Status of QCD

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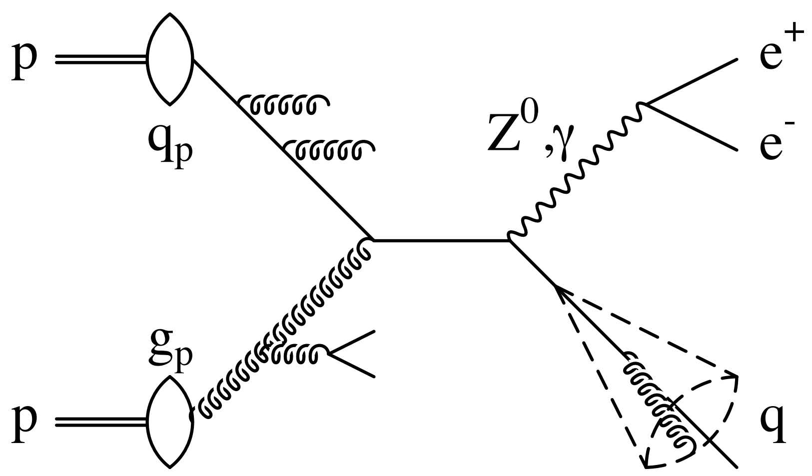
QCD at High Energy Colliders

- QCD: successful theory of strong interactions
- QCD is omnipresent in high energy collisions



QCD effects
- initial state: parton distributions
- final state: jets
- hard scattering matrix elements
- higher order corrections
- multiple radiation

- Understanding of QCD mandatory for
  - Interpretation of collider data
  - Precision studies
  - Searches for new physics

Topics

1. Jets and event shapes
2. Multiparticle production at NLO
3. Precision observables at NNLO
4. Parton distributions
5. Infrared structure and resummation
6. Multi-loop results

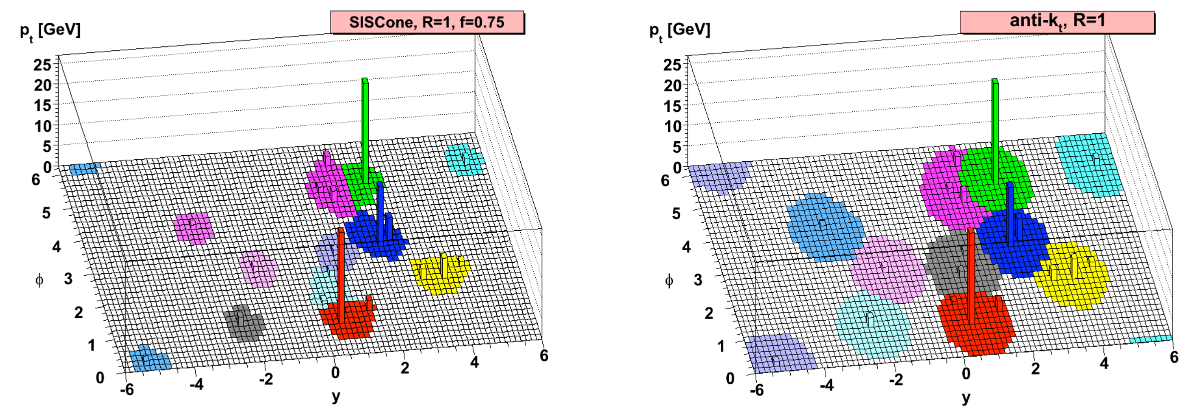
Jets and event shapes

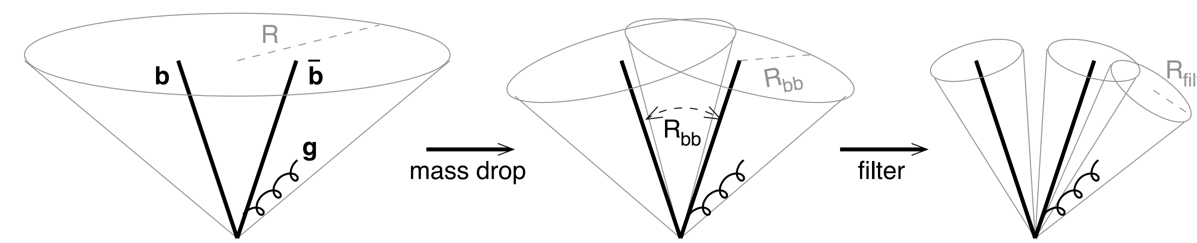
Jets

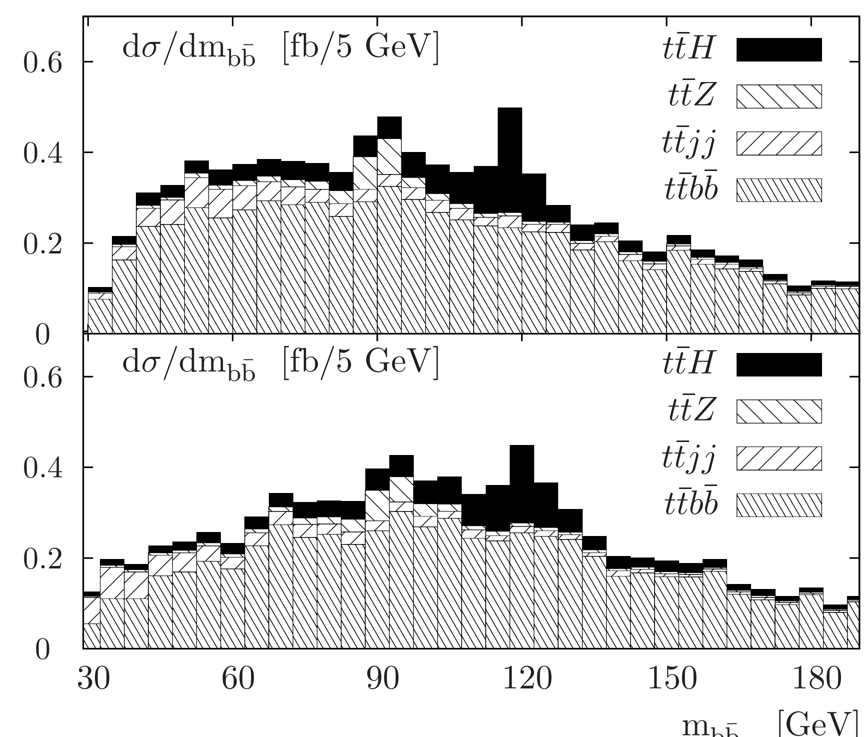
- final-state signature of quark and gluon production
- defined through a jet algorithm
  - infrared-safety: jets insensitive to collinear and soft radiation
  - cone algorithms
    - intuitive picture, numerically fast
    - splitting and merging nearby cones: infrared sensitivity
    - IR-safe formulation: SISCone (G. Salam, G. Soyez)
  - recombination algorithms (e.g. Durham k_T)
    - less intuitive, numerically slow
    - infrared safety ensured
    - cone-formed jets obtained by anti-k_T algorithm (M. Cacciari, G. Salam)
    - fast implementation: FastJet (M. Cacciari, G. Salam)

Jets as analysis tool

- Jet catchment area (M. Cacciari, G. Salam)
  - allows study of outside-jet activity (underlying event)
- Jet substructure
  - aim: reconstruction of boosted massive particles decaying into jets
  - all decay products inside one 'fat jet'
  - resolve decay products by jet substructure
    (J. Butterworth, A. Davison, M. Rubin, G. Salam)
  - reconstruction of $t\bar{t}H$ final states becomes feasible
    (T. Plehn, G. Salam, M. Spannowsky)

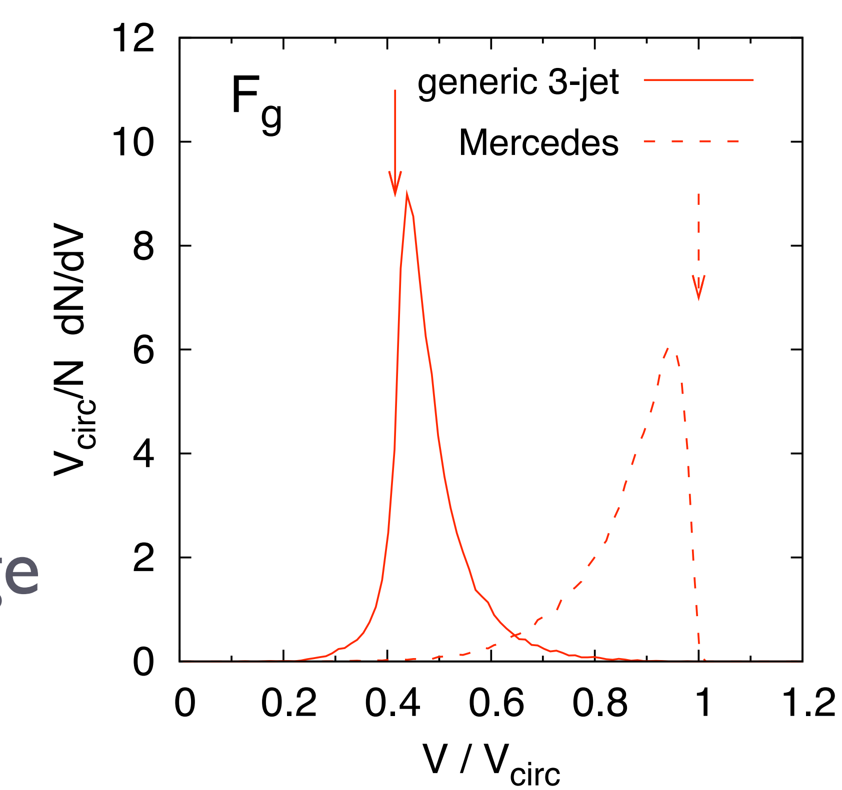






Event shapes

- characterize geometical properties of hadronic final state
  - e.g. thrust: momentum flow along event axis $T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|} \right)$
  - ideal case: global variables (depend on all hadrons in event)
- extensively measured at LEP for precision QCD studies
  - measurements of α_s
  - resummation effects
  - power corrections
- event shapes at hadron colliders
  (A. Banfi, G. Salam, G. Zanderighi)
  - defined using transverse momenta only
  - tools for model-independent searches
  - global variables difficult due to finite coverage
  - extensive classification of different variables



Multiparticle production at NLO

Multi-particle production

Multi-particle final states (with jets, leptons, photons) very frequent at colliders

- decay signatures of short-lived massive particles (e.g. top quarks)
- are signal and background for new particle searches
- want reliable predictions

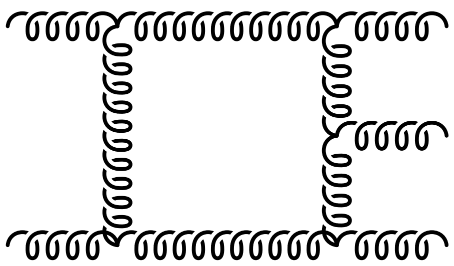
Why NLO?

