}(N) + \log \hat{\sigma}(N)}$$

Saddle point approximation

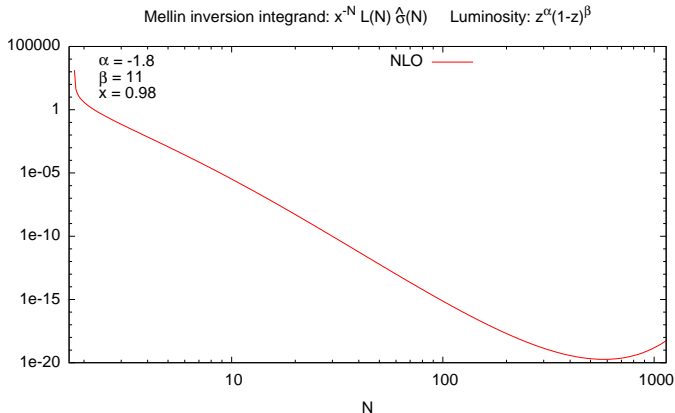
$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{N \log \frac{1}{x} + \log \mathcal{L}(N) + \log \hat{\sigma}(N)}$$

By saddle point approximation the dominant contribution is given by the position of the saddle.

Saddle point approximation

$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{N \log \frac{1}{x} + \log \mathcal{L}(N) + \log \hat{\sigma}(N)}$$

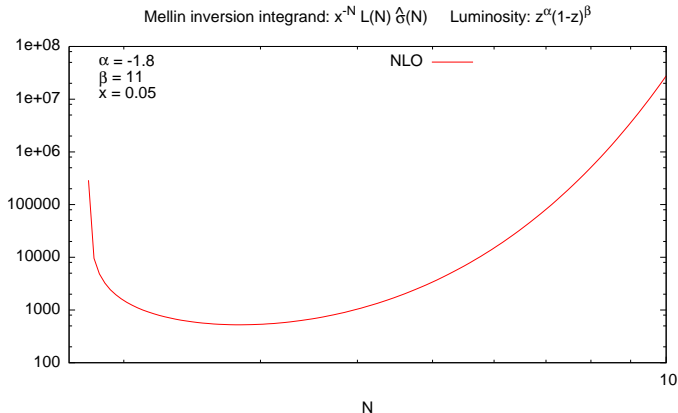
By saddle point approximation the dominant contribution is given by the position of the saddle.



Saddle point approximation

$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{N \log \frac{1}{x} + \log \mathcal{L}(N) + \log \hat{\sigma}(N)}$$

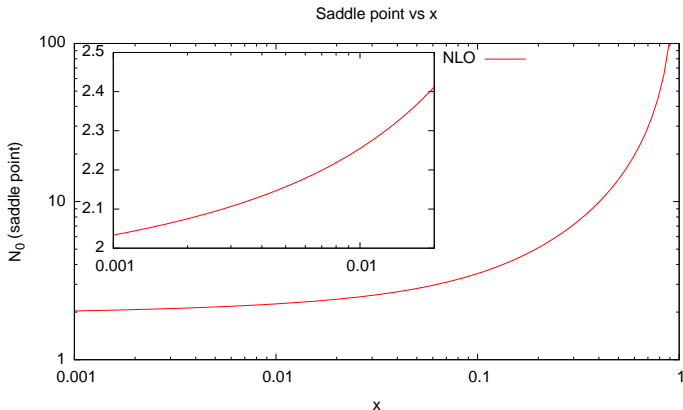
By saddle point approximation the dominant contribution is given by the position of the saddle.



Saddle point approximation

$$\sigma(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{N \log \frac{1}{x} + \log \mathcal{L}(N) + \log \hat{\sigma}(N)}$$

By saddle point approximation the dominant contribution is given by the position of the saddle.



Relevance of resummation

If we require: log contribution $> 60\%$ of the total

If we require: log contribution $> 60\%$ of the total

$$\Rightarrow N \gtrsim 2.1$$

Relevance of resummation

If we require: log contribution $> 60\%$ of the total

$$\Rightarrow N \gtrsim 2.1 \quad \Rightarrow x \gtrsim 0.002$$

If we require: log contribution $> 60\%$ of the total

$$\Rightarrow N \gtrsim 2.1 \quad \Rightarrow x \gtrsim 0.002$$

Much smaller than expected!

If we require: log contribution $> 60\%$ of the total

$$\Rightarrow N \gtrsim 2.1 \quad \Rightarrow x \gtrsim 0.002$$

Much smaller than expected!

Resummation can be relevant for values of x as small as 0.002

$$\hat{\sigma}^{\text{res}}(N)$$

Borel prescription (1)

$$\hat{\sigma}^{\text{res}}(N) = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \left(\bar{\alpha} \log \frac{1}{N} \right)^k, \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1}\left[\left(\bar{\alpha} \log \frac{1}{N}\right)^k\right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1} \left[\left(\bar{\alpha} \log \frac{1}{N} \right)^k \right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Treat the divergent series $\mathcal{M}^{-1}(\hat{\sigma}^{\text{res}}(N))$ with Borel method:*

* Proposed by R.Abbate, S.Forte, G.Ridolfi, J.Rojo, M.Ubiali

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1} \left[\left(\bar{\alpha} \log \frac{1}{N} \right)^k \right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Treat the divergent series $\mathcal{M}^{-1}(\hat{\sigma}^{\text{res}}(N))$ with Borel method:*

$$\sum_{k=0}^{\infty} a_k t^k \stackrel{\text{Borel}}{=} \int_0^{+\infty} dw e^{-w} \sum_{k=0}^{\infty} \frac{a_k}{k!} (wt)^k$$

