The FONLL General-Mass scheme in DIS and the Les Houches HQ benchmarks

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HEAVY QUARKS SCHEMES IN DIS

Heavy quark schemes in perturbative QCD

Heavy quark schemes in DIS have received a lot of attention in the recent years, specially due to its impact in the determination of PDFs and correspondingly in LHC physics.
Two very well-understood schemes:

1. Assume heavy quarks effectively massless above $Q^2 = m_H^2 \rightarrow$ Massless scheme, valid for $Q^2 \gg m_H^2$
2. Heavy quarks retain their mass for all $Q^2 \rightarrow$ Massive scheme, valid for $Q^2 \sim m_H^2$

However, their matching in a unique GM-VFN scheme valid at all Q^2 is much more challenging

Matched calculations

Matched calculations aim to combined mass effects in the massive scheme with log resummation in the massless scheme

Several proposals in the literature, including:

1. ACOT: Used in CTEQ family of PDF analysis since 2006 (Collins and Tung 86, Aivazis et al 93, Collins 98). Several variants: Simplified ACOT (S-ACOT), S-ACOT-χ
2. Thorne-Roberts: Used in MSTW family of PDF analysis (Thorne and Roberts 98, Thorne 06)
3. FONLL: originally formulated for hadronic collisions (Cacciari, Greco and Nason 98), recently applied to DIS (Forte, Laenen, Nason, Rojo 10). Currently being implemented in the NNPDF family of PDF analysis
4. BMSN (Buza et al 96): Used in the ABKM08 (Alekhin et al 09) analysis
   Resummation of $\ln Q^2/m^2$ terms not included
   The use of $N_f = 3$ PDFs is required in this scheme

THE FONLL APPROACH

Heavy quarks in deep-inelastic scattering,
Stefano Forte, Eric Laenen, Paolo Nason, Juan Rojo
arXiv:1001.2312 [hep-ph]
Nucl.Phys.B834:116-162,2010

FONLL in a nutshell

- Express the massive result $F^{(n_l)}$ in terms of the massless PDFs and α_s (non trivial from $\mathcal{O}\left(\alpha_s^2\right)$)

$$F^{(n_l)}(x,Q^2) = x\int_x^1 \frac{dy}{y} \sum_{i=q,\bar{q},g} B_i\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

- Define massless limit of the massive computation as

$$F^{(n_l,0)}(x,Q^2) \equiv x\int_x^1 \frac{dy}{y} \sum_{i=q,\bar{q},g} B_i^{(0)}\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

$$\lim_{m\to 0}\left[B_i\left(x,\frac{Q^2}{m^2}\right) - B_i^{(0)}\left(x,\frac{Q^2}{m^2}\right)\right] = 0$$

- The FONLL approximation is then

$$F^{\rm FONLL}(x,Q^2) \equiv F^{(d)}(x,Q^2) + F^{(n_l)}(x,Q^2),$$
$$F^{(d)}(x,Q^2) \equiv \left[F^{(n_l+1)}(x,Q^2) - F^{(n_l,0)}(x,Q^2)\right]$$

Important technical advantage: PDFs and α_s expressed always in the (n_l+1) scheme

FONLL in a nutshell

- Far from threshold, $Q^2 \gg m^2$ $F^{(n_l,0)}(x,Q^2) \sim F^{(n_l)}(x,Q^2) \to$ the massless computation recovered

$$F^{\rm FONLL}(x,Q^2) \sim F^{(n_l+1)}(x,Q^2)$$

- Near threshold the "difference term" is formally higher order but unreliable, so one can correct it by mass suppressed terms, using for example a damping factor (FONLL default)

$$F^{(d,\,th)}(x,Q^2) \equiv f_{\rm thr}(x,Q^2)F^{(d)}(x,Q^2), \quad f_{\rm thr}(x,Q^2) = \Theta(Q^2-m^2)\left(1-\frac{Q^2}{m^2}\right)^2 ,$$

or some form of $\chi-$scaling,

$$F^{(d,\,\chi)}(x,Q^2) \equiv F^{(d)}(x,Q^2) = x\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

$$F^{(d,\,\chi,v2)}(x,Q^2) \equiv F^{(d)}(\chi(x,Q^2),Q^2), \quad \chi = x\left(1+\frac{4m^2}{Q^2}\right).$$

The choice of threshold prescription represent an intrinsic ambiguity of the matching procedure. Can this ambiguity be minimized?

Perturbative ordering in FONLL

Three FONLL schemes for different ordering of the perturbative expansion can be defined:

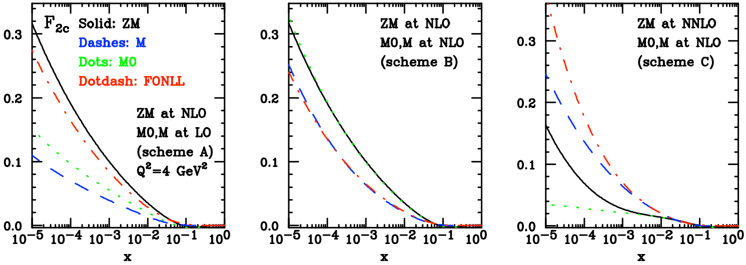
1. Scheme A $\rightarrow \mathcal{O}(\alpha_s)$ in massless and in massive
2. Scheme B $\rightarrow \mathcal{O}(\alpha_s)$ in massless and $\mathcal{O}(\alpha_s^2)$ in massive
3. Scheme C $\rightarrow \mathcal{O}(\alpha_s^2)$ in massless and in massive

In any of the three schemes, any threshold prescription can be implemented
These schemes can be related to existing approaches

1. Scheme A is identical to S-ACOT
2. Scheme B was formulated with similar scope as TR (use the information from the $\mathcal{O}(\alpha_s^2)$ massive computation in a NLO GM-VFN scheme), but they turn to be different
3. Scheme C should be S-ACOT at NNLO?