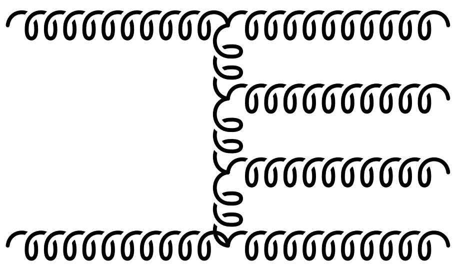
- reduce uncertainty of theory prediction
  - reliable normalization
  - reliable error estimate
- jet algorithm dependence
- effects of extra radiation

NLO calculations

Require two principal ingredients (here: pp → 3j)

- one-loop matrix elements
  - explicit infrared poles from loop integral
  - known for all 2 → 2 processes
  - known for many 2 → 3 processes
  - current frontier 2 → 4: major challenge
- tree-level matrix elements
  - implicit poles from soft/collinear emission





Combining virtual corrections and real emission

- extract process-independent implicit poles from real emission
  - residue subtraction (S. Frixione, Z. Kunszt, A. Signer)
  - dipole subtraction (S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi)
  - antenna subtraction (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover; A. Daleo, D. Maitre, TG)

NLO: multi-leg one-loop amplitudes

- Challenges of one-loop multileg-amplitudes
  - complexity: number of diagrams, number of scales
  - stability: linear dependence among external momenta
- General structure

$$\mathcal{A} = \sum_i d_i \text{ Box}_i + \sum_i c_i \text{ Triangle}_i + \sum_i b_i \text{ Bubble}_i + \sum_i a_i \text{ Tadpole}_i + R$$

- Enormous recent progress
  - tensor reduction and form factor decomposition
    (A. Denner, S. Dittmaier; T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, C. Schubert)
  - unitarity and multi-particle cuts to fix coefficients
    (Z. Bern, L. Dixon, D. Dunbar, D.A. Kosower; R. Britto, F. Cachazo, B. Feng; P. Mastrolia; D. Forde; S. Badger)
  - reduction at integrand level (G. Ossola, C. Papadopoulos, R. Pittau)
  - numerical D-dimensional unitarity (R.K. Ellis, W. Giele, Z. Kunszt, K. Melnikov)

Automating NLO calculations

Real radiation: based on LO event generators

- dipole subtraction
  - SHERPA (T. Gleisberg, F. Krauss)
  - MadDipole (R. Frederix, N. Greiner, TG)
  - TeVJet (M. Seymour, C. Tevlin)
  - AutoDipole (K. Hasegawa, S. Moch, P. Uwer)
  - Helac/Phegas (M. Czakon, C.G. Papadopoulos, M. Worek)
- residue subtraction
  - MadFKS (R. Frederix, S. Frixione, F. Maltoni, T. Stelzer)
- extensive libraries in existing NLO packages
  - MCFM (J. Campbell, R.K. Ellis)
  - NLOJET++ (Z. Nagy, Z. Trocsanyi)

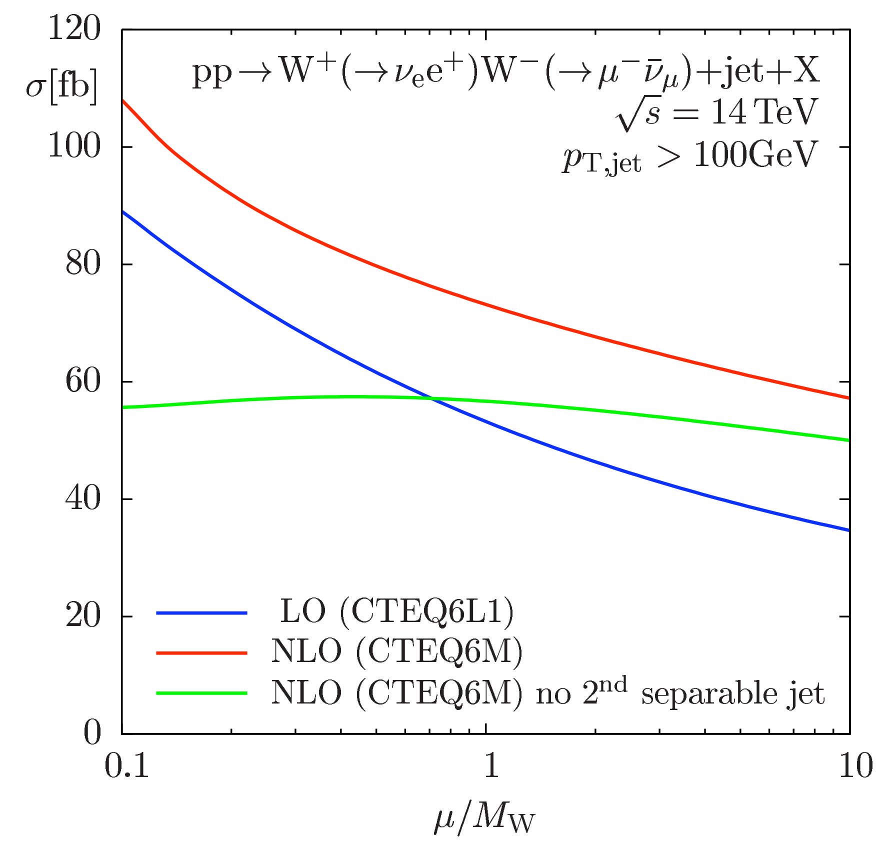
Automating NLO calculations

Virtual corrections: implementations

- semi-numerical form factor decomposition: GOLEM
  (T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, T. Reiter)
- unitarity and multi-particle cuts: BlackHat
  (C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D.A. Kosower, D. Maitre)
- reduction at integrand level: CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
- generalized D-dimensional unitarity: Rocket (W. Giele, G. Zanderighi)
- several more packages in progress (A. Lazopoulos; W. Giele, Z. Kunszt, J. Winter)

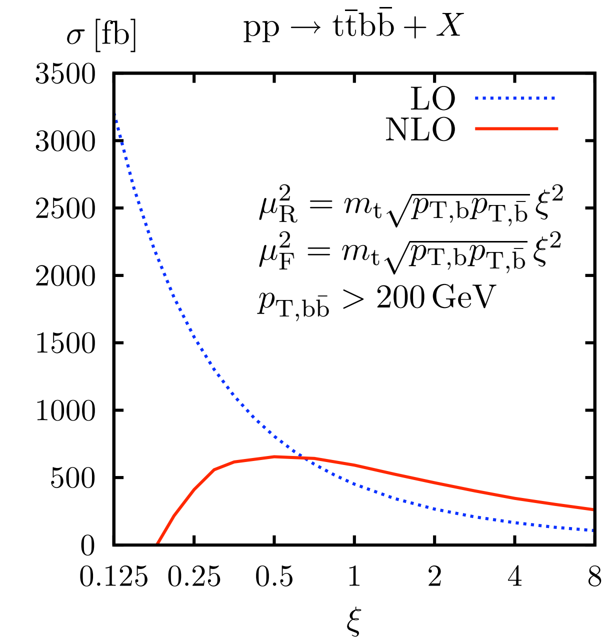
NLO: multi-leg processes

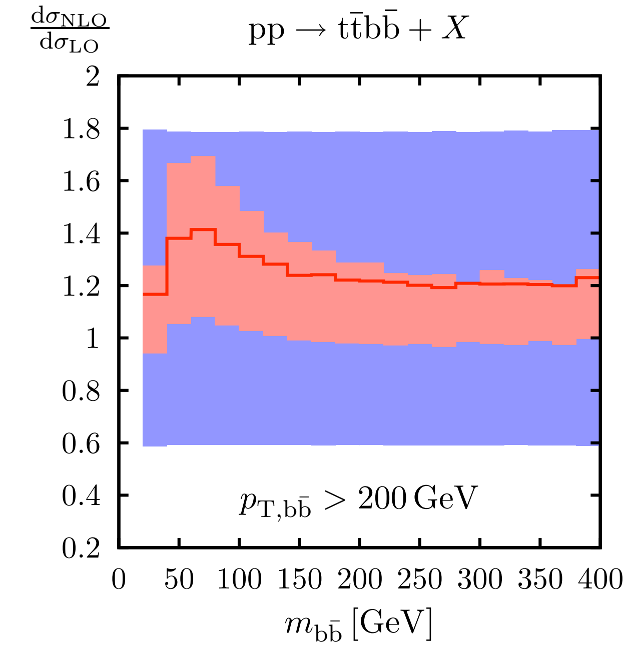
- recent results for 2 →3 processes
  - $pp \to VV + j$ (S. Dittmaier, S. Kallweit, P. Uwer; J. Campbell, R.K. Ellis, G. Zanderighi; T. Binoth, T. Gleisberg, S. Karg, N. Kauer, G. Sanguinetti)
  - $pp \to H + 2j$ (J. Campbell, R.K. Ellis, G. Zanderighi; M. Ciccolini, A. Denner, S. Dittmaier; S. Badger, E.W.N. Glover, P. Mastrolia, C. Williams)
  - $pp \to VVV$ (A. Lazopoulos, K. Melnikov, F. Petriello; G. Bozzi, F. Campanario, V. Hankele, D. Zeppenfeld; T. Binoth, G. Ossola, C. Papadopoulos, R. Pittau)
- vector boson fusion processes
  - VBFNLO (D. Zeppenfeld et al.)



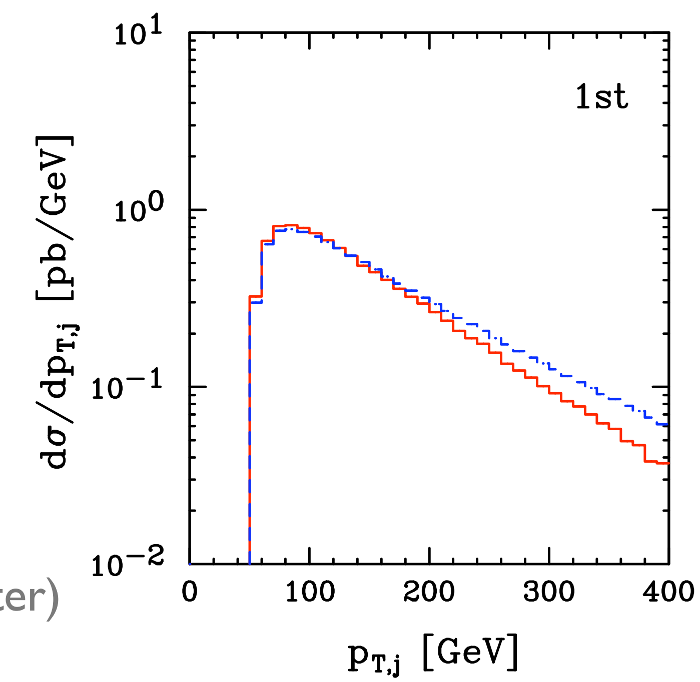
NLO multileg: $t\bar{t}$ + $b\bar{b}$ and $t\bar{t}$ + 2j

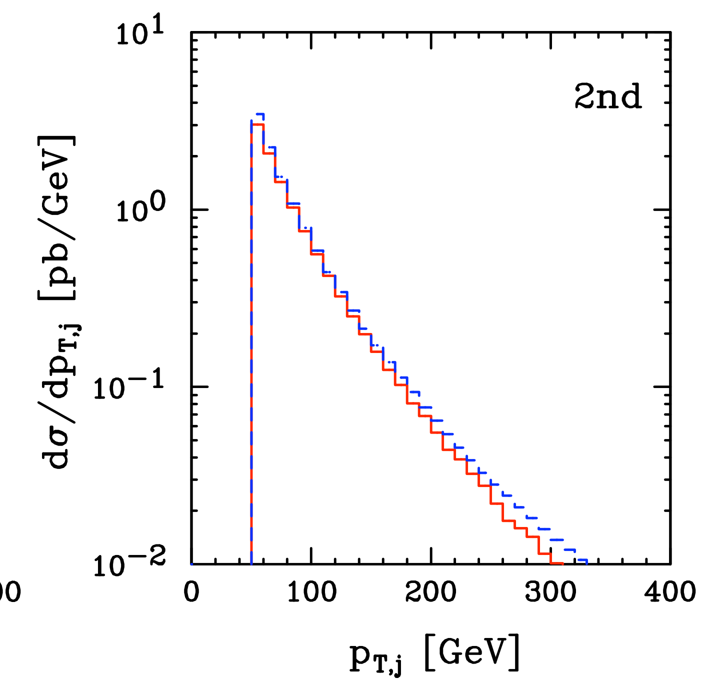
- $pp \longrightarrow t\bar{t}b\bar{b}$: 2 →4 including masses
  - using numerical tensor reduction
    (A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini)
  - using CutTools/Helac
    (G. Bevilacqua, M. Czakon, C. Papadopoulos, M. Worek)
  - NLO corrections affect shape
  - background to Higgs studies





- $pp \longrightarrow t\bar{t} + 2j$
  - using CutTools/Helac
    (G. Bevilacqua, M. Czakon, C. Papadopoulos, M. Worek)