* Proposed by R.Abbate, S.Forte, G.Ridolfi, J.Rojo, M.Ubiali

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1} \left[\left(\bar{\alpha} \log \frac{1}{N} \right)^k \right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Treat the divergent series $\mathcal{M}^{-1}(\hat{\sigma}^{\text{res}}(N))$ with Borel method:*

$$\sum_{k=0}^{\infty} a_k t^k \stackrel{\text{Borel}}{=} \int_0^{+\infty} dw e^{-w} \sum_{k=0}^{\infty} \frac{a_k}{k!} (wt)^k$$

- the inner sum converges

* Proposed by R.Abbate, S.Forte, G.Ridolfi, J.Rojo, M.Ubiali

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1} \left[\left(\bar{\alpha} \log \frac{1}{N} \right)^k \right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Treat the divergent series $\mathcal{M}^{-1}(\hat{\sigma}^{\text{res}}(N))$ with Borel method:*

$$\sum_{k=0}^{\infty} a_k t^k \stackrel{\text{Borel}}{=} \int_0^{+\infty} dw e^{-w} \sum_{k=0}^{\infty} \frac{a_k}{k!} (wt)^k$$

- the inner sum converges
- the integral diverges (the series is not Borel-summable)

* Proposed by R.Abbate, S.Forte, G.Ridolfi, J.Rojo, M.Ubiali

Borel prescription (1)

$$\mathcal{M}^{-1}[\hat{\sigma}^{\text{res}}(N)] = \sum_{k=0}^{\infty} h_k(\bar{\alpha}) \mathcal{M}^{-1} \left[\left(\bar{\alpha} \log \frac{1}{N} \right)^k \right], \quad \bar{\alpha} = 2\beta_0\alpha_s$$

Treat the divergent series $\mathcal{M}^{-1}(\hat{\sigma}^{\text{res}}(N))$ with Borel method:*

$$\sum_{k=0}^{\infty} a_k t^k \stackrel{\text{Borel}}{=} \int_0^{+\infty} dw e^{-w} \sum_{k=0}^{\infty} \frac{a_k}{k!} (wt)^k$$

- the inner sum converges
- the integral diverges (the series is not Borel-summable)
- proposed solution: cut-off C in the integral

* Proposed by R.Abbate, S.Forte, G.Ridolfi, J.Rojo, M.Ubiali

Borel prescription (2)

$$\hat{\sigma}_{\text{BP}}(z, C) = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{d\xi}{\Gamma(\xi + 1)} \frac{[(1-z)^{\xi-1}]_+}{\sqrt{z}^\xi} \int_0^C \frac{dw}{\bar{\alpha}} e^{-\frac{w}{\bar{\alpha}}} \hat{\sigma}^{\text{res}} \left(e^{-\frac{w}{\bar{\alpha}\xi}} \right)$$

Borel prescription (2)

$$\hat{\sigma}_{\text{BP}}(z, C) = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{d\xi}{\Gamma(\xi + 1)} \frac{[(1-z)^{\xi-1}]_+}{\sqrt{z}^\xi} \int_0^C \frac{dw}{\bar{\alpha}} e^{-\frac{w}{\bar{\alpha}}} \hat{\sigma}^{\text{res}} \left(e^{-\frac{w}{\bar{\alpha}\xi}} \right)$$

Remarks

- resummed expression at parton level

Borel prescription (2)

$$\hat{\sigma}_{\text{BP}}(z, C) = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{d\xi}{\Gamma(\xi + 1)} \frac{[(1-z)^{\xi-1}]_+}{\sqrt{z}^\xi} \int_0^C \frac{dw}{\bar{\alpha}} e^{-\frac{w}{\bar{\alpha}}} \hat{\sigma}^{\text{res}} \left(e^{-\frac{w}{\bar{\alpha}\xi}} \right)$$

Remarks

- resummed expression at parton level
- asymptotic to the original divergent series

Borel prescription (2)

$$\hat{\sigma}_{\text{BP}}(z, C) = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{d\xi}{\Gamma(\xi + 1)} \frac{[(1-z)^{\xi-1}]_+}{\sqrt{z}^\xi} \int_0^C \frac{dw}{\bar{\alpha}} e^{-\frac{w}{\bar{\alpha}}} \hat{\sigma}^{\text{res}} \left(e^{-\frac{w}{\bar{\alpha}\xi}} \right)$$

Remarks

- resummed expression at parton level
- asymptotic to the original divergent series
- parameter C to estimate ambiguity

Borel prescription (2)

$$\hat{\sigma}_{\text{BP}}(z, C) = \frac{1}{2\pi i} \oint_C \frac{d\xi}{\Gamma(\xi + 1)} \frac{[(1-z)^{\xi-1}]_+}{\sqrt{z}^\xi} \int_0^C \frac{dw}{\bar{\alpha}} e^{-\frac{w}{\bar{\alpha}}} \hat{\sigma}^{\text{res}} \left(e^{-\frac{w}{\bar{\alpha}\xi}} \right)$$