$F_{2c}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{2c}(x, Q^2)$:



In FONLL scheme B ZM∼M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL∼Massive
Greatly reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

In all schemes mass-suppressed corrections are important even at moderate Q^2

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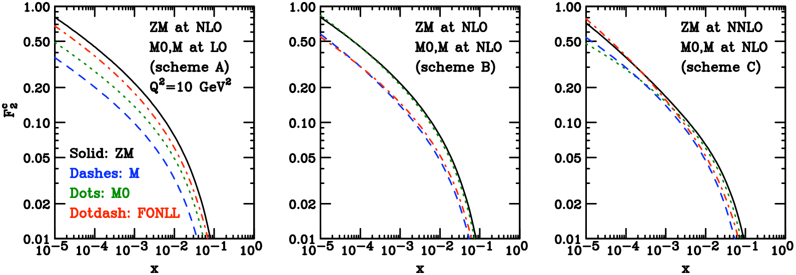

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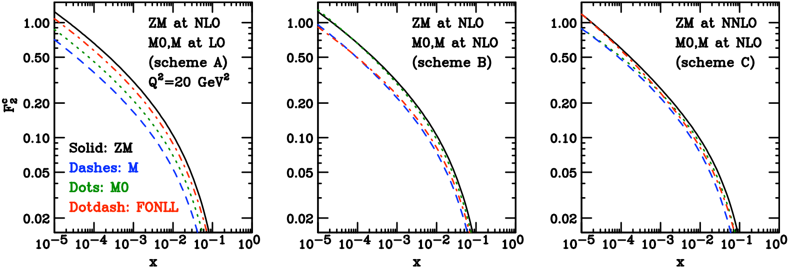

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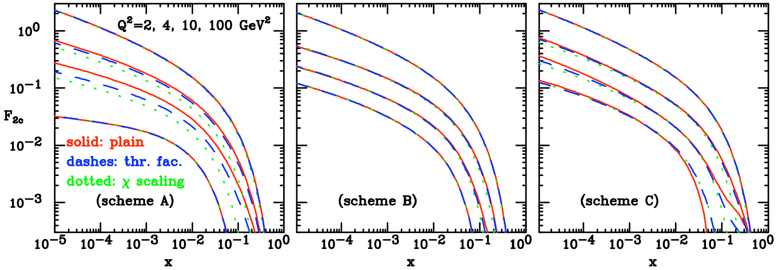
$F_{2c}(x, Q^2)$ in FONLL - threshold prescriptions

The FONLL result for $F_{2c}(x, Q^2)$ with different threshold prescriptions



In FONLL the ambiguity due to choice of (arbitrary) threshold prescription present in scheme A dissapears in scheme B

This threshold ambiguity can be as large as the resummation itself

FONLL - Summary

- The FONLL approach provides a simple, flexible and practically viable implementation of heavy quark effects in DIS
- FONLL allows for the combination of fixed order heavy quark emission terms with the all–order resummation of collinear logs which appear at scales much larger than the heavy quark mass.
- A significant feature is that the perturbative order at which the fixed–order and resummed results are obtained can be chosen independently of each other in the most suitable way
- By exploiting this flexibility, we have defined scheme B, which thanks to the use of the α_S^2 massive coefficients cures the threshold ambiguities which affect scheme A $\rightarrow$ FONLL-B is A NLO GM-VFN scheme without (almost) threshold ambiguity
- FONLL can be easily generalized to arbitrary perturbative orders, as shown by scheme C, a relatively simple NNLO GM-VFN scheme

THE LES HOUCHES HEAVY QUARK BENCHMARKS STUDY

The SM and NLO Multileg Working Group: Summary report, arXiv:1003.1241
Extended version in preparation

Les Houches HQ Benchmark settings

- A unique set of PDFs is used → We adopt the Les Houches toy PDF set, evolved in Q^2 to NLO and NNLO using the HOPPET program (G. P. Salam and J. Rojo, Comput. Phys. Commun. 180 (2009) 120)
- $\alpha_s\left(Q^2\right)$ obtained from exact solution of the RG equations from $\alpha_s\left(Q^2 = m_c^2 = 2\,\mathrm{GeV}^2\right) = 0.35$
- The charm quark is the only heavy quark ($m_b \to \infty$)
- F_{2c} (F_{Lc}) defined as the contribution to F_2 (F_L) when light quark charges set to zero
- Compare F_{2c} and F_{Lc} for the different GM-VFN schemes for different values of Q^2, from 4 GeV^2 to 100 GeV^2
  Here concentrate on F_{2c} - see extras for F_{Lc}
- Comparisons available between ACOT, TR and FONLL
  Inclusion of the BMSN (used in ABKM08) in progress

Results: F_{2c} in FONLL vs. S-ACOT

The $\chi-$scaling threshold prescription used in S-ACOT-χ can be implemented in two alternative ways (with the difference being subleading)

- $x \to \chi$ replacement only inside convolutions

$$F^{(\chi)}(x,Q^2) = x \int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y}, \alpha(Q^2)\right) f(y,Q^2),$$