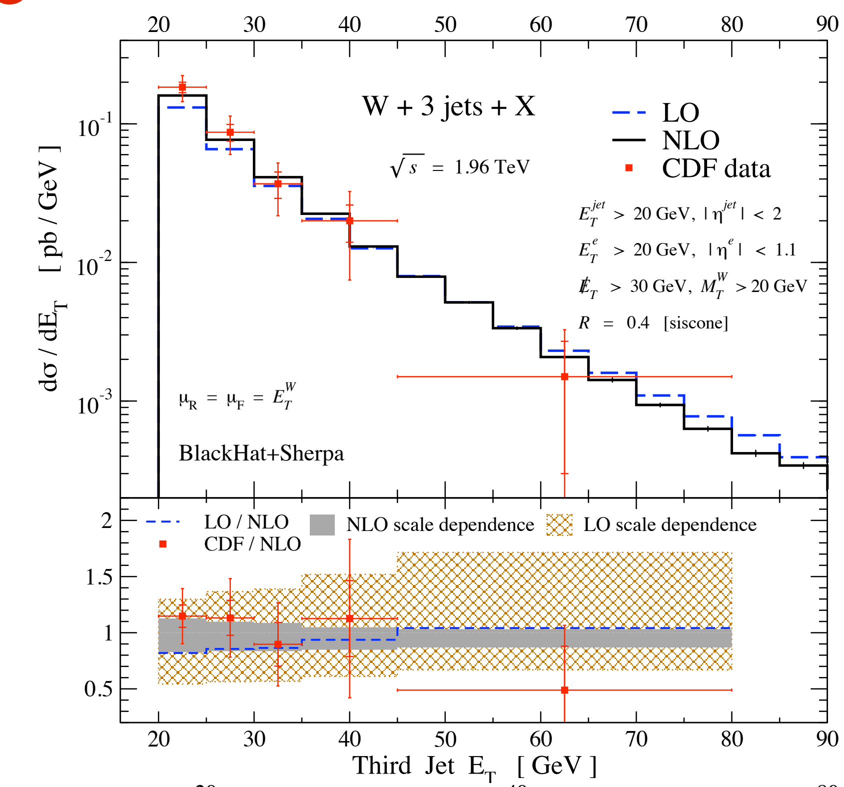


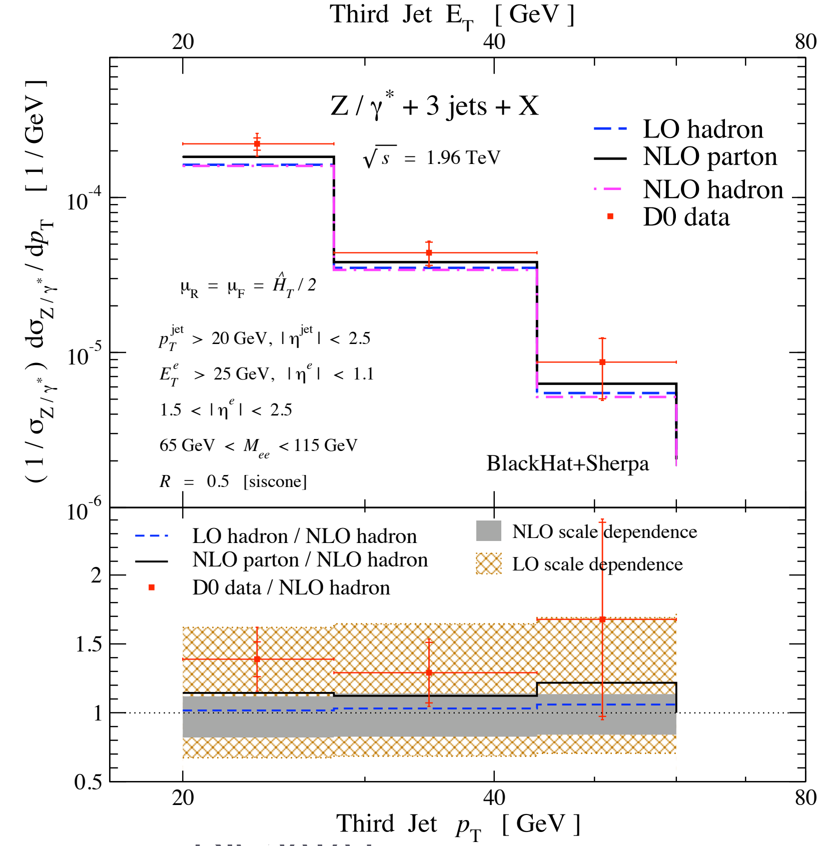


- in progress: $pp \longrightarrow b\bar{b}b\bar{b}$
  (T. Binoth, N. Greiner, A. Guffanti, J.P. Guillet, T. Reiter, J. Reuter)

NLO multileg: $W^\pm + 3j$, $Z^0 + 3j$

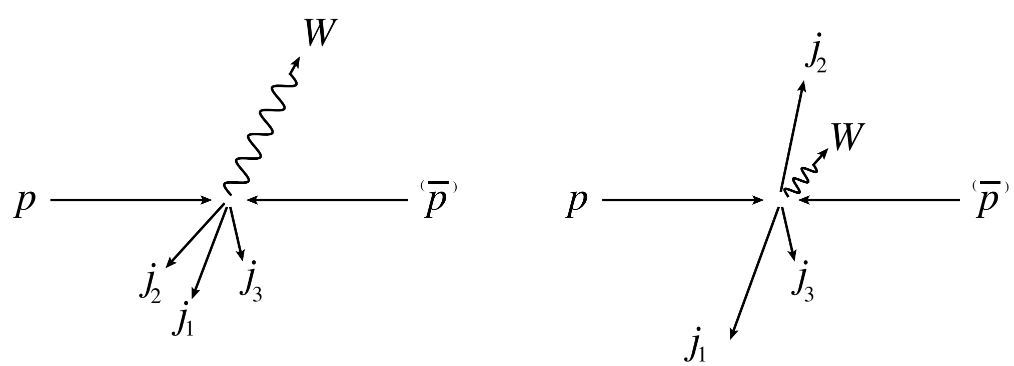
- Calculations of $W^\pm$ + 3j
  - Blackhat + Sherpa (C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre)
  - Rocket (R.K. Ellis, K. Melnikov, G. Zanderighi)
- excellent description of Tevatron data
  - moderate corrections
  - precise prediction
  - rich phenomenology
    - scale setting
    - final state correlations between lepton and jets
- Calculation of Z^0 + 3j
  - Blackhat + Sherpa

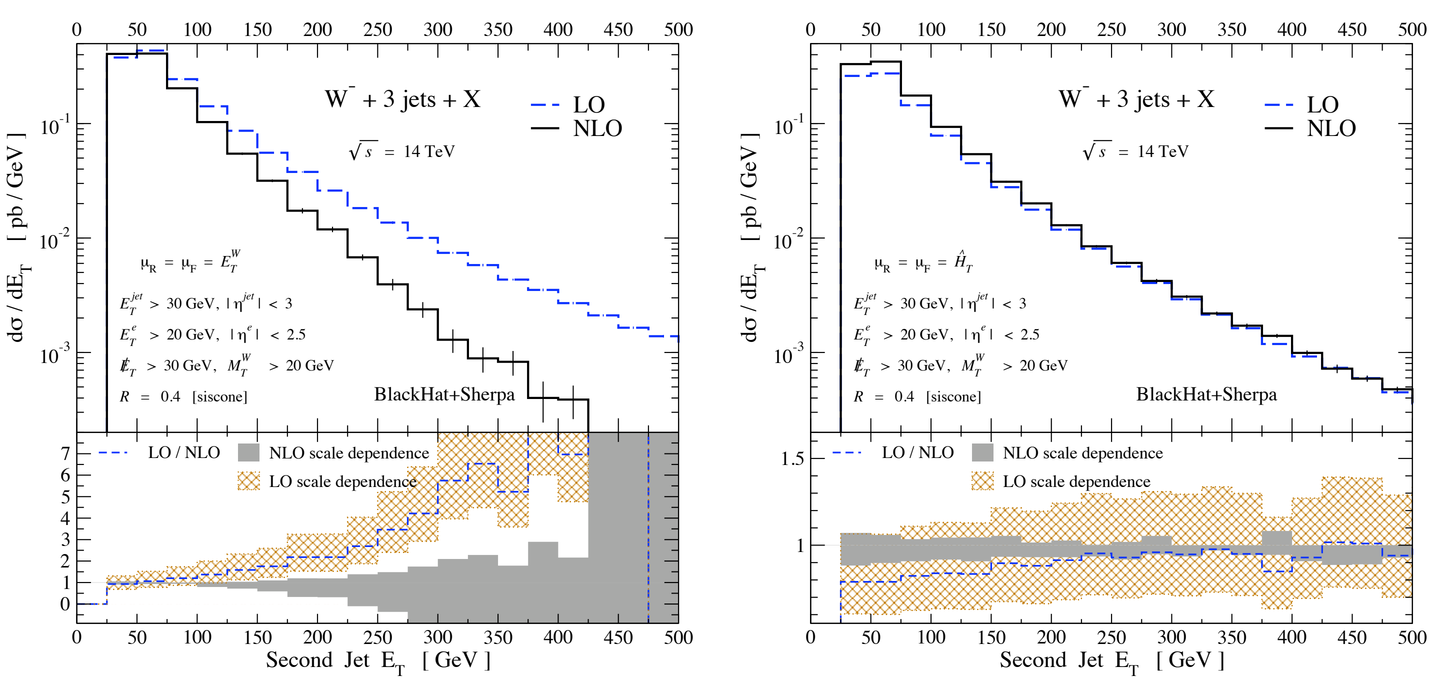




NLO multileg: $W^{\pm}$ + 3j

- Scale choice in multileg processes (Blackhat+Sherpa)
  - $\mu = E_{TW}$ is inappropriate: poor perturbative convergence
  - $\mu = H_T$ accounts for full hard final state dynamics





Allows optimization of LO studies

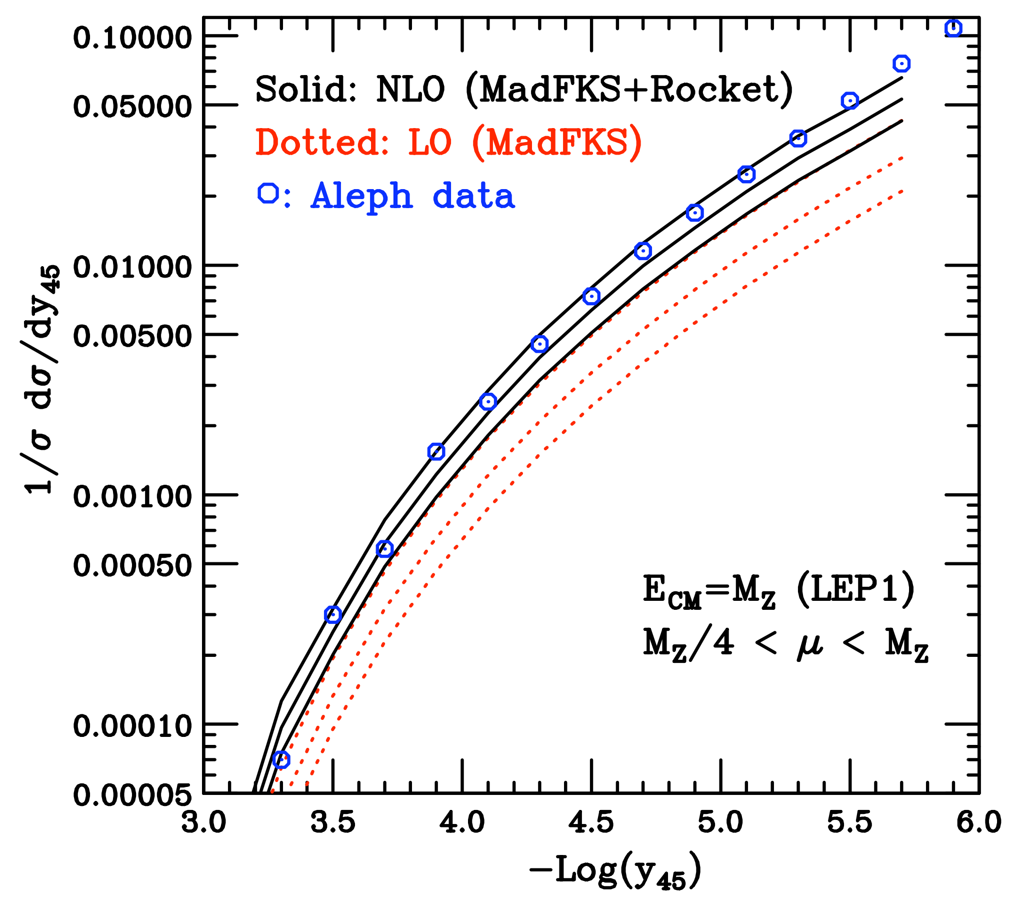
- local choice at each vertex
- important for background studies
- e.g. pp → W(→τν) + 3j for SUSY

(K. Melnikov, G. Zanderighi)

NLO multileg: $e^+e^- \to 5$ jets

- one-loop amplitudes are crossing of hadron-collider V+3j
  - computed using Rocket package
- real radiation from MadGraph
  - subtraction using MadFKS
- improved scale dependence
- better agreement with data
- possibly new extraction of α_s

R. Frederix, S. Frixione, K. Melnikov, H. Stenzel, G. Zanderighi



Precision observables at NNLO

Precision observables

Processes measured to few per cent accuracy

- $e^+e^- \to 3$ jets
- 2+1 jet production in deep inelastic scattering
- jet production at hadron colliders
- vector boson production at hadron colliders
- vector boson plus jet production at hadron colliders
- top quark pair production at hadron colliders

Processes with potentially large perturbative corrections

- Higgs production at hadron colliders
- vector boson pairs at hadron colliders

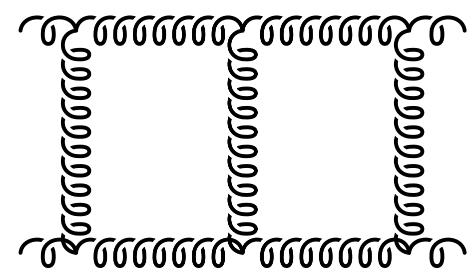
Require NNLO corrections for

- meaningful interpretation of experimental data
- precise determination of fundamental parameters