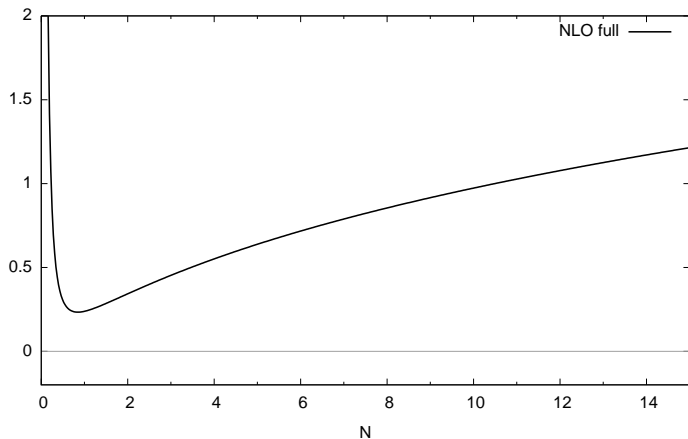
Remarks

- resummed expression at parton level
- asymptotic to the original divergent series
- parameter C to estimate ambiguity
- cut-off related to the inclusion of higher-twist terms

$$e^{-\frac{C}{\bar{\alpha}}} \simeq \left(\frac{\Lambda^2}{Q^2} \right)^{C/2}$$

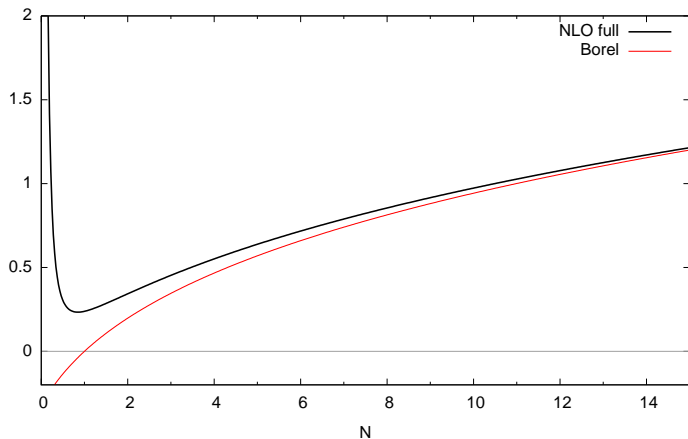
Comparison with fixed order: Drell-Yan $q\bar{q}$ at NLO

$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$



Comparison with fixed order: Drell-Yan $q\bar{q}$ at NLO

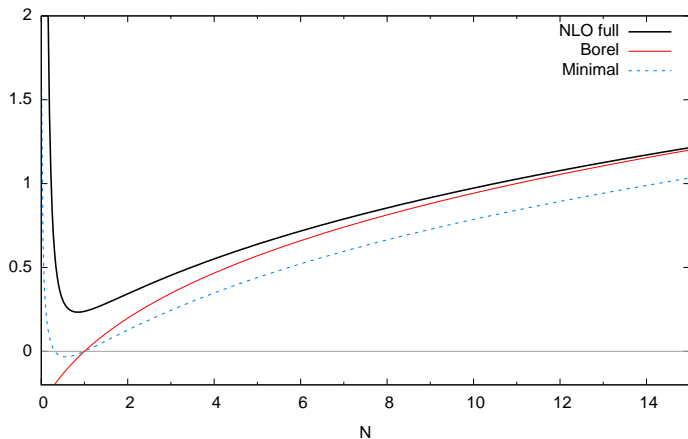
$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$



Comparison with fixed order: Drell-Yan $q\bar{q}$ at NLO

$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$

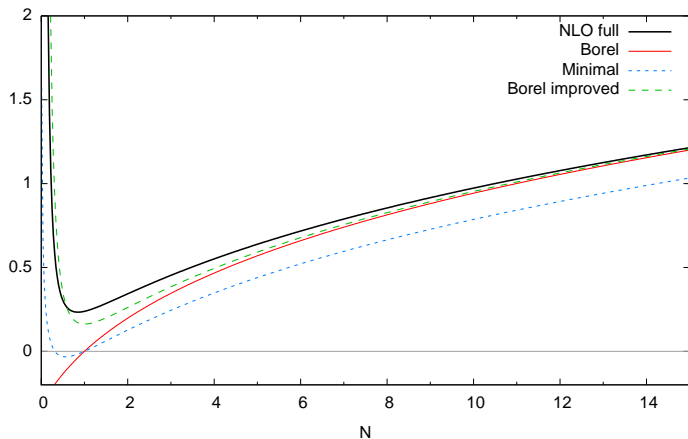
$$\left[\frac{\log \log \frac{1}{z}}{\log \frac{1}{z}} \right]_+$$



Comparison with fixed order: Drell-Yan $q\bar{q}$ at NLO

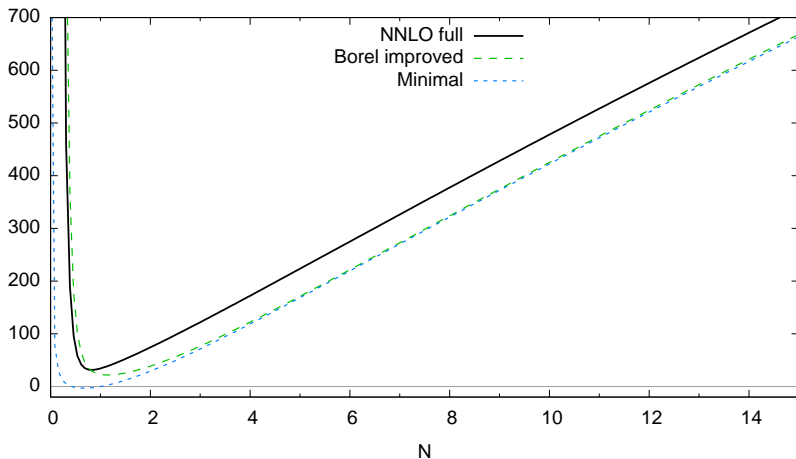
$$\frac{4\alpha_s C_F}{\pi} \left\{ \left[\frac{\log(1-z)}{1-z} \right]_+ - \frac{\log \sqrt{z}}{1-z} - \frac{(1+z)}{2} \log \frac{1-z}{\sqrt{z}} + \left(\frac{\pi^2}{12} - 1 \right) \delta(1-z) \right\}$$