- $x \to \chi$ replacement in the structure function argument

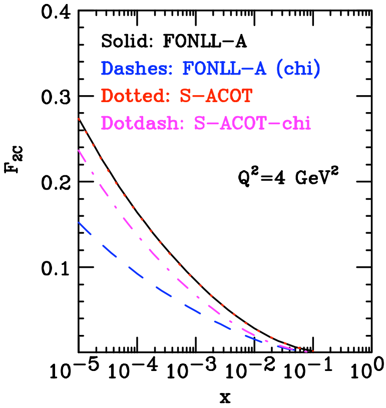
$$F^{(\chi,v2)}(x,Q^2) = \chi(x,Q^2) \int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y}, \alpha(Q^2)\right) f(y,Q^2),$$

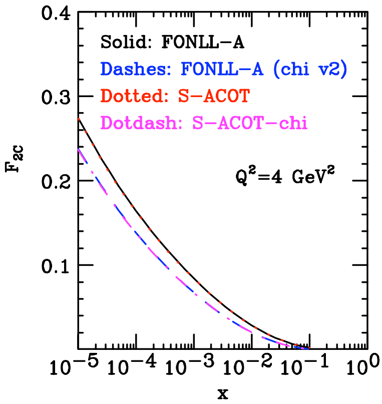
$$\chi(x,Q^2) = x\left(1 + \frac{4m^2}{Q^2}\right).$$

$F^{(\chi)}(x,Q^2)$ used in CTEQ6.6, while $F^{(\chi,v2)}(x,Q^2)$ implemented in MSTW2008

Results: F_{2c} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A
S-ACOT-χ is identical to FONLL scheme A with χ scaling (v2)





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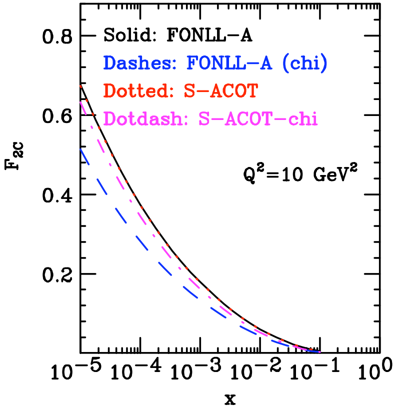
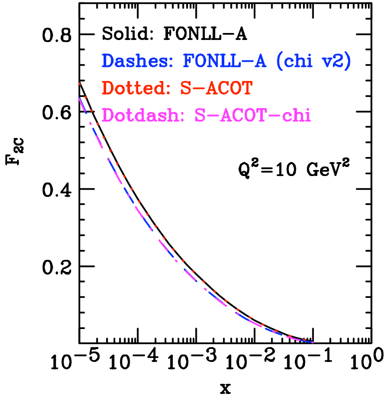
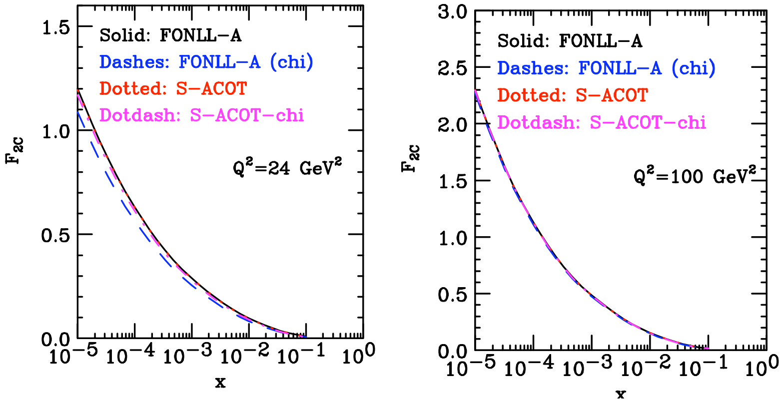




Results: F_{2c} in FONLL vs. S-ACOT

As Q^2 increases all schemes are identical (threshold effects negligible)

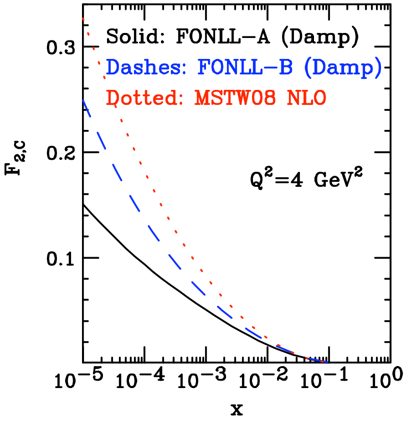


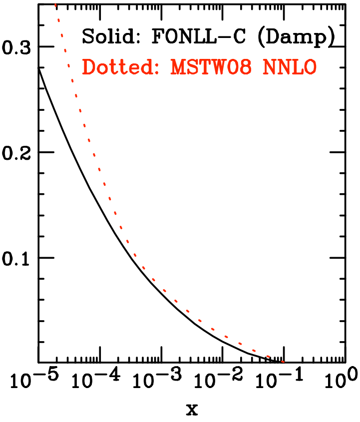
Results: F_{2c} in FONLL vs. S-ACOT

- FONLL-A (plain) is identical to S-ACOT (both for F_{2c} and for F_{Lc})
- FONLL-A is identical to S-ACOT-χ once the proper threshold prescription is adopted
- The S-ACOT-χ numbers provided by F. Olness use a different $\chi-$scaling than the ones used in the CTEQ6.6 fit (P. Nadolsky)
- It is crucial to carefully state the threshold prescription used in each case $\rightarrow$ In FONLL scheme A (and in S-ACOT) the effect of the threshold prescription can be as large as the resummation itself
- The default threshold prescription used in FONLL (damping factor) falls between the two implementations of $\chi-$scaling