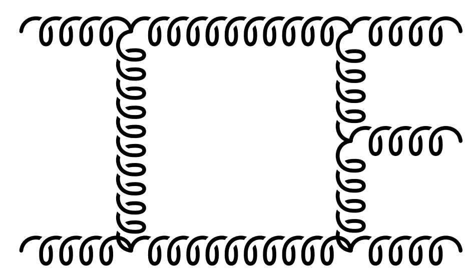
NNLO calculations

Require three principal ingredients (here: pp → 2j)

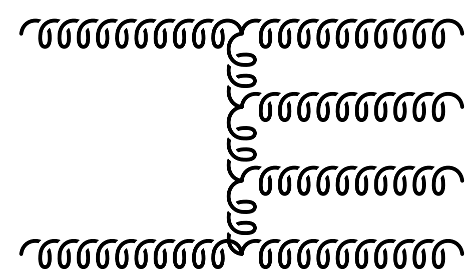
- two-loop matrix elements
  - explicit infrared poles from loop integral
  - known for all massless 2 → 2 processes



- one-loop matrix elements
  - explicit infrared poles from loop integral
  - and implicit poles from soft/collinear emission
  - usually known from NLO calculations



- tree-level matrix elements
  - implicit poles from two partons unresolved
  - known from LO calculations



need method to extract implicit poles

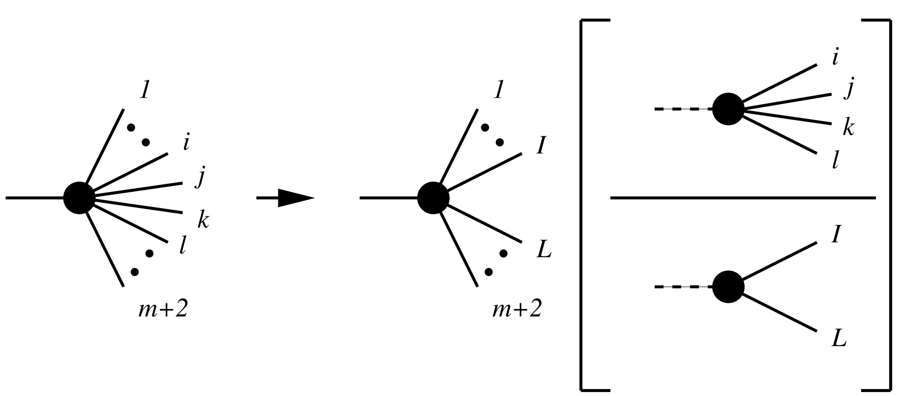
NNLO calculations: real radiation

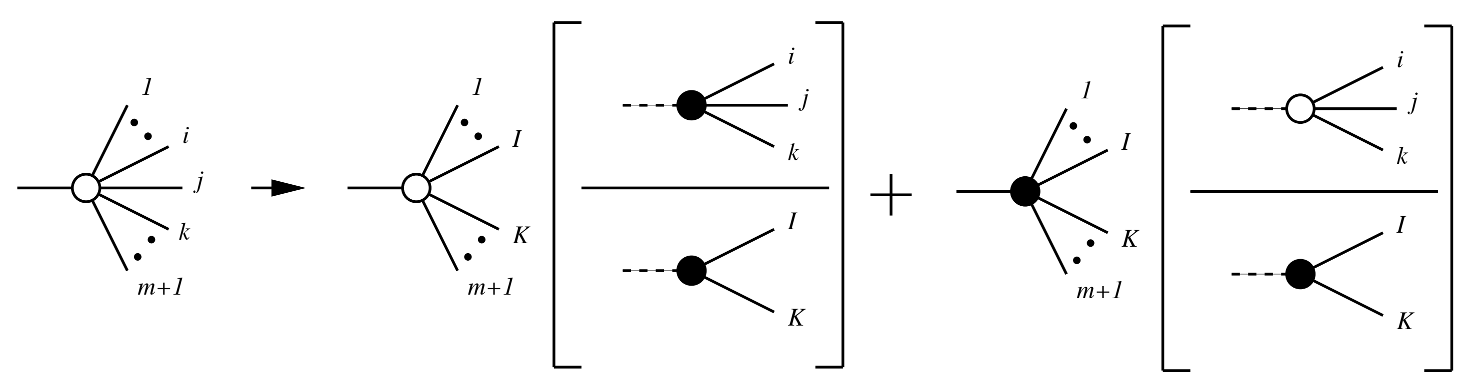
Technical challenge: real radiation for arbitrary final state cuts

- two unresolved partons at tree level, one parton at one loop
- infrared limits are process-independent

Solutions

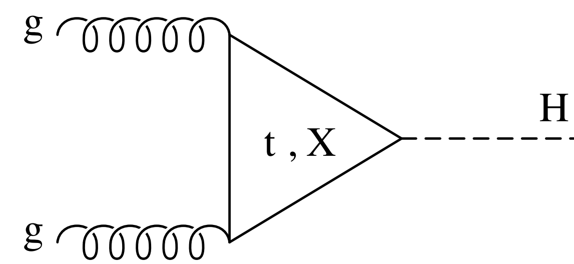
- sector decomposition: expansion in distributions, numerical integration
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
- subtraction: approximation in all unresolved limits, analytical integration
  - several well-established methods at NLO
  - NNLO for specific hadron collider processes: q_T subtraction (S. Catani, M. Grazzini)
  - NNLO for e^+e^- processes : antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)



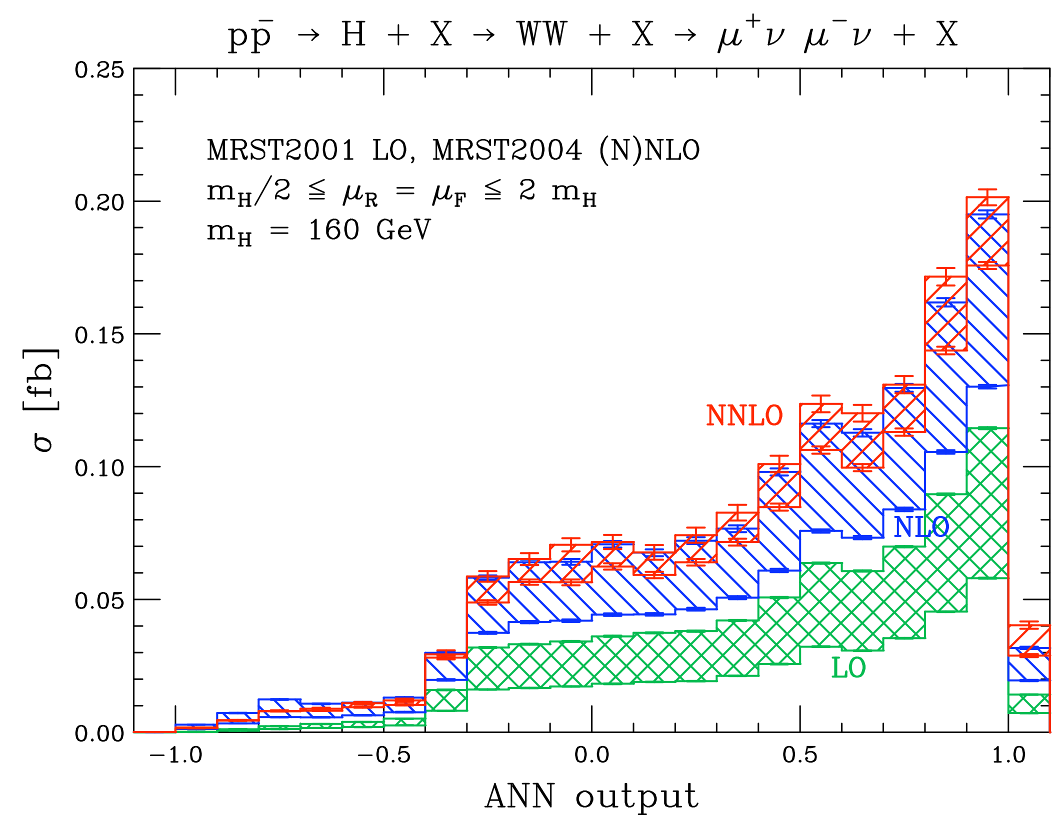


Higgs boson production at NNLO

Dominant production process: gluon fusion



- exclusive calculations to NNLO, including H decay to γγ or VV
  - using sector decompostion
    (C. Anastasiou, K. Melnikov, F. Petriello)
  - using q_T-subtraction (S. Catani, M. Grazzini)
- Higgs at Tevatron: H → WW → lν lν
  - all distributions to NNLO
    (C. Anastasiou, G. Dissertori, M. Grazzini, F. Stöckli, B. Webber)
  - cuts on jet activity
  - neural-network output to NNLO

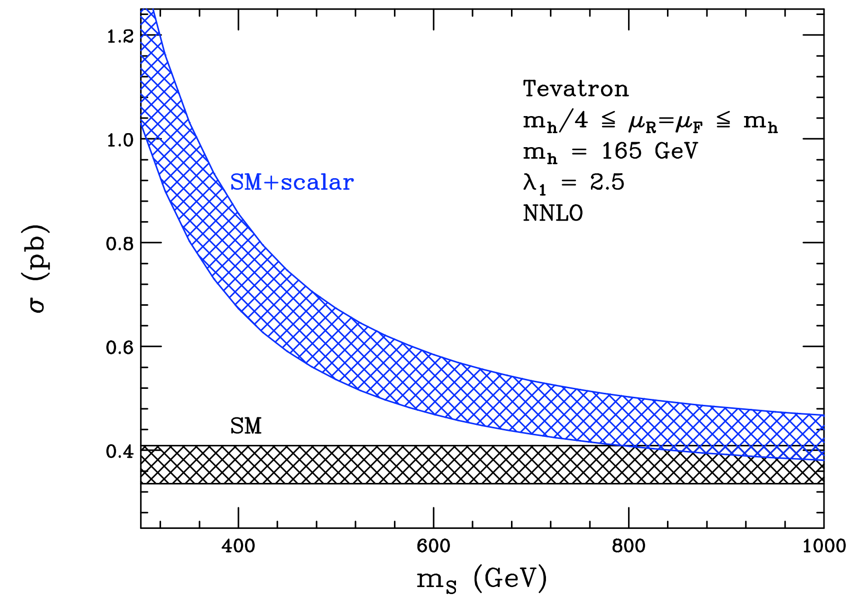


- precise prediction requires mixed QCD/EW corrections
  (C. Anastasiou, R. Boughezal, F. Petriello)

Higgs boson production

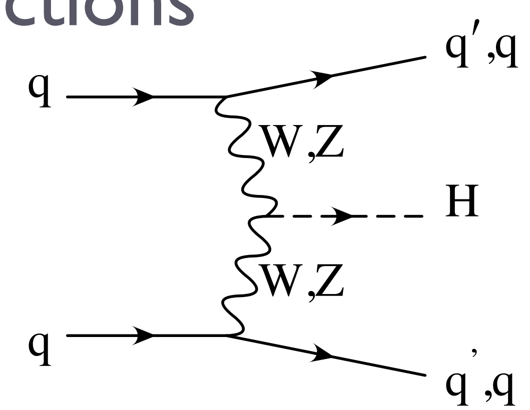
Gluon fusion cross section: sensitive to new physics effects

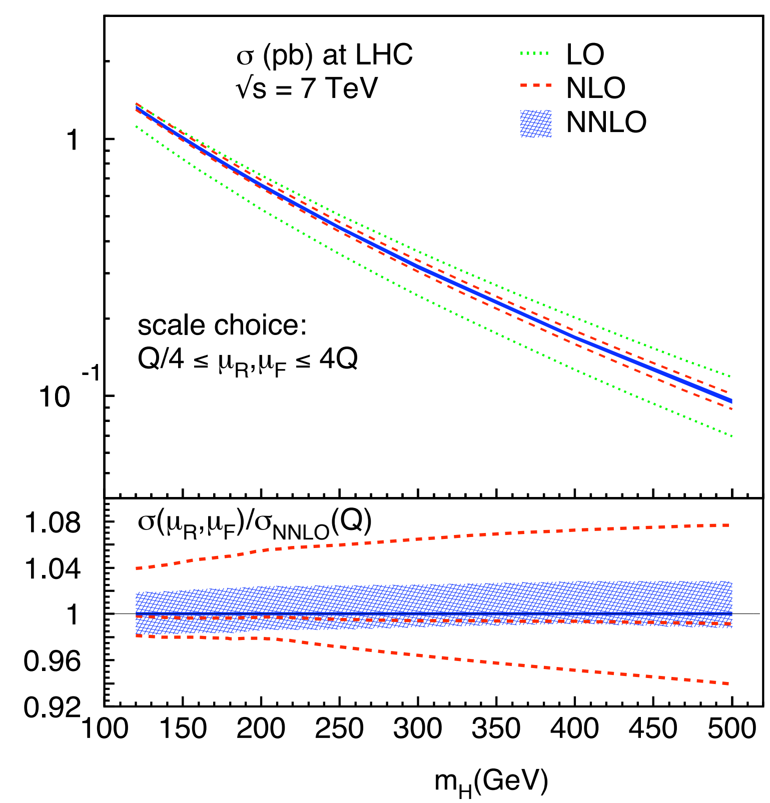
- squark/gluino loops
  (C. Anastasiou, S. Beerli, A. Daleo; M. Mühlleitner, H. Rzehak, M. Spira)
- extra heavy quark families
  (C. Anastasiou, R. Boughezal, E. Furlan)
- colour-octet scalars (R. Boughezal, F. Petriello)