$$\left[\frac{\log \log \frac{1}{z}}{\log \frac{1}{z}} \right]_+$$



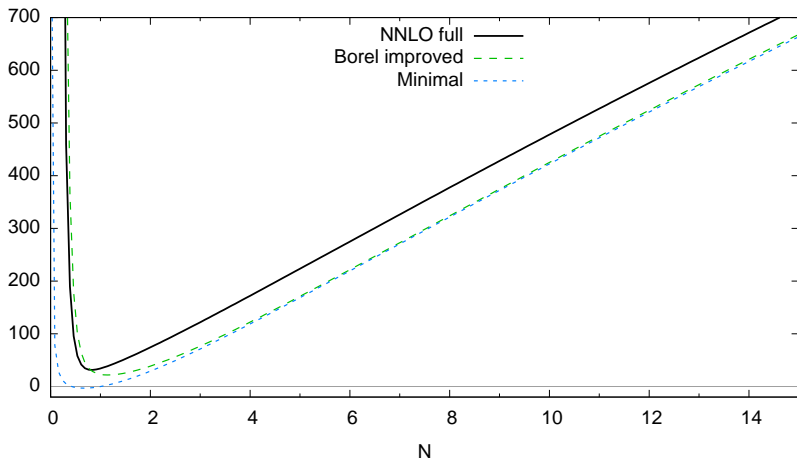
Comparison with fixed order: Drell-Yan $q\bar{q}$ at NNLO

Drell-Yan partonic $q\bar{q}$. Order α_s^2 Mellin transform



Comparison with fixed order: Drell-Yan $q\bar{q}$ at NNLO

Drell-Yan partonic $q\bar{q}$. Order α_s^2 Mellin transform



Discrepancy due to terms like $\log^k(1-z) \rightarrow \frac{\log^k N}{N} \Rightarrow$ subleading

Subleading terms

- Minimal prescription

$$\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}$$

- Minimal prescription

$$\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}$$

- Borel prescription improved

$$\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}$$

- Minimal prescription

$$\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}$$

- Borel prescription improved

$$\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}$$

- There are subleading terms which are important

$$\log^k(1-z) \quad \text{and similar}$$

and which are not (or only partially) included in the resummed

- Minimal prescription

$$\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}}$$

- Borel prescription improved

$$\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}$$

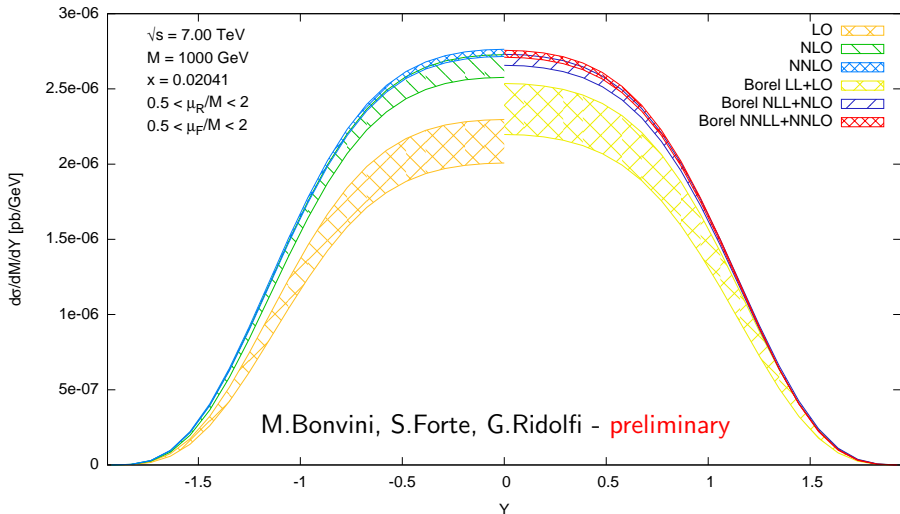
- There are subleading terms which are important

$$\log^k(1-z) \quad \text{and similar}$$

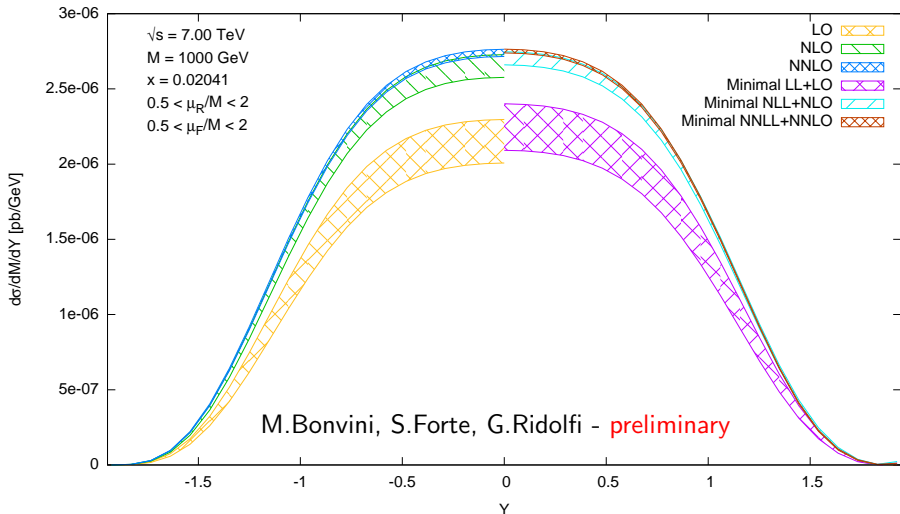
and which are not (or only partially) included in the resummed