Results: F_{2c} in FONLL vs. MSTW08

With default threshold prescriptions:





Results: F_{2c} in FONLL vs. MSTW08

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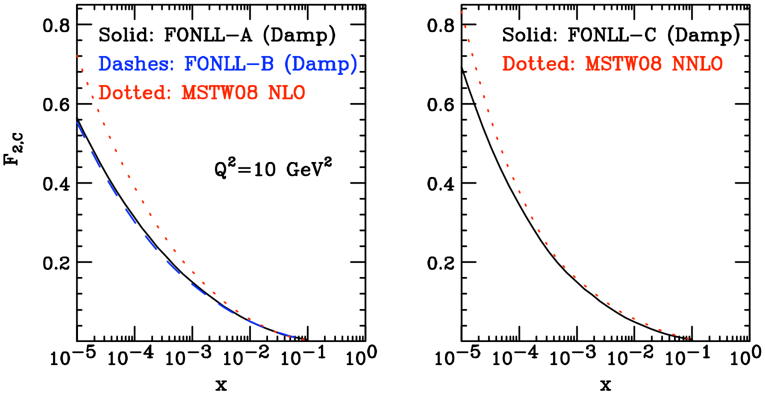
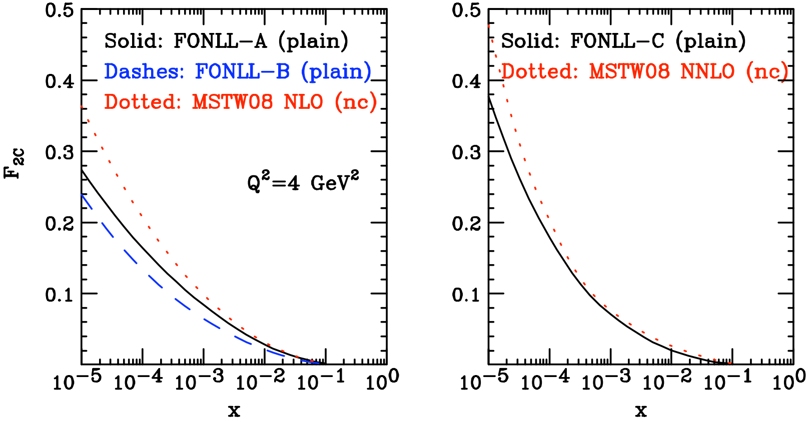


Results: F_{2c} in FONLL vs. MSTW08

With threshold prescriptions switched off:



Results: F_{2c} in FONLL vs. MSTW08

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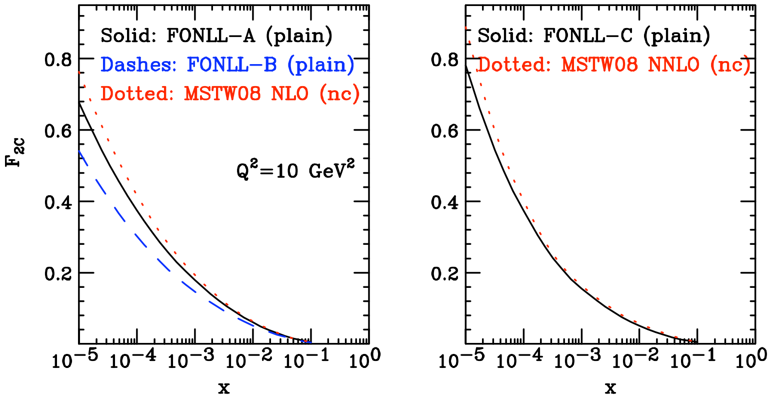
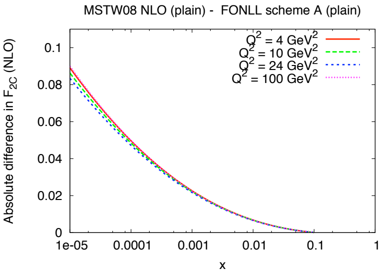
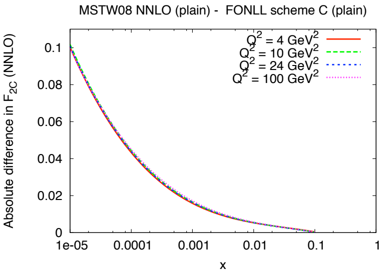


Results: F_{2c} in FONLL vs. MSTW08

The only difference for $F_{2c}(x, Q^2)$ between FONLL scheme A (and scheme C) and MSTW08 NLO (and NNLO) (without threshold prescriptions) is a Q^2−independent matching term f in MSTW08:

$$\text{FONLL-A - MSTW08-NLO} = f\left(x, \alpha_s\left(m_c^2\right)\right)$$





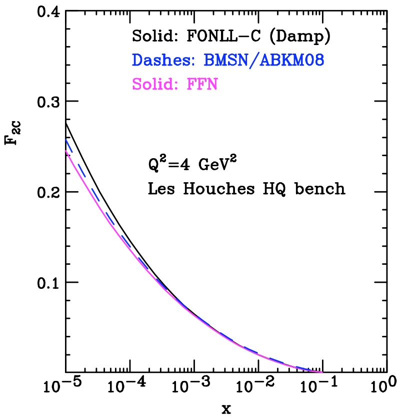
The same conclusions holds for S-ACOT vs. MSTW08 NLO