Higgs production in vector boson fusion

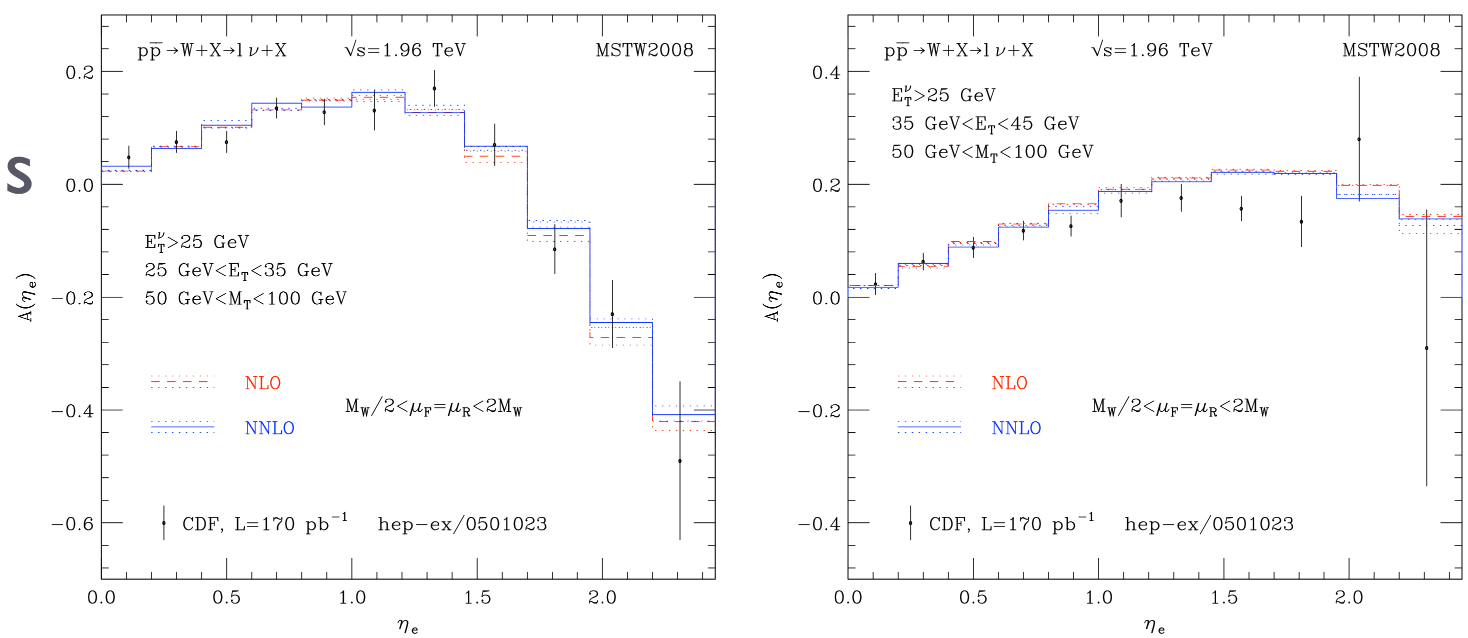
- factorizable NNLO corrections
  (P. Bolzoni, F. Maltoni, S. Moch, M. Zaro)
- SUSY loop corrections
  (M. Rauch, W. Hollik, T. Plehn, H. Rzehak)





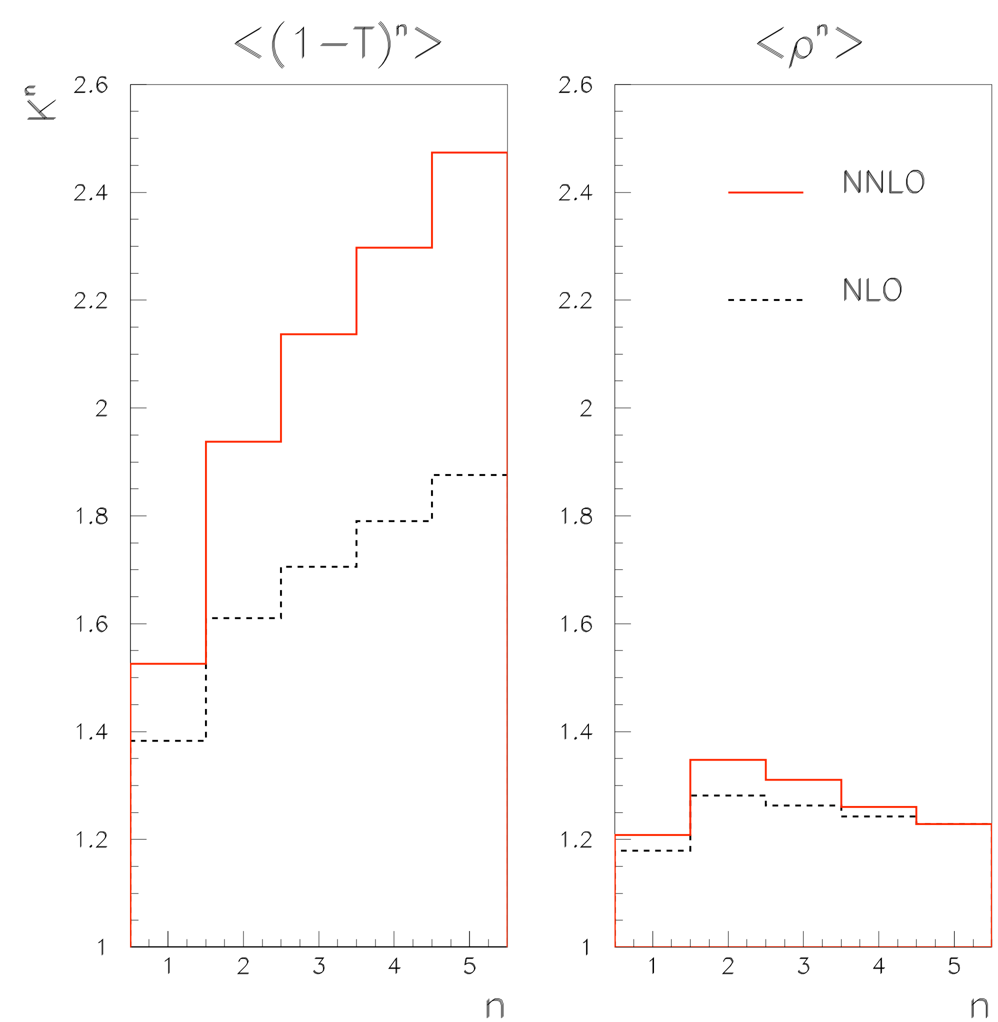
Vector boson production at NNLO

- Inclusive NNLO coefficient functions known for long time
- Recent results: fully exclusive calculations
  - parton-level event generator
    - using sector decomposition (K. Melnikov, F. Pertriello)
    - using q_T subtraction (S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini)
  - including vector boson decay
  - allowing arbitrary final-state cuts
- Application: lepton charge asymmetry (S. Catani, G. Ferrera, M. Grazzini)
  - small NNLO corrections
  - determine quark distributions



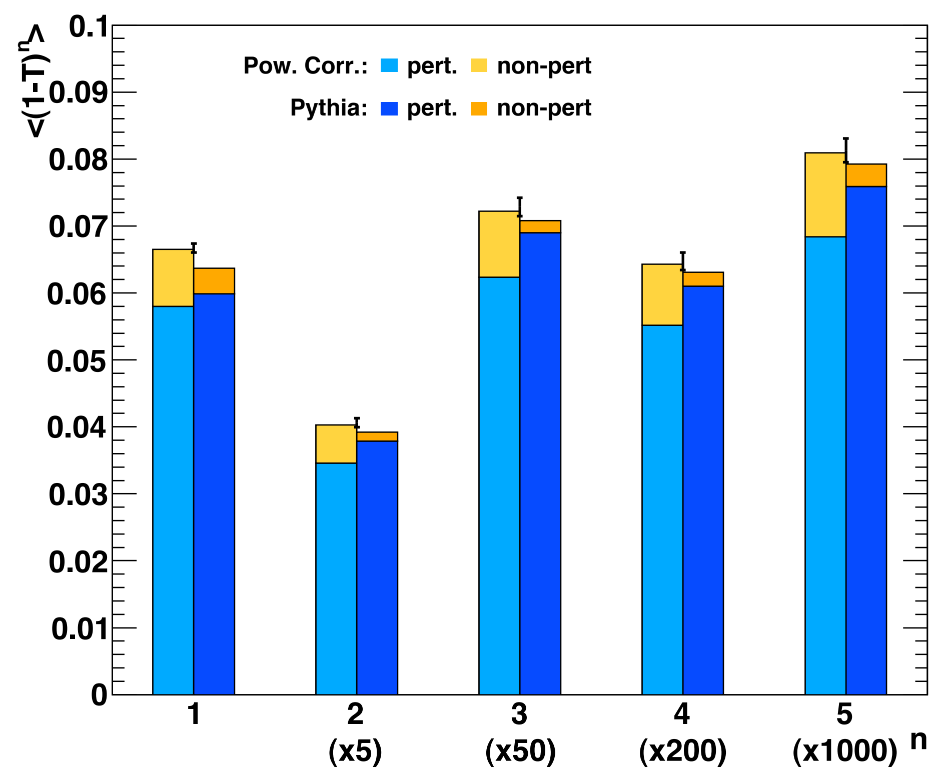
Jet production at NNLO: e^+e^- collisions

- Two calculations of NNLO corrections to $e^+e^- \to$ 3 jets
  - using antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
  - as parton-level event generator
  - allow evaluation of event shape distributions/moments and jet rates
- NNLO corrections differ substantially between observables
  - e.g. moments of thrust/heavy jet mass
  - improved consistency among shapes
- NNLO scale uncertainty
  - few per cent for most shape variables
  - one per cent for three-jet rate
- use to extract α_s from LEP data



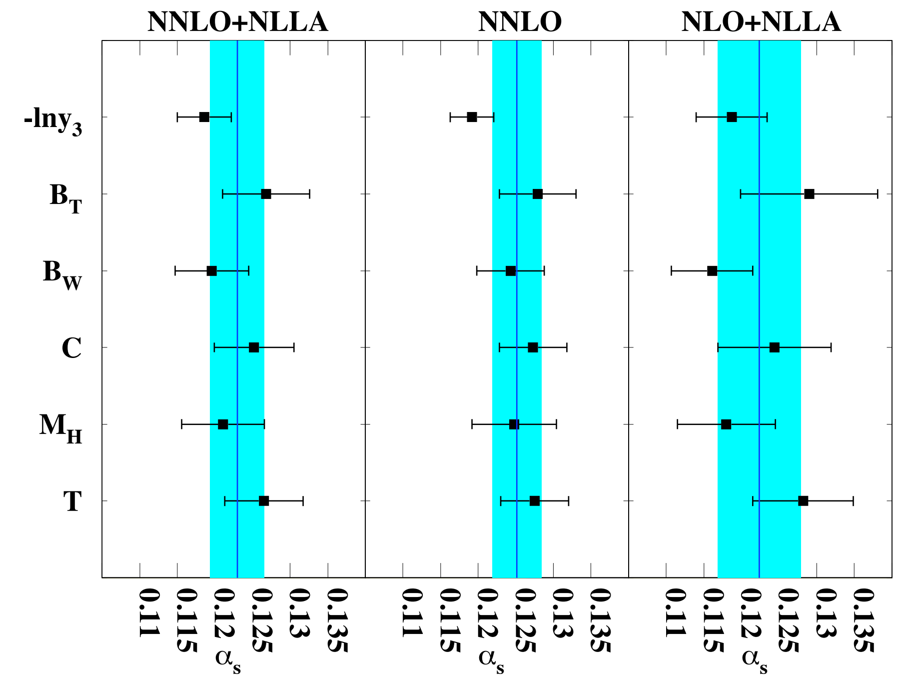
Jet production at NNLO: e^+e^- collisions