Impact in phenomenology

DY rapidity distribution. Collider: pp Subprocess: Z+gamma

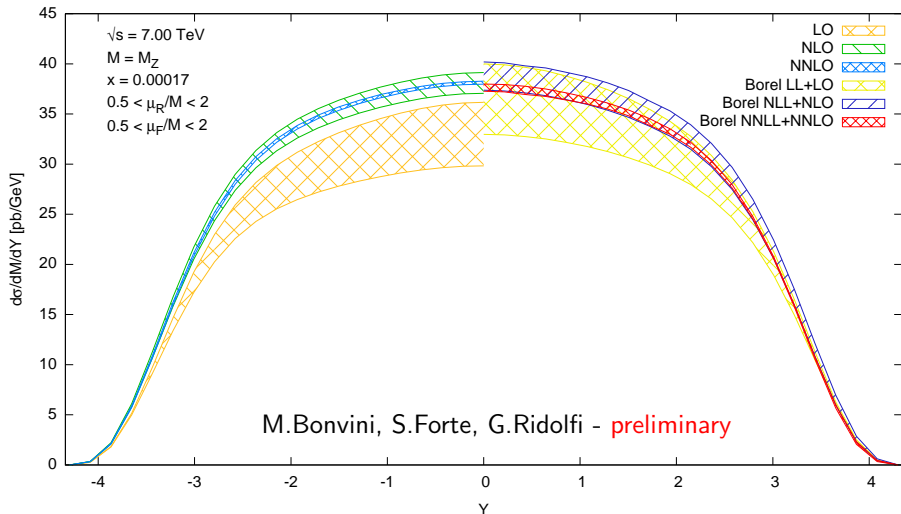


DY rapidity distribution. Collider: pp Subprocess: Z+gamma



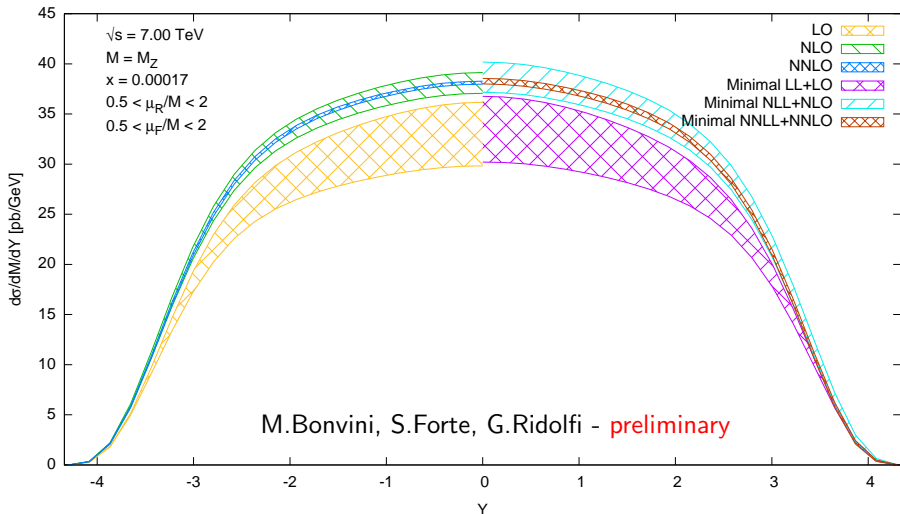
Rapidity distribution: Z at LHC (cteq6.6 pdfs used)

DY rapidity distribution. Collider: pp Subprocess: Z+gamma

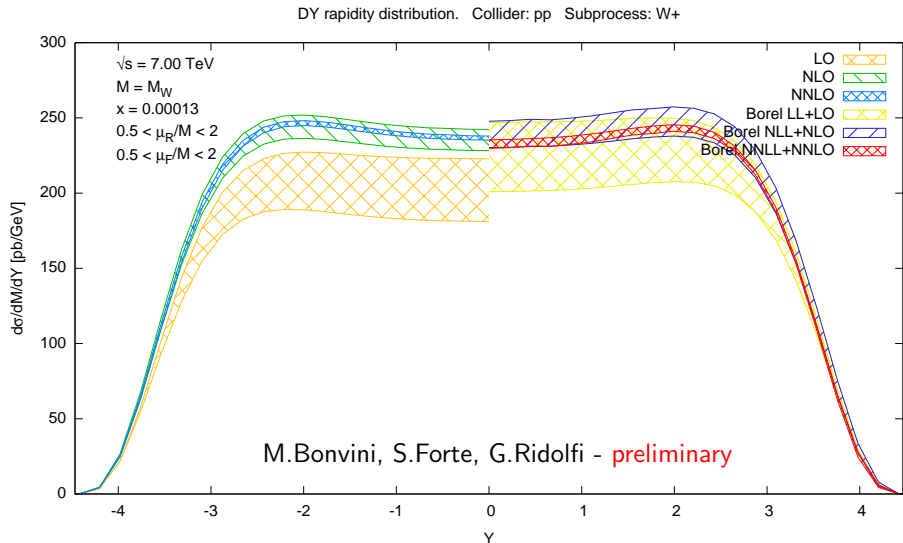


Rapidity distribution: Z at LHC (cteq6.6 pdfs used)