Results: F_{2c} in FONLL vs. MSTW08

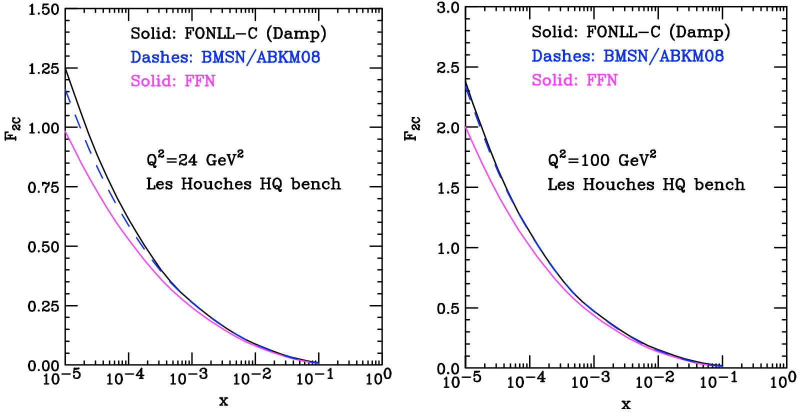
Summary of the TR/MSTW08 vs. FONLL comparison

- FONLL scheme B was formulated with a similar motivation than TR $\rightarrow$ Use all information from the $\mathcal{O}\left(\alpha_s^2\right)$ massive computation in the NLO GM-VFN scheme
- In practice, since TR freeze their $\mathcal{O}\left(\alpha_s^2\right)$ term at $Q^2 = m_c^2$, for F_{2c} TR and FONLL-B turn out to be alternative schemes
- TR NLO is S-ACOT/FONLL-A plus the constant (subleading) term, and shares with these schemes the large dependence on the choice of (arbitrary) threshold prescription (unlike FONLL-B which is unaffected by this choice of prescription)
- Similar conclusions for TR NNLO and FONLL-C: identical up to a Q^2-independent subleading term
- For F_{Lc} instead the TR ordering leads to similar results between FONLL-B and MSTW08.

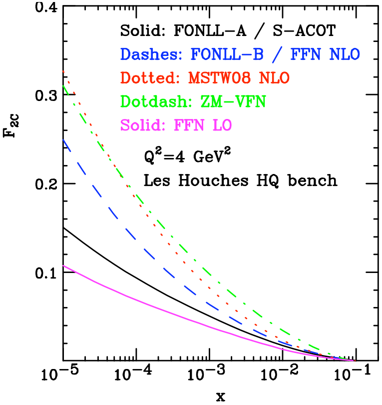
Results: F_{2c} in FONLL vs. BMSN/ABKM08

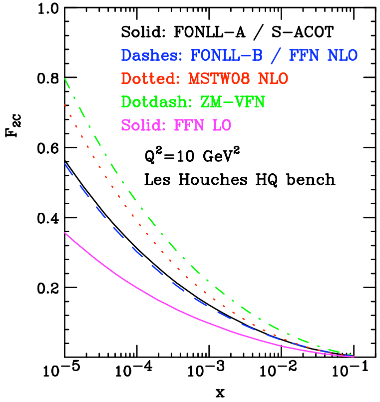


Results: F_{2c} in FONLL vs. BMSN/ABKM08



LH HQ benchmarks: F_2^c NLO schemes summary





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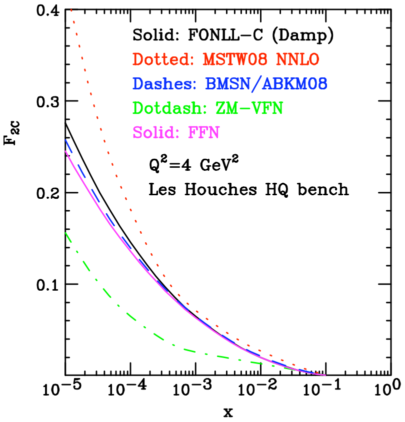
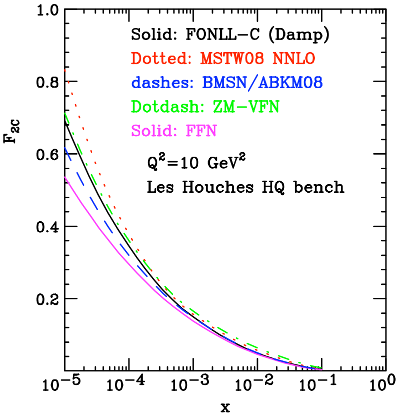




SUMMARY AND FUTURE WORK

Summary

The Les Houches benchmark study on heavy quarks GM-VFN schemes has allowed to identify and quantify similarities and differences between schemes:

1. FONLL-A (plain) is identical to S-ACOT, and FONLL-A (χ,v2) is identical to S-ACOT-χ (Olness), both for F_{2c} and F_{Lc}
2. The only difference between FONLL-A (plain) (and S-ACOT) and MSTW08 NLO for F_{2c} is a Q^2-independent matching term present in MSTW08
3. The only difference between FONLL-C (plain) and MSTW08 NNLO for F_{2c} is a Q^2-independent matching term present in MSTW08
4. FONLL scheme B is a completely different scheme from MSTW08 NLO for F_{2c}. In particular, unlike S-ACOT or MSTW08, it is independent of the threshold prescription
5. On the other hand, due to the TR ordering, FONLL scheme B(C) is very close to MSTW08 NLO(NNLO) for F_{Lc}

Outlook

More work is still required: From the theoretical point of view:

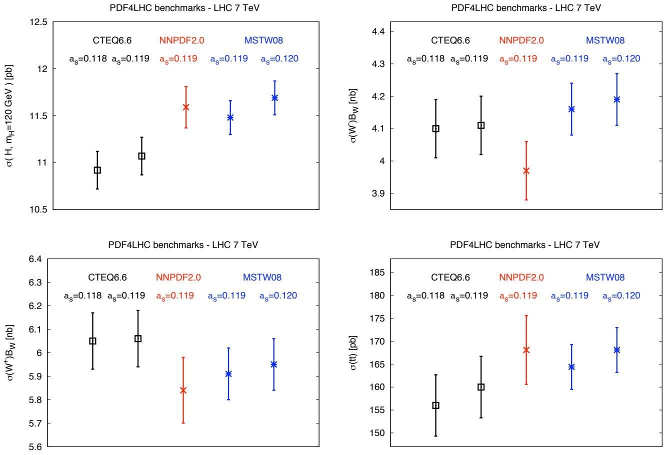
- CTEQ is planning NNLO PDF fits $\rightarrow$ would S-ACOT at NNLO be identical to FONLL scheme C?
- MSTW has studied variations of his GM-VFN scheme with respect his default values (R. Thorne, PDF4LHC DESY 10/09) $\rightarrow$ How these new settings affect the benchmark comparisons?
- Inclusion of the BMSN scheme (used in the latest ABKM08 analysis) in the benchmark comparison

From the phenomenological point of view:

- Impact of different GM-VFN schemes in the determination of PDF with benchmark-like settings
- Impact of different GM-VFN schemes in relevant LHC observables

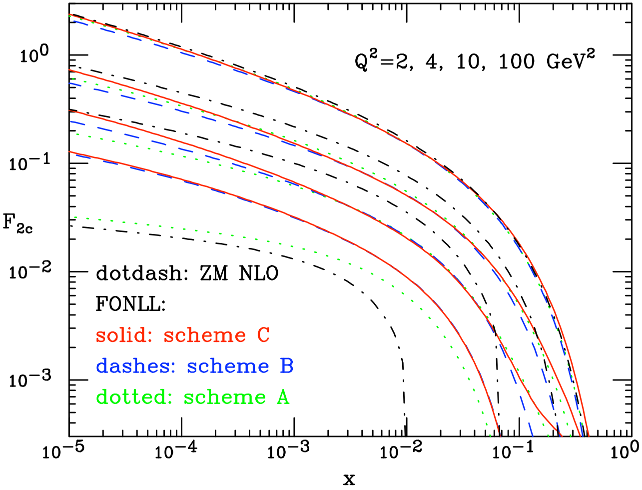
Outlook

The impact of HQ corrections at LHC 7 TeV is likely within 1–sigma range

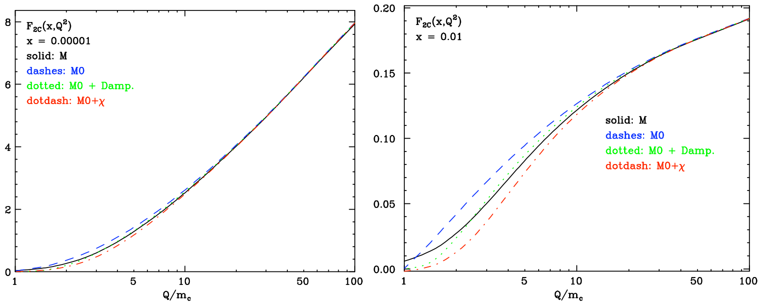


EXTRA MATERIAL

$F_{2c}(x, Q^2)$ in FONLL - Summary



Threshold prescriptions



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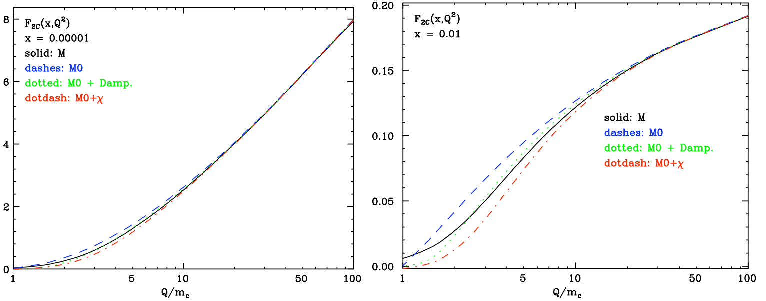

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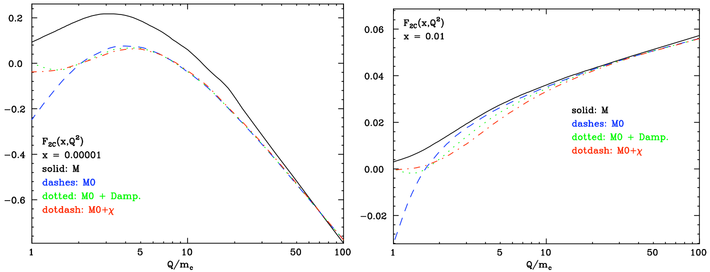

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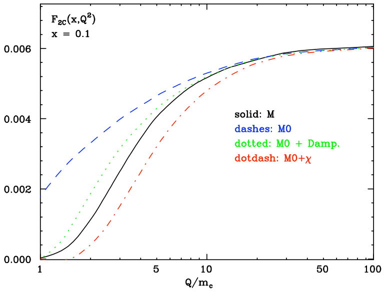
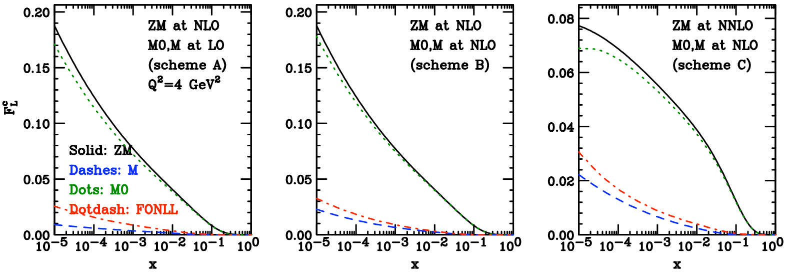