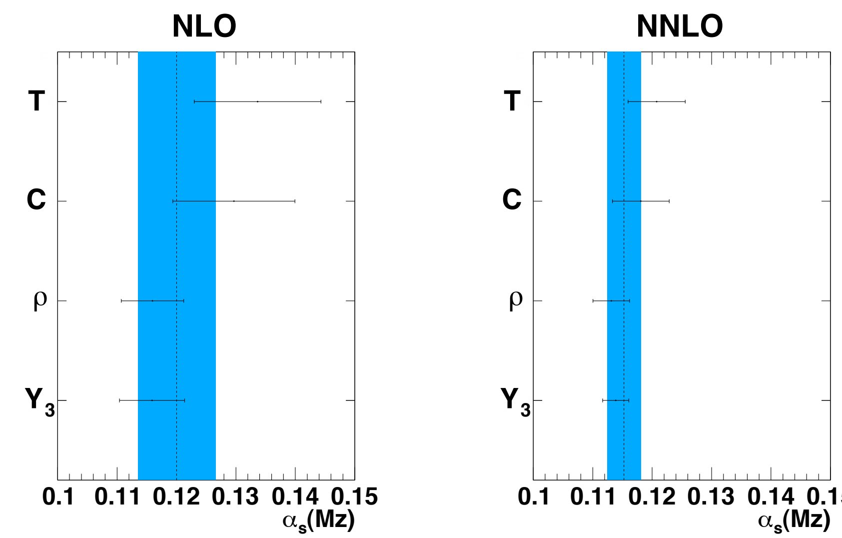
- Event shapes: matching NNLO onto resummation in 2-jet limit
  - NNLO+NLLA for all shapes (G. Luisoni, H. Stenzel, TG)
  - NNLO+N^3LLA for thrust (T. Becher, M. Schwartz)
- hadronization corrections
  - from LL parton-shower event generators
  - from renormalon-based dispersive model (Y. Dokshitzer, G. Marchesini, B. Webber)
  - extend dispersive model to NNLO (M. Jaquier, G. Luisoni, TG)
  - power corrections differ substantially

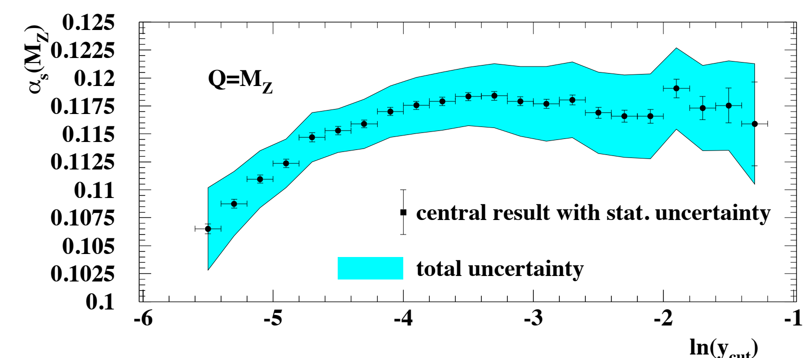


α_s (M_Z) from NNLO jet observables

- event shapes at NNLO+NLLA
  - JADE (S. Bethke, S. Kluth, C. Pahl, J. Schieck)
    - 0.1172 ± 0.0006 (st) ± 0.0040 (sy) ± 0.0030 (th)
  - ALEPH (G. Dissertori, A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, G. Luisoni, H. Stenzel, TG)
    - 0.1224 ± 0.0009 ± 0.0015 ± 0.0035
- thrust at NNLO+N^{3}LLA (T. Becher, M. Schwartz)
  - 0.1172 ± 0.0010 ± 0.0014 ± 0.0012
- thrust: NNLO+dispersive model (R. Davison, B. Webber)
  - 0.1164 ± 0.0027
- moments: NNLO+dispersive model
  - JADE/OPAL (M. Jaquier, G. Luisoni, TG)
    - 0.1153 ± 0.0017 (exp) ± 0.0023 (th)
- three-jet rate at NNLO
  - ALEPH (G. Dissertori et al.)
    - 0.1175 ± 0.0020 (exp) ± 0.0015 (th)

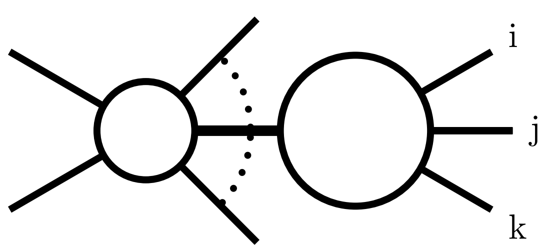




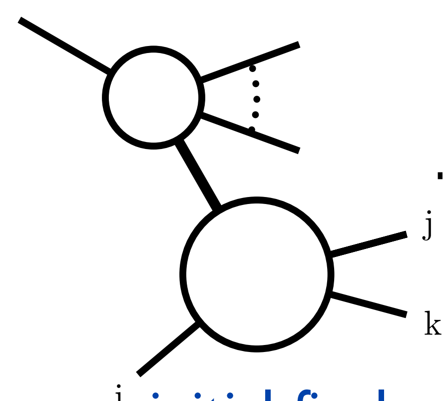


NNLO jet cross sections at hadron colliders

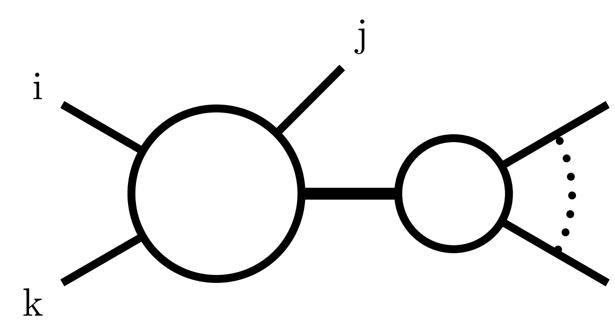
- two-loop matrix elements known for:
  - two-jet production
    (C. Anastasiou, E.W.N. Glover, C. Oleari, M.E. Tejeda-Yeomans; Z. Bern, A. De Freitas, L.J. Dixon)
  - vector-boson-plus-jet production (E. Remiddi, TG)
  - (2+1) jet production in DIS (E.W.N. Glover, TG)
- antenna subtraction formalism at NNLO: radiators in initial state
  - final-final antennae known
  - initial-final antennae derived recently: sufficient for (2+1) jets in DIS
    (A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG)
  - initial-initial antennae in progress (R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann)



final-final



initial-final



initial-initial

Towards top quark pairs at NNLO

- two-loop matrix elements: $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - known in large-energy limit (M. Czakon, A. Mitov, S. Moch)
  - quark-initiated process: known numerically (M. Czakon)
  - quark-initiated process: fermionic contributions and leading-colour terms confirmed analytically (R. Bonciani, A. Ferroglia, D. Maitre, C. Studerus, TG)
  - one-loop self-interference known (J. Körner, Z. Merebashvili, M. Rogal; C. Anastasiou, M. Aybat)
- one-loop and tree-level matrix elements known from NLO corrections to $pp \to t\bar{t} + j$ (S. Dittmaier, P. Uwer, S. Weinzierl)
- requires: method to handle NNLO real radiation
  - including hadronic initial states and massive particles
  - developments towards massive antenna subtraction (A. Gehrmann-De Ridder, M. Ritzmann)

Parton distributions

Parton distributions

Determine parton distributions from global fit to collider data

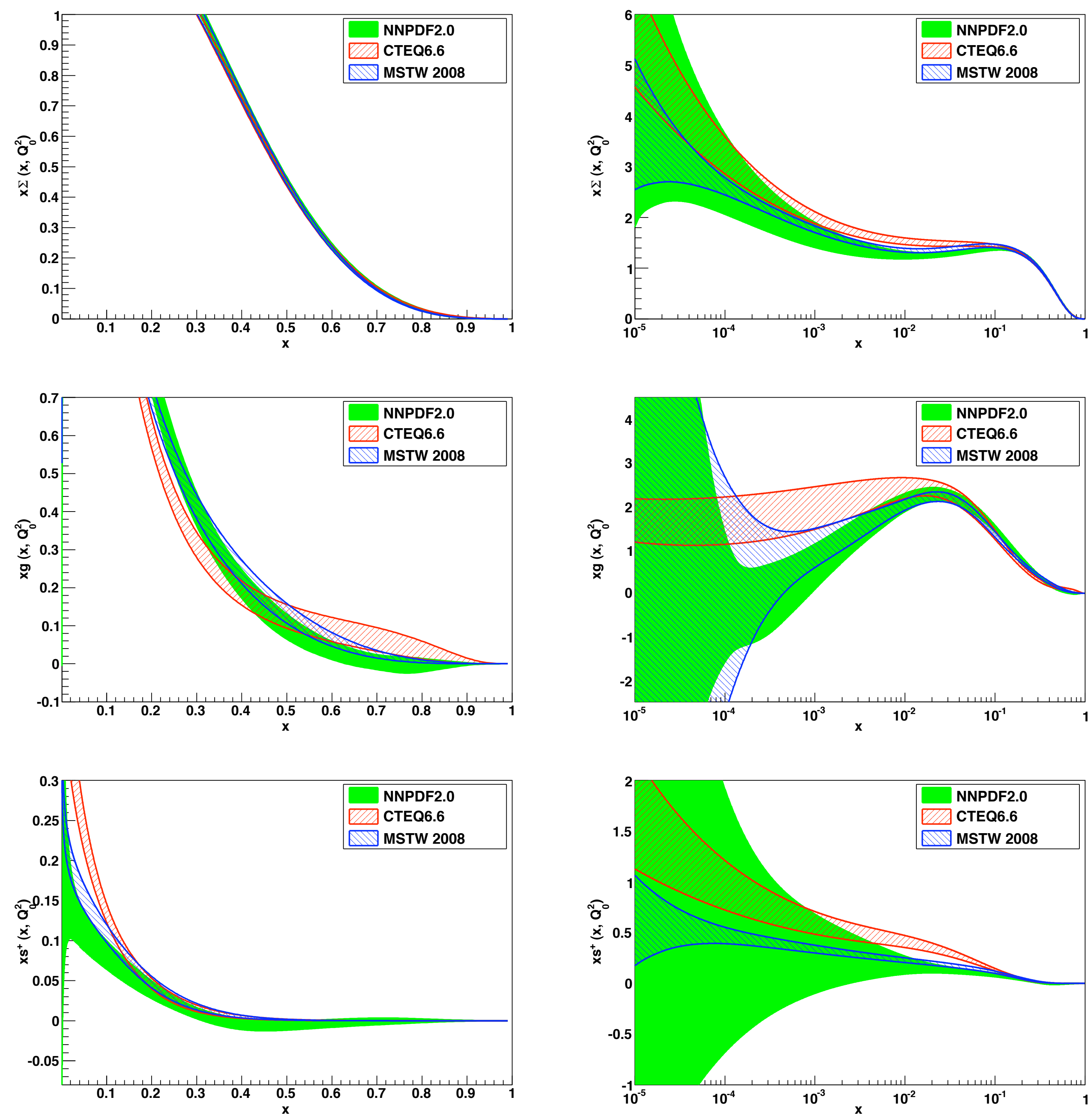
- data sets
  - DIS (fixed target, HERA)
  - heavy quark production in DIS (HERA)
  - Drell-Yan (fixed target, Tevatron)
  - vector boson production (Tevatron)
  - jet production (HERA, Tevatron)
- at LO, NLO, NNLO
- require DGLAP splitting functions
  - known to NNLO (S. Moch, J. Vermaseren, A. Vogt)
- require hard coefficient functions
  - known to NNLO for DIS, Drell-Yan, heavy quarks in DIS (W.L. van Neerven et al.; C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello; I. Bierenbaum, J. Blümlein, S. Klein)
  - known to NLO for jet production
- must incorporate experimental and theoretical errors

Parton distributions: global fits

- MSTW: LO/NLO/NNLO (A.D. Martin, W.J. Stirling, R. Thorne, G. Watt)
  - error propagation through eigenvectors of covariance matrix
  - allow systematic comparison of different orders
- CTEQ: LO/NLO (J. Pumplin, J. Huston, H.L. Lai, P. Nadolsky, W.K. Tung, C.P. Yuan)
  - special focus on Tevatron jet data
  - also provide effective LO* distributions for event generators
- JR: NNLO (P. Jiminez-Delgado, E. Reya)
  - based on radiative parton model (valence-like partons at low scales)
- NNPDF: NLO (R. Ball, L. DelDebbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali)
  - fully flexible initial distributions through neural network parametization
  - error treatment through multiple fits to data replicas
- ABKM: NNLO (S. Alekhin, J. Blümlein, S. Klein, S. Moch)
  - extensive comparison of different heavy quark prescriptions