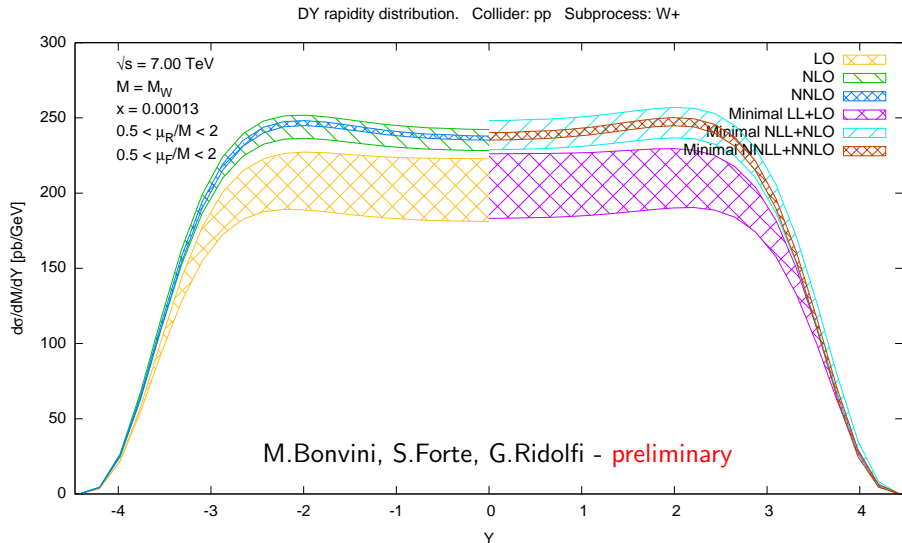
DY rapidity distribution. Collider: pp Subprocess: Z+gamma



Rapidity distribution: W^+ at LHC (cteq6.6 pdfs used)

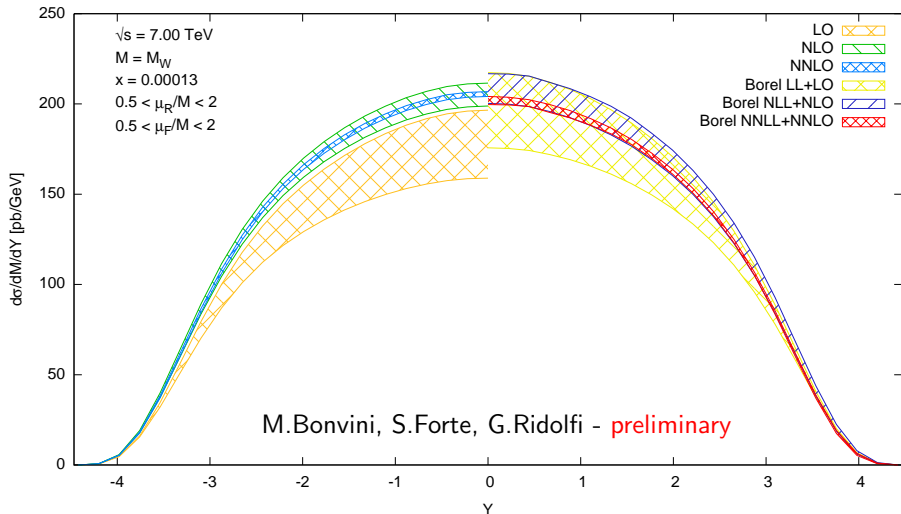


Rapidity distribution: W^+ at LHC (cteq6.6 pdfs used)



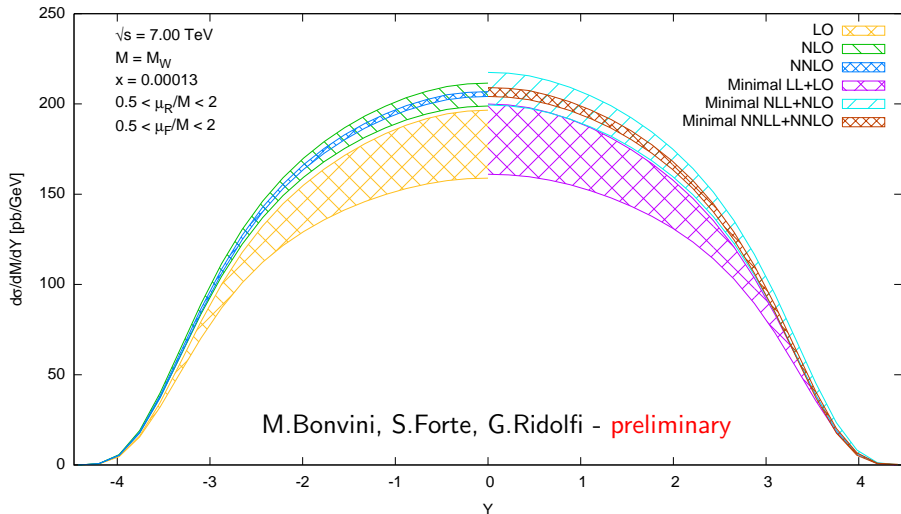
Rapidity distribution: W^- at LHC (cteq6.6 pdfs used)

DY rapidity distribution. Collider: pp Subprocess: W^-



Rapidity distribution: W^- at LHC (cteq6.6 pdfs used)

DY rapidity distribution. Collider: pp Subprocess: W^-



New results

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription
- New phenomenological results: rapidity distributions

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription
- New phenomenological results: rapidity distributions
- Ambiguity estimation

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription
- New phenomenological results: rapidity distributions
- Ambiguity estimation

Outlook

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription
- New phenomenological results: rapidity distributions
- Ambiguity estimation

Outlook

- Include subdominant (intermediate- z) contributions (next talk)

New results

- Quantitative evaluation of x for which resummation is important: much smaller than expected
- Improved Borel prescription
- New phenomenological results: rapidity distributions
- Ambiguity estimation

Outlook

- Include subdominant (intermediate- z) contributions (next talk)
- Data...

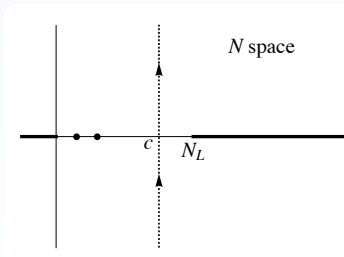
Backup slides

Minimal prescription

Proposed by S.Catani, M.L.Mangano, P.Nason, L.Trentadue:

$$\sigma_{\text{MP}}(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN x^{-N} \mathcal{L}(N) \hat{\sigma}^{\text{res}}(N)$$

with $c < N_L$, as in the figure.



Minimal prescription

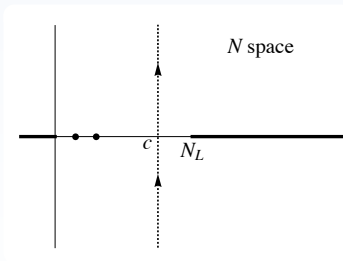
Proposed by S.Catani, M.L.Mangano, P.Nason, L.Trentadue:

$$\sigma_{\text{MP}}(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN x^{-N} \mathcal{L}(N) \hat{\sigma}^{\text{res}}(N)$$

with $c < N_L$, as in the figure.

Good properties:

- well defined for all x
- exact for invertible functions
- asymptotic to the original divergent series



Minimal prescription

Proposed by S.Catani, M.L.Mangano, P.Nason, L.Trentadue:

$$\sigma_{\text{MP}}(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN x^{-N} \mathcal{L}(N) \hat{\sigma}^{\text{res}}(N)$$

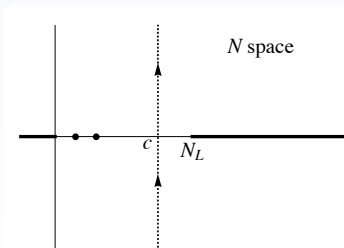
with $c < N_L$, as in the figure.

Good properties:

- well defined for all x
- exact for invertible functions
- asymptotic to the original divergent series

But...

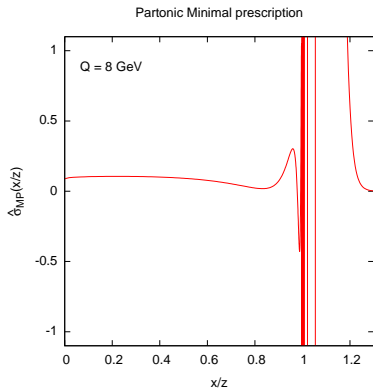
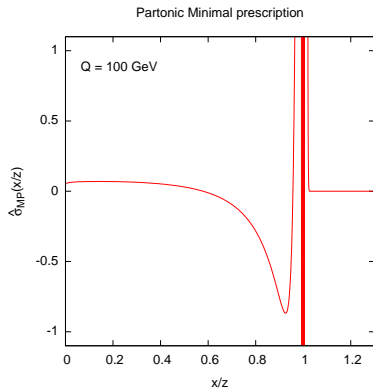
- a non-physical region of the parton cross-section contributes
- difficult numerical implementation



Minimal prescription: non-physical contribution

$$\sigma_{\text{MP}}(x) = \int_0^1 \frac{dz}{z} \mathcal{L}(z) \underbrace{\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \left(\frac{x}{z}\right)^{-N} \hat{\sigma}^{\text{res}}(N)}_{\hat{\sigma}_{\text{MP}}(x/z)}$$

The integral extends from 0 to 1, not from x to 1!



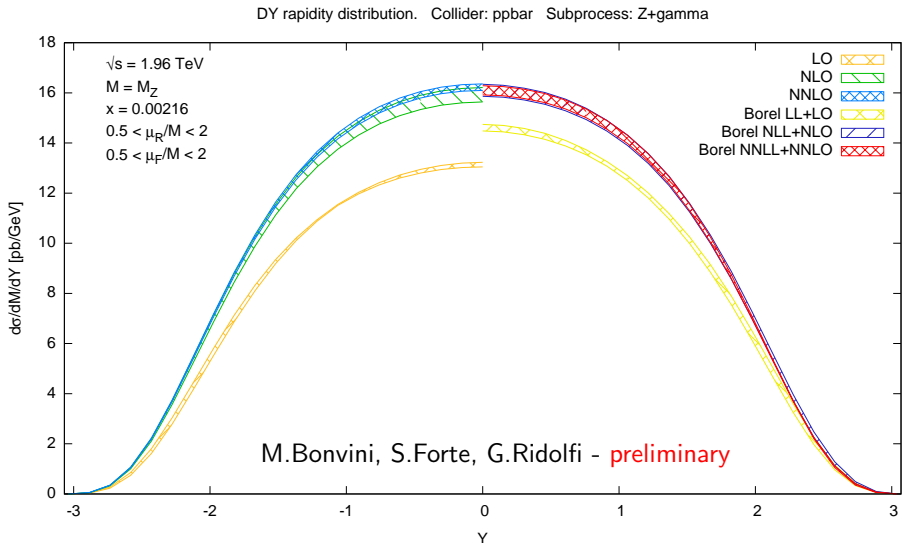
$$\int_0^1 dz z^{N-1} \left[\frac{\log^k(1-z)}{1-z} \right]_+ = \frac{1}{k+1} \sum_{j=0}^{k+1} \binom{k+1}{j} \Gamma^{(j)}(1) \Gamma(N) \Delta^{(k+1-j)}(N)$$

$$\int_0^1 dz z^{N-1} \left[\frac{\log^k(1-z)}{1-z} \right]_+ = \frac{1}{k+1} \sum_{j=0}^{k+1} \binom{k+1}{j} \Gamma^{(j)}(1) \Gamma(N) \Delta^{(k+1-j)}(N)$$

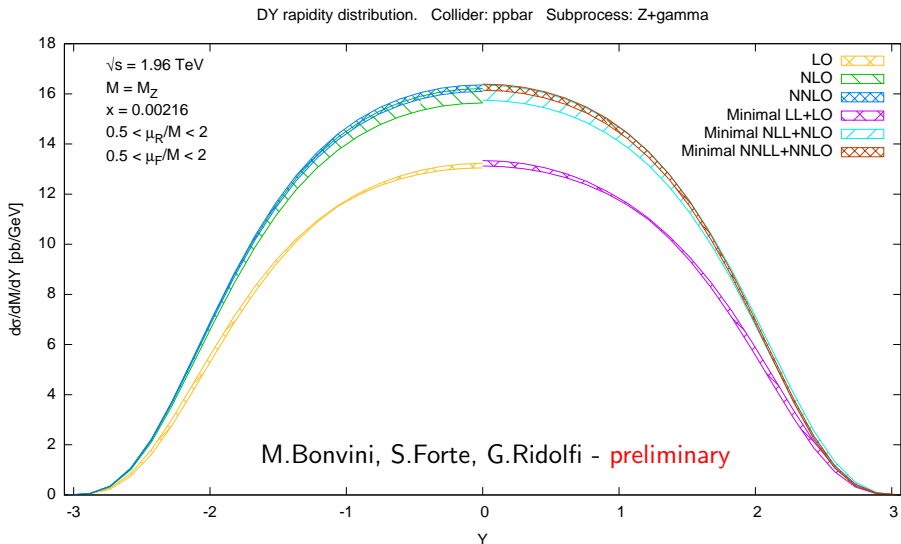
in the large N limit

$$\begin{aligned} &\simeq \frac{1}{k+1} \sum_{j=0}^{k+1} \binom{k+1}{j} \Gamma^{(j)}(1) \log^{k+1-j} \frac{1}{N} \\ &= \frac{\Gamma^{(k+1)}(1)}{k+1} + \int_0^1 dz z^{N-1} \left[\frac{\log^k \log \frac{1}{z}}{\log \frac{1}{z}} \right]_+ \end{aligned}$$

Rapidity distribution: Z at Tevatron (cteq6.6 pdfs used)

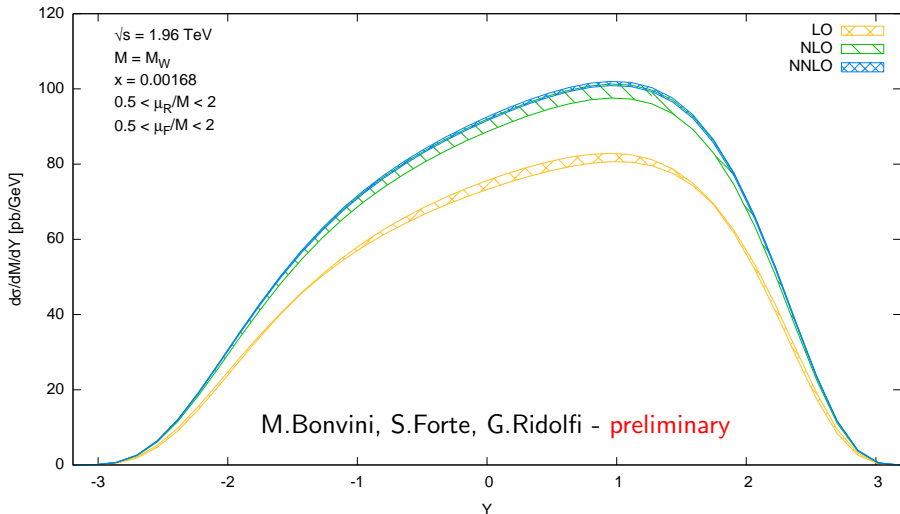


Rapidity distribution: Z at Tevatron (cteq6.6 pdfs used)



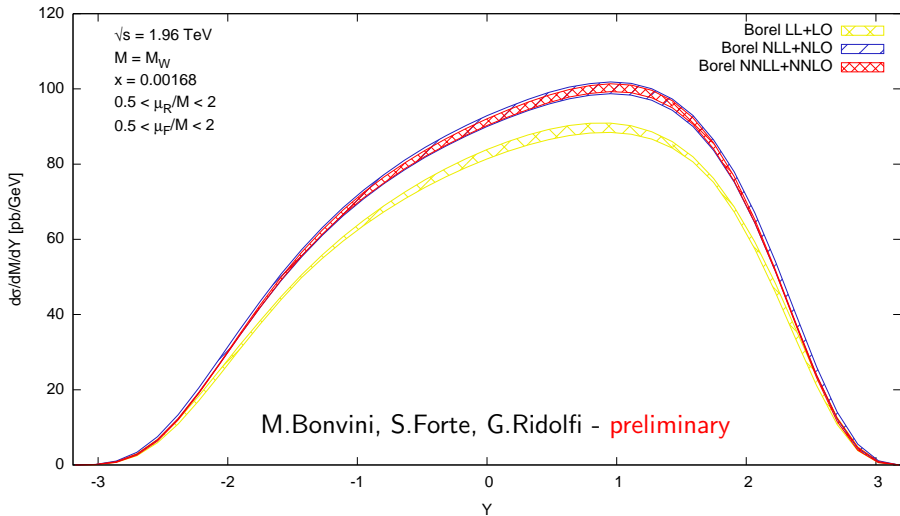
Rapidity distribution: W^+ at Tevatron (cteq6.6 pdfs used)

DY rapidity distribution. Collider: ppbar Subprocess: W+



Rapidity distribution: W^+ at Tevatron (cteq6.6 pdfs used)

DY rapidity distribution. Collider: ppbar Subprocess: W+



Rapidity distribution: W^+ at Tevatron (cteq6.6 pdfs used)

DY rapidity distribution. Collider: ppbar Subprocess: W+