$F_{Lc}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{Lc}(x, Q^2)$



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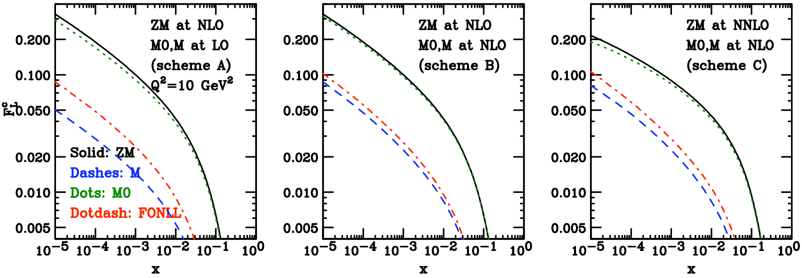

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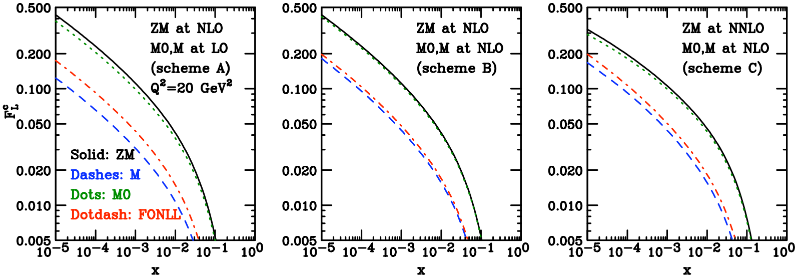

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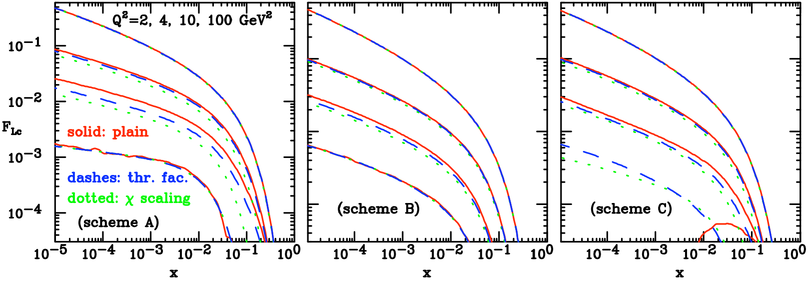

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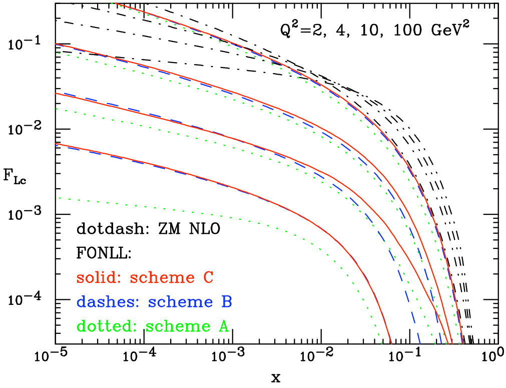
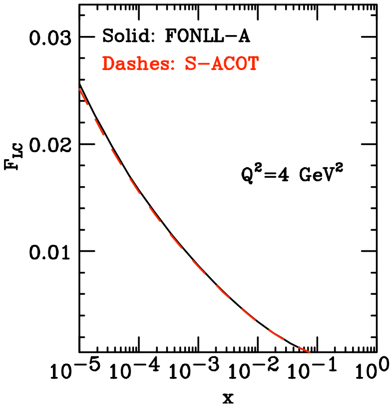
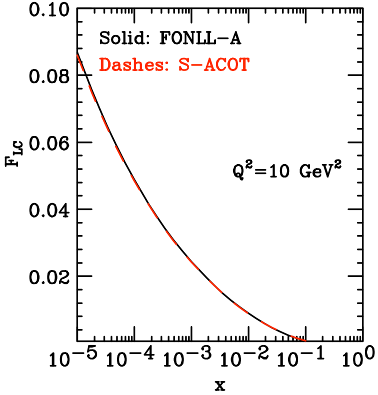


The massless is very far from FONLL even at large Q^2 for $F_{Lc}\left(x, Q^2\right)$

Results: F_{Lc} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A also for F_{Lc}





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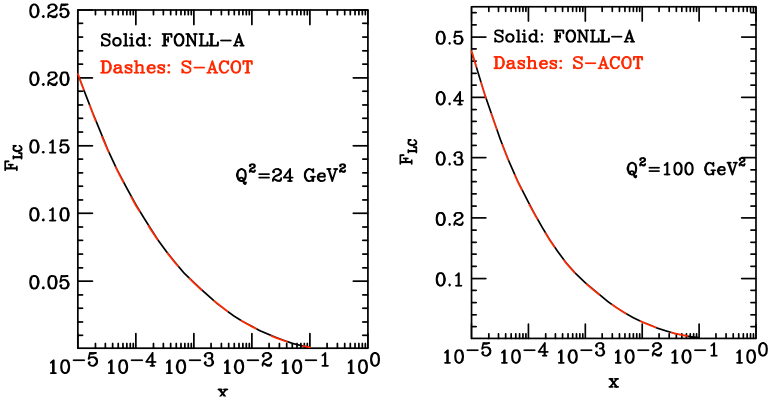
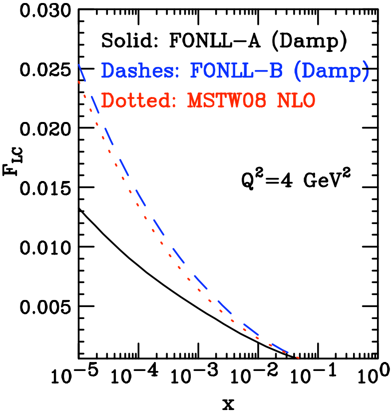