Parton distributions: global fits

- Comparison of recent fits
  - quark distributions determined precisely at large x
  - gluon distribution uncertain within 10% at large x, differences between fits
  - large errors for $x < 10^{-3}$
  - consistency within errors
  - tendency towards low α_s, e.g. α_s=0.1135±0.0014 (ABKM)



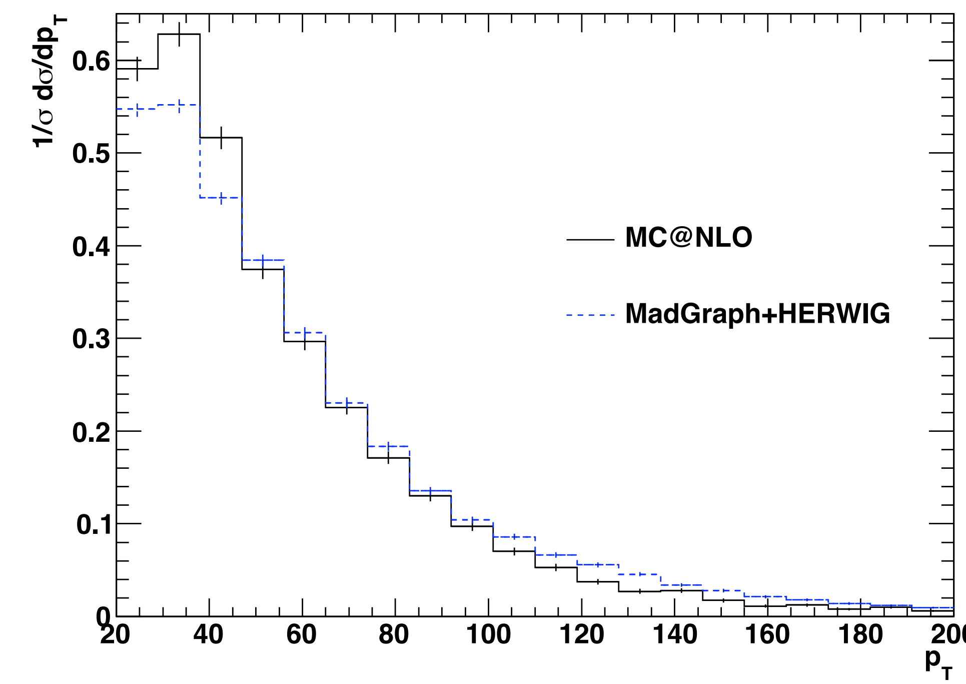
Infrared Structure and Resummation

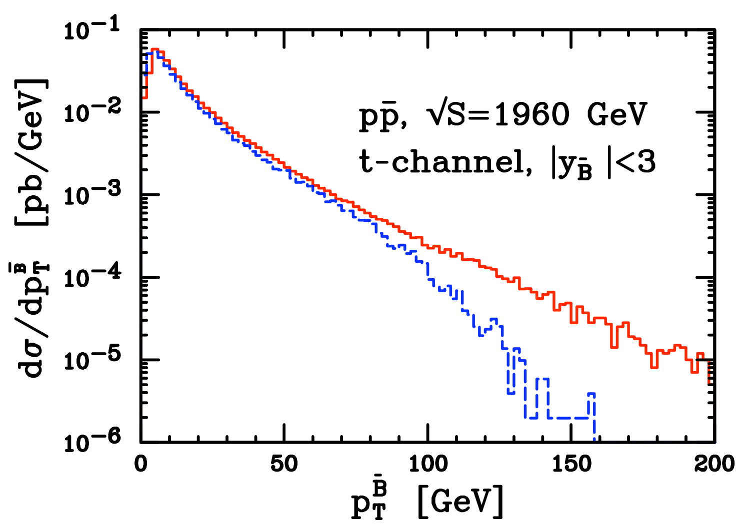
Resummation of large logarithms

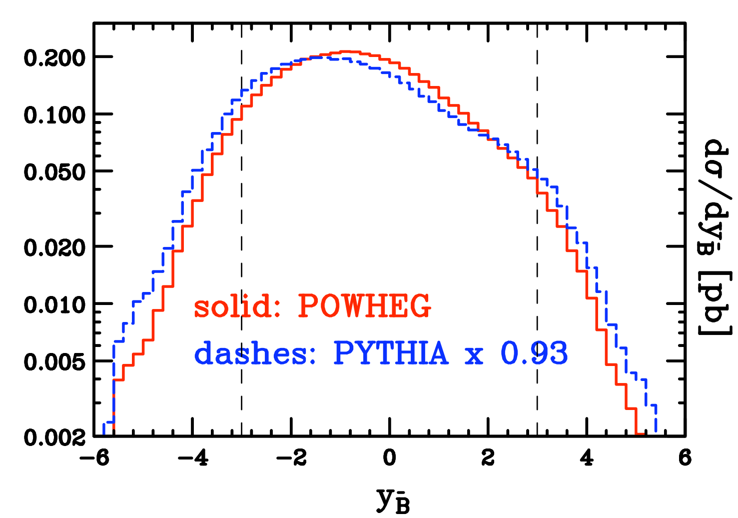
- QCD perturbative expansion in α_s
  - reliable, if process is defined by single hard scale
  - problematic, if several largely different scales contribute
    - encounter large logarithms at each perturbative order (e.g. $\log^n (p_T/M_W)$)
  - problematic, if specific restrictions apply to final state
    - phase space for soft radiation is limited (e.g. $\log^n (1-x)$ in DIS, Drell-Yan)
  - problematic if final state multiplicity is unrestricted
    - large corrections in high energy limit (e.g. $\log^n x$ in DIS: BFKL)
- need for rearranging perturbative series
  - resummation of large logarithmic corrections to all orders
- leading logarithmic resummation
  - parton shower based event generators (HERWIG, PYTHIA, SHERPA,...)

Resummation and fixed order

- Combining NLO and parton shower generators
  - MC@NLO approach (B. Webber, S. Frixione)
    - $W^{\pm}t$ production, including interference effects (C. White, S. Frixione, E. Laenen, F. Maltoni)
    - $H^{\pm}t$ production (C. Weydert, S. Frixione, M. Herquet, M. Klasen, E. Laenen, T. Plehn, G. Stavenga, C. White)
  - POWHEG approach (S. Frixione, P. Nason, C. Oleari)
    - single top production (S. Alioli, P. Nason, C. Oleari, E. Re)
    - Higgs in vector boson fusion (P. Nason, C. Oleari)
    - general framework: POWHEG box







Understanding infrared structure

Infrared poles in loop amplitudes ↔ logarithms in real emission

- predict IR poles at two loop from resummation
  (S.Catani; G. Sterman, M. Tejeda-Yeomans)
- predict large-x resummation coefficients for DIS, DY, Higgs production from IR poles of three-loop from factors (S. Moch, J. Vermaseren, A. Vogt)
- IR poles in QCD ↔ UV poles in soft-collinear effective theory (SCET)
  - pole structure from multiplicative renormalization in SCET (T. Becher, M. Neubert)

$$|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = \lim_{\epsilon\to 0} \boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)\,|\mathcal{M}_n(\epsilon,\{\underline{p}\})\rangle$$

  - all-order conjecture: IR poles of massless multi-loop multi-leg determined by cusp anomalous dimension and collinear anomalous dimensions
    (T. Becher, M. Neubert; L. Dixon, E. Gardi, L. Magnea)

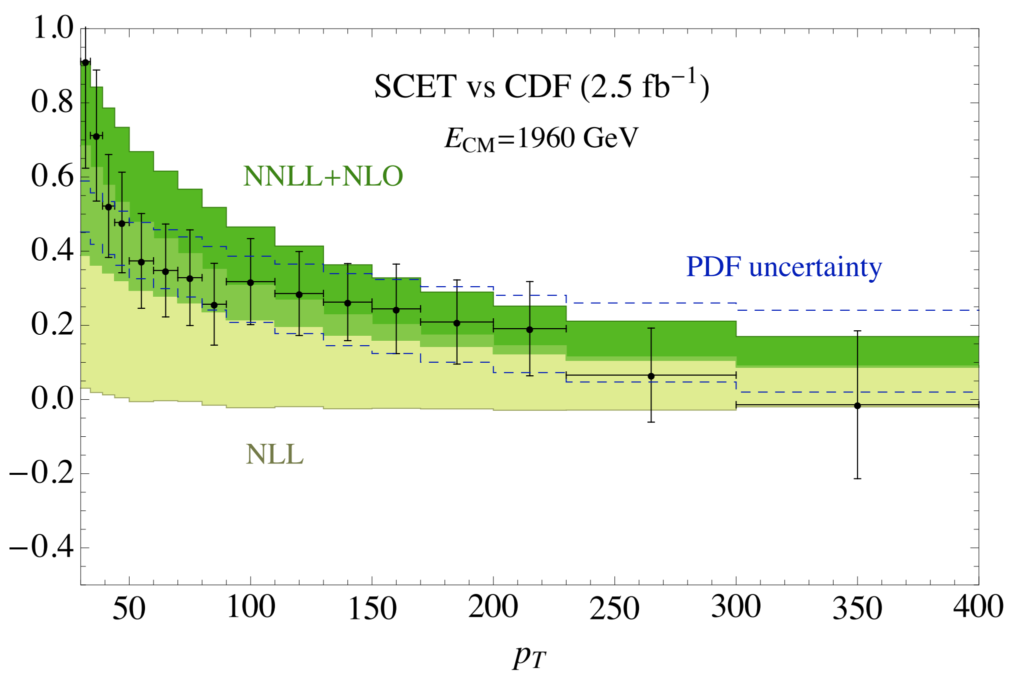
$$\boldsymbol{Z}(\epsilon,\{\underline{p}\},\mu) = \mathbf{P}\exp\left[\int_\mu^\infty \frac{\mathrm{d}\mu'}{\mu'}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu')\right], \quad \boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

Resummation in SCET

General structure of resummation (J. Collins, G. Sterman, D. Soper)

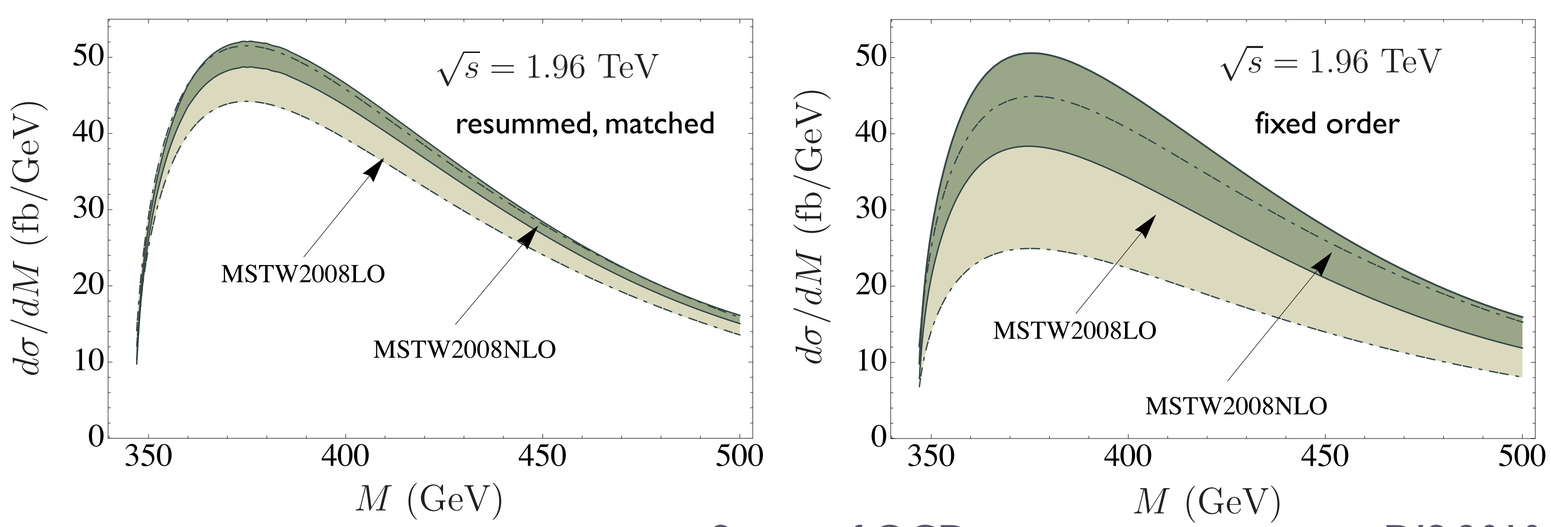
σ = (hard) × (soft) × (jet functions - final state collinear) × (parton distributions)

- only hard coefficient is process dependent
- SCET identifies each element with an operator or a non-local function (A. Idilbi, X. Ji, F. Yuan; T. Becher, M. Neubert, B. Pecjak)
  - each element has natural scale (hard, collinear, soft)
  - compute anomalous dimensions
  - resum large logarithms by solving evolution equations
- applications, matched onto fixed order
  - thrust in e^+e^- (T. Becher, M. Schwartz)
  - inclusive Drell-Yan and Higgs production (A. Idilbi, X. Ji, F. Yuan; V. Ahrens, T. Becher, M. Neubert, L.L. Yang)
  - inclusive photons (T. Becher, M. Schwartz)
- open issues
  - jet production, radiation off incoming partons



Top quark pair production: resummation

- Soft-gluon resummation: previously NLO+NLL
- infrared structure more involved with massive quarks
  - multi-particle correlations appear (A. Ferroglia, M. Neubert, B. Pecjak, L.L. Yang)
  - can predict IR poles to two-loop order (M. Beneke, M. Czakon, P. Falgari, A. Mitov)
- recent progress: NLO+NNLL
  - dominant contributions derived previously (S. Moch, P. Uwer; N. Kidonakis)
  - full corrections obtained from massive soft anomalous dimensions (M. Czakon, A. Mitov, G. Sterman)
  - and using SCET (V. Ahrens, A. Ferroglia, M. Neubert, B. Pecjak, L.L. Yang)



Multi-loop results

Multi-loop: motivation

Need corrections beyond NNLO to:

- analyze very precisely measured quantites (e.g. sum rules)
- describe processes with slowly converging perturbative description (e.g. quarkonia, Higgs production)
- understand infrared structure of QCD at high orders
- extract resummation coefficients

Mutli-loop results

Loop amplitudes: Frontier of complexity

- One loop: 2 → n, limit: numerical stability, evaluation time (recently accomplished: various 2 → 4 processes)
- Two loops: 2 → 2, limit: several scales in loop integrals
- Three loops: 1 → 2, limit: integrals
- Four loops: 1 → 1, limit: integrals

Multi-loop integral techniques

- Reduction to master integrals (K. Chetyrkin, F. Tkachov)
- Mellin-Barnes integration (V.A. Smirnov, B. Tausk)
- Differential equations (A. Kotikov; E. Remiddi, TG)
- Sector decomposition (T. Binoth, G. Heinrich)

Multi-loop: techniques

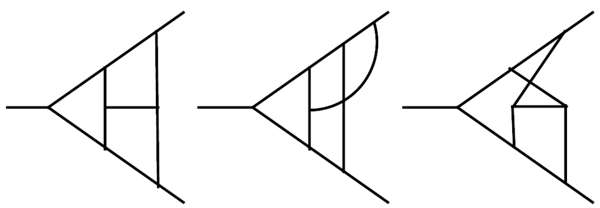
Reduction to master integrals

- exploit linear relations among loop integrals
- must solve large linear systems of equations ($\lesssim$ 1'000'000)
- exploit lexicographic ordering (S. Laporta)
- Public computer algebra implementations:
  - AIR in Maple (C. Anastasiou, A. Lazopoulos)
  - FIRE in Mathematica (A.V. Smirnov)
  - Reduze - in C++, with parallel computing option (C. Studerus)
- any multi-loop integral expressed through few master integrals

Multi-loop: techniques

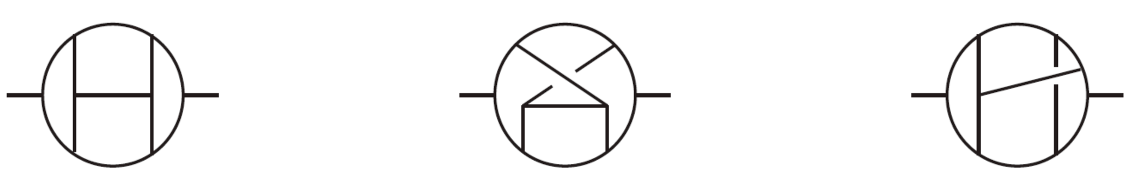
Computing master integrals

- Mellin-Barnes method
  - Evaluation of integrals: numerical or analytical
  - Recent results: three-loop massless vertex integrals



G. Heinrich, D. Kosower, T. Huber, D. Maitre
R. Lee, V.A. Smirnov, A.V. Smirnov

- Glue-and-cut method
  - Exploit relations between different topologies
  - Recent results: four-loop massless propagator integrals



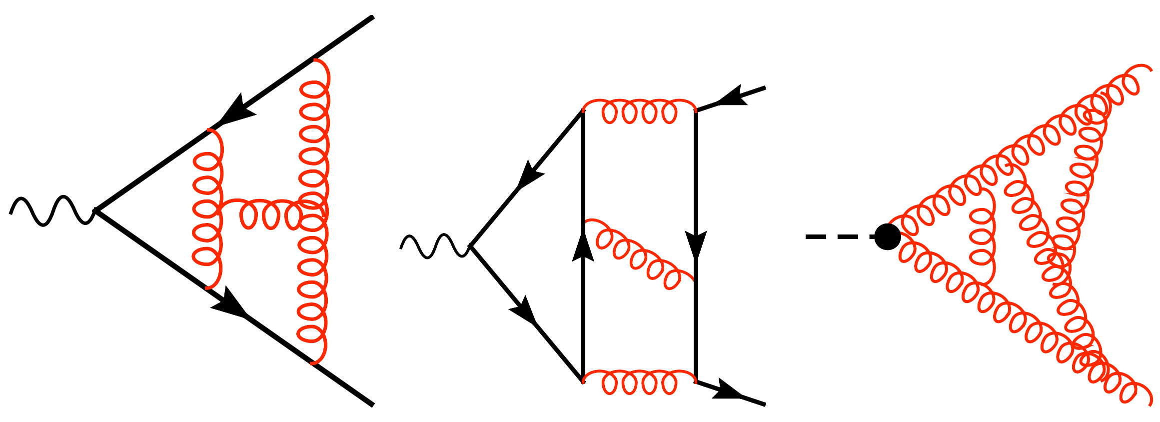
P. Baikov, K. Chetyrkin
A.V. Smirnov, M. Tentyukov

- Sector decomposition (numerical)

Three-loop form factors

Vertex functions: $\gamma^* \to q\bar{q}$ and $H \to gg$

- simplest infrared-divergent QCD objects
- allow study of infrared structure of QCD
- determine resummation coefficients
- enter benchmark reactions: DIS, Drell-Yan, Higgs production
- computed recently to three loops



P. Baikov, K. Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser
E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus, TG

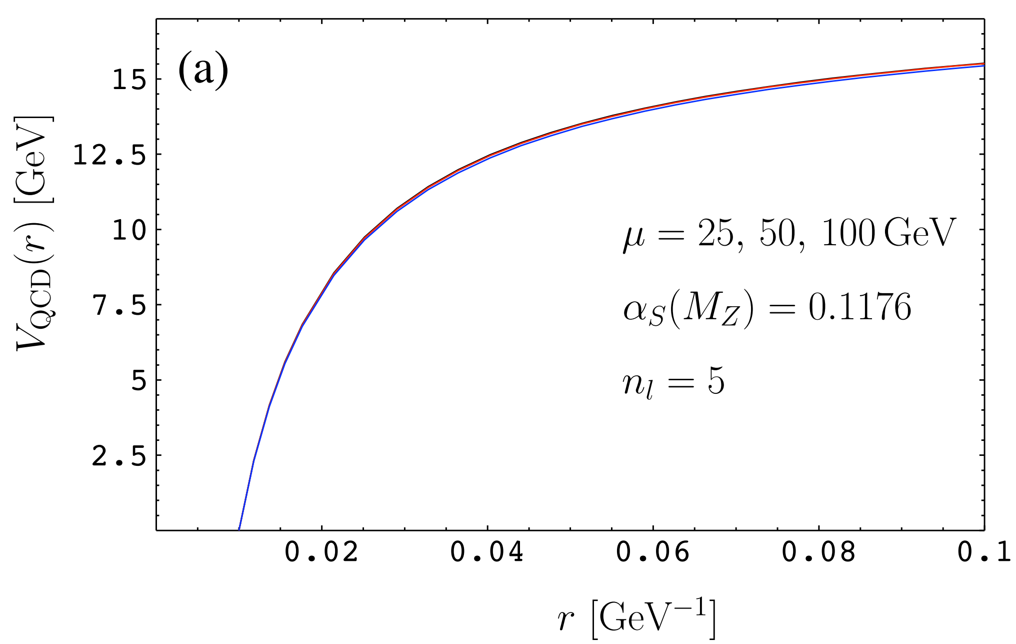
Three-loop static quark potential

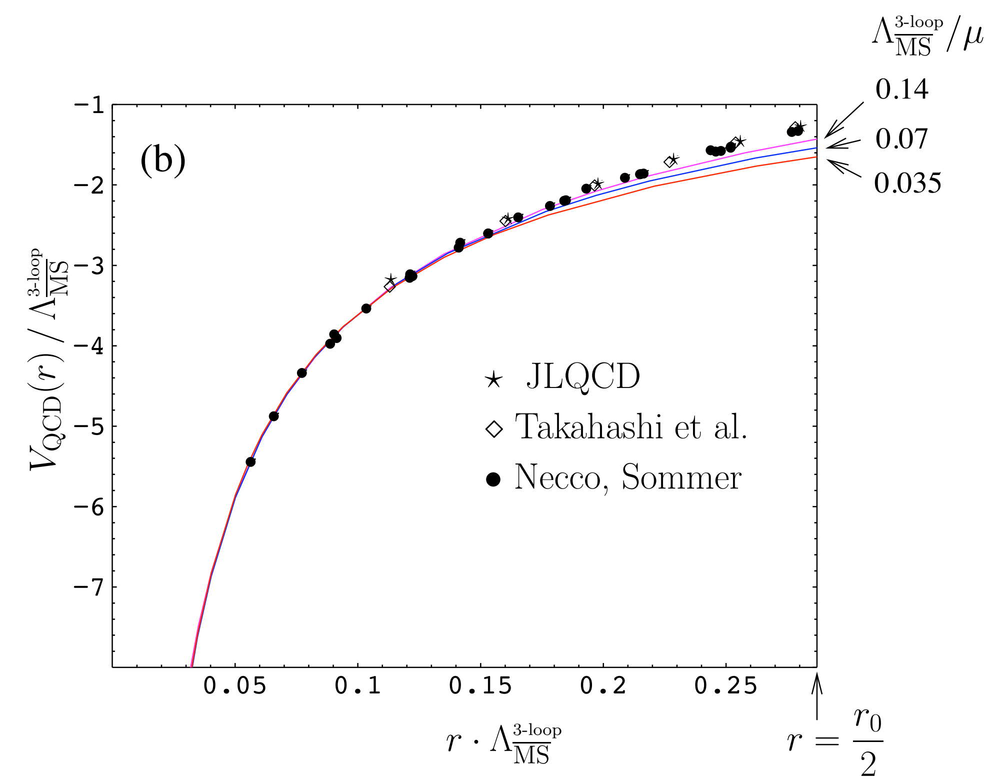
Potential formed by infinitely heavy $Q\bar{Q}$ pair

- heavy quark mass determination from sum rules
- description of top quark pair production at threshold

$$V(|\vec{q}\,|) = -\frac{4\pi C_F \alpha_s}{\vec{q}^{\,2}}\left[1 + \frac{\alpha_s}{4\pi}a_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s}{4\pi}\right)^3\left(a_3 + 8\pi^2 C_A^3 \ln\frac{\mu^2}{\vec{q}^{\,2}}\right) + \cdots\right]$$

A.V. Smirnov, V.A. Smirnov, M. Steinhauser
C. Anzai, Y. Kiyo, Y. Sumino





Four-loop two-point functions

- Important observables expressed as two-point functions
  - total decay rates
  - DIS sum rules
  - moments of structure functions
- recent four-loop results (P. Baikov, K. Chetyrkin, J.H. Kühn)
  - hadronic R-ratio and τ-decay rate →
  - polarized Bjorken sum rule

$$\alpha_s(M_Z)|_Z = 0.1190 \pm 0.0026,$$

$$\alpha_s(M_Z)|_\tau = 0.1202 \pm 0.0019$$

$$C^{Bjp} = 1 - a_s + (-4.583 + 0.3333\, n_f)\, a_s^2 + a_s^3 \left(-41.44 + 7.607\, n_f - 0.1775\, n_f^2\right) + \left(-479.4 + 123.4\, n_f - 7.697\, n_f^2 + 0.1037\, n_f^3\right) a_s^4.$$

  - Adler function and Crewther relation

Conclusions

QCD crucial for successful LHC physics

- parton distributions
- jets and event shapes as observables and tools
- understanding signals and backgrounds

QCD is getting ready for future challenges

- improved jet algorithms and event shape definitions
- enormous progress on NLO multi-leg calculations
- first NNLO calculations for precision observables
- turn into NNLO parton distributions
- understanding the all-order infrared structure
- landmark results at three and four loops