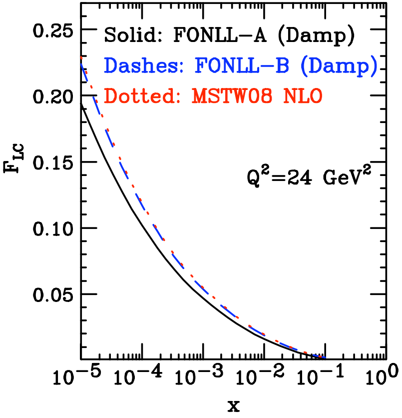
Results: F_{Lc} in FONLL vs. MSTW08

With default threshold prescriptions:



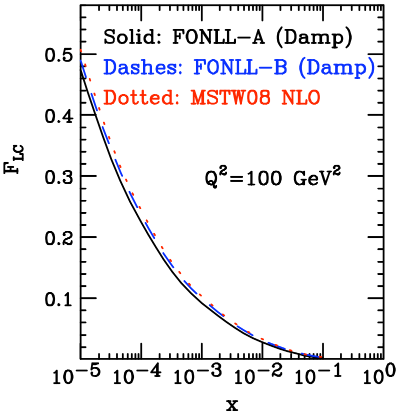
Results: F_{Lc} in FONLL vs. MSTW08

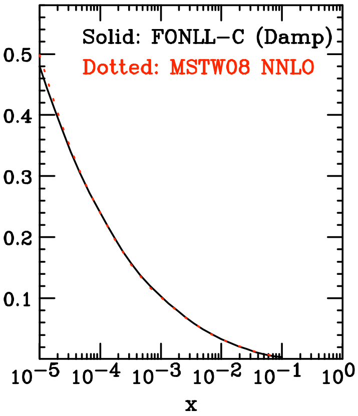
With default threshold prescriptions:



Results: F_{Lc} in FONLL vs. MSTW08

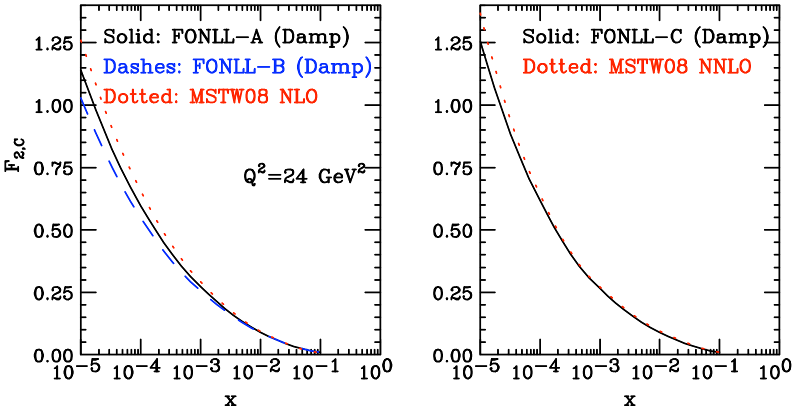
With default threshold prescriptions:





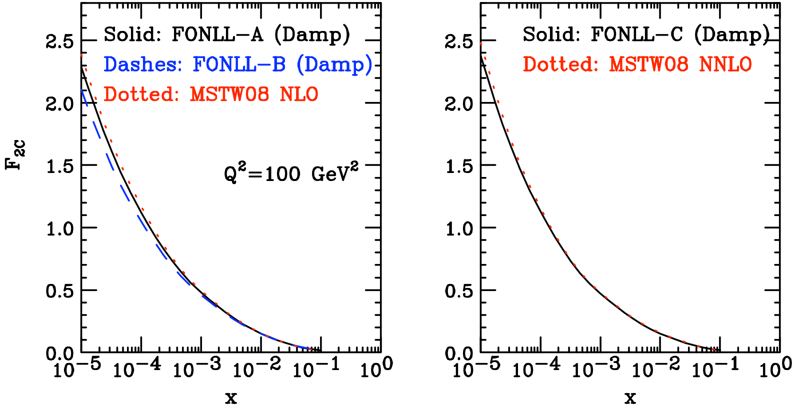
FONLL vs. MSTW08

With default threshold prescriptions:



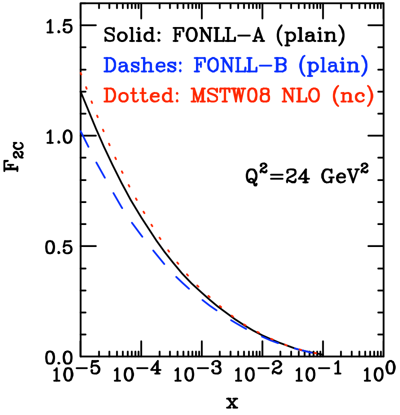
FONLL vs. MSTW08

With default threshold prescriptions:



FONLL vs. MSTW08

With threshold prescriptions switched off:



FONLL vs. MSTW08

With threshold prescriptions switched off